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Hybrid Control for Vision Based Cart-Inverted Pendulum System

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Abstract—**This paper presents a vision based Cart-Inverted Pendulum (CIP) system under a hybrid feedback configuration: the continuous cart's position measured by encoder and the delayed & sampled inverted pendulum's upper coordinates, obtained from a visual sensor. The challenge here is to stabilize the CIP from a big inclined initial angle by using a low cost CCD camera. Under this scheme, we propose a hybrid control which consists in a Jumping-up (Bang-Bang) control and a two causal stabilization loops control: the first one (inner loop) realizes a linearization and the stabilization control of the pendulum based on an innovative Piecewise Continuous Reduced Order Luenberger Observer (PCROLO) coupled with a linearization module, the second one (the outer loop) realizes a Lyapunov based control for the unstable internal system (refer to the book of Isidori [12]) with lower dynamics than that of the pendulum. This hybrid control method is capable of balancing the CIP system within small cart's displacement. Performances issues of the proposed method are illustrated by the experimental figures and videos.**

I. INTRODUCTION

During the last few years, there has been a considerable amount of interest in the control of vision based under-actuated mechanical systems. The interest comes from the need of supervision in remote control especially via Internet based network, more flexible contactless wiring and improved signal/noise ratio. Various models of vision based underactuated mechanical control have been reported in attempt to improve the visual servoing's performance.

Recalling [11] the visual servoing term is defined as using visual feedback to control a robot. For example visual (image based) features such as points, lines and regions can be used to enable the alignment of a manipulator/gripping mechanism with an object. Vision is a part of a control system providing feedback. However, traditionally visual sensing and manipulation are combined in an open-loop configuration, 'looking' and 'moving', or just for visualizations and animations purposes referring to pendulum control in remote control laboratory [2], [16]. Recently, visual supervision has been gradually combined in the closed control loop particularly for cart-inverted pendulum control such as in [7], [20]. Unfortunately there is no real successful application reported on controlling the cart's position and pendulum's angle by visual servoing till now. Only a fuzzy-logic based controller was reported in [14], but only for controlling a rotary pendulum near the unstable equilibrium zone not exceeding ±5°. For larger deviations, the system turned out to be too slow to compensate. This control was limited in time on a few seconds in keeping the pendulum upright. And in [8], a just-in-time human simulated method was developed to stabilize a two-link Direct Drive Arm-pendulum system. The direction of the pendulum movement is restricted on tangential plane for the trajectory of the tip of second link. This human learning and memory based fuzzy control can stabilize the inverted pendulum only for seconds ($\leq$ 16 s) and with larger angular position oscillations ±26°, just like humans.

Analyzing the difficulties of previous vision based research works related to CIP control; it seemed that the camera signal has not been sufficiently exploited. The problem is that these sensors often deliver sampled and delayed signals due to their digital nature and computation-transfer time (image processing) respectively. Our challenge here is to consider the low cost CCD cameras as contact-less pendulum sensor to stabilize the CIP jumping from a big angular position with a big time delay.

Therefore, our efforts have been focused on the development of an accurate observer using the theory of Piecewise Continuous Systems (PCS) [13] to develop a PCROLO. This kind of PCS is continuous controlled hybrid systems with independent switching and controlled input [13], [18]. Considering the sampled delayed camera's measurements (pendulum's angular position) as autonomous switching and controlled impulse, we estimate the present continuous pendulum's angular position and angular speed. With the improved pendulum's angular position and the estimated angular speed, we can construct our control methods. The research work presented here is an extension and development of the preceding works [13], [18], [19].

The paper is organized as follows: section II presents the description and modeling of the real vision based CIP system, particularly the way to calculate the delayed and sampled pendulum's angular position. A hybrid control consists of the Jumping-up control and the two causal stabilization loops control under a logic-based switch mechanism is proposed in section III. And the PCROLO is

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presented in detail. In section IV, the illustrated experimental results can be found. Finally section V includes some conclusion and perspectives remarks.

II. Description and Modeling

A. Description of Vision based System

The vision based cart-inverted pendulum experimental system illustrated in Fig. 1, contains the following four parts:

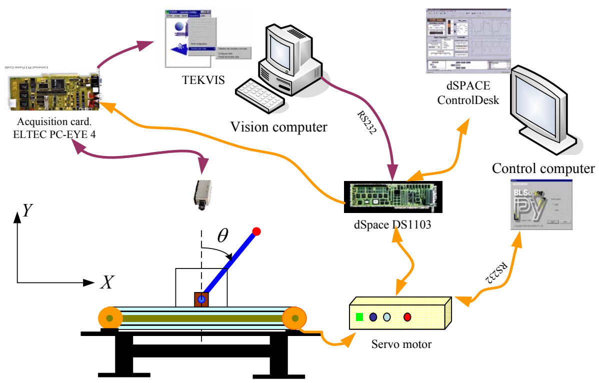


Fig. 1. Vision based inverted pendulum control system

1) The mechanical system: The system is composed of an aluminum chassis, enabling only 48 cm displacement and a plastic inverted pendulum mounted on the mobile cart. Between the pendulum and the cart, an installed shock absorber prevents the pendulum from completely falling to horizontal position. It allows the pendulum to have a maximum angle 50° with respect to the vertical position.

2) The controller: The controller is implemented on the dSpace based Digital Signal Processing card DS1103 via ControlDesk integrated with Matlab/Simulink. We can model, supervise and develop directly control methods for the real-time system by the benefits of the access to the control card's variables. The control signals are sent to a power amplifier via ± 10 V DAC.

3) The actuator: This part is composed of a AC servo motor (SANYO DENKI PY2A015A3-A3). It's driven by a dSpace computer input/output card via the power amplifier supplied with 240 V. The AC motor delivers a nominal couple of 3.0 Nm with a power of 200 W. The platform returns the cart's continuous position through encoders.

4) The vision system: Instead of using hi-tech digital cameras capable of higher sampling rates (up to 2000 frames/sec), higher spatial resolution (up to 4000 × 4000 pixels) and improved signal/noise ratio than their analog counterparts, we used a low cost IR CCD (Jai M50 IR) camera with a sampling rate of 25 frames/sec and a low resolution of 640 × 480 pixels in non-interlaced mode.

It is linked to a vision computer which constitutes an image acquisition card ELTEC PC-EYE 4 and an image processing software TEKVIS. This system is used to determine the pendulum's upper tip (x_C, y_C) coordinates and transmits them to the control computer via the RS-232 serial communication port. In the near future, we extend to transfer this structure by the Internet based network control. The measurements of the camera are available at a sampling rate of T=40 ms (acquisition-processing-transfer time). And in order to facilitate the vision sensing, an infrared LED has been added on the upper tip of pendulum.

In the first phase and in order to synchronize the camera with the control algorithm, the camera is triggered by an external periodical pulse signal, generated via the dSpace card with a sampling period equal the acquisition-processing-transfer time.

As soon as the control computer receives the pendulum's coordinates, a four step "TSAI" calibration method [10] is carried out in the control system. It supports not only the real inverted pendulum top position but also the compensation of the deformations caused by the camera's lenses.

B. Modeling Vision Based System

By referring to the methods proposed in [5], [6], the cart-inverted pendulum can be modeled as follows

$$\ddot{x} = (-\dot{x} + k_m u)/\tau_m \tag{1}$$

$$\ddot{\theta} = -2\zeta\omega_n\dot{\theta} + \omega_n^2 \sin\theta - K\cos\theta\ddot{x} \tag{2}$$

with: $\omega_n = \sqrt{mgl/(J + ml^2)}$ natural frequency,

$\zeta = B_r/[2\omega_n(J + ml^2)]$ damping ratio,

$K = ml/(J + ml^2)$ the gain

where: u, k_m and τ_m the control input tension, overall gain and time constant of the cart & motor system. And θ the angle of the pendulum with y-axis, l the length from the pendulum's center of gravity to the pivot, m the mass of the pendulum, B_r the viscous damping constant between the pendulum and the cart, g the gravitational acceleration, $J=(ml^2)/3$ the pendulum momentum of inertia and x the cart's position on x-axis.

C. Pendulum's Angle Computation (θ)

According to our platform structure, the only accessible information is the coordinate (x_C, y_C) of the pendulum's upper tip projection on the x-y plane via the vision system and the cart's continuous position x. In these conditions, according to Fig. 1, θ can be computed as follows

$$\theta = \sin^{-1}[(x_C - x)/2l]. \tag{3}$$

In our case, only sampled and delayed measurements of the camera are available

$$(x_C(kt_e - T_e), y_C(kt_e - T_e)) = (x_{C,k-q}, y_{C,k-q}) \tag{4}$$

with: t_e the camera's sampling cycle, and $T_e=qt_e$,
where: k, q are integers,

T_e represents the delay time corresponding the time necessary for data acquisition, processing and transfer.

In our vision system, t_e=40 ms and $q=1$.

Finally, from (3) and (4), θ_{k-1} is computed, the pendulum's angle feedback obtained in a sampled and delayed format.

III. Hybrid Stabilization Control

The main vision based hybrid control architecture, which is illustrated in Fig. 2, consists a Jumping-up control and a

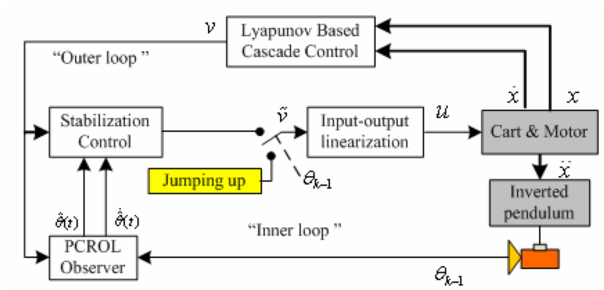


Fig. 2. Vision based hybrid control architecture

two loops cascade stabilization control. The commutation between the Jumping-up control and the two loops stabilization control is switched under a logic-based switch mechanism. The stabilization two cascade loops includes the inner loop (a linearization and the stabilization control of the pendulum based on PCROLO), and the outer loop (a Lyapunov based control for the unstable internal system having lower dynamics than that of the pendulum).

A. Jumping up control

Normally, the proposed two loops cascade stabilization control method is valid for a big inclined initial pendulum angle ($\theta = 50°$), but in the reason of the used motor's limited torque support in our real CIP system, we have to add a Jumping-up (Bang-Bang) control. The energy based swing-up control used in [1], [3], [4], [17] or like [15] a PD position controller to swing up the pendulum from the downward position to upward position can not be used here. This is because these controllers introduce an oscillation effect to move the pendulum to upright. But in our systems, because of the circular shock absorber, the pendulum's angular position is limited above the horizontal plane.

The proposed Jumping-up control's main idea is simple. It comes from a basic everyday life example: considering a person leaned against a backward seat in a car, applying an appropriate accelerating ($t \in [0, t_1]$), null ($t \in]t_1, t_2]$) and decelerating ($t \in]t_2, T]$) acceleration to the car, the person's body will incline forward. Inspired from this experience, we consider the cart of CIP system as the car and the pendulum as the person's body on the car. To facilitate the balancing CIP system, the ideal situation at the end of the Jumping-up control is that the cart's position $x(T)=0$ and speed $\dot{x}(T)=0$ under $x(0)\neq 0$.

We note $x_0 = x(0)$, $x_T = x(T)$ and $\varepsilon = \mathrm{sign}(x(0))$ for a simplification sake.

From (1), we introduce $\tilde{v} = (-\dot{x} + k_m u)/\tau_m$, which leads to

$$\ddot{x} = \tilde{v}\,. \tag{5}$$

--First step: $t \in [0, t_1]$, with $x_0 \neq 0$ and $\tilde{v} = -\varepsilon M$, $M > 0$. By integration of $\ddot{x} = \tilde{v}$, we have

$$x_{t_1} = x_0 - \varepsilon M t_1^2 \text{ and } \dot{x}_{t_1} = -\varepsilon M t_1\,.$$

--Second step: $t \in]t_1, t_2]$, with $\tilde{v} = 0$. We have

$$x_{t_2} = x_0 + \varepsilon M t_1^2/2 - \varepsilon M t_1 t_2 \text{ and } \dot{x}_{t_2} = \dot{x}_{t_1}\,.$$

--Third step: $t \in]t_2, T]$, with $x_T = 0$, $\tilde{v} = \varepsilon\alpha M$, $\alpha > 0$. We have the relations

$$t_1 = \alpha(T - t_2) \text{ and}$$
$$M = \varepsilon x_0 / [T(t_1 + \alpha t_2) - t_1^2 + \alpha t_2^2 + \alpha T^2/2] > 0\,.$$

By adjusting the parameters (x_0, T, α, t_2) experimentally and considering the constraint on the position of the cart, we attempt to jumping up the pendulum to the unstable equilibrium under the constraint of the pendulum's maximum angular position.

The switching is activated when the visual feedback shows that the pendulum is close to the upright position ($|\theta_{k-1}| \leq 0.2$). This logic-based mechanism is realized by a RS trigger. We choose a big switch value for θ to compensate the camera sensor's big time delay to produce a smoother switch under small pendulum's small angular position θ and speed $\dot{\theta}$. This strategy has been tested successfully in the real vision based CIP system.

B. Inner Loop: PCROLO Based Stabilization Control

Under the assumption that the pendulum's angular position θ and velocity $\dot{\theta}$ are well estimated via PCROLO from the visual feedback θ_{k-1}, the stabilization of the pendulum, from (2, 5) a stable inverted pendulum dynamics can be imposed by introducing a new control v, a new gain K_2, a new natural frequency Ω_n and a new damping ratio Z defined as follows

$$-2\zeta\omega_n\dot{\theta} + \omega_n{}^2 \sin\theta - K\cos\theta\tilde{v} = -2Z\Omega_n\dot{\theta} - \Omega_n^2\theta + K_2 v\,. \tag{6}$$

From (6), we obtain the relation equation between $\tilde{v}$ and v

$$\tilde{v} = [-2(\zeta\omega_n - Z\Omega_n)\dot{\theta} + \Omega_n^2\theta + \omega_n^{\ 2}\sin\theta - K_2 v]\big/ K\cos\theta \quad (7)$$

with $|\theta| < \pi/2$.

After the transformation, one gets

$$\ddot{x} = [-2(\zeta\omega_n - Z\Omega_n)\dot{\theta} + \Omega_n^2\theta + \omega_n^{\ 2}\sin\theta - K_2 v]\big/ K\cos\theta\ , \quad (8)$$

$$\ddot{\theta} = -2Z\Omega_n\dot{\theta} - \Omega_n^2\theta + K_2 v\ . \quad (9)$$

From (9), the linearized pendulum's state space model is

$$\dot{\Theta}(t) = A\Theta(t) + Bv(t) \quad (10)$$

with $A = \begin{bmatrix} 0 & 1 \\ -\Omega_n^2 & -2Z\Omega_n \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ K_2 \end{bmatrix}$ and $\Theta(t) = \begin{bmatrix} \theta(t) \\ \dot{\theta}(t) \end{bmatrix}$.

The problem is to estimate the continuous position $\theta(t)$ and velocity $\dot{\theta}(t)$ from the visual feedback θ_{k-1}. In order to solve this problem, we develop a specific PCROL observer. This observer combines a Reduced Order Discrete Luenberger Observer (RODLO) and two Piecewise Continuous Systems (PCS) as defined in [13].

A PCS $\Sigma(\{kt_e\}, A, B, C)$ is symbolized by Fig. 3. It is

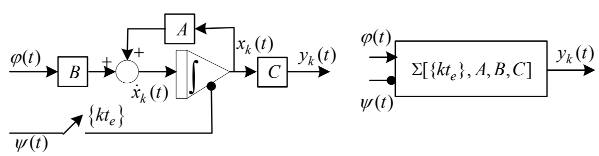


Fig. 3. a) PCS detailed structure b) PCS symbolic representation

characterized by a first continuous input *φ(t)*, a second input *ψ(t)* sampled at discrete instants kt_e, three matrices *A*, *B*, *C*, a state vector *x(t)* and an output vector *y(t)*.

In these conditions, the functioning equations defined as

$$\begin{aligned} & x_k^0 = x_k(0) = \psi(kt_e), && \forall t \in \{kt_e\} \\ & x_k(t) = \exp(A(t-kt_e))x_k^0 + \int_{kt_e}^{t} \exp A(t-\tau)B\varphi(\tau)d\tau, && \forall t \in \left]kt_e, (k+1)t_e\right[\\ & y_k(t) = Cx_k(t). && \forall t \in [0, +\infty[\end{aligned} \quad (11)$$

In order to observe $\Theta(t)$, choose *A* and *B* as in (10) and $C = I_2$

According to Fig. 4, the PCROLO is constructed as follows:

1) First step: PCS I

Using the PCS I, with $\varphi(t) = v(t)$ and $\psi(t) = 0$, one obtains

$$M_{k-1}(t) = \int_{(k-1)t_e}^{t} \exp A(t-\tau)Bv(\tau)d\tau\ .$$

By sampling (Zero-Order-Holder) at each kt_e, we have

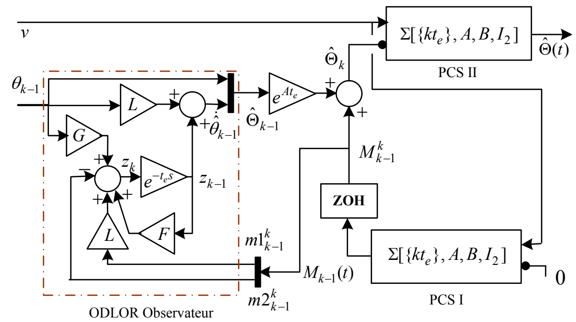


Fig. 4. Piecewise Continuous Reduced Order Luenberger Observer

$$M_{k-1}^k = \int_{(k-1)t_e}^{kt_e} \exp A(kt_e - \tau)Bv(\tau)d\tau = \begin{bmatrix} m1_{k-1}^k & m2_{k-1}^k \end{bmatrix}^T .$$

2) Second step: RODLO

Θ_{k-1} is estimated by a RODLO defined as

$$z_k = Fz_{k-1} + G\theta_{k-1} + (m2_{k-1}^k - Lm1_{k-1}^k)\ ,$$
$$z_{k-1} = \hat{\dot{\theta}}_{k-1} - L\theta_{k-1}$$

where *F*, *G* and *L* are defined from $\exp(At_e) = \begin{bmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{bmatrix}$ as:

$F = (f_{22} - Lf_{12})$,

$G = (f_{22} - Lf_{12})L + (f_{21} - Lf_{11})$,

$L = (f_{22}/f_{12}) \in R$ (Maximize RODLO's convergence speed).

Estimating $\dot{\theta}_{k-1}$ by $\hat{\dot{\theta}}_{k-1} = z_{k-1} + L\theta_{k-1}$, one gets $\hat{\Theta}_{k-1}$, then $\hat{\Theta}_k$, by integration of (10) on the time interval $[(k-1)t_e, kt_e[$

$$\hat{\Theta}_k = \exp(At_e)\hat{\Theta}_{k-1} + M_{k-1}^k\ .$$

3) Third step: PCS II

Using the PCS II, with $\varphi(t) = v(t)$ and $\psi(t) = \hat{\Theta}_k$, one has

$$\hat{\Theta}(t) = \exp(At)\hat{\Theta}_k + \int_{kt_e}^{t} \exp A(t-\tau)Bv(\tau)d\tau\ . \quad (12)$$

C. *Outer Loop: Lyapunov Function Based Control*

The main idea is to introduce an angular reference θ_{ref} as an intermediate for the pendulum angular position in the aim of ensuring the stability of the vision based CIP system and coupling the control *u*.

Referring to [9], [12], we apply the quasi-steady state assumption in (8) and (9), the inverted pendulum system is brought to $\ddot{\theta} = \dot{\theta} = 0$, and $\theta = \theta_{ref}$. Thus the simplified dynamic system becomes

$$\ddot{x} = \tilde{v} = [\omega_n^{\ 2}\tan(\theta_{ref})]\big/ K\ , \quad (13)$$

$$v = (\Omega_n^2 \theta_{ref}) \big/ K_2 \, . \tag{14}$$

Based on this internal simplified pendulum system, a Lyapunov candidate function is defined as follows

$$V(x, \dot{x}) = (\chi x^2 + \delta \dot{x}^2) \big/ 2, \text{ with: } \chi, \delta \geq 0.$$

In order to ensure the Lyapunov derivation's negativity

$$\dot{V}(x, \dot{x}) = \dot{x}(\chi x + \delta \ddot{x}) = \dot{x}(\chi x + \delta \tilde{v}) < 0 \, ,$$

we choose a particular function which stabilizes its non-minimum internal dynamics defined as follows

$$\chi x + \delta \tilde{v} = -\mu [1 - e^{-(\chi x^2 + \delta \dot{x}^2)/2}] \dot{x}, \quad \mu > 0 \, . \tag{15}$$

Therefore, the negativity of the chosen Lyapunov function is assured, and from (15), one gets the control $\tilde{v}$ defined as follows

$$\tilde{v} = -[\chi x + \mu (1 - e^{-(\chi x^2 + \delta \dot{x}^2)/2}) \dot{x}] \big/ \delta \, . \tag{16}$$

Then, replacing (16) in (13), we compute the intermediate value θ_{ref}, and by substituting θ_{ref} in (14), we have

$$v = \tan^{-1} \left\{ -K[\chi x + \mu (1 - e^{-(\chi x^2 + \delta \dot{x}^2)/2}) \dot{x}] \big/ (\delta \omega_n{}^2) \right\} \Omega_n^2 \big/ K_2 \, .$$

Finally from (1, 16), u can be calculated as

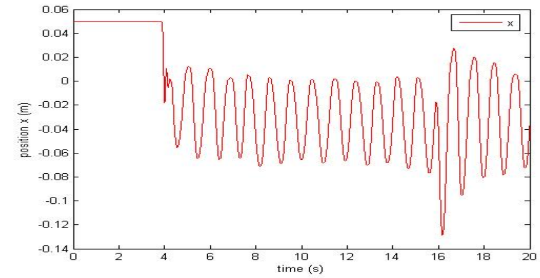
$$u = -\left\{ \tau_m \chi x + [\tau_m \mu (1 - e^{-(\chi x^2 + \delta \dot{x}^2)/2}) - \delta] \dot{x} \right\} \big/ k_m \delta \, .$$

IV. Experimental Results

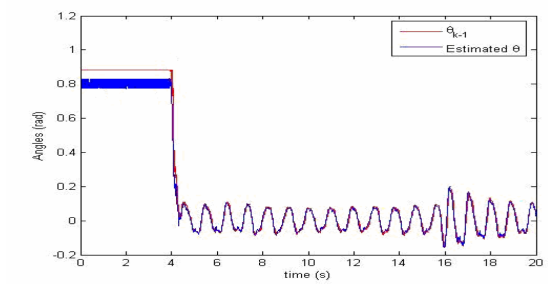
In the real vision based system: the mass of the plastic pendulum is 38 g and its length is 30.5 cm, the viscous friction between the pendulum and the cart is supposed small. Therefore, according to the modeling procedure, the inverted pendulum is characterized by $\zeta = 0.002$ and $\omega_n = 6.781$. The cart & motor system is characterized with $k_m = 2.92$ and $\tau_m = 0.008$. For the Jumping-up controller we select $\alpha = 1$, $T = 0.14$ s, $t_1 = t_2 = 0.07$ s and $x_0 = 0.05$ cm. And for the stabilization controller part, we choose $K_2 = \omega_n^2$, $\xi = 1.2$, $\Omega_n = \omega_n$, $\chi = 100$, $\delta = 10$ and $\mu = 4$.

Note that the initial pendulum angular position $\theta_0 = 0.873 \text{ rd} = 50°$. So, we test the proposed control method on the real vision based CIP system under an initial condition $(x, \dot{x}, \theta, \dot{\theta})_0 = (0.05, 0, 0.873, 0)$.

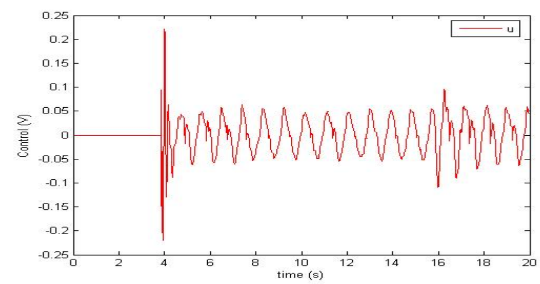
Experimental results are depicted in Fig. 5 under the presence of a small perturbation (i.e. hit on the top of the pendulum). Fig. 5(a, b) show the results of the cart's



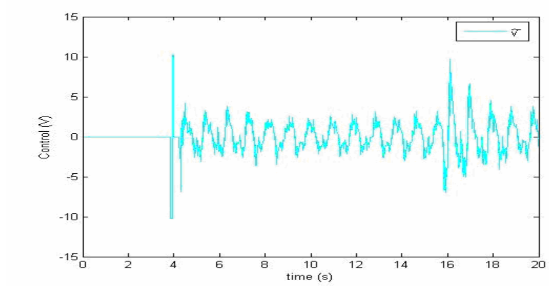
(a) displacement x



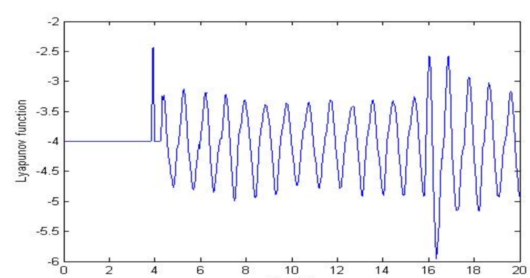
(b) θ_{k-1} & $\hat{\theta}(t)$



(c) control u



(d) control $\tilde{v}$



(e) Lyapunov function $\dot{V}(x, \dot{x})$

Fig. 5. Vision based hybrid control experimental resultats

displacement limited in 10 cm and the pendulum's angular position remaining in ±5° and the PCROLO gives precise and robust estimations. And Fig. 5(c, d, e) show the corresponding controls u, $\tilde{v}$ and $\dot{V}(x,\dot{x})$. The result of $\tilde{v}$ illustrates well the principle of Jumping-up control and then the stabilization control under the logic-based switch mechanism. From this figure, we remark clearly that the switching instant is activated around about 4.2 s. And the proposed Lyapunov function assures the negativity of $\dot{V}(x,\dot{x})$. It verifies very well the control strategy: the guarantee stability of the whole vision based CIP systems.

In spite of small oscillations, the experimental tests demonstrate that the proposed method is able to maintain the pendulum angular position upright, and keeping the cart's position around zero for $t \in [t, +\infty[$ normally. And it's important to note that the Matlab/Simulink numerical result hasn't oscillations effects. More experimental results and the illustrative videos are available on http://www-lagis.univ-lille1.fr/~wang/Research_eng.html. In these videos, it's important to note that we cover the camera's visual feedback with a paper to end up the experimental demonstrations.

V. Conclusion and Perspectives

This article supports a direct and efficient method for controlling a vision based CIP system with a big time delay and a big inclined initial angular position. And this underactuated mechanical system is nonlinear with non-minimum internal dynamics.

The appeared oscillations in the stabilization control are probably due to inaccurate viscous friction identification between the cart and the pendulum, the not ignoring calibration error of the used low cost CCD camera (Jai M50 IR) and the non-minimum internal zero dynamics of the cart-pendulum system.

The experimental results and videos demonstrate the effectiveness and robustness of our method. And also with authors experiences, the implementation of the vision based CIP systems on the dSpace based Digital Signal Processing card via ControlDesk integrated with Matlab/Simulink platform is convenient in real system's design, supervision and control.

In the near future, we plan to reduce the oscillations appeared in the stabilization control. And we also extend to apply the proposed principle of the control method to a vision based spherical pendulum on an x-y robot.

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