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Fault Class Isolation in Linear Systems with Unknown Inputs

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Abstract— **In fault isolation, it is often necessary to first make a decision on which class of fault has occurred. Such a decision requires isolating the occurred fault class amongst different possible fault classes. We shall refer to this problem as Fault Class Isolation (FCI), and this problem is the subject of this paper. In order to address the FCI problem, a general methodology is proposed. As an application of the methodology, FCI schemes using unknown input observers are designed to achieve the isolation between actuator and sensor class of faults for certain linear systems with unknown inputs. The results demonstrate that the FCI between actuator and sensor class of faults can be achieved under certain conditions. Examples are provided to show the effectiveness of the proposed FCI schemes.**

I. INTRODUCTION

In the context of fault diagnosis, a typical control system can be shown as in Figure 1. The system includes actuators, sensors, and a process to be controlled. Therefore, faults are often put into three classes, i.e., actuator faults, sensor faults, and process component faults. Many fault detection and isolation schemes have been proposed for certain class of faults. For examples, actuator fault diagnosis schemes were proposed in [1], [2], [3], sensor fault diagnosis schemes were designed in [4], [5], and process component fault diagnosis schemes were presented in [6], [7].

A common assumption in the above cited references is that the fault class is known. However, given a control system possibly subject to faults, it is often the case that the fault class is not known a priori. In fact, how to determine the fault class is an important part of fault isolation.

Although FCI is often needed, little research is devoted to it. In fact, not such result has been found in the literature. This observation motivated the research in this paper, which aims to raise the FCI problem and provide a general methodology to solve it.

The purpose of this paper is to propose the FCI problem and present methods that can be used to solve it. Specifically, it is intended to propose schemes to determine which classes of faults have occurred, that is, whether actuator faults, sensor faults, or process component faults have occurred.

The remainder of the paper is arranged as follows: In Section 2, a FCI problem is formulated, and a general methodology to solve the problem is provided. In Section 3, the general methodology proposed in Section 2 is applied to the FCI problem of a class of linear systems with unknown inputs, and FCI schemes are proposed based unknown input observer (UIO) design. In Section 4, two examples are provided to show the effectiveness of the proposed FCI schemes and simulation results are presented. Finally, concluding remarks are made in the last section.

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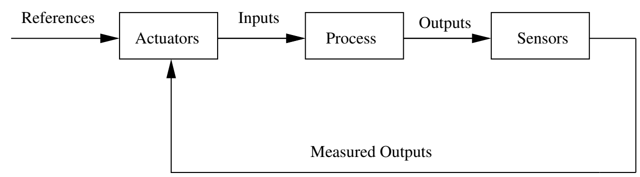

Fig. 1. A typical control system

II. FCI PROBLEM AND GENERAL METHODOLOGY

With the goal of FCI in mind, an immediate temptation is trying to formulate and solve the following problem.

General FCI Problem: Given a control system as in Figure 1, determine the possible classes of faults, and once a fault is detected, identify to which class the fault belongs.

If the problem could be solved, FCI amongst the three most common classes of faults, i.e., actuator faults, sensor faults, and process component faults, would be achieved in a perfect way. However, one may soon find that the problem is too general to be solved because the number of measurements provided by sensors will not be enough when the number of actuators and process components exceeds the number of sensors.

In order to formulate a tractable fault class isolation problem, some assumptions have to be made. In this paper, the following FCI problem is formulated.

Problem of FCI Between Two Fault Classes (FCI2): Given a control system as in Figure 1, assume that only one class of faults among two possible fault class can occur. Then the problem is to identify the fault class after fault detection.

According to the problem, there are three cases in total, i.e., FCI between actuator and sensor faults; FCI between actuator and process component faults; and FCI between sensor and process component faults.

A general methodology that will be used to solve the above formulated problem is proposed as follows.

General Methodology: Design one residual or a bank of residuals such that at least one residual is insensitive to one

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class of faults but sensitive the other class of faults. If this very residual is zero (small), the fault class is isolated as the one whose residual is insensitive to. If it is not zero (small), the other class is isolated as the fault class.

In this paper, only FCI between actuator and sensor faults is considered because the methodology can be used in the other two cases as well.

III. FCI IN A CLASS OF LINEAR SYSTEMS WITH UNKNOWN INPUTS

As an application of the general methodology proposed in the previous section, the FCI between actuator and sensor faults is considered for linear systems with unknown inputs described as below

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Dd(t) \\ y(t) &= Cx(t) \end{aligned} \quad (1)$$

where $x(t)$, $y(t)$, $u(t)$ are the system state vector, output vector and input vector respectively, and $x(t) \in R^n$, $y(t) = (y_1(t) \ \cdots \ y_p(t))^T$, $u(t) = (u_1(t) \ \cdots \ u_m(t))^T$. $d(t)$ is the unknown input vector and $d(t) \in R^q$. A, B, C and D are all known matrices.

Because FCI is performed after fault detection, a fault detection scheme is needed. In the following subsection, a fault detection scheme based on UIO design is provided.

For convenience, define $u^H(t)$ as the desired input vector when all actuators are healthy, which is computed according to the controller design.

A. Fault Detection Using UIO Design

The observer proposed in [9] is used here for fault detection.

$$\begin{aligned} \dot{z}_D(t) &= N_D z_D(t) + G_D u^H(t) + L_D y(t) \\ \hat{x}_D(t) &= z_D(t) - E_D y(t) \end{aligned} \quad (2)$$

where N_D, G_D, L_D, M_D are defined as

$$\begin{aligned} N_D &= M_D A - K_D C \\ L_D &= K_D(I + CE_D) - M_D A E_D \\ G_D &= M_D B, \ M_D = I + E_D C \end{aligned} \quad (3)$$

where E_D and K_D are chosen by the designers, and I is an identity matrix in $R^{n \times n}$.

Remark 1: If actuator faults are present, $u(t)$ is no longer available. However, $u^H(t)$ is always available. This is the reason $u^H(t)$ is used in the UIO design.

In order to ease the design difficulty of the above observer, according to [10], the following linear matrix inequality (LMI) is introduced for computing $\bar{Y}_D$, $\bar{K}_D$, and P_D.

$$\begin{aligned} &((I + U_D C)A)^T P_D + P_D(I + U_D C)A \\ &+(V_D CA)^T \bar{Y}_D^T + \bar{Y}_D(V_D CA) \\ &-C^T \bar{K}_D^T - \bar{K}_D C < 0 \end{aligned} \quad (4)$$

where $U_D = -D(CD)^+$ and $V_D = I - (CD)(CD)^+$ with $(CD)^+ = ((CD)^T(CD))^{-1}(CD)^T$.

Once $\bar{Y}_D$ and $\bar{K}_D$ are obtained, E_D and K_D can be computed as

$$\begin{aligned} E_D &= -D(CD)^+ + P_D^{-1}\bar{Y}_D(I - (CD)(CD)^+), \\ K_D &= P_D^{-1}\bar{K}_D. \end{aligned} \quad (5)$$

The following result has been proved in [10].

Theorem 1: Assume that CD is of full column rank, then the UIO given by (2) and (3) exists if and only if the LMI defined by (4) has a feasible solution of $\bar{Y}_D$, $\bar{K}_D$ and a symmetric matrix $P_D > 0$.

If the system (1) admits the UIO given by (2) and (3), a residual used for fault detection can be defined as $r_D(t) = \|y(t) - C\hat{x}_D(t)\|$, and the detection can be achieved using the following logic.

1) Choose a threshold for $r_D(t)$.
2) If $r_D(t)$ stays below the threshold, no fault has occurred. Otherwise, faults have been occurred.

In order to apply the general methodology, UIO design is used to generate the desired residuals. The following two cases are considered:

- Case A: Design a UIO such that it is insensitive to both $u(t)$ and $d(t)$.
- Case B: Design a bank of UIOs such that at least one UIO is insensitive to both actuator faults and $d(t)$.

Case A is considered in this subsection, while Case B will be studied in the following subsection.

B. FCI in Case A

In this case, by modifying the UIO given by (2) and (3), a UIO with the following structure is proposed.

$$\begin{aligned} \dot{z}_A(t) &= N_A z_A(t) + L_A y(t) \\ \hat{x}_A(t) &= z_A(t) - E_A y(t) \end{aligned} \quad (6)$$

where N_A and L_A are defined as

$$\begin{aligned} N_A &= M_A A - K_A C \\ L_A &= K_A(I + CE_A) - M_A A E_A \end{aligned} \quad (7)$$

with $M_A = I + E_A C$, and E_A and K_A are chosen by the designers.

As in the precious subsection, the following LMI is introduced for computing $\bar{Y}_A$, $\bar{K}_A$, and P_A.

$$\begin{aligned} &((I + U_A C)A)^T P_A + P_A(I + U_A C)A \\ &+(V_A CA)^T \bar{Y}_A^T + \bar{Y}_A(V_A CA) \\ &-C^T \bar{K}_A^T - \bar{K}_A C < 0 \end{aligned} \quad (8)$$

where $U_A = -\bar{D}(C\bar{D})^+$ and $V_A = I - (C\bar{D})(C\bar{D})^+$ with $(C\bar{D})^+ = ((C\bar{D})^T(C\bar{D}))^{-1}(C\bar{D})^T$ with $\bar{D} = (B \ D)$.

Once $\bar{Y}_A$ and $\bar{K}_A$ are obtained, E_A and K_A can be computed as

$$\begin{aligned} E_A &= -\bar{D}(C\bar{D})^+ + P_A^{-1}\bar{Y}_A(I - (C\bar{D})(C\bar{D})^+), \\ K_A &= P_A^{-1}\bar{K}_A. \end{aligned} \quad (9)$$

The following result can be derived easily using Theorem 1.

Theorem 2: Assume that $C\bar{D}$ is of full column rank, then the UIO given by (6) and (7) exists if and only if the LMI defined by (8) has a feasible solution of $\bar{Y}_A$, $\bar{K}_A$ and a symmetric matrix $P_A > 0$.

If the system (1) admits the UIO given by (6) and (7), a residual used for FCI can be defined as $r_A(t) = \|y(t) - C\hat{x}_A(t)\|$. Using the UIO given by (2) and (3) and the UIO given by (6) and (7), a FCI scheme for the problem of FCI2 can be proposed as follows.

UIO Based FCI2 Scheme For Case A:

1) Generate $r_D(t)$ using (2) and (3).
2) Generate $r_A(t)$ using (6) and (7).
3) Monitor $r_D(t)$ and $r_A(t)$.
4) If $r_D(t)$ becomes nonzero (or no longer small), faults are detected.
5) After faults are detected, monitor $r_A(t)$ only.
6) If $r_A(t)$ becomes nonzero (or no longer small), sensor faults have occurred. Otherwise, actuator faults have occurred.

It should be pointed out that the scheme is based on the general methodology and only works under the assumptions that only one class of faults can occur and that only actuator and sensor faults are the possible classes of faults.

C. FCI in Case B

In order to use the proposed scheme in the previous subsection, the existence of the UIO given by (6) and (7) is required. For some systems, the condition that $C\bar{D}$ is of full column rank is too strong to be satisfied, or the LMIs may not have a feasible solution. For such systems, it is desired to achieve FCI under weaker conditions. On the other hand, it is often that only some actuators are actually faulty, it is not necessary that the UIOs are designed to be insensitive to all the inputs in $u(t)$, and instead, the UIOs are only required to be insensitive to certain number of inputs in $u(t)$. Based on the above arguments, Case B is considered in this subsection.

For convenience, let ϕ denote the empty set, and 2^S denote the set consisting of all subsets of $S = \{1, 2, \cdots, m\}$. Denote $B = (b_1 \ \cdots \ b_m)$. For any $s = \{i_1, \cdots, i_l\} \in 2^S$ with $i_1 \leq \cdots \leq i_l, 1 \leq l \leq m$,, denote $B_s = (b_{i_1} \ \cdots \ b_{i_l})$, and define $\bar{B}_s$ as the matrix consisting of the remaining columns of B after removing the columns of B_s from B. Similarly, denote $u_s(t) = (u_{i_1}(t) \ \cdots \ u_{i_l}(t))^T$ and $\bar{u}_s(t)$ is defined in a similar manner as $\bar{B}_s$. Define $u_s^H(t)$ and $\bar{u}_s^H(t)$ in the same way as $u_s(t)$ and $\bar{u}_s(t)$

For any $s = \{i_1, \cdots, i_l\} \in 2^S$ with $1 \leq l \leq m$, system (1) can be rewritten as

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \bar{B}_s\bar{u}_s(t) + B_s u_s(t) + Dd(t), \\ y(t) &= Cx(t). \end{aligned} \tag{10}$$

Define $\bar{D}_s = (B_s \ D)$ and $\bar{d}(t) = (u_s^T(t) \ d^T(t))^T$, by modifying the detection UIO, the following UIO can be proposed.

$$\begin{aligned} \dot{z}_s(t) &= N_s z_s(t) + \bar{G}_s\bar{u}_s^H(t) + L_s y(t) \\ \hat{x}_s(t) &= z_s(t) - E_s y(t) \end{aligned} \tag{11}$$

where $N_s, \bar{G}_s, L_s, M_s$ are defined as

$$\begin{aligned} N_s &= M_s A - K_s C, \\ \bar{G}_s &= M_s\bar{B}_s, \\ L_s &= K_s(I + CE_s) - M_s A E_s, \\ M_s &= I + E_s C. \end{aligned} \tag{12}$$

As in Subsection A, the following linear matrix inequality (LMI) is introduced for computing $\bar{Y}_s$, $\bar{K}_s$, and P_s.

$$\begin{aligned} &((I + U_sC)A)^T P_s + P_s(I + U_sC)A \\ &+(V_sCA)^T\bar{Y}_s^T + \bar{Y}_s(V_sCA) \\ &-C^T\bar{K}_s^T - \bar{K}_sC < 0 \end{aligned} \tag{13}$$

where $U_s = -\bar{D}_s(C\bar{D}_s)^+$ and $V_s = I - (C\bar{D}_s)(C\bar{D}_s)^+$ with $(C\bar{D}_s)^+ = ((C\bar{D}_s)^T(C\bar{D}_s))^{-1}(C\bar{D}_s)^T$.

With $\bar{Y}_s$, $\bar{K}_s$, and P_s, K_s and E_s can be computed as

$$\begin{aligned} K_s &= P_s^{-1}\bar{K}_s, \\ E_s &= -\bar{D}_s(C\bar{D}_s)^+ \\ &\quad +P_s^{-1}\bar{Y}_s(I - (C\bar{D}_s)(C\bar{D}_s)^+). \end{aligned} \tag{14}$$

With K_s and E_s, all the observer gain matrices can be computed using (12).

According to Theorem 1, the following theorem can also be proved.

Theorem 3: For a given set $s = \{i_1, \cdots, i_l\} \in 2^S$ with $1 \leq l \leq m$, assume that $C\bar{D}_s$ is of full column rank, then the UIO given by (11) and (12) exists if and only if the LMI defined by (13) has a feasible solution of $\bar{Y}_s$, $\bar{K}_s$ and a symmetric matrix $P_s > 0$.

Assume that there exists an integer l_0 such that $C\bar{D}_s$ is of full column rank and the LMI defined by (13) has a feasible solution of $\bar{Y}_s$, $\bar{K}_s$ and a symmetric matrix $P_s > 0$ for all sets of the form $s = \{i_1, \cdots, i_{l_0}\} \in 2^S$. Let l_{max} be the largest one amongst such integers.

If $l_{max} = m$, Case A can be solved. If $l_{max} < m$, the FCI scheme for Case A can no longer be applied because the UIO given by (6) and (7) does not exist. Actually, if $l_{max} < m$, in order to solve the problem of FCI2, Case B, instead of Case A, should be considered. In order to achieve FCI2, it is assumed that the number of faults n_f is less than l_{max}.

For all sets of the form, $s = \{i_1, \cdots, i_{l_{max}}\} \in 2^S$, if l is replaced by l_{max}, a bank of UIOs of the form (11) and (12) can be designed. Actually, there are $C_m^{l_{max}}$ such UIOs in total. Using all the designed UIOs, a bank of residuals can be defined as $r_s(t) = \|y(t) - C\hat{x}_s(t)\|$ with $s = \{i_1, \cdots, i_{l_{max}}\} \in 2^S$.

Now, using the UIO given by (2) and (3) and the UIOs given by (11) and (12) for all sets of the form $s = \{i_1, \cdots, i_{l_{max}}\} \in 2^S$, a FCI2 scheme can be proposed as follows.

UIO Based FCI2 Scheme For Case B:

1) Generate $r_D(t)$ using (2) and (3).
2) Generate all the residuals $r_s(t)$ using (11) and (12) for all sets of the form $s = \{i_1, \cdots, i_{l_{max}}\} \in 2^S$.

3) Choose thresholds for $r_D(t)$ and all $r_s(t)$, and monitor them.
4) If $r_D(t)$ becomes larger than its chosen threshold, faults are detected.
5) After faults are detected, monitor all $r_s(t)$ only.
6) If all residuals $r_{s_0}(t)$ go beyond their thresholds, sensor faults have occurred. Otherwise, actuator faults have occurred.

Remark 2: Although how to chosen suitable thresholds is not trivial task, our focus is on FCI and the design of schemes for FCI.

Remark 3: It is obvious that the scheme proposed for Case B can be applied to more classes of systems than the one for Case A. However, an additional condition on the number of faults is required, which is $n_f \leq l_{max}$. This is not a severe limitation because many existing fault isolation results in the literature only consider the single fault case, which satisfies the condition automatically.

Remark 4: It should be pointed out that sliding mode observers ([4], [8]) can also be used to solve the problem in a very similar way. It is also possible to combine the general methodology in this paper with the geometric approach in [11] to solve the FCI problem formulated for a class of nonlinear system with unknown inputs. Since the main purpose of this paper is to raise the problem of FCI, present a general methodology to solve the FCI problem, and provide a means as to how to implement the methodology using UIO design based approach, these and other possible extensions are left as future research topics.

IV. EXAMPLES AND SIMULATION RESULTS

Two examples are given in this section. The first one is taken as the inverted pendulum with a cart example studied in [8] to show the effect of the FCI scheme proposed for Case A. The FCI scheme is designed based on a local linear model of the inverted pendulum with a cart, but tested on its nonlinear system model. In order to test the FCI scheme proposed for Case B, the local linear model of the inverted pendulum example is used by adding an additional input manually. Here, both the design and the test of the FCI scheme for Case B are carried out based only on the linearized model with an added input.

A. FCI Scheme For Case A: The inverted Pendulum With A Cart

The inverted pendulum with a cart example in [8] is used here to show the effect of the FCI scheme proposed for Case A, which is described by the following state space model.

$$\begin{aligned}
\dot{x}_1(t) &= x_3(t) \\
\dot{x}_2(t) &= x_4(t) \\
\dot{x}_3(t) &= \frac{1}{3.735 - 0.615cos^2(x_2(t))}(-6.3x_3(t) \\
&+ 0.0283x_4(t)cos(x_2(t)) \\
&- 6.0266sin(x_2(t))cos(x_2(t)) \\
&+ 0.1953x_4^2(t)sin(x_2(t)) + u(t)) \\
\dot{x}_4(t) &= \frac{1}{0.062 - 0.0102cos^2(x_2(t))}(-0.009x_3(t) \\
&+ 1.9151sin(x_2(t)) - 0.3242x_3(t)cos(x_2(t)) \\
&- 0.0102sin(x_2(t))cos(x_2(t))x_4^2(t) \\
&- 0.0523cos(x_2(t))u(t)) \qquad (15)
\end{aligned}$$

where $x_1(t) = x(t), x_2(t) = \theta(t), x_3(t) = \dot{x}(t), x_4(t) = \dot{\theta}(t)$.
According to [8], via linearizing the above system around $x_1(t) = x_2(t) = x_3(t) = x_4(t) = 0$, a linear model can be derived as

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad (16)$$

where

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -1.9333 & -1.9872 & 0.0091 \\ 0 & 36.9711 & 6.2589 & -0.1738 \end{pmatrix},$$

$$B = \begin{pmatrix} 0 \\ 0 \\ 0.3205 \\ -1.0095 \end{pmatrix},$$

and a full state feedback can be designed to assign closed-loop system poles at $\{-4.2, -4.4, -4.6, -4.8\}$.

To test the fault detection effects, only $x_1(t), x_2(t), x_3(t)$ are assumed to be available. Hence,

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

and $y(t) = Cx(t)$. The fault detection and fault class isolation schemes are designed based on the linearized model (16) with $y(t) = Cx(t)$. Because the unknown input vector term $d(t)$ is absent, the following observer is designed for fault detection

$$\begin{aligned}
\dot{z}_D(t) &= Az_D(t) + Bu^H(t) + K_D(y(t) - Cz_D(t)) \\
\hat{x}_D(t) &= z_D(t) \qquad (17)
\end{aligned}$$

where

$$K_D = \begin{pmatrix} 1.0000 & 0.0034 & 0.9940 \\ 0.0079 & 2.2260 & 0.0162 \\ -0.0059 & -1.9164 & -1.6870 \\ 0.0054 & 37.3842 & 6.2620 \end{pmatrix},$$

and $u^H(t) = 41.2181y_1(t) + 171.6711y_2(t) + 43.1215y_3(t) + 29.3803x_4(t)$, which can assign closed-loop system poles at $\{-4.2, -4.4, -4.6, -4.8\}$ when the system is free of faults (note that all states are assumed available for the controller design). It is easy to see that the above observer is a special case of the UIO given by (2) and (3) with $E_D = 0$.

By letting $\bar{D} = B$, it can be checked the LMI (8) is feasible, and the UIO given by (6) and (7) can be designed using the Matlab LMI toolbox as

$$\begin{aligned}
\dot{z}_A(t) &= N_A z_A(t) + L_A y(t) \\
\hat{x}_A(t) &= z_A(t) - E_A y(t) \qquad (18)
\end{aligned}$$

The observer gain matrices are not provided because of lack of space.

The detection observer and the UIO for fault class isolation designed based on the linearized model (16) are applied to the nonlinear system model given by (15) in the simulations. The proposed FCI scheme for Case A is first tested for an actuator fault ($u(t) = u^H(t) + 0.1(t-5)$ when $t \geq 5$), and the simulation results are shown in Figure 2. It is tested also for a sensor fault ($y(t) = x_1(t) + 0.1(t-5)$ when $t \geq 5$), and the simulation results are presented in Figure 3.

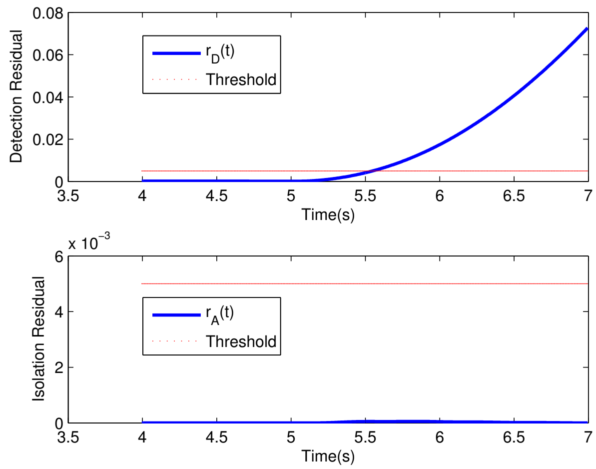

Fig. 2. The simulation results in the presence of an actuator fault

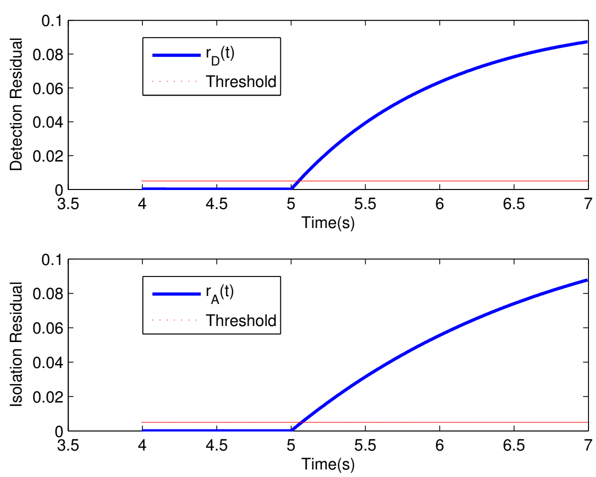

Fig. 3. The simulation results in the presence of a sensor fault

In both Figure 2 and Figure 3, faults are detected because both the detection residuals $r_D(t)$ exceed the threshold and keep going up. It is also easy to see that the FCI residual $r_A(t)$ in Figure 2 remains small, while the residual $r_A(t)$ in Figure 3 exceeds the threshold and goes up. Therefore, using UIO Based FCI Scheme For Case A, it can be concluded that the results in Figure 2 indicate that an actuator fault has occurred while those in Figure 3 indicate that sensor faults have occurred. The decisions are correct, and the FCI scheme for Case A isolated the occurred fault class successfully.

B. FCI Scheme For Case B

In order to test the FCI scheme for Case B, an additional input is added to the local linear model (16), which lead to the following system

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{19}$$

where A is the same as in the previous subsection, $u(t) = (u_1(t)\ u_2(t))^H$, and

$$B = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0.3205 & -1.0095 \\ -1.0095 & 0.3205 \end{pmatrix},$$

and a full state feedback is designed to assign closed-loop system poles at $\{-4.2, -4.4, -4.6, -4.8\}$.

Again, to test the fault detection effects, it is assumed that

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix},$$

and $y(t) = Cx(t)$.

The fault detection and FCI schemes are designed based on the linearized model (19) with $y(t) = Cx(t)$. Because the unknown input vector term $d(t)$ is absent, the following observer is designed for fault detection

$$\begin{aligned} \dot{z}_D(t) &= Az_D(t) + Bu^H(t) + K_D(y(t) - Cz_D(t)) \\ \hat{x}_D(t) &= z_D(t) \end{aligned} \tag{20}$$

where

$$K_D = \begin{pmatrix} 1.0000 & 0.0034 & 0.9940 \\ 0.0079 & 2.2260 & 0.0162 \\ -0.0059 & -1.9164 & -1.6870 \\ 0.0054 & 37.3842 & 6.2620 \end{pmatrix},$$

and $u^H(t) = -K_c x(t)$ with

$$K_c = \begin{pmatrix} -6.2139 & -61.8228 & -9.1570 & -9.6138 \\ -22.1595 & -16.7789 & -9.8895 & -2.8542 \end{pmatrix},$$

which can assign closed-loop system poles at $\{-4.2, -4.4, -4.6, -4.8\}$ when the system is free of faults (note again that all states are assumed available for the controller design).

By letting $\bar{D} = B$, it is easy to check that rank $C\bar{D} = 1$, which implies that the UIO given by (6) and (7) can no longer be designed because E_A does not exist such that $E_A C\bar{D} = -\bar{D}$. Hence, for system (19), it is impossible to consider Case A, instead, Case B should be considered. In such a case, it is assumed only a single fault can occur.

Because there are only two inputs, it is easy to see that s is either $\{1\}$ or $\{2\}$. For simplicity and with a slight abuse of notation, s is replaced by 1 or 2 when it is either $\{1\}$ or $\{2\}$.

Define $\bar{D}_1 = B_1$ and $\bar{D}_2 = B_2$, according to the UIO given by (11) and (12), note that $\bar{u}_1^H(t) = u_2^H(t)$ and $\bar{u}_2^H(t) = u_1^H(t)$, the following UIOs can be designed.

$$\begin{aligned} \dot{z}_1(t) &= N_1 z_1(t) + \bar{G}_1 u_2^H(t) + L_1 y(t) \\ \hat{x}_1(t) &= z_1(t) - E_1 y(t) \end{aligned} \quad (21)$$

and

$$\begin{aligned} \dot{z}_2(t) &= N_2 z_2(t) + \bar{G}_2 u_1^H(t) + L_2 y(t) \\ \hat{x}_2(t) &= z_2(t) - E_2 y(t) \end{aligned} \quad (22)$$

The proposed FCI scheme for Case B is tested for an actuator fault ($u(t) = u^H(t) + 0.1(t-5)$ when $t \geq 5$), and the simulation results are shown in Figure 4.

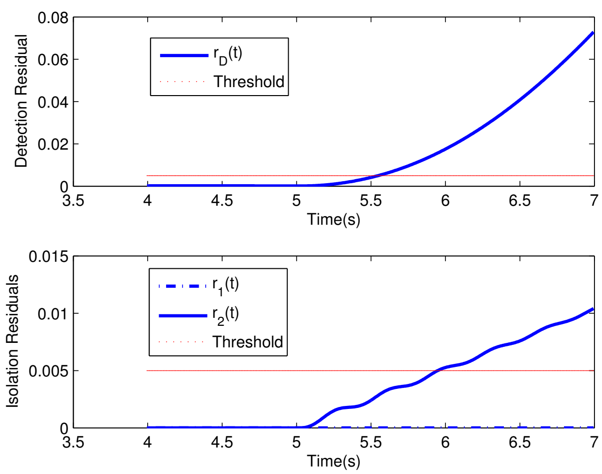

Fig. 4. The simulation results in the presence of an actuator fault

In Figure 4, a fault is detected because the detection residual $r_D(t)$ exceeds the threshold and keeps going up. It is also easy to see that the FCI residual $r_1(t)$ remains small, while the residual $r_2(t)$ exceeds the threshold and keeps going up. Therefore, using UIO Based FCI Scheme For Case B, it can be concluded that an actuator fault, rather than a sensor fault, has occurred.

V. CONCLUSIONS AND FUTURE WORK

A. Conclusions

In this paper, the problem of FCI was raised. A feasible FCI problem was formulated, and a general methodology for the formulated problem was proposed. As an application of the general methodology, UIO based FCI schemes were proposed for the FCI between actuator and sensor faults for two cases, which are Case A and Case B defined in Section 3. The simulation results showed both schemes were able to achieve FCI successfully. Because the scheme for Case B requires weaker conditions, it is more promising to be used to the FCI problems between component and sensor faults and those component and actuator faults.

B. Future Work

Using the general methodology, many other FCI schemes can be designed using different types of observer or output estimator based design techniques. Sliding mode observers ([4], [8]), the geometric approach based observers ([11]), and output estimators ([3], [12]) are only a few possible examples that can be used to design FCI schemes. Moreover, the methodology is not confined to linear systems, it is expected that it can be applied to nonlinear systems as well, which is another future research topic. Because model based FCI has been paid very little attention, many FCI problems need to be studied and solved in the future.

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REFERENCES

[1] H. Wang and S. Daley, "Actuator fault diagnosis: an adaptive observer-based technique," *IEEE Trans. on Automatic Control,* vol. 41, No.7, pp. 1073-1078, 1996.

[2] Weitian Chen and M. Saif, "Fault detection and isolation based on novel unknown input observer design," *Proceedings of the ACC,* Minneapolis, Minnesota, USA, June 2006, pp.5129-5134.

[3] Weitian Chen and M. Saif, "High-order sliding-mode differentiator based actuator fault diagnosis for linear systems with arbitrary relative degree and unmatched unknown inputs," *Proceedings of the CDC,* San Diego, CA, USA, 2006, pp. 1153-1158.

[4] C. P. Tan and C. Edwards, "Sliding mode observers for robust detection and reconstruction of actuator and sensor faults," *International Journal of Robust and Nonlinear Control,* vol. 13, pp. 443-463, 2003.

[5] R. Mattone and A. De Luca, "Nonlinear Fault Detection and Isolation in a Three-Tank Heating System," *IEEE Trans. on Control Systems Technology,* Vol. 14, No. 6, pp.1158 - 1166, 2006.

[6] T. Marcu and L. Mirea, "Robust detection and isolation of process faults using neural networks," *IEEE Control Systems Magazine,* Vol. 17, No. 5, pp. 72 - 79, 1997.

[7] X. Zhang, M. M. Polycarpou, and T. Parisini, "A robust detection and isolation scheme for abrupt and incipient faults in nonlinear systems," *IEEE Trans. on Automatic Control,* vol. 47, No.4, pp. 576-593, 2002.

[8] C. Edwards, S. K. Spurgeon, and R. J. Patton, "Sliding mode observers for fault detection and isolation," *Automatica,* vol. 36, No 4, pp. 541-553, 2000.

[9] M. Darouach, M. Zasadzinski, and S.J. Xu, Full-order observers for linear systems with unknown inputs, *IEEE Trans. on Automatic Control,* vol. 39, No. 3, pp. 607-609, 1994.

[10] Weitian Chen and M. Saif, "Unknown Input Observer Design for a Class of Nonlinear Systems: an LMI Approach," *Proceedings of the ACC,* Minneapolis, Minnesota, USA, June 2006, pp. 834 - 838.

[11] Claudio De Persis and A. Isidori, "A geometric approach to nonlinear fault detection and isolation," *IEEE Transactions on Automatic Control,* vol. 46, No.6, pp. 853 - 865, 2001.

[12] Weitian Chen and M. Saif, "Output estimator based fault detection, isolation and estimation for systems with unmatched unknown inputs ," *IEEE Joint CCA/CACSD/ISIC*, Munich, Germany, pp. 1753 - 1758, 2006.