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Self-sorting in a swarm of heterogeneous agents

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Abstract— **Sorting of heterogeneous units is a self-organized behavior which is seen in many biological systems. One of the best examples of such systems is a system of biological cells of heterogeneous types that has the ability to self-organize into specific formations, form different types of organs and, ultimately, develop into a living organism. Earlier research in this area has indicated that such self-sorting behaviors in biological cells and tissues are made possible because of difference in the adhesivity between different types of cells or tissues. Inspired by this differential adhesivity model, this paper presents a decentralized approach based on differential artificial potential to achieve the self sorting behavior in a swarm of heterogeneous robotic agents. The method is based on the proposition that agents of different types experience different magnitude of potential while they are interacting with agents of different types. An analysis of the system with the proposed approach in Lyapunov sense is carried out for stability. Extensive simulation studies and numerical analysis suggest that the proposed method would always lead a population of heterogeneous agents closer to the sorted or segregated configuration.**

I. INTRODUCTION

Formation control of multiple autonomous vehicles has received attention of several researchers working in the area of mobile robotics because of its potential applications in a number of fields including cooperated search and rescue operation, surveillance, reconnaissance, and boundary protection. Advancement in communication and sensing technologies, and in computing resources have made it possible to coordinate the movement of several autonomous vehicles working cooperatively to achieve certain mission. One of the very first applications of formation control of multiple agents was behavioral simulation of flocks of birds, herd of animals and schools of fish for computer graphics by Reynolds [17]. He stated three simple behaviors that lead to flocking in birds and fish: collision avoidance, velocity matching, and flock centering (in decreasing order of precedence). The biggest merit of Reynolds' approach was that these behaviors were based on observations of local environment and interactions on a local scale that could be fully implemented in individual agents. These local interactions among agents resulted in global flocking, schooling, and herding behaviors which were totally scalable.

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Drawing inspiration from Reynolds' approach, many researchers have focused on designing decentralized controller for achieving flocking behavior. The examples include behavior-based methods [2], leader-follower technique [5], [6], method based on formation constraint and virtual leaders/beacons [7], Lyapunov function based methods [13], [14], [15], [16]. The concept of artificial potential has been used in robotics by many researchers. For example, artificial potential has been used for path planning [24], manipulator control [23], robot navigation [12], and obstacle avoidance [11], and multi-robot formation control [13], [15].

Obtaining a desired shape and pattern of the formation can be critical for a mission relying on coordinated action by multiple mobile agents. For example, if a large number of robots need to be deployed to perform complicated tasks such as surveillance of large area, perimeter protection of vital installation, or surrounding site of a chemical or hazardous waste spill, the robots must be able to autonomously organize themselves in certain formation, pattern, or shape. In many situations, it may not be possible to integrate all the capabilities, sensing or actuation, required for different kinds of tasks in an individual robot. Accordingly, the robots may have heterogeneous abilities for sensing and actuation that will enable them to perform specific tasks. Heterogeneous robots must be able to self-organize themselves in a mission specific manner to carry out tasks assigned to them. The main contribution of this paper is synthesis and analysis of a controller that allows the robots to segregate or sort so that they form separate groups comprising of homogeneous robots.

II. SEGREGATION AND SORTING BEHAVIORS IN BIOLOGY

Sorting is a phenomenon which is seen in several biological systems. Examples include brood sorting by ants [4], segregation in amphibian larvae based on kinship [10], and aggregation /segregation behaviors in cockroaches [1] based on odors of strains. Sorting of cells based on their types and functionalities is one of the best examples of sorting in biological systems. Cell sorting is one of the basic phenomenon which leads to formation of patterns and organs in living organisms. Study of formation of patterns in living organisms is called morphogenesis. The mechanisms by which these patterns form can provide valuable insights for distributed problem solving strategies. Most of the strategies or models in literature that can explain formation of patterns rely on differential attraction/inhibition. For example, Swindale's model [20] accounts for formation of ocular dominance stripes in visual cortex based on local activation and lateral inhibition

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(LALI) mechanism [8] for like type of synapses, and local inhibation and lateral activation (LILA) [3], [20], the reverse, for the unlike type of synapses. Reaction-diffusion [22] is a model to mathematically represent the transport phenomenon in biological and natural systems. This model tries to explain the interaction of particles with the environment and their motion in space. In early 1990s, Graner and Glazier [9] proposed a lattice based modified version of large-Q Potts model with differential adhesivity to explain and simulate the sorting of a mixture of two types of biological cells. In fact, it has been long known [19] that it is the difference in intercellular adhesivity that leads to sorting in cells. The final state of cell configuration is achieved when the overall surface energy is globally minimized. Based on this principle, Steinberg [19] postulated that cells are sorted i.e., two types of cellular units A and B are segregated when:

$$W_{AB} < \frac{(W_{AA} + W_{BB})}{2} \tag{1}$$

where W_{AA} and W_{BB} represent the work of cohesion between particles or cells of same types (i.e., between types A & A, and B & B respectively), and W_{AB} represents the work of adhesion between cells of types A and B. The method for self-sorting in artificial mobile agents presented in this paper is motivated by this differential adhesivity phenomenon observed in biological systems that leads to sorting.

III. Problem Formulation

The group of mobile agents consists of N fully actuated agents, each of whose dynamics is given by the double integrator:

$$\begin{aligned} \dot{q}_i &= p_i \\ \dot{p}_i &= u_i(t) \qquad i = 1, 2, \ldots, N \end{aligned} \tag{2}$$

where q_i and p_i are m-dimensional position and velocity vectors respectively of agent i. The group of mobile agents, considered in this paper, consists of two different types of agents: type A and type B. The number of agents of type A is N_A and that of type B is N_B such that $N = N_A + N_B$. The objective of this paper is to synthesize a controller that can asymptotically flock and separate the robots of type A and type B into two different groups (referred to as *sorting* or *segregation*). Agents are said to flock (asymptotically) when all agents achieve the same velocity vector, distances between the agents are stabilized, and no collisions occur. Let us try to precisely define the term *Segregation*. A group of agents of types A and B are said to be *segregated* if there exists a hyperplane that separates the two different types of agents. Alternatively:

$$\exists \mathbf{e} : \langle (q_j - q_i), \mathbf{e} \rangle \geq 0 \qquad \forall i \in A \quad \& \quad j \in B \tag{3}$$

where $\langle \mathbf{a}, \mathbf{b} \rangle$ means inner product between vectors **a** and **b**. This implies that every agent of type A is on one side of the hyperplane, and that of type B is on the other side of the hyperplane. For the sake of simplicity, we relax this definition and alternatively define *Segregation* to be a configuration of agents where the average distance between the agents of like types (type A or type B) is less than the average distance of agents between the unlike types (between agents of type A and type B). Alternatively,

$$r^{AA}_{avg} < r^{AB}_{avg}, \quad r^{BB}_{avg} < r^{AB}_{avg} \tag{4}$$

where r^{XY}_{avg} is the average distance between agents of types X and Y.

IV. Control Law Formulation

This section presents the control law which causes a population of heterogeneous agents to asymptotically flock as well as segregate. For a system of N mobile agents with N_A agents of type A and N_B agents of type B, following feedback control law is considered:

$$u_i = -\sum_{j \in N_i} \nabla_{q_i} V_{ij}(\|q_j - q_i\|) - a \sum_{j \in N_i} (p_i - p_j) \tag{5}$$

where u_i is the control input to the agent i, N_i is the set of agents in the neighborhood of agent i, $V_{ij}(\|q_j - q_i\|)$ is the artificial potential of interaction between agents i and j, $\|q_j - q_i\|$ is the norm of vector $(q_j - q_i)$ representing the euclidean distance between agent i and j, and ∇_{q_i} is the gradient with respect to coordinates of agent i i.e., q_i. First term in equation (5) represents the gradient of potential function, and the second term represents damping and causes the agents to match their velocities with each other. The artificial potential is a non-negative function of relative distances between a pair of neighbors given by $V_{ij}(q_i, q_j) : R^{2m} \rightarrow R_{\geq 0}$. Artificial potential function, V_{ij} due to interaction between two agents i and j can be expressed [13] as:

$$V_{ij} = \begin{cases} a\left(ln(q_{ij}) + \frac{d_0}{q_{ij}}\right) & \text{if} \quad 0 \leq q_{ij} \leq d_1 \\ a\left(ln(d_1) + \frac{d_0}{d_1}\right) & \text{if} \quad q_{ij} > d_1 \end{cases} \tag{6}$$

where, a is a scalar control gain, and $q_{ij} = \|q_j - q_i\|$. The parameters d_0 and d_1 respectively represent the inter-agent distance below which (i.e. when $q_{ij} < d_0$) the interaction force is repulsive (negative) and above which (i.e. when $q_{ij} > d_1$) the interaction force is zero. Figure 1 shows the potential function plotted against the inter-agent distance. As indicated in the figure, the potential becomes minimum when the inter-agent distance is d_0. The interaction between agents happen with the help of sensing or communication devices. The parameter d_1, then, can be regarded as the sensing or communication range. Without loss of generality, in this paper the parameter d_1 is considered infinity so that each agent can interact with the rest of the agents. Results presented in this paper remain unaffected for finite d_1 if connectedness of the underlying graph of the system is assumed. Equation (6), under such a condition, can be simply written as:

$$V_{ij} = a\left(ln(q_{ij}) + \frac{d_0}{q_{ij}}\right) \tag{7}$$

The basis for controller synthesis in this paper is the parameter d_0. Since there are two types of mobile agents

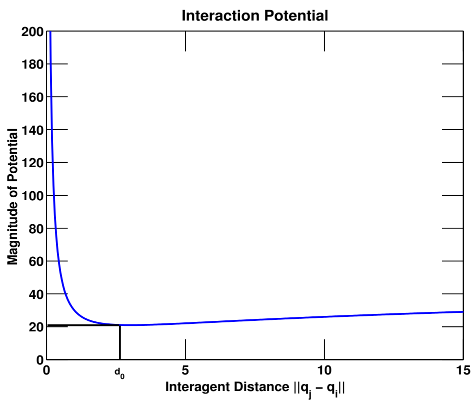

Fig. 1. Interaction Potential versus Inter-Agent Distance

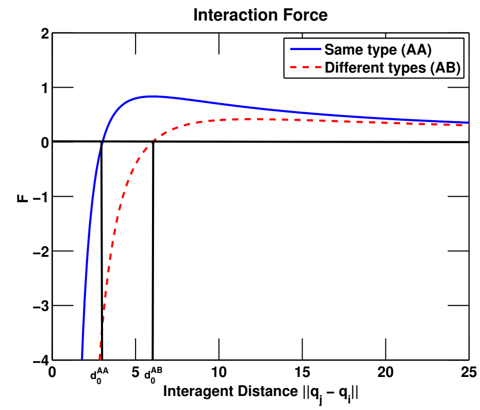

Fig. 2. Interaction Force between Agents

involved in the system, there are three different kinds of artificial potentials involved: a) Potentials arising due to interaction between types A and A, b) Potentials arising due to interaction between types B and B, and c) Potentials arising due to interaction between types A and B. In this paper, potentials arising due to interaction between the same types are considered to be same, i.e., $V_{ij}^{AA} = V_{ij}^{BB}$. The term V_{ij}^{AA} is given by:

$$V_{ij}^{AA} = a\left(ln(q_{ij}) + \frac{d_0^{AA}}{q_{ij}}\right) \tag{8}$$

and the term V_{ij}^{AB} is given by:

$$V_{ij}^{AB} = a\left(ln(q_{ij}) + \frac{d_0^{AB}}{q_{ij}}\right) \tag{9}$$

The control law for segregation can be achieved when:

$$d_0^{AA} = d_0^{BB} < d_0^{AB} \tag{10}$$

Figure 2 shows the plot of force of interaction due to similar types and due to dissimilar types of robots versus interagent distance when the condition for segregation controller (10) is met. In this case, it can be seen that the interaction force between agents of same types is greater than that of force between agents of different types at any given distance. Hence, this method of segregation, based on differential potential, is analogous to Steinberg's [19] explanation of cell sorting based on differential adhesiveness (see Equation (1)).

V. Controller Analysis

In this section, we carry out an analysis of convergence and stability properties of the system of multiple agents obeying dynamics given by Equation (2) under control law given by Equation (5). In order to carry out stability analysis of the collective motion of agents, the following positive definite function can be chosen as the Lyapunov function:

$$\phi(\mathbf{q},\mathbf{p}) = V(\mathbf{q}) + \frac{1}{2}\mathbf{p}^T\mathbf{p} \tag{11}$$

where $\mathbf{q} \in R^{mN}$ is stacked position vector of all agents, $\mathbf{p} \in R^{mN}$ is stacked velocity vector of all agents, and $V(\mathbf{q}) : R^{mN} \rightarrow R_{\geq 0}$ is the total potential energy of the system.The total potential of the system consists of three parts: 1) Potential due to interaction of agents of type A, 2) Potential due interactions amongst agents of types A and B, and 3) Potential due to interaction of agents of type B. This can be written as:

$$\begin{aligned} V(\mathbf{q}) &= \mathbf{V_{AA}}(\mathbf{q}) + \mathbf{V_{BB}}(\mathbf{q}) + \mathbf{V_{AB}}(\mathbf{q}) \\ &= \frac{1}{2}\sum_{i\in A}\sum_{j\in A, j\neq i} V_{ij}(\|q_j - q_i\|) + \sum_{i\in A}\sum_{j\in B} V_{ij}(\|q_j - q_i\|) \\ &+ \frac{1}{2}\sum_{i\in B}\sum_{j\in B, j\neq i} V_{ij}(\|q_j - q_i\|) \end{aligned} \tag{12}$$

The collective dynamics of the system can be given by:

$$\dot{\mathbf{q}} = \mathbf{p} \tag{13}$$

$$\dot{\mathbf{p}} = -\nabla V(\mathbf{q}) - \hat{L}(\mathbf{q})\mathbf{p} \tag{14}$$

where $\hat{L}(\mathbf{q}) \in R^{mN\times mN}$ is m-dimensional graph Laplacian (see reference [15]). Among other important properties of graph Laplacian matrix $\hat{L}(\mathbf{q})$, it is a positive semi-definite matrix.

Lemma 5.1: Consider a system of N mobile agents. Each of the agents follows dynamics given by Equation (2), and with feedback control law given by Equation (5). For any initial condition belonging to the level set of $\phi(\mathbf{q},\mathbf{p})$ given by $\Omega_C = \{(\mathbf{q},\mathbf{p}) : \phi(\mathbf{q},\mathbf{p}) \leq C)\}$ with $C > 0$, and when the underlying graph of the system is connected and cohesive, then the system asymptotically converges to an invariant set $\Omega_I \subset \Omega_C$ such that the points in Ω_I have a velocity that is bounded and velocity of all agents match.

Differentiating $\phi(\mathbf{q},\mathbf{p})$ with respect to time and using Equation (14) one gets:

$$\begin{aligned} \dot{\phi}(\mathbf{q},\mathbf{p}) &= \mathbf{p}^T\nabla V(\mathbf{q}) + \mathbf{p}^T\dot{\mathbf{p}} \\ &= \mathbf{p}^T\nabla V(\mathbf{q}) + \mathbf{p}^T(-\nabla V(\mathbf{q}) - \hat{L}(\mathbf{q})\mathbf{p}) \\ &= -\mathbf{p}^T\hat{L}(\mathbf{q})\mathbf{p} \leq 0 \end{aligned} \tag{15}$$

since $\hat{L}(\mathbf{q})$ is a positive semi-definite matrix. From Lasalle's Invariance Principle, all solutions of the system starting in Ω_C will converge to the largest invariant set

$\Omega_I = \Big\{(\mathbf{q},\mathbf{p}) \in \Omega_C : \dot{\phi}(\mathbf{q},\mathbf{p}) = 0)\Big\}$, and this happens when the velocity of all agents match. For a detailed proof of this lemma, please see references [13], [15]. Also, equilibrium condition is achieved when $\dot{\mathbf{p}} = 0$. Since there are no external forces acting on the agents, the velocities of all agents become zero (the center of mass of the system does not move), i.e. $\mathbf{p} = \mathbf{0}$. Hence, equilibrium is achieved when the total potential of the system is at extremum. This leads to the following proposition.

Proposition 5.2: When control actions lead to the global minimization of potential, then the system is segregated if the condition for segregation controller (10) is applied.

Since each term of $V(\mathbf{q})$ is a non-negative term, the global minimum is reached when each individual term of the expression (12) is minimum (assuming each term is independent of the other). This happens when:

$$\begin{aligned} r_{ij} &= \|q_i - q_j\| = d_0^{AA} \qquad \forall i \in A, j \in A \\ r_{ij} &= \|q_i - q_j\| = d_0^{BB} \qquad \forall i \in B, j \in B \\ r_{ij} &= \|q_i - q_j\| = d_0^{AB} \qquad \forall i \in A, j \in B \end{aligned} \tag{16}$$

Hence if condition (10) for controller is applied, we will have the segregation given by condition (4).

However, the system of agents with a global minimum configuration where all individual potentials are minimum is not achievable. Problems if this nature, referred to as graph embeddings [18], have been extensively studied in topological graph theory. More likely, the system will reach a local minimum condition which is given by:

$$\nabla V(\mathbf{q}) = 0 \tag{17}$$

The partial derivative given by equation (17) for an agent $i \in A$ is given by the equation:

$$\begin{aligned} \nabla_{q_i} V(\mathbf{q}) = \sum_{j\in A, j\neq i} a \left[\frac{1}{\|q_i - q_j\|} - \frac{d_0^{AA}}{\|q_i - q_j\|^2}\right] \frac{(q_j - q_i)}{\|q_i - q_j\|} \\ + \sum_{j\in B} a \left[\frac{1}{\|q_i - q_j\|} - \frac{d_0^{AB}}{\|q_i - q_j\|^2}\right] \frac{(q_j - q_i)}{\|q_i - q_j\|} = 0 \end{aligned} \tag{18}$$

Equation (18) is also a force balance equation for an agent $i \in A$ and can be re-written as:

$$\sum_{j\in A, j\neq i} F_{ij}^{AA}(q_j - q_i) + \sum_{j\in B} F_{ij}^{AB}(q_j - q_i) = 0 \tag{19}$$

where

$$\begin{aligned} F_{ij}^{AA} &= \left[\frac{1}{\|q_i - q_j\|^2} - \frac{d_0^{AA}}{\|q_i - q_j\|^3}\right] \\ F_{ij}^{AB} &= \left[\frac{1}{\|q_i - q_j\|^2} - \frac{d_0^{AB}}{\|q_i - q_j\|^3}\right] \end{aligned} \tag{20}$$

If we sum up equation (19) for all $i \in A$, and noting that $F_{ij}^{AA} = F_{ji}^{AA}$, then we will obtain the following equation:

$$\sum_{i\in A}\sum_{j\in B} F_{ij}^{AB}(q_j - q_i) = 0 \tag{21}$$

Equation (21) leads to the following proposition:

Proposition 5.3: If we consider one dimensional case, i.e., $q, p \in R$, then the system of heterogeneous swarming agents following dynamics (2) and control law (5) flock together such that the average distance between the agents of different types is greater than or equal to the parameter d_0^{AB}, i.e., $r_{avg}^{AB} \geq d_0^{AB}$.

This proposition can be proved using equation (21) in the following manner. Let us write:

$$f_{ij}^{AB} = F_{ij}^{AB}(q_j - q_i) \tag{22}$$

Let us assume that out of $n_A n_B$ possible terms of f_{ij}^{AB}, m are the terms for which $q_{ij} = r_{ij} \geq d_0^{AB}$ and n ($m+n = n_A n_B$) are the terms for which $q_{ij} = r_{ij} < d_0^{AB}$. Hence, equation (21) can be written as:

$$\sum_{k=1}^{m} f_k^{AB} + \sum_{k=1}^{n} f_k^{AB} = 0 \tag{23}$$

Let us assume that $d_0^{AB} + x_c$ is the mean distance for the terms for which $r_{ij} \geq d_0^{AB}$, and $d_0^{AB} - x_{c'}$ is the mean distance for the terms for which $r_{ij} < d_0^{AB}$. Hence, equation (23) can be written as:

$$m\left(f^k|_{d_0^{AB}+x_c}\right) + n\left(f^k|_{d_0^{AB}-x_{c'}}\right) = 0 \tag{24}$$

Subsituting from equations (20) and (22):

$$\frac{m}{(d_0^{AB}+x_c)^2}x_c - \frac{n}{(d_0^{AB}-x_{c'})^2}x_{c'} = 0 \tag{25}$$

and since $(d_0^{AB}+x_c) \geq (d_0^{AB}-x_{c'})$, we have from equation (25):

$$m x_c \geq n x_{c'} \tag{26}$$

The average distance between agents of type A and type B is given by:

$$\begin{aligned} r_{avg}^{AB} &= \frac{1}{m+n}\left[m(d_0^{AB}+x_c) + n(d_0^{AB}-x_{c'})\right] \\ &= d_0^{AB} + \frac{1}{m+n}(m x_c - n x_{c'}) \geq d_0^{AB} \end{aligned} \tag{27}$$

d_0^{AB} is a design parameter that can be chosen to be arbitrarily large value, and since r_{avg}^{AB} is always greater than d_0^{AB}, r_{avg}^{AB} can be made arbitrarily high.

Let us now examine the two-dimensional case when $q, p \in R^2$. Since F_{ij}^{AB} is a scalar quantity, in two dimensional case, equation (21) can be equivalently written into the following two scalar equations:

$$\sum_{i\in A}\sum_{j\in B} F_{ij}^{AB} x_{ij} = 0 \tag{28}$$

$$\sum_{i\in A}\sum_{j\in B} F_{ij}^{AB} y_{ij} = 0 \tag{29}$$

where $x_{ij} = (x_j - x_i)$ and $y_{ij} = (y_j - y_i)$, x_i and y_i are coordinates along X and Y axis for agent i. Hence $q_{ij} =$

$\sqrt{(x_{ij}^2 + y_{ij}^2)}$. Let us write equations (28) and (29) as the following:

$$\sum_{k:F_k \geq 0} F_k x_k + \sum_{k:F_k < 0} F_k x_k = 0 \tag{30}$$

$$\sum_{k:F_k \geq 0} F_k y_k + \sum_{k:F_k < 0} F_k y_k = 0 \tag{31}$$

If we assume:

$$\sum_{k:F_k \geq 0} F_k x_k = c_1 \tag{32}$$

then

$$\sum_{k:F_k < 0} F_k x_k = -c_1 \tag{33}$$

and similarly,

$$\sum_{k:F_k \geq 0} F_k y_k = c_2 \tag{34}$$

$$\sum_{k:F_k < 0} F_k y_k = -c_2 \tag{35}$$

Then from Cauchy-Schwarz inequality relation, equations (32) - (35), and noting that $q_k = \sqrt{x_k^2 + y_k^2)}$:

$$\|\mathbf{F}\|_{F_k \geq 0} \|\mathbf{Q^1}\| \geq \sqrt{c_1^2 + c_2^2} = c \tag{36}$$

$$\|\mathbf{F}\|_{F_k < 0} \|\mathbf{Q^2}\| \geq \sqrt{c_1^2 + c_2^2} = c \tag{37}$$

where $\mathbf{F} = \{F_1, F_2 ...\}$, $\mathbf{Q} = \{q_1, q_2 ...\}$, and $c \geq 0$.

Again from Cauchy-Shwarz inequality relations and equations (36) and (37), it is evident that:

$$0 \leq \sum_{F_k \geq 0} F_k q_k \leq c \tag{38}$$

$$-c \leq \sum_{F_k < 0} F_k q_k \leq 0 \tag{39}$$

From inequalities (38) and (39), we can write:

$$-c \leq \sum_{F_k \geq 0} F_k q_k + \sum_{F_k < 0} F_k q_k \leq c \tag{40}$$

Using similar analysis used to prove Lemma (5.3), we can show that:

$$m x_c - n x_{c'} \geq -c \tag{41}$$

where m is the number of terms for which $f_k \geq 0$, and n is the number of terms for which $f_k < 0$. This leads to the following:

$$r_{avg}^{AB} \geq d_0^{AB} - \frac{c}{m+n} \tag{42}$$

The value c in the above inequality is bounded because the quantities c_1 and c_2 in equations (32) and (34) are bounded. Since c is bounded, we can always choose d_0^{AB} to be arbitrarily large making r_{avg}^{AB} to be arbitrarily large.

VI. Simulation Results and Discussions

Extensive simulations were carried out to verify the results obtained in the previous sections. In the simulations, following parameters were assumed :
$d_0^{AA} = d_0^{BB} = 3$, and $d_0^{AB} = 6$

Figure 3 shows the configuration of a population of 20 agents (10 each of type A and B) in a 2D space at different times during the simulation. The agents started off at a random configuration, and control law given by equation 5 based on differential potential was applied to the agents. The final configuration at time T=750 sec shows that the agents of types A and B form two separate groups.

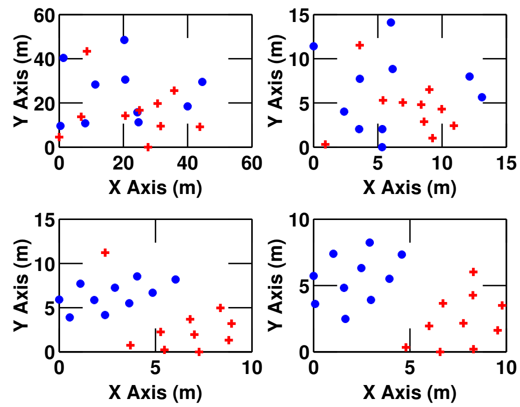

Fig. 3. Configurations of Agents at Times T=0 (top left), T=250 sec (top right), T=500 (bottom left), and T=750 sec (bottom right)

Figure (4) shows the plot of average distances between agents of types A and A (r_{avg}^{AA}), B and B (r_{avg}^{BB}), and A and B (r_{avg}^{AB}) versus time for the above simulation. At the final configuration, the average distances r_{avg}^{AA}, r_{avg}^{BB}, and r_{avg}^{AB} were found out to be 3.21, 3.22, and 6.97 respectively, which clearly shows that the population was segregated based on the condition given by (4).

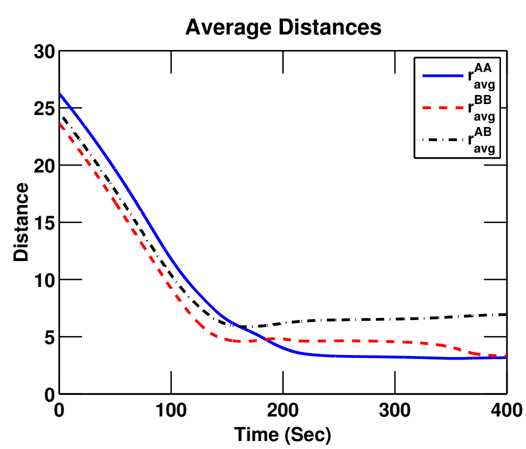

Fig. 4. Average Distances Between Agents of Types A and A (r_{avg}^{AA}), B and B (r_{avg}^{BB}), and A and B (r_{avg}^{AB})

The results given above were for just one simulation run. In order to verify that the method presented in this paper leads to segregation in general in a population of heterogeneous agents, an extensive simulation study was carried out in which more than 100 runs were performed. Figure 5 shows the average distance between agents at steady-state. In each of the simulation runs, the population of agents consisting of types A and B was initialized in a random configuration obtained via uniform distribution of agents in 2D space, and number of agents of type A and B were each chosen randomly between values 5 and 15. Each of the runs was carried out for 500 seconds of simulation time. The average distances between agents shown in the figure are calculated at the steady (final) state. It can be easily seen that the average distance between agents of type A (r_{avg}^{AA}) and average distance agents of type B (r_{avg}^{BB}) is less than the average distance between agents of type A and B (r_{avg}^{AB}) for each of the simulation runs. Also, it is evident from the figure that r_{avg}^{AB} is always greater than the parameter d_0^{AB} supporting our result from proposition 5.3. Moreover, in each of the simulation runs, the agents were completely segregated and satisfied the separating hyperplane condition of segregation given by equation (3).

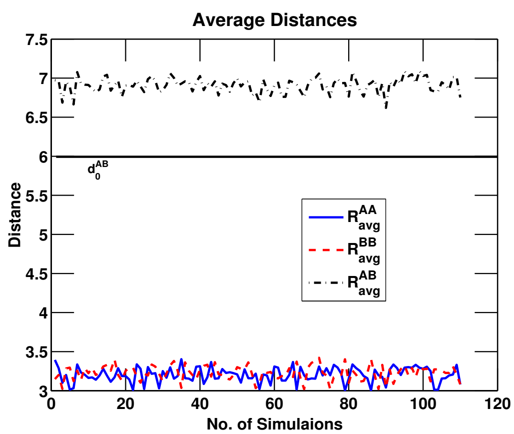

Fig. 5. Average Distance between Agents

The method presented in this paper is equally applicable to more than two types of agents and in higher dimensional space.

VII. Conclusions

The paper presents a decentralized method to achieve self organized bahavior of sorting or segregation in a population of heterogeneous agents. The method is based on the concept of differential artificial potential. The paper presents the stability analysis of a population of agents in Lyapunov framework, and lays down an analytical foundation for synthesis of controllers for self-sorting in artificial potential function framework. Specifically, condition for the synthesis of controllers for sorting is analyzed in one and two dimensional space. Extensive simulation studies verify the results obtained in this paper, and shows the effectiveness of the proposed method in achieving the sorting behavior in a swarm of heterogeneous agents.

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