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A New Disturbance Observer for Non-minimum Phase Linear Systems

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***Abstract*— Motivated by the fact that the application of the disturbance observer (DOB) approach has been limited to minimum phase systems, we propose a new DOB configuration for non-minimum phase systems. The proposed configuration introduces a new filter, which corresponds to the Q-filter of the classical linear DOB, in a different place of the inner-loop. This new configuration enables an application of DOB idea to the non-minimum phase systems. After analyzing robust internal stability of the proposed configuration, we present a synthesis methodology of the filter based on the $\mathcal{H}_\infty$ synthesis technique.**

I. Introduction

Disturbance observer (DOB) approach [1], [4], [7–10] as a tool for disturbance attenuation has been widely employed in the industry. Versatility of DOB comes from, without doubt, its simplicity and powerful ability to attenuate disturbances and compensate plant uncertainties. An interesting feature of DOB is that it can be used as an inner-loop controller so that the real uncertain plant in the presence of disturbances is forced to behave like the nominal plant in disturbance-free environment. As a result, DOB can be combined with any (pre-existing) outer-loop controller that is designed without considering plant uncertainties and disturbances. However, applications of the DOB approach have been limited to the minimum phase systems or systems having no zero dynamics. (It is proved in [11] that the minimum phaseness of the plant is one of the necessary conditions for internal stability in case of the classical DOB approach.)

The goal of the paper is to propose a new DOB configuration that can be applied to non-minimum phase linear systems. Although there exist some research works on DOB for non-minimum phase systems [2], [3], [13], [15], [16], they seem to have difficulties in handling general cases. In fact, the application of [13] is limited to those disturbances that come from a known external system (*i.e.*, exosystem), and [2] and [16] deal with non-minimum phase systems simply by inverting only the invertible (stable minimum phase) part. A disturbance observer proposed in [3] is limited to the estimation of disturbances without considering the stability of the closed-loop system, and the approach of [15] is limited to the case where the plant model does not have uncertainty. In contrast to those previous works, we present a new DOB configuration that can be applied to fairly general non-minimum phase systems. A systematic design methodology, specialized to the proposed DOB structure, is also given with the help of the $\mathcal{H}_\infty$ synthesis technique.

The paper is organized as follows. In Section II, a new DOB configuration is presented and robust stability and performance recovery criterion are given in terms of a new filter $\Theta(s)$. Section III provides a design procedure for $\Theta(s)$ based on the $\mathcal{H}_\infty$ synthesis technique. A motivation of the proposed DOB configuration is presented in Section IV and a conclusion is given in Section V.

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II. A New DOB Configuration for Non-minimum Phase Systems

Fig. 1 is the proposed new DOB configuration. In the figure, $P(s)$ is the single-input single-output linear time-invariant plant of our interest. We assume that $P(s)$ is unknown but is an element of the set $\mathcal{P}$ that is a collection of strictly proper rational transfer functions. $P_n(s)$ represents a nominal model of the real plant $P(s)$, and $P_n(s)$ is also contained in $\mathcal{P}$. An outer-loop controller $C(s)$ is assumed to be designed *a priori* for the nominal model $P_n(s)$. This implies that the nominal closed-loop system (*i.e.*, $\frac{P_n(s)C(s)}{1+P_n(s)C(s)}$) is stable and has satisfactory performances for a certain control goal. However, due to the uncertainty of the real plant and the external disturbance d, the actual closed-loop performance could be degraded or, even worse, the stability could be lost. The role of the inner-loop controller (*i.e.*, systems represented by $\frac{P_n\Theta}{1+P_n\Theta}$ and $\frac{\Theta}{1+P_n\Theta}$, with a certain filter $\Theta(s)$, in Fig. 1) is to maintain the stability of the actual closed-loop system and preserve the nominal performance even in the presence of the plant uncertainty and disturbance d.

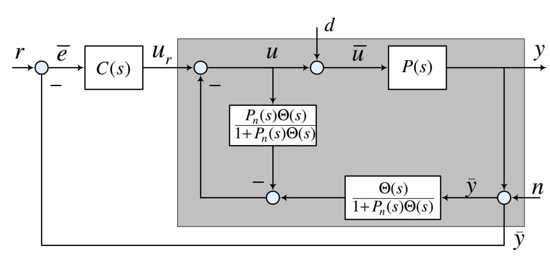

Fig. 1. The proposed DOB configuration (the shaded block) with an outer-loop controller $C(s)$. The signals r, d, and n represent the reference command, the unknown external disturbance, and the measurement noise, respectively. y is the output of the closed-loop system while u is the feedback control to the plant.

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From Fig. 1, the plant output y is expressed as

$$y(s) = \frac{(P_n\Theta + 1)PC}{1 + PC + (1 + P_nC)P\Theta} r(s) + \frac{P}{1 + PC + (1 + P_nC)P\Theta} d(s) - \frac{PC + (1 + P_nC)P\Theta}{1 + PC + (1 + P_nC)P\Theta} n(s). \quad (1)$$

For the time being, let us assume that all the transfer functions above are stable. In addition, it is natural to assume that there exists a frequency ω_L such that the measurement noise $n(j\omega)$ is significant in the frequency range (ω_L, ∞) while the disturbance $d(j\omega)$ and the reference $r(j\omega)$ are significant on $(0, \omega_L)$. Moreover, suppose that the filter $\Theta(s)$ is designed so that $|P(j\omega)\Theta(j\omega)|$ has very large magnitude in the low frequency range $(0, \omega_L)$, that is,

$$|P(j\omega)\Theta(j\omega)| > W_1, \quad \forall\omega \in (0, \omega_L),$$

for a sufficiently large $W_1 > 0$. Then, we obtain from (1) that

$$y(j\omega) \approx \frac{P_nC(j\omega)}{1 + P_nC(j\omega)} r(j\omega), \qquad \forall\omega \in (0, \omega_L). \quad (2)$$

This implies that, even when there exist plant uncertainty and input disturbance, the steady-state performance of the actual closed-loop system is recovered to the nominal one in the absence of disturbance. (More rigorous justification of performance recovery for the conventional DOB can be found in [11].) This property will be called as *performance recovery* throughout the paper. In order to enjoy this performance recovery, the filter $\Theta(s)$ should be designed such that the robust internal stability of the closed-loop system (Fig. 1) is ensured while $|P(j\omega)\Theta(j\omega)|$ is sufficiently large in the low frequency range.

To inspect the internal stability, nine transfer functions from $[r, d, n]^T$ to $[\bar{e}, \bar{u}, \bar{y}]^T$ in Fig. 1 are computed by

$$\frac{1}{\Delta_{cl}(s)} \begin{bmatrix} 1 + P\Theta, & -P, & -1 \\ (P_n\Theta + 1)C, & 1, & -(1 + P_nC)\Theta - C \\ (P_n\Theta + 1)PC, & P, & 1 \end{bmatrix}$$

where $\Delta_{cl}(s) = (1 + PC) + (1 + P_nC)P\Theta$. If the above nine transfer functions are stable for all $P(s) \in \mathcal{P}$, then the closed-loop system is said to be *robustly internally stable*.

Now let us write P, P_n, C, and Θ as ratios of coprime polynomials, that is, $P(s) = N(s)/D(s)$, $P_n(s) = N_n(s)/D_n(s)$, $C(s) = N_c(s)/D_c(s)$, and $\Theta(s) = N_\theta(s)/D_\theta(s)$. Then, it can be shown in a similar way to [5, p. 37, Theorem 1] that the closed-loop system is internally stable if and only if the characteristic polynomial

$$\delta(s) := D_\theta(D_nD_cD + D_nN_cN) + N_\theta(D_nD_cN + N_nN_cN) \quad (3)$$

is Hurwitz. We summarize the discussions so far in the following.

Theorem 1: Let $\mathcal{P}$ be a set of strictly proper rational transfer functions. The closed-loop system in Fig. 1 is robustly internally stable if and only if $\delta(s)$ is Hurwitz for all $P(s) \in \mathcal{P}$. ♢

Unfortunately Theorem 1 is not convenient to design $\Theta(s)$ for robust internal stability. To overcome this difficulty, a more viable but sufficient condition is given as follows.

Theorem 2: The closed-loop system in Fig. 1 is robustly internally stable if the following conditions hold.

(H1) $P_nC/(1 + P_nC)$ is stable,

(H2) $P\Theta/(1 + P\Theta)$ is stable for all $P(s) \in \mathcal{P}$,

(H3) $\left\| \frac{C(P-P_n)}{(1+P_nC)(1+P\Theta)} \right\|_\infty < 1$ for all $P(s) \in \mathcal{P}$. ♢

Proof: By manipulating the equation (3), we obtain

$$\begin{aligned} \delta(s) &= (D_nD_c + N_nN_c)(D_\theta D + N_\theta N) \\ &\quad + D_\theta N_c(ND_n - N_nD) \\ &= (D_nD_c + N_nN_c)(D_\theta D + N_\theta N)(1 + \Phi(s)) \end{aligned} \quad (4)$$

where $\Phi(s) := \frac{D_\theta N_c(ND_n - N_nD)}{(D_nD_c+N_nN_c)(D_\theta D+N_\theta N)}$. Note that, from assumptions (H1) and (H2), $(D_nD_c + N_nN_c)$ and $(D_\theta D + N_\theta N)$ are Hurwitz, which implies that $\Phi(s)$ is stable. Moreover, assumption (H3) implies that

$$\left\| \frac{\frac{N_c}{D_c}\left(\frac{N}{D} - \frac{N_n}{D_n}\right)}{\left(1 + \frac{N_n}{D_n}\frac{N_c}{D_c}\right)\left(1 + \frac{N}{D}\frac{N_\theta}{D_\theta}\right)} \right\|_\infty = \|\Phi\|_\infty < 1.$$

Thus, it follows from the Nyquist criterion or the small-gain theorem [14] that $(1 + \Phi(s))^{-1}$ is stable. This implies that the denominator of $(1 + \Phi(s))^{-1}$ is Hurwitz, and thus, $\delta(s)$ is Hurwitz for all $P(s) \in \mathcal{P}$. ■

Assumption (H1) is nothing but the stability requirement of the nominal closed-loop system with an outer-loop controller $C(s)$. Therefore, with such a $C(s)$ designed, the remaining task is to design $\Theta(s)$ such that the assumptions (H2) and (H3), which play the role of *robust stability criterion*, are satisfied with the following *performance recovery criterion*:

(H4) $|P(j\omega)\Theta(j\omega)| > W_1, \quad \forall\omega \in (0, \omega_L),\ \forall P(s) \in \mathcal{P}$,

where W_1 is a sufficiently large positive constant. However, the conditions (H2), (H3), and (H4) should be satisfied for all $P(s) \in \mathcal{P}$, which may make it still difficult to obtain $\Theta(s)$.

In the next section, by restricting our interest to a multiplicative perturbation model of the plant, we present a systematic way to design $\Theta(s)$ by virtue of the $\mathcal{H}_\infty$ synthesis tool.

Remark 1: If the plant uncertainty is very small, then it is enough to have $P_n\Theta/(1 + P_n\Theta)$ be stable and $|P_n(j\omega)\Theta(j\omega)| > W_1$ on $(0, \omega_L)$ because the assumptions (H2), (H3), and (H4) still hold with $P(s)$ sufficiently close to $P_n(s)$. ♢

III. Construction of $\Theta(s)$ for the Plant having Multiplicative Uncertainty

Throughout this section, we assume that the uncertain plant transfer function belongs to a set $\mathcal{P} := \{P = (1 + \Delta W_2)P_n : \|\Delta(s)\|_\infty \leq 1, \Delta(s)$ is stable and rational$\}$, where $P_n(s)$ is a strictly proper rational transfer function, and $W_2(s)$ is a fixed stable (possibly improper) transfer function such that every $P(s)$ in $\mathcal{P}$ is strictly proper and $|W_2(j\omega)| < 1$ on $(0, \omega_L)$[1].

Theorem 3: Let $\tilde{W}_1(s)$ be a stable transfer function such that

$$|\tilde{W}_1(j\omega)| \geq \frac{W_1}{1 - |W_2(j\omega)|} + 1, \quad \forall \omega \in (0, \omega_L). \quad (5)$$

Then, the conditions (H2), (H3), and (H4) hold if $\Theta(s)$ stabilizes[2] P_n and satisfies

$$\left\| \tilde{W}_1 \frac{1}{1 + P_n\Theta} \right\|_\infty < 1, \quad (6)$$

$$\left\| \left| W_2 \frac{P_nC}{1 + P_nC} \frac{1}{1 + P_n\Theta} \right| + \left| W_2 \frac{P_n\Theta}{1 + P_n\Theta} \right| \right\|_\infty < 1. \quad (7)$$

$\diamondsuit$

Although the conditions (6) and (7) may look more complicated than (H2), (H3), and (H4), these conditions do not include the statement 'for all $P(s) \in \mathcal{P}$'. Moreover, they are in the standard form frequently used in the $\mathcal{H}_\infty$ synthesis solver (*e.g.*, MATLAB Robust Control Toolbox). For convenience, define

$$L = P_n\Theta, \quad S = \frac{1}{1 + P_n\Theta},$$

$$T = \frac{P_n\Theta}{1 + P_n\Theta} \quad \text{and} \quad \tilde{W}_2 = \frac{P_nC}{1 + P_nC} W_2.$$

Then, (6) and (7) are rewritten as

$$\|\tilde{W}_1 S\|_\infty < 1, \quad (8)$$

$$\||\tilde{W}_2 S| + |W_2 T|\|_\infty < 1, \quad (9)$$

respectively. It will be shown in the proof that (8) implies (H4) while (9) implies (H2) and (H3). Thus, (8) and (9) are called 'performance recovery condition' and 'robust stability condition', respectively.

Proof of Theorem 3: (H4) follows from

$$\begin{aligned} \|\tilde{W}_1 S\|_\infty < 1 &\Leftrightarrow \left| \frac{\tilde{W}_1}{1 + L} \right| < 1, \quad \forall \omega \\ &\Leftrightarrow |\tilde{W}_1| < |1 + L| \leq 1 + |L|, \quad \forall \omega \\ &\Rightarrow \frac{W_1}{|1 + \Delta W_2|} \leq |\tilde{W}_1| - 1 < |L|, \quad \text{on } (0, \omega_L) \\ &\Leftrightarrow W_1 < |1 + \Delta W_2| \, |P_n\Theta| = |P\Theta|, \quad \text{on } (0, \omega_L). \end{aligned}$$

On the other hand, (7) implies that $\|W_2 T\|_\infty < 1$ and

$$\left\| \frac{\tilde{W}_2 S}{1 - |W_2 T|} \right\|_\infty < 1. \quad (10)$$

Since Θ stabilizes P_n and $\|W_2 T\|_\infty < 1$, it follows from [5, p. 53, Theorem 1] that Θ stabilizes every plant in $\mathcal{P}$, which implies the assumption (H2). Now, from (10), it follows that

$$\left\| \frac{\tilde{W}_2 S}{1 + \Delta W_2 T} \right\|_\infty < 1.$$

Thus, we have

$$\begin{aligned} &\left\| \frac{\tilde{W}_2 \frac{1}{1+P_n\Theta}}{1 + \Delta W_2 \frac{P_n\Theta}{1+P_n\Theta}} \right\|_\infty < 1 \\ \Leftrightarrow &\left\| \tilde{W}_2 \frac{1}{1 + P_n\Theta + \Delta W_2 P_n\Theta} \right\|_\infty < 1 \\ \Leftrightarrow &\left\| \frac{P_nC}{1 + P_nC} \frac{W_2}{1 + P_n(1 + \Delta W_2)\Theta} \right\|_\infty < 1 \\ \Leftrightarrow &\left\| \frac{C}{1 + P_nC} \frac{P_n W_2}{1 + P\Theta} \right\|_\infty < 1. \end{aligned}$$

Since $\|\Delta\|_\infty \leq 1$, we obtain

$$\left\| \frac{C}{1 + P_nC} \frac{P_n \Delta W_2}{1 + P\Theta} \right\|_\infty < 1.$$

Thus, since Δ is arbitrary, this implies assumption (H3). $\diamondsuit$

Remark 2: A remark on the feasibility of (8) and (9) is in order. Firstly, since the property (5) is needed only on $(0, \omega_L)$, the transfer function $\tilde{W}_1(j\omega)$ is usually chosen so that it is small at high frequencies. Otherwise, (8) is not likely to hold since $S(j\omega) \approx 1$ at high frequencies. Secondly, $W_2(s)$ should be chosen such that $P_n(s)C(s)W_2(s)$ is either strictly proper or biproper and less than unity at high frequencies. To see this, observer that, since $P_n(s)$ is strictly proper, $S(j\omega) \approx 1$ and $\tilde{W}_2(j\omega) \approx P_nCW_2(j\omega)$ at high frequencies. This, together with (9), implies that $|P_nCW_2(j\omega)| < 1$ at high frequencies. Finally, $|W_2(j\omega)|$ should be less than unity at low frequencies because $T(j\omega) \approx 1$ there and (9) should be satisfied. (This is a reasonable assumption because the plant model is relatively accurate at low frequencies.) $\diamondsuit$

With $\tilde{W}_1$ and W_2 as in Remark 2, the standard $\mathcal{H}_\infty$ solver can be used for finding a suitable $\Theta(s)$ that stabilizes the nominal plant P_n and meets (8) and (9). Once such a $\Theta(s)$ is found, then, under the condition (H1), the closed-loop system in Fig. 1 is robustly internally stable and performance recovery criterion (H4) is met .

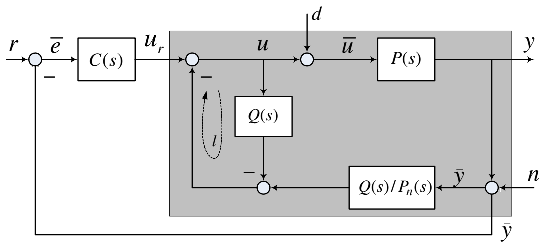
IV. Behind-the-scenes of Introducing $\Theta(s)$

The new configuration of Fig. 1 was inspired by an idea that the zero dynamics of the plant can be changed by a parallel connection of a certain filter, and this preliminary idea is refined to yield the proposed configuration of Fig. 1. In this section, some intuition for the new DOB configuration is discussed with the comparison to the classical DOB configuration.

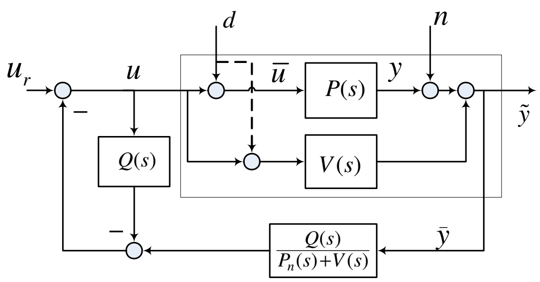
The classical DOB configuration is illustrated in Fig. 2(a), which has been actively studied in, e.g., [4], [7–12]. It features the existence of the so-called 'Q-filter' $Q(s)$, which is a stable low-pass filter. From Fig. 2(a), the plant output y

[1]This is quite a standard and reasonable assumption since plant uncertainty $W_2(j\omega)$ is relatively small at low frequencies.

[2]$\Theta(s)$ is said to stabilize $P_n(s)$ if the unity feedback system composed of $P_n(s)$ and $\Theta(s)$ is stable, *i.e.*, the transfer function $\frac{P_n\Theta}{1+P_n\Theta}$ is stable.

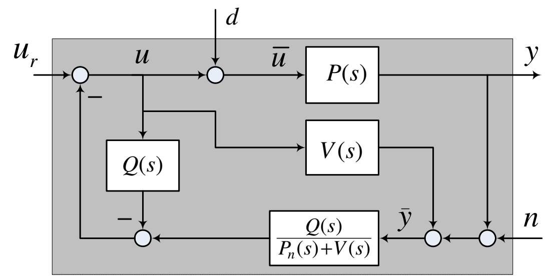


(a)

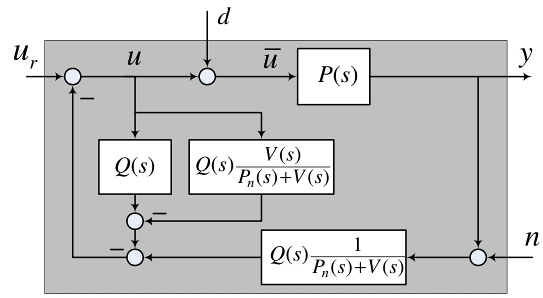


(b)

Fig. 2. (a) The classical DOB configuration. (b) Inspiration for the proposed DOB configuration of Fig. 1.



(a)



(b)

Fig. 3. Intermediate diagrams towards Fig. 1.

is obtained as

$$y(s) = \frac{P_n P C}{P_n(1+PC)+Q(P-P_n)} r(s) + \frac{P_n P(1-Q)}{P_n(1+PC)+Q(P-P_n)} d(s) - \frac{P(Q+P_n C)}{P_n(1+PC)+Q(P-P_n)} n(s).$$

Since the low-pass filter $Q(j\omega) \approx 1$ and the measurement noise $n(j\omega) \approx 0$ in the low frequency range $(0, \omega_L)$ with a certain constant $\omega_L > 0$, the above equation becomes

$$y(j\omega) \approx \frac{P_n C}{1+P_n C}(j\omega) r(j\omega), \qquad \forall \omega \in (0, \omega_L).$$

This implies that, assuming that all the transfer functions are stable, the closed-loop system with the DOB behaves as if it were the disturbance-free nominal one in the frequency range $(0, \omega_L)$.

Here, the role of the $Q(s)$ is to make the transfer function $Q(s)P_n^{-1}(s)$ proper so that it is implementable and to avoid the algebraic loop ℓ in Fig. 2(a). In addition, the cutoff frequency of the low-pass filter $Q(s)$ should be chosen larger than ω_L.

Regarding the robust internal stability of Fig. 2(a), it has been pointed out in [11] that one of the necessary conditions for the internal stability is that the polynomial

$$p_s(s) := N(s)\left(D_c D_n(s) + N_c N_n(s)\right) \qquad (11)$$

is Hurwitz (with the same notation used in Section II). This in turn implies that the (uncertain) plant must be of *minimum phase* because $N(s)$ is the numerator of $P(s)$.

In order to overcome the restriction of the minimum phaseness, a new configuration is devised as in Fig. 2(b). In the figure, we imagine that the plant is $P(s) + V(s)$ rather than $P(s)$, with a certain filter $V(s)$, which is motivated by the fact that the zero dynamics of $P(s)$ is easily affected by adding $V(s)$. Therefore, under the assumption that $P(s) + V(s)$ is of minimum phase, we then construct the classical DOB like in Fig. 2(b) as if the plant were $P(s) + V(s)$. In fact, this approach makes sense when the signal d enters $V(s)$ as well (see the dotted line in Fig. 2(b)), which is not the case. Although d cannot be injected into $V(s)$ in practice because d is unknown, we can approximately assume that there exists the dotted line in Fig. 2(b), if $|V(j\omega)|$ is small in the low frequency range where d is significant. Assuming this, we consider the configuration in Fig. 3(a) instead of that in Fig. 2(b), both of which are approximately equivalent. Then, Fig. 3(b) is obtained from Fig. 3(a). Finally, by letting $V(s) = 1/\Theta(s)$ and by eliminating the Q-filter $Q(s)$ in Fig. 3(b), we obtain the proposed configuration of Fig. 1. Therefore, $\Theta(s)$ should be designed to be a proper transfer function such that $|\Theta(j\omega)|$ is large at low frequencies. Here we could remove $Q(s)$ because the function $\Theta(s)$ in Fig. 1 plays the same role as $Q(s)$, that is, $\Theta(s)$ makes the transfer function $1/(1 + P_n(s)\Theta(s))$ proper and avoids the algebraic loop.

The benefit of the newly proposed configuration is obviously seen in (4), where one can find the polynomial $(D_\theta D + N_\theta N)$. This polynomial, in fact, plays the role of

$N(s)$ in $p_s(s)$ of (11), but now there is some freedom of designing $D_\theta(s)$ and $N_\theta(s)$ (*i.e.*, designing $\Theta(s)$) so that it becomes Hurwitz although $N(s)$ is not Hurwitz.

V. Conclusions

We have proposed a new DOB configuration that can deal with the non-minimum phase linear systems. The proposed method can also be applied to minimum phase systems, but the classical DOB approach is simpler for such cases. We also presented a design methodology of the key component $\Theta(s)$ using the $\mathcal{H}_\infty$ synthesis technique.

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