2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

Fault diagnosis in nonlinear systems: An application to a three-tank system

J. Juan Rincon-Pasaye, Rafael Martinez-Guerra and Alberto Soria-Lopez
Departamento de Control Automatico, Cinvestav-IPN
A. P. 14-740, Av. IPN 2508, Mexico, D. F., 07360, Mexico
Email: jrincon{rguerra}@ctrl.cinvestav.mx, betosoria@gmail.com

***Abstract*— The fault diagnosis problem for nonlinear systems is treated, some results based on a differential algebraic approach are used in order to determine fault diagnosability with the minimum number of measurements from the system. Two schemes of nonlinear observers are used for reconstructing the fault signals for comparison purposes, one of them being a reduced order observer and the other a sliding mode observer. The methodology was tested in a real time implementation of the three-tank system for which a previous identification of uncertain parameters is realized in order to improve fault estimations.**

Keywords: Fault diagnosis, reduced-order observer, sliding mode observer, differential algebra.

I. INTRODUCTION

The fault diagnosis problem has been studied for more than three decades, many papers regarding this problem can be found in the literature, see for instance [1], [2], [3], [5]. For nonlinear systems a variety of approaches have been proposed. Some model-based approaches can be found, such as the approaches based upon differential geometric methods [4], [15]. On the other hand, alternative approaches have been proposed based on an algebraic and differential framework [6]-[13]. These approaches consist in the observation of the dynamics of the fault variables, which are defined as uncertain inputs [7].

This paper deals with nonlinear systems diagnosis and the goal is to find malfunctions in the system, based on input-output measurements. The outputs are mainly measured signals obtained from sensors, their number is important in order to know whether a system is diagnosable or not. This question has been solved in terms of the differential transcendence degree concept in [14] and [20].

The fault diagnosis problem is considered as the problem of observing the fault signals. So the diagnosability of a systems is given by the so-called *algebraic observability condition* [7], [8] of the fault. In this paper, two schemes of observers are proposed in order to estimate the fault signals, one of them is a reduced-order observer based on a free-model approach and the other is a sliding-mode observer based on a partial change of coordinates. Both schemes possess asymptotic convergence properties and are relatively easy to design when the algebraic observability condition is available.

Even though the algebraic approach has been applied to the fault diagnosis problem for almost one decade, there are not reported works containing real-time applications based in this theoretical framework. The Amira DTS200 interconnected three-tank system [17] has been considered for the experimental fault diagnosis study, we can see for instance [12], [15] and [16] even recently, one work based in the geometric approach has been reported [15]. We can also mention one previous work with the three-tank system using the differential algebraic approach [12], in that work the authors only report a numerical simulation study and not a real-time experiment, also, they only solve the simplest case in which three measured outputs are available to estimate two faults, that is to say, they do not analyze the minimal number of measurements to attack the diagnosis problem as we do in this work.

This paper is organized as follows: In section II some definitions and examples in a differential algebraic framework are given. In section III the minimal number of measurements that one need to make a system diagnosable in terms of the differential transcendence degree concept is given. An asymptotic reduced-order observer for the fault signals is presented in section IV. In section V an asymptotic sliding-mode observer is given. In section VI the three-tank system model is described, the minimal measurements condition is evaluated in three different cases, the reduced-order and the sliding-mode observers are designed and an identification strategy for the uncertain parameters is given. In section VII the experimental results are shown for the fault and state estimation with two different observers. Finally, in section VIII the paper is closed with some concluding remarks.

II. SOME DEFINITIONS

Some basic definitions are introduced. Further details can be found in [6]-[10] and references therein.

Definition 1 Let $\mathcal{L}$ and $\mathcal{K}$ be differential fields. A differential field extension $\mathcal{L}/\mathcal{K}$ is given by $\mathcal{K}$ and $\mathcal{L}$ such that: 1) $\mathcal{K}$ is a subfield of $\mathcal{L}$ and; 2) the derivation of $\mathcal{K}$ is the restriction to $\mathcal{K}$ of the derivation of $\mathcal{L}$.

Example 1 $R < e^t > /R$ is a differential field extension, where $R \subseteq R < e^t >$. e^t being a solution of $\dot{x} - x = 0$.

Definition 2 Let $\xi = (\xi_1, \xi_2, \ldots, \xi_n)$ be a set of elements of $\mathcal{L}$. If it satisfies an algebraic differential equation $P\left(\xi, \dot{\xi}, \ddot{\xi} \ldots\right) = 0$ with coefficients in $\mathcal{K}$ it is called $\mathcal{K}$-differentially algebraically dependent. Otherwise, ξ is called differentially $\mathcal{K}$-algebraically independent.

Definition 3 Any set of elements of $\mathcal{L}$ which is differentially $\mathcal{K}$-algebraically independent and maximal with respect to inclusion forms a differential transcendence basis of $\mathcal{L}/\mathcal{K}$. Two such basis have the same cardinality. This is called the *differential transcendence degree* of $\mathcal{L}/\mathcal{K}$ and denoted by $diff\ tr\ d^{\circ}\ \mathcal{L}/\mathcal{K}$

Definition 4 A fault is a not permitted deviation of at least one characteristic property or parameter of any process in relation to the development of the same parameter under normal conditions. Faults are defined as transcendent elements over $\mathcal{K}\langle u\rangle$, therefore, a system with the presence of faults is a differential transcendental extension, denoted as $\mathcal{K}\langle u,f,y\rangle/\mathcal{K}\langle u\rangle$, where f is a fault vector and its time derivatives.

Definition 5 Let $\mathcal{G}$, $\mathcal{K}\langle u\rangle$ be differential fields. A dynamics with faults is a finitely generated differential algebraic extension $\mathcal{G}/\mathcal{K}\langle u,f\rangle$, $\mathcal{G}=\mathcal{K}\langle u,f,\xi\rangle$, $\xi\in\mathcal{G}$. Any element of $\mathcal{G}$ satisfies an algebraic differential equation with coefficients over $\mathcal{K}$ in the components of u, f and their time derivatives.

Definition 6 (Algebraic observability condition) A fault $f\in\mathcal{G}$ is said to be diagnosable if it is possible to estimate the fault from the available measurements of the system, i.e., f is diagnosable if it is algebraically observable over $\mathbb{R}\langle u,y\rangle$.

Let us consider the class of nonlinear systems with faults described by the following equation

$$\left\{\begin{array}{l} \dot{x}(t)=A(x,\bar{u}) \\ y(t)=h(x,u) \end{array}\right. . \tag{1}$$

Where $x=(x_1,\ldots,x_n)^T\in\mathbb{R}^n$ is a state vector, $u=(u_1,\ldots,u_m)\in\mathbb{R}^m$ is a known input vector, $f=(f_1,\ldots,f_\mu)\in\mathbb{R}^\mu$ is an unknown input vector, $\bar{u}=(u,f)\in\mathbb{R}^{m+\mu}$, $y(t)\in\mathbb{R}^p$ is the output vector. A and h are assumed to be analytical vector functions.

Example 3 Let us consider the system

$$\left\{\begin{array}{rl} \dot{x}_1= & x_1x_2+f+u \\ \dot{x}_2= & x_1 \\ y= & x_2 \end{array}\right. . \tag{2}$$

Since f satisfies the differential algebraic equation $f-\ddot{y}+y\dot{y}+u=0$, then, the system (2) is diagnosable and the fault can be reconstructed from the knowledge of u, y and their time derivatives.

Remark 1 The diagnosability condition is independent of the observability of a system.

Example 4 Let us consider the system

$$\left\{\begin{array}{rl} \dot{x}_1= & x_1x_2+f+u \\ \dot{x}_2= & x_1 \\ \dot{x}_3= & x_3f+u \\ y= & x_2 \end{array}\right. . \tag{3}$$

In this case f is diagnosable. However, x_3 is not algebraically observable.

III. On the number of faults and measurements

The following results from the theory of differential algebraic field extensions are useful to determine whether a fault can be reconstructed from the knowledge of inputs and outputs:

Property 1 [19] Let $\mathcal{K}$, $\mathcal{L}$, $\mathcal{M}$, be differential fields such that $\mathcal{K}\subset\mathcal{L}\subset\mathcal{M}$. Then

$$diff\ tr\ d^o(\mathcal{M}/\mathcal{K})=diff\ tr\ d^o(\mathcal{M}/\mathcal{L})+diff\ tr\ d^o(\mathcal{L}/\mathcal{K}) \tag{4}$$

Property 1 is an important tool to proof theorems 1 and 2.

Theorem 1 [8] Assume that the system (1) is diagnosable, then the number of faults is less or equal to the number of outputs, i.e.

$$\mu\le p \tag{5}$$

■

Another important result in the diagnosis problem is given as follows.

Theorem 2 [14] The system (1) is diagnosable if and only if

$$diff\ tr\ d^{\circ}\ \mathcal{K}\langle u,y\rangle/\mathcal{K}\langle u\rangle=\mu \tag{6}$$

■

IV. Reduced-order observer

Let consider system (1). The fault vector f is unknown and it can be assimilated as a state with uncertain dynamics. Then, in order to estimate it, the state vector is extended to deal with the unknown fault vector. The new extended system is given by

$$\begin{array}{c} \dot{x}(t)=A(x,\bar{u}) \\ \dot{f}=\Omega(x,\bar{u}) \\ y(t)=h(x,u) \end{array} \tag{7}$$

where $\Omega(x,\bar{u})$ is a bounded uncertain function.

Note that a classic Luenberger observer can not be constructed because the term $\Omega(x,\bar{u})$ is unknown. Then, the above problem is overcome by using a reduced order uncertainty observer in order to estimate the failure variable f [6].

Next Lemma describes the construction of a proportional reduced order observer for (7).

Lemma 1 If the following hypotheses are satisfied:

H1: $\Omega(x,\bar{u})$ is bounded, i.e., $|\Omega(x,\bar{u})|\le N$.

H2: $f(t)$ is algebraically observable over $k\langle u,y\rangle$.

H3: γ is a C^1 real-valued function.

Then the system

$$\dot{\hat{f}}\ =\ K\left(f-\hat{f}\right) \tag{8}$$

is an asymptotic reduced order observer for system (7), where $\hat{f}$ denotes the estimate of fault f and $K\in\mathbb{R}^+$ determines the desired convergence rate of the observer. ■

The proof is omitted and it can be followed as in [21]

Remark 1. Sometimes the output time derivatives (which are unknown), appear in the algebraic equation of the fault, then it is necessary to use an auxiliary variable to avoid using them.

Corollary The dynamic system (8) along with

$$\dot{\gamma}=\psi(x,\bar{u},\gamma),\ \text{with}\ \gamma_0=\gamma(0)\ \text{and}\ \gamma\in C^1 \tag{9}$$

constitute a proportional asymptotic reduced order fault observer for the system (7), where γ is a change of variable which depends on the estimated fault $\hat{f}$, and the state variables. ■

V. SLIDING-MODE OBSERVER

Consider the nonlinear system with faults given by (1), assuming that the system satisfies definition 6, that is to say, the fault vector f is algebraically observable over $\mathbb{R}\langle u, y\rangle$ and therefore satisfies a differential algebraic polynomial

$$\bar{\psi}(f, y, \dot{y}, \ddot{y}, ..., \overset{(r)}{y}, u, \dot{u}, ...) = 0 \tag{10}$$

Where r is the maximum order of the output time derivatives.

Introducing the following change of coordinates

$$\eta_1 = y,\ \eta_2 = \dot{y},\ ..., \eta_r = \overset{(r-1)}{y}. \tag{11}$$

Then in a domain D where $\partial\bar{\psi}/\partial \overset{(r)}{y}$ is invertible, the corresponding input-output representation of (1) and (10) can be rewritten as follows

$$\begin{array}{c} \dot{\eta}_1 = \eta_2 \\ \dot{\eta}_2 = \eta_3 \\ \cdots \\ \dot{\eta}_r = \Phi(\eta_1, \eta_2, ...\eta_r, u, \dot{u}, .. \overset{(r-1)}{u}) \end{array} . \tag{12}$$

Where $\Phi(.)$ is considered as an unmodelled dynamics.

The observer structure. The following system is a sliding-mode asymptotic observer for the system (12).

$$\begin{array}{c} \dot{\hat{\eta}}_1 = \hat{\eta}_2 + m_1 sign(y - \hat{y}) \\ \cdots \\ \dot{\hat{\eta}}_{r-1} = \hat{\eta}_r + m_2 sign(y - \hat{y}) \\ \dot{\hat{\eta}}_r = m_r sign(y - \hat{y}) \\ \text{with } \hat{y} = \hat{\eta}_1 \end{array} \tag{13}$$

where $m_j > 0$, $\forall j = 1, ..., r$, and

$$sign(y - \hat{y}) = \begin{cases} 1 & if\ (y - \hat{y}) > 0 \\ -1 & if\ (y - \hat{y}) < 0 \\ undefined & if\ (y - \hat{y}) = 0 \end{cases} .$$

Then returning to the original coordinates and taking into account (10), the fault can be estimated from

$$\bar{\psi}(\hat{f}, \hat{\eta}, \dot{\hat{\eta}}, \ddot{\hat{\eta}}, ..., \overset{(r)}{\hat{\eta}}, u, \dot{u}, ...) = 0 \tag{14}$$

Observer Convergence Analysis. The goal of implementing an observer with guarantied convergence properties in the general case is a complicated task, so in the following part the case $r = 2$ is considered, and with $m_1 = m\tau^{-1}, m_2 = m_1^2$, the observer structure is

$$\begin{array}{c} \dot{\hat{\eta}}_1 = \hat{\eta}_2 + m\tau^{-1} sign(y - \hat{y}) \\ \dot{\hat{\eta}}_2 = m^2\tau^{-2} sign(y - \hat{y}) \end{array} . \tag{15}$$

In order to analyze the convergence properties of the proposed observer in this case, let the error estimation dynamics be defined as

$$\begin{array}{c} e_1 = \eta_1 - \hat{\eta}_1 \\ e_2 = \dfrac{\eta_2 - \hat{\eta}_2}{m} \end{array} . \tag{16}$$

It follows that the estimation error vector $e = \begin{bmatrix} e_1 & e_2 \end{bmatrix}^T$ verify the ordinary differential equation

$$\dot{e} = A_{\bar{\mu}} e - K sign(Ce + \delta) + \Delta s \tag{17}$$

Where $\bar{\mu} > 0$ is a regularizing parameter, and

$$A_{\bar{\mu}} = \begin{bmatrix} -\bar{\mu} & m \\ 0 & -\bar{\mu} \end{bmatrix},\ K = m\tau^{-1}\begin{bmatrix} 1 \\ \tau^{-1} \end{bmatrix}, \\ C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

and $\Delta s = \begin{bmatrix} \bar{\mu} e_1 \\ \frac{1}{m}\Phi + \bar{\mu} e_2 \end{bmatrix}$ is an uncertainty term.

Assumption A1. There exist nonnegative constants L_{0s}, L_{1s}, such that the following generalized quasi-Lipschitz condition holds

$$\|\Delta s\| \leq L_{os} + (L_{1s} + \|A_{\bar{\mu}}\|)\, \|e\| .$$

Assumption A2. There exists an additive output bounded noise δ, that is $y = \eta_1 + \delta$, and

$$\|\delta\|_\Lambda^2 := \delta^T \Lambda \delta \leq (\delta^+)^2 < \infty,$$

where the positive definite matrix $\Lambda = \Lambda^T > 0$ is a normalizing matrix (because different components of the output may have a different physical nature).

Assumption A3. There exists a positive definite matrix $Q_0 = Q_0^T > 0$, such that the following matrix Riccati equation

$$PA_\mu + A^T P + PRP + Q = 0$$

with

$$R := \Lambda_s^{-1} + 2\, \|\Lambda_s\|\, L_{1s} I,\ \ \Lambda_s = \Lambda_s^T > 0, \\ Q = Q_0 + 2(L_{1s} + \|A_{\bar{\mu}}\|^2) I$$

has a positive definite solution $P = P^T > 0$.

Assumption A4. The gain matrix K is selected as $K = kP^{-1}C^T$, where $k > 0$.

Theorem 3. If the assumptions *A1* to *A4* are satisfied then

$$\left[1 - \frac{\tilde{\mu}}{V(e)}\right]_+ \to 0 \tag{18}$$

Where $V(e) = \|e\|_P^2 := e^T P e$,

$$\tilde{\mu} = \tilde{\mu}(k) := \left(\frac{\rho(k)}{\sqrt{(k\alpha_P)^2 + \rho(k)\alpha_Q} + k\alpha_P}\right)^2, \\ \rho(k) := 2\, \|\Lambda_s\|\, L_{0s}^2 + 4k\sqrt{n\, \|\Lambda^{-1}\|}\delta^+, \\ \alpha_P := \lambda_{\min}(P^{-1/2} C^T C P^{-1/2}) \geq 0, \\ \alpha_Q := \lambda_{\min}(P^{-1/2} Q^T Q P^{-1/2}) > 0$$

and the function $[\cdot]_+$ is defined as follows

$$[x]_+ = \begin{cases} x & if\ x \geq 0 \\ 0 & if\ x < 0 \end{cases} . \ ■$$

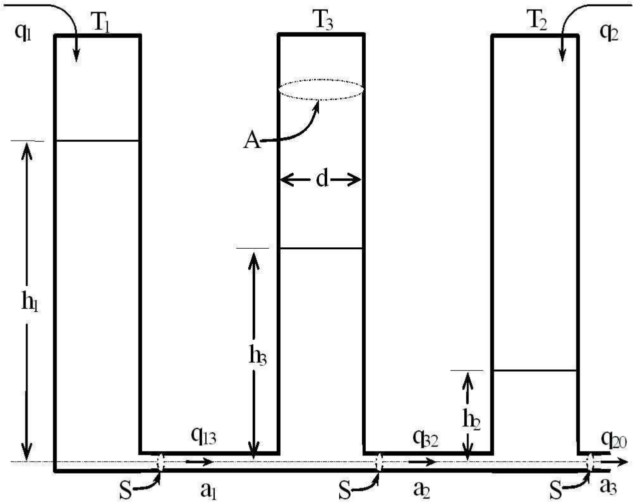

Fig. 1. Schematic diagram of the three-tank system

The proof of this theorem is omitted and can be followed using some ideas given in [18].

Remark 2. The theorem 3 states that the weighted estimation error norm $V(e)$ actually converges to the zone $\tilde{\mu}$ asymptotically. In other words, it is ultimately bounded.

VI. Application to the three-tank system

A. Description of the three-tank system

The Amira DTS200 is described in figure 1. The corresponding nominal model is given by the following system [17]

$$\begin{array}{l} \dot{x}_1 = \frac{1}{A}(q_1 - q_{13}) \\ \dot{x}_2 = \frac{1}{A}(q_2 + q_{32} - q_{20}) \\ \dot{x}_3 = \frac{1}{A}(q_{13} - q_{32}) \end{array} \tag{19}$$

where the state vector is chosen as $\left[\begin{array}{ccc} x_1 & x_2 & x_3 \end{array}\right] = \left[\begin{array}{ccc} h_1 & h_2 & h_3 \end{array}\right]$, being h_i the level in the tank i. A is the transversal constant section of any of the identical tanks, and q_{ij} represents the water flow from tank i to tank j, $(i, j \in \{1, 2, 3\})$ which according to the generalized Torricelli's rule, valid for laminar flow

$$q_{ij} = a_i S \ sign(\Delta h_{ij}) \sqrt{2g \left|\Delta h_{ij}\right|} \tag{20}$$

with $q_{20} = a_2 S \ \sqrt{2gh_2}$.

Where $\Delta h_{ij} \stackrel{\triangle}{=} h_i - h_j$, S is the transversal area of the pipe that interconnects the tanks (see figure 1) and a_i are the output flow coefficients,which are not exactly known, so they are considered as uncertain parameters.

The system (19) has four state regions in which the corresponding model is differentiable, in this work $x_1 \geq x_3 \geq x_2$ is the only considered region of operation.

B. Considered faults and measurements

The nominal model (19) is transformed into the following, where two additive faults f_1 and f_2 ($\mu = 2$) are considered in the actuators that control the input flows $u_1 = q_1$, $u_2 = q_2$.

$$\begin{array}{l} \dot{x}_1 = \frac{1}{A}(u_1 - q_{13} + f_1) \\ \dot{x}_2 = \frac{1}{A}(u_2 + q_{32} - q_{20} + f_2) \ . \\ \dot{x}_3 = \frac{1}{A}(q_{13} - q_{32}) \end{array} \tag{21}$$

C. Diagnosability analysis

In order to determine what is the minimum number of measurements to be considered for fault reconstruction, we have to evaluate the algebraic observability condition for the faults in different cases of measurement availability. We only consider the region of operation $x_1 \geq x_3 \geq x_2$,

1) Case 0: The easiest case (and the only one reported in previous works [12]) takes place when we can measure the complete state vector, that is to say, we have three outputs: $y_1 = x_1$, $y_2 = x_2$, $y_3 = x_3$; in this case, from (21) we have

$$f_1 = A\dot{y}_1 + a_1 S\sqrt{2g\,(y_1 - y_3)} - u_1 \tag{22}$$

$$f_2 = A\ \dot{y}_2 - a_3 S\sqrt{2g\,(y_3 - y_2)} + a_2 S\sqrt{2gy_2} - u_2 \tag{23}$$

it is clear from (22) and (23) that the system is diagnosable, with $p = 3$ and $\mu = 2$ (see theorem 1).

2) Case 1: Elimination of x_1. In this case we consider only the two measurable outputs: $y_2 = x_2$ and $y_3 = x_3$. By taking into account the third state equation from (21), which can be rewritten as

$$A\dot{y}_3 = a_1 S\sqrt{2g\,(x_1 - y_3)} - a_3 S\sqrt{2g\,(y_3 - y_2)}, \tag{24}$$

we get

$$x_1 = y_3 + \frac{1}{2ga_1^2 S^2}\left(A\dot{y}_3 + a_3 S\sqrt{2g\,(y_3 - y_2)}\right)^2 \tag{25}$$

Then, by replacing x_1 in (22) we obtain a set of two differential equations with coefficients in $\mathbb{R}\langle u, y\rangle$ with two unknowns f_1 and f_2, this means that $\mathit{diff\ tr\ d}^\circ\ \mathbb{R}\langle u, y\rangle/\mathbb{R}\langle u\rangle = 2$ and therefore, from theorem 2, the faults are diagnosable with the two considered outputs.

3) Case 2: Elimination of x_2. In this case we consider only the two measurable outputs: $y_1 = x_1$ and $y_3 = x_3$. By taking into account (24) we obtain

$$x_2 = y_3 - \frac{1}{2ga_3^2 S^2}\left(-A\ \dot{y}_3 + a_1 S\sqrt{2g\,(y_1 - y_3)}\right)^2 \tag{26}$$

By replacing x_2 in (23) in a similar way we can obtain that $\mathit{diff\ tr\ d}^\circ\ \mathbb{R}\langle u, y\rangle/\mathbb{R}\langle u\rangle = 2$ and from theorem 2, the faults are diagnosable with the two considered outputs.

4) Case 3: Elimination of x_3. In this case we consider only the two measurable outputs: $y_1 = x_1$ and $y_2 = x_2$. By taking into account equation (22) we get

$$x_3 = y_1 - \frac{1}{2ga_1^2 S^2}\left(-A\ \dot{y}_1 + f_1 + u_1\right)^2 \tag{27}$$

By replacing x_3 in (23) we can obtain that $\mathit{diff\ tr\ d}^\circ\ \mathbb{R}\langle u, y\rangle/\mathbb{R}\langle u\rangle = 2$ and therefore, from theorem 2, the faults are diagnosable with only the two considered outputs.

D. Fault Reconstruction

The reduced-order and the sliding-mode observers have been used to obtain effective fault estimations [20], [21], as well as they can be used to estimate time derivatives as follows.

Reduced order observer. Let us consider the following time derivative to be estimated

$$\eta = \dot{y}. \tag{28}$$

According to (8), we propose the observer structure

$$\dot{\hat{\eta}} = K(\eta - \hat{\eta}) \tag{29}$$

introducing the change of variable

$$\hat{\eta} = \gamma + Ky \tag{30}$$

and from (29) and (30) we can get $\dot{\gamma} = -K\hat{\eta}$, then again from (30)

$$\dot{\gamma} = -K\gamma - K^2 y \tag{31}$$

then (31) together with (30) constitute an asymptotic estimator for η.

Sliding-Mode Observer. We introduce the following change of variables: $\eta_1 = y$, $\eta_2 = \dot{\eta}_1$, then we obtain the following observer

$$\left.\begin{aligned} \dot{\hat{\eta}}_1 &= \hat{\eta}_2 + m\tau^{-1}sign(y - \hat{\eta}_1) \\ \dot{\hat{\eta}}_2 &= m^2\tau^{-2}sign(y - \hat{\eta}_1) \end{aligned}\right\} \tag{32}$$

which can be used to estimate η_2 from the knowledge of y.

E. Identification of the uncertain parameters

As it was mentioned, the output flow coefficients a_i are not exactly known, but as it can be easily verified, they are algebraically identifiable [12], that is to say, they satisfy differential algebraic equations in $\mathbb{R}\langle u, y\rangle$. Indeed by considering available the complete state vector ($y_1 = x_1$, $y_2 = x_2$, $y_3 = x_3$), in the region of operation, we can obtain from (19) the following relationships:

$$a_1 = \frac{q_1 - A\dot{y}_1}{S\sqrt{2g(y_1 - y_3)}}, \tag{33}$$

$$a_2 = \frac{q_1 + q_2 - A\left(\dot{y}_1 + \dot{y}_2 + \dot{y}_3\right)}{S\sqrt{2gy_2}}, \tag{34}$$

$$a_3 = \frac{q_1 - A\left(\dot{y}_1 + \dot{y}_3\right)}{S\sqrt{2g(y_3 - y_2)}}. \tag{35}$$

VII. Experimental Results

We verified the real time performance of the proposed estimators by using the Amira DTS200 system. The known parameter values for the utilized system are: $A = 0.0149$ m^2, $S = 5 \times 10^{-5}$ m^2 and the unknown parameters are a_1, a_2 and a_3. The sample time in all the experiments was 0.001 s. The experimental results are described as follows

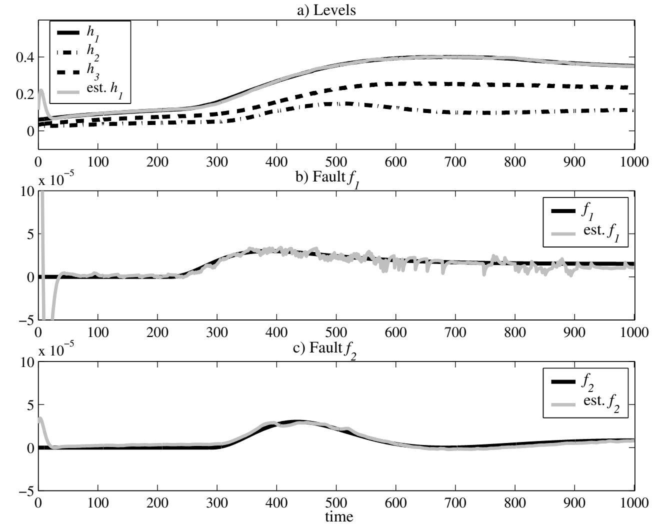

Fig. 2. Fault diagnosis for unknown h_1 using the reduced-order observer: a) Levels. b) Actual and estimated f_1. c) Actual and estimated f_2.

A. Identification results

With no presence of faults, equations (33), (34) and (35) along a period of $1000s$ were computed while maintaining $q_1 = 0.000025$ m^3/s and $q_2 = 0.000020 m^3/s$, for 1000 s, the estimated values for the flow parameters are

$$\hat{a}_1 = 0.418,\ \hat{a}_2 = 0.789,\ \hat{a}_3 = 0.435. \tag{36}$$

B. Fault estimation results

In all the experiments described in this subsection the input flows were maintained constant as $q_1 = 0.00002$ m^3/s and $q_2 = 0.000015$ m^3/s, also two faults were artificially generated through the following expressions:

$$f_1 = 0.00005\left[1 + \sin\left(0.2te^{-0.01t}\right)\right]\mathcal{U}(t - 220),$$
$$f_2 = 0.00005\left[1 + \sin\left(0.05te^{-0.001t}\right)\right]\mathcal{U}(t - 300),$$

where $\mathcal{U}(t)$ is the unit step function.

The two proposed schemes for fault estimation were evaluated for x_1 not measurable.

The two outputs $y_2 = x_2$ and $y_3 = x_3$ were taken into consideration, for this reason an estimation for the unknown state x_1 was necessary to be obtained. In figure 2 we show the resulting estimations achieved with the reduced-order observer. A low-pass filter was necessary in order to reduce the effect of the measurement noise, here we propose a second-order Butterwort filter whose transfer function is given by

$$G_f(s) = \frac{1}{32s^2 + 8s + 1}. \tag{37}$$

The gain values chosen for the fault observers were $k_1 = 2$, $k_2 = 2$ and for x_1, $k_{x_1} = 0.3$. As we can observe, the estimation results with this scheme are good (see figure 2). A sliding-mode observer was also tested in this case. In figure 3 the corresponding results achieved with the sliding-mode observer are shown. It is worth to mention that with this

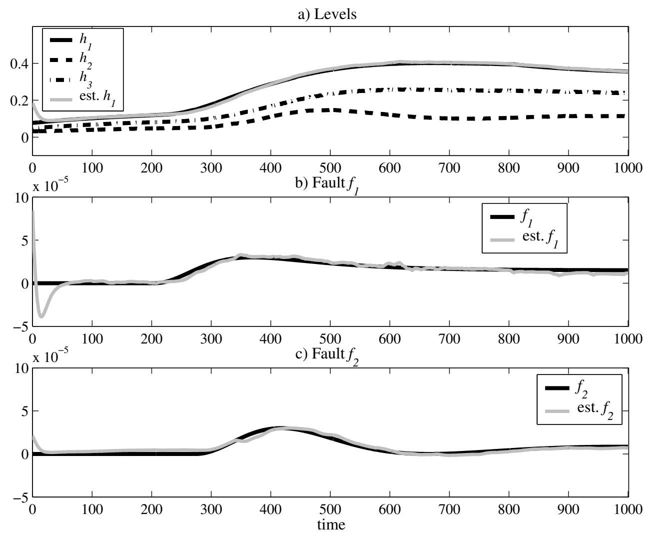

Fig. 3. Fault diagnosis for unknown h_1 using the sliding-mode observer: a) Levels. b) Actual and estimated f_1. c) Actual and estimated f_2.

observer it was not necessary to include the reducing noise filter providing the inherent robustness of this observer. The gain values chosen for the fault observers were $\tau_1 = \tau_2 = 1$, $m_1 = m_2 = 0.1$ and for x_1 : $\tau_{x_1} = 1$, $m_{x_1} = 0.1$. As we can observe, this scheme also provides good estimation results (see figure 3).

VIII. Concluding Remarks

The differential algebraic approach for fault estimation was presented. The usefulness of theorem 2 was shown in the determination of the minimal number of measurements needed for fault diagnosis in the Amira DTS-200 three-tank system ($p = 2$), this allowed the estimation of two simultaneous faults with less measurements than in previous reported works. The theoretical and simulation results were tested in a real-time experimental setting. The experimental results for the two different observers showed similar performance, nevertheless the proposed sliding-mode observer is more robust against measurement noise, as it was expected.

References

[1] Alcorta García E. and Frank P, "Deterministic nonlinear observer-based approaches to fault diagnosis: a survey", *Control Eng. Pract.* **no. 5**, pp 663-670, 1997.

[2] Frank P. and Ding X. "Survey of robust residual generation and evaluation methods in observer-based fault detection systems", *Journal of Process Control*, **no. 7**, pp 403-424. 1977

[3] Willsky A., "A survey of design methods in observer-based fault detection systems", *Automatica*, **no. 12**, pp 601-611, 1976

[4] De Persis C. and Isidori, "A geometric approach to nonlinear fault detection and isolation", *IEEE Transactions on Automatic Control*, vol. 46, **no. 6**, pp 853-865, 2001

[5] Massoumnia, Verghese G. and Willsky A., "Failure detection and identification", IEEE Transactions on Automatic Control, vol. 34, pp 316-321, 1989.

[6] Martínez-Guerra R. and Mendoza Camargo J., "Observers for a class of Liouvillian and non-differentially flat systems", *IMA Journal of Mathematical Control and Information*, **no. 21**, pp 493-509, 2004.

[7] Martínez-Guerra R. and Diop S., "Diagnosis of nonlinear systems using an unknown-input observer: an algebraic and differential approach", *IEE Proc. Control Theory Appl.*, vol 151, **no. 1**, pp 130-135, February 2004.

[8] Diop S. and Martínez-Guerra R., "An algebraic and data derivative information approach to nonlinear system diagnosis", in *Proceedings of the European Control Conference (ECC)*, Porto, Portugal, pp. 2334-2339, 2001.

[9] Martínez-Guerra R. González-Galán A, Juárez L. and Cruz-Victoria J., "Diagnosis for a class of non differentially flat and Liouvillian systems", *IMA Journal of Mathematical Control and Information*, vol. 24, pp 177-195, 2007.

[10] Diop S. and Martínez-Guerra R., "On an algebraic and differential approach of nonlinear systems diagnosis", in *Proceedings of the 40th IEEE Conference on Decision and Control*, Orlando Florida, USA, pp. 585-589, December 2001.

[11] Fliess M., Join C. and Mounier H., "An introduction to nonlinear fault diagnosis with an application to a congested internet router", *Advances in Communication Control Networks, C. T. Abdallah, J. Chiasson (Eds), Lecture Notes, Conf. Inf. Sci.*, Springer, Berlin, vol. 308, pp. 327-343, 2005.

[12] Join C., Sira-Ramírez H. and Fliess M., "Control of an uncertain three tank system via on-line parameter identification and fault detection", In *Procedings of the 16th IFAC World Congress*, Prague, July 2005.

[13] Fliess M., Join C. and Sira-Ramírez H., "Robust residual generation for nonlinear fault diagnosis: an algebraic setting with examples", *International Journal of Control*, vol. 14, **no. 77**, 2004.

[14] Cruz-Victoria J. and Martínez-Guerra R., "Fault reconstruction using differential algebraic methods", *Journal (WSEAS) Transactions on Systems*, Issue 12, vol. 4, **no. 8**, pp 2269-2276, December 2005.

[15] Join C., Ponsart J.-C., Sauter D. and Theilliol D. "Nonlinear filter design for fault diagnosis: application to the three-tank system", IEE Proc. Control Theory Appl., vol. 152, No. 1, pp 55-64, 2005.

[16] Theilliol D., Noura H., Ponsart J.-C., "Fault diagnosis and accommodation of a three-tank system based on analytical redundancy", ISA Transactions, no. 41, 2002.

[17] Amira DTS200: "*Laboratory setup three tank system*", Amira Gmbh, Duisburgh, Germany, 1996

[18] Martínez-Guerra R., Aguilar R., and Poznyak A., "A new robust sliding-mode observer for monitoring in chemical reactors". Journal of Dynamic Systems, Measurement and Control, vol. 126, pp. 473-478, 2004.

[19] Kolchin E., *Differential Algebra and Algebraic Groups*. New York: Academic Press, 1973.

[20] Rincón-Pasaye J. J. and Martínez-Guerra R., "Fault diagnosis by means of sliding mode techniques ". In *Proceedings of the 4th International Conference on Electrical and Electronics Engineering (ICEEE07)*, pp 218-221, Mexico City, September 2007

[21] Martínez-Guerra R., Cruz-Victoria J. , and Rincón-Pasaye J. J. "On nonlinear systems diagnosis using differential and algebraic methods", *Journal of the Franklin Institute*, to appear 2007.