2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

ThB12.5

Robust Inverse Control for Combustion Engine Test Benches

Engelbert Gruenbacher#, L. del Re+

Abstract— **In this paper the application of a robust inverse tracking method to the test bench control in order to achieve a high tracking performance is presented. This controller consists of a feedforward part which is the inverse realization of the approximate model of the combustion engine test bench and a robustifying feedback controller, which is for compensating the approximation error or unknown input disturbances. The robustifying controller is simply an extension of a central robust stabilizing controller which usually is the solution of a Hamilton Jacobi inequality. To this end, an iterative way to find a solution of this partial differential equation is applied here. Finally, the presented tracking controller for the combustion engine test bench is compared to a standard decoupled control strategy in a simulation environment.**

I. INTRODUCTION

Robust tracking control strategy consisting of a feedforward control part and a feedback control have recently been published (see [3]). In the present paper this theory will be applied to the tracking control of a combustion engine test bench. Combustion engine test benches are commonly used to parameterize the engine control unit (ECU) of combustion engines. To do so, real vehicle load patterns are simulated on the engine test bench. Therefore, the trajectories measured once in a real car have to be reconstructed on the test bench. Hence, it is necessary to track given patterns of engine speed and engine torque.

The problem of controlling combustion engine test benches is by far not a new application. In [1] a robust MIMO controller is applied to control the test bench. Another strategy is used in [2] where adaptive methods are implemented in order to deal with the often unknown behavior of combustion engines. The main challenge of test bench control is to achieve good tracking performance, even if the test bench system, especially the combustion engine test bench behavior, is not well known. In this paper we do a total different approach. We assume a rough model of the system and we develop an inverse controller for this rough model. Model uncertainties are then considered by an additional robustifying feedback controller. As it will be shown in simulation this approach is quite powerful and results in a very good performance.

The paper is organized as follows: in the next chapter we shortly introduce the reader to the mathematical model of the combustion engine test bench. Then the main theoretical results of the paper about robust tracking (see [3]) will be presented and repeated. In section IV the nonlinear robust tracking controller is calculated. Finally, the controller will be applied in simulation. Conclusions including future aspects will complete the paper.

#,+ Engelbert Gruenbacher and Prof. Luigi del Re are with the Institute for Design and Control of Mechatronical Systems, Johannes Kepler University Linz, Altenbergerstraße 69, A-4020 Linz, Austria `[engelbert.gruenbacher,luigi.delre]@jku.at`

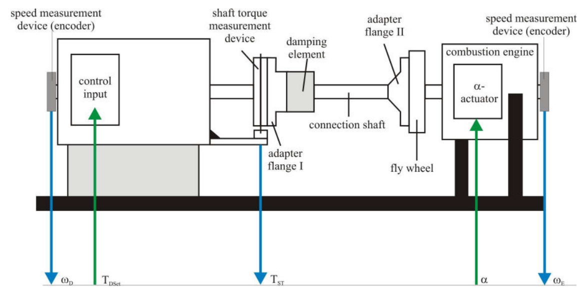

Fig. 1. Engine test bench system

II. MATHEMATICAL MODEL OF A COMBUSTION ENGINE TEST BENCH

A typical combustion engine test bench system is illustrated in Figure 1. The main parts of such a dynamical engine test bench are the dynamometer, the connection shaft and the combustion engine. Considering the torque of the combustion engine and the air gap torque of the dynamometer as the inputs to the mechanical part of the engine test bench system, the model description can be reduced to a lumped engine connected to the dynamometer inertia by a damped torsional flexibility (see [6]). Hence, the model of the mechanical part of the test bench system is

$$\begin{aligned} \Delta\dot{\varphi} &= \omega_E - \omega_D && \text{(1a)} \\ \dot{\omega}_E &= {\theta_E}^{-1}\left(T_{Edyn} - c\Delta\varphi - d\left(\omega_E - \omega_D\right)\right) && \text{(1b)} \\ \dot{\omega}_D &= {\theta_D}^{-1}\left(c\Delta\varphi + d\left(\omega_E - \omega_D\right) - T_{DSet}\right) && \text{(1c)} \end{aligned}$$

where θ_E is the inertia of the combustion engine, θ_D the inertia of the dynamometer, ω_E and ω_D are the engine and the dynamometer speed, c is the spring constant and d the damping constant. Finally, T_{Edyn} and T_{DSet} are the torque of the combustion engine and the air gap torque of the dynamometer respectively.

The most critical part of the system is the combustion engine. Due to the complexity of the ECU (including switching, dead zones, delays, scheduling) we will not concentrate on a detailed combustion engine model, but on a very simple system description containing some uncertainties which can be generated with much less effort. In fact, this means that the engine system behavior is approximated by a so called extended Hammerstein system (see [5]), *i.e.*

$$\begin{aligned} \dot{T}_E &= -\left(a_0 + a_1\omega_E + a_2\omega_E^2\right)T_E && \text{(2a)} \\ &\quad + m\left(\omega_E, T_E, \alpha_{acc}\right) \\ T_{Edyn} &= T_E && \text{(2b)} \end{aligned}$$

where $m(\omega_E, T_E, \alpha_{acc})$ is a static nonlinear map which is continuous but not continuously differentiable.

The composite model of the engine test bench is the result of the connection between the mechanical part of system (1) and the engine model (2). With T_{E0}, $\Delta\varphi_0$, ω_{E0}, ω_{D0} and the corresponding α_{acc0} defining the operating point and ΔT_E, $\max(\Delta\varphi)$, $\Delta\omega_E$ and $\Delta\omega_D$ defining the maximum expected distance from the operating point we get

$$\begin{aligned} \dot{x}_1 &= -\left(\tilde{a}_0 + \tilde{a}_1 x_3 + \tilde{a}_2 x_3^2\right) x_1 - \tilde{a}_3 x_3 - \tilde{a}_4 x_3^2 + v && \text{(3a)} \\ \dot{x}_2 &= \beta\left(x_3 - x_4\right) && \text{(3b)} \\ \dot{x}_3 &= {\theta_E}^{-1}\left(c\beta^{-1}x_1 - c\beta^{-1}x_2 - d\left(x_3 - x_4\right)\right) && \text{(3c)} \\ \dot{x}_4 &= {\theta_D}^{-1}\left(c\beta^{-1}x_2 + d\left(x_3 - x_4\right) - \tilde{T}_{DSet}\right) && \text{(3d)} \end{aligned}$$

where

$$\begin{aligned} x_1 &= \frac{T_E - T_{E0}}{\Delta T_E}, \qquad x_2 = \frac{\Delta\varphi - \Delta\varphi_0}{\max(\Delta\varphi)} \\ x_3 &= \frac{\omega_E - \omega_{E0}}{\Delta\omega_E}, \qquad x_4 = \frac{\omega_D - \omega_{D0}}{\Delta\omega_D} \end{aligned}$$

are the scaled state variables and

$$\begin{aligned} v = \hat{m}(x_3, x_1, \alpha_{acc}) &= \frac{m(\omega_E, T_E, \alpha_{acc})}{\Delta T_{E0}} \\ &\quad - \frac{m(\omega_{E0}, T_{E0}, \alpha_{acc0})}{\Delta T_{E0}}\Big|_{\substack{\omega_E \to x_3 \\ T_E \to x_1}} \\ \tilde{T}_{DSet} &= \frac{T_{Dset} - T_{E0}}{\Delta\omega_E} \end{aligned}$$

are the scaled nonlinear static input map and the scaled dynamometer torque. Furthermore, $\tilde{a}_0, \tilde{a}_1, \tilde{a}_2, \tilde{a}_3$ and $\tilde{a}_4$ are the scaled system parameters of the system and $\beta = \frac{\Delta\omega}{\max(\Delta\varphi)}$. Consider system (3) and notice that it possesses two control inputs (v and $\tilde{T}_{DSet}$), which are not the physical control inputs of the test bench. The physical inputs are the accelerator pedal and the dynamometer torque. Hence, the nonlinear static map in the combustion engine path $v = \hat{m}(x_3, x_1, \alpha_{acc})$ has to be locally inverted with respect to the real input α_{acc} which is done by an approximative inversion. On this account the nonlinear static map is not exactly compensated what yields an uncertainty. Additionally to this uncertainty, the dynamical behavior of the accelerator pedal actuator, consisting of a delay time and linear dynamics, is not considered in the system description (3). However, the error caused by the approximation and by neglecting the accelerator pedal actuator affects the system in the same direction as the input and, hence, it is possible to add a multiplicative error model as shown in Figure 2. Likewise it is possible to consider the neglected dynamics of the dynamometer (see Figure 2). Furthermore, the system possesses four outputs, since we assume that the full state is measurable, hence $y = x$. The performance variable is

$$z = \left(h(x) \;\; u\right)' \qquad (4)$$

where $h(x)$ will be discussed below.

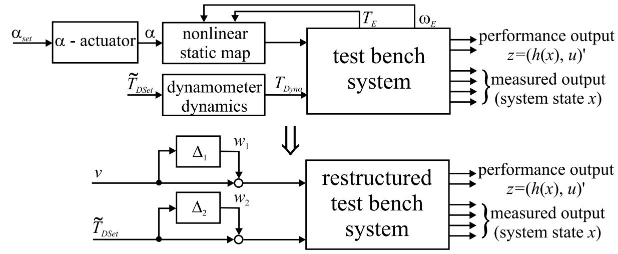

Fig. 2. Error model of the system

III. PREMILIARIES AND THEORETICAL ASPECTS

A. Problem Statement

Consider a nonlinear uncertain system that has well a defined relative degree:

$$\begin{aligned} \dot{x} &= f(x) + g(x)(u + w) && \text{(5a)} \\ y &= h_y(x) && \text{(5b)} \\ z &= h_z(x) + d(x)u && \text{(5c)} \end{aligned}$$

where $x \in R^n$ is the state, $u \in R^m$ the control input, $y \in R^p$ the output variable and $z \in R^p$ the performance variable for a stabilizing optimal L_2 or suboptimal H_∞ controller (see section [4]) and $w \in R^m$ is the disturbance input. Note by this definition it is possible to consider input disturbances caused *e.g.* by actuator uncertainties as well as model uncertainties.

For system (5) we assume that $x = 0$ is an equilibrium point, i.e. $f(0) = 0$, and that $h_2(0) = 0$. Moreover, we also assume that $f(x)$ is a smooth vector field and that $g(x)$, $h_z(x)$ and $h_y(x)$ are smooth mappings. Without loss of generality it is assumed that $d(x)'h_z(x) = 0$. Finally, assume that $g(x)$ and $d(x)$ has full column rank. In particular, $d(x)'d(x) = R(x) > 0$, for each x.

For $w = 0$ we assume that there exists a feedforward control law that guarantees perfect tracking $y = \tilde{y}$ for any reference output $\tilde{y}$ as long as $\tilde{x} \in X \subset R$ ($\tilde{x}$ is the corresponding desired state trajectory), $\dot{\tilde{x}}$ exists, and the initial state is well known. This feedforward control law, called $k_{ff}\left(\dot{\tilde{x}}, \tilde{x}\right)$, can be calculated using different methods (see [7], [8], [11] and [9]). It is obvious that if $w \neq 0$ the feedforward control law fails and a tracking error will occur. Hence, we want to solve a problem which is called **R**obustifying the **F**eedforward **C**ontrol (RFC) problem.

B. Definitions and Preliminaries

Definition 1 (RFC problem): Consider the nonlinear system with matched disturbances (5) and the feedforward control law $k_{ff}\left(\dot{\tilde{x}}, \tilde{x}\right)$. Find an additional feedback controller $k_{fb}(\tilde{x}, x)$ such that the control law

$$u = k_{ff}\left(\dot{\tilde{x}}, \tilde{x}\right) + k_{fb}(\tilde{x}, x) \qquad (6)$$

with

$$k_{fb}(\tilde{x}, x)\big|_{x=\tilde{x}} = 0$$

asymptotically stabilizes the closed loop system $\forall \tilde{x} \in X$ and the state error

$$e = x - \tilde{x} \qquad (7)$$

is bounded in L_2 as long as the input disturbance is bounded in L_2.

Remark 1: For validating the performance of the controller that satisfies the RFC problem, we introduce an additional performance variable $z_e = h_e(e)$ where $h_e(e)$ is the state error depending output map.

Definition 2 (Gradient difference field): The gradient difference field in x is defined as difference of the gradients of a scalar function $V(x)$ at two different points x and $x+e$ for $x, e \in R^n$.

$$\Gamma_x(e) = \left.\frac{\partial V(x)}{\partial x}\right|_{x+e} - \left.\frac{\partial V(x)}{\partial x}\right|_{x} \tag{8}$$

Lemma 1: For any point x the gradient difference field $\Gamma_x(e)$ defines the gradient of a local scalar function $V_x(e)$ (scalar function of e which is locally varying with x.

$$\Gamma_x(e) = \frac{\partial V_x(e)}{\partial e}$$

Lemma 2: If $V(x)$ is a scalar convex function in the convex set $x \in C \subseteq X$ then the local scalar function $V_x(e)$ from Lemma 1 is convex, too. Furthermore with $V_x(0) = 0$ the function is greater than or equal to zero.

For the proof of these lemmas we want to refer to [3]

Remark 2: The local scalar function can be calculated using the path independent line integral

$$V_x(e) = \int_0^e \Gamma_x(e)\, de. \tag{9}$$

Setting $V_x(0) = 0$ ensures that the scalar function is positive and if it is convex it is positive definite. Hence, this function may be a candidate Lyapunov function.

Definition 3: The distribution of the error vector field along a given trajectory caused by an imperfect feedforward tracking law is called the feedforward tracking error vector field and is defined as

$$l_{\tilde{x},\dot{\tilde{x}}}(e) = f(\tilde{x}+e) + g(\tilde{x}+e)\, k_{ff}\left(\dot{\tilde{x}}, \tilde{x}\right) - \dot{\tilde{x}}$$

Definition 4: The maximum eigenvalue of the input weighting matrix is defined as follows

$$\overline{\lambda}_R = \max_{x \in X} \lambda_{\max}(R(x))$$

C. Solution of the RFC problem

Consider a system given by (5) and assume that a stabilizing controller with guaranteed robustness bounds exists. As shown in [12], [4] and also in [10] such a controller for a given system (5) is

$$u = -R(x)^{-1} g(x)' \frac{\partial V(x)}{\partial x}' \tag{10}$$

where $V(x)$ is a control Lyapunov function which guarantees robustness and which may be the solution of the Hamilton Jacobi Bellman (HJB) for the L_2 optimal control law or the Hamilton Jacobi Isaac for the suboptimal H_∞ control law. For the latter approaches it has been shown that robustness (see [13] for linear systems and [4] for nonlinear systems) in terms of a maximum H_∞ attenuation level from the input disturbance w to the performance output variable z can be guaranteed and calculated *a priori*.

D. Sufficient conditions for solving the RFC problem

Assume that there exists a stabilizing control law (10) which is either an optimal L_2 controller or a suboptimal H_∞ controller. Thus there exists a solution - called $V(x)$ - of the HJB or of the HJI for which it should be assumed that $V(x)$ is strictly convex and its Hessian matrix does not vanish.

$$\frac{\partial^2 V(x)}{\partial x^2} > 0, \qquad \forall x \in X. \tag{11}$$

Since $V(x)$ is a convex function we can now apply Lemma 2 and the Remark 2 to define a local scalar positive definite function $V_x(e)$ by using the gradient difference field (see Definition 2). The following theorem is then sufficient for solving the RFC problem.

Theorem 1: If there exists a positive definite scalar function $V(x)$ which obeys (11) such that the controller (10) guarantees robustness and if there exist positive constants $k > 0$, $\kappa \geq 1$ and $0 < \alpha < \kappa / \left(k \bar{\lambda}_R\right)$ such that the scalar function $V_{\tilde{x}}(e)$ satisfies

$$\begin{aligned} \frac{m}{\alpha} \left\| \frac{\partial V_{\tilde{x}}(e)}{\partial \tilde{x}} \right\|_2 + \frac{1}{\alpha} \frac{\partial V_{\tilde{x}}(e)}{\partial e} l_{\tilde{x},\dot{\tilde{x}}}(e) + \frac{1}{2} h_e(e)' h_e(e) \\ -k \frac{\partial V_{\tilde{x}}(e)}{\partial e} g(\tilde{x}+e)\, g(\tilde{x}+e)' \frac{\partial V_{\tilde{x}}(e)}{\partial e}' < 0 \end{aligned} \tag{12}$$

for all $x \in X$, for a proper but constraint error region $\|e\| < E$ and for $m \geq \max_{\forall t} \left\| \dot{\tilde{x}}(t) \right\|_2$, then a robust tracking controller which solves the RFC problem is defined by

$$k_{fb}(\tilde{x}, x) = -\tilde{R}(x)^{-1} r_{\tilde{x}}(e) \tag{13}$$

with

$$r_{\tilde{x}}(e) = g(\tilde{x}+e)' \frac{\partial V_{\tilde{x}}(e)}{\partial e}'$$
$$\tilde{R}(x) = R(x)/\kappa$$

Theorem 2: If there exists a solution for (12) with $\alpha = \kappa / \left(2k\bar{\lambda}_R\right)$ then the controller (13) ensures an H_∞ attenuation level $\|T_{z_e,w}\|_\infty \leq \gamma$, where

$$\gamma = \frac{\bar{\lambda}_R \sqrt{2k}}{\kappa} \tag{14}$$

For the proof it should be referred to [3].

IV. Design of a robust tracking controller

As shown in Figure 3 the following tracking controller consists of a feedforward part which is an inverse controller and a robustifying feedback part. The output of the inverse controller is added to the output of the robust controller and the sum is used for the approximated inversion of the nonlinear static map. Since the inversion is only an approximated inversion, the feedback controller has to be sufficiently robust. In the following we will assume that the

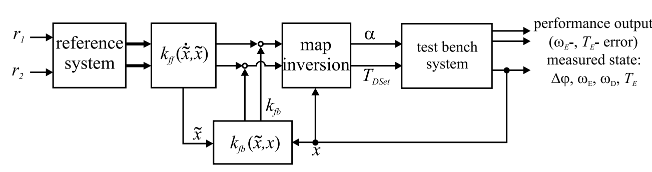

Fig. 3. Structure of the robust tracking controller

state variables of the system are fully available.

Remark 3: It should be stressed that the engine torque, which is a state of the system, is in practice not measurable.

However, a thorough discussion on the technological and theoretical problems underneath the torque computation or estimation is out of the scope of the present paper.

A. Inverse control of the combustion engine test bench

For the system inversion the system (3) is brought into control normal form. Therefore the outputs to be tracked have to be defined. In this application the outputs of the system are the engine torque (rescaled variable x_1) and the engine speed (rescaled variable x_3). Using the regular transformation matrix

$$\begin{pmatrix} z_{11} \\ z_{21} \\ z_{22} \\ \eta \end{pmatrix} = \begin{pmatrix} x_1 \\ x_3 \\ \frac{c}{\theta_E \beta}(x_1 - x_2) - \frac{d}{\theta_E}(x_3 - x_4) \\ x_2 \end{pmatrix}$$

the system is transformed into the control normal form

$$\begin{aligned}
\dot{z}_{11} &= -\left(\tilde{c}_0 + \tilde{c}_1 z_{22} + \tilde{c}_2 z_{22}^2\right) z_{11} - \gamma_1 z_{21} - \gamma_2 z_{21}^2 + v \\
\dot{z}_{21} &= z_{22} \\
\dot{z}_{22} &= -\frac{c}{\theta_E \beta}\left(\tilde{c}_0 + \tilde{c}_1 z_{22} + \tilde{c}_2 z_{22}^2 + \frac{d}{\theta_D} - \frac{c}{d}\right) z_{11} \\
&\quad -\frac{c}{\theta_E \beta}\left(\gamma_1 z_{21} + \gamma_2 z_{21}^2\right) + \frac{c}{\theta_E \beta} v - \frac{d}{\theta_E \theta_D} T_{DSet} \\
&\quad -\frac{\left(d^2 \theta_D - c\theta_E \theta_D + d^2 \theta_E\right)}{d\theta_E \theta_D} z_{22} + \frac{c^2}{d\theta_E \beta}\eta \\
\dot{\eta} &= \frac{1}{d}\left(c\left(z_{11} - \eta\right) - \theta_E \beta z_{22}\right)
\end{aligned}$$

The inverse realization can now be easily calculated. The inputs of the inverse system are the desired engine torque, the derivative of the desired engine torque, the engine speed and the first and the second derivative of the engine speed. Thus, a necessary condition for the reference trajectory of the engine torque is that it is differentiable and a necessary condition for the reference trajectory of the engine torque is that it is twice differentiable. The output of the inverse system is the control input, which yields perfect tracking if the initial state is well known and the system is exactly described by the mathematical model.

$$\begin{aligned}
\tilde{v} &= \dot{\bar{t}}_E + \left(\tilde{c}_0 + \tilde{c}_1 \bar{\omega}_E + \tilde{c}_2 \bar{\omega}_E^2\right) \bar{t}_E + \gamma_1 \bar{\omega}_E + \gamma_2 \bar{\omega}_E^2 \\
\tilde{T}_{DSet} &= \frac{c\theta_D}{\beta d}\dot{\bar{t}}_E - \frac{\theta_E \theta_D}{d}\ddot{\bar{\omega}}_E + \frac{c}{\beta}\left(1 - \frac{c\theta_D}{d^2}\right)\bar{t}_E \\
&\quad -\left(\theta_E + \theta_D - \frac{c\theta_E \theta_D}{d^2}\right)\dot{\bar{\omega}}_E + \frac{c^2 \theta_D}{d^2 \beta}\hat{\eta}
\end{aligned}$$

where $\bar{t}_E$ is the desired normalized engine torque, $\dot{\bar{t}}_E$ its derivative, $\bar{\omega}_E$ is the desired normalized engine speed, $\dot{\bar{\omega}}_E$ the first and $\ddot{\bar{\omega}}_E$ the second derivative, and $\hat{\eta}$ the state of the inverse system (zero dynamics)

$$\dot{\hat{\eta}} = \frac{c}{d}\hat{\eta} - \frac{c}{d}\bar{t}_E - \frac{c\theta_E \beta}{d}\dot{\bar{\omega}}_E.$$

B. Reference system

A reference system is important if the reference trajectory is not known *a priori* since for inverse control it is necessary to know the derivatives of the desired trajectories. The reference system must imply the performance limits of the system in order to desire feasible trajectories. Furthermore, the order of the reference system depends on the relative degree of the test bench system which is 3. For our application we choose a linear and decoupled reference system which *e.g.* is

$$\begin{aligned}
\dot{\tilde{z}}_{11} &= -\tilde{a}_1 \tilde{z}_{11} + \tilde{a}_1 r_1 \\
\bar{t}_E &= \tilde{z}_{11}
\end{aligned}$$

for the combustion engine torque and

$$\begin{aligned}
\dot{\tilde{z}}_{21} &= \tilde{z}_{22} \\
\dot{\tilde{z}}_{22} &= -\tilde{a}_2 \tilde{z}_{21} - \tilde{a}_3 \tilde{z}_{22} + \tilde{a}_2 r_2 \\
\bar{\omega}_E &= \tilde{z}_{21}
\end{aligned}$$

for the engine speed where $\tilde{a}_1$, $\tilde{a}_2$, $\tilde{a}_3$ define the dynamics of the reference system taking into account the physical performance limits of the system. For a detailed discussion of a reference system including the performance limits the reader is referred to [15]. r_1 and r_2 are the inputs of the reference system.

The reference system defines the reference trajectory of the first three states of the system in normal form. The reference trajectory of the fourth state $\tilde{\eta}$ is calculated by the differential equation

$$\dot{\tilde{\eta}} = \frac{c}{d}\tilde{\eta} - \frac{c}{d}\tilde{z}_{11} - \frac{c\theta_E \beta}{d}\tilde{z}_{22}$$

Using the reference system the inverse control law yields

$$\begin{aligned}
\tilde{v} &= -\tilde{a}_1 \tilde{z}_{11} + \tilde{a}_1 r_1 + \left(\tilde{c}_0 + \tilde{c}_1 \tilde{z}_{21} + \tilde{c}_2 \tilde{z}_{21}^2\right) \tilde{z}_{11} \\
&\quad +\gamma_1 \tilde{z}_{21} + \gamma_2 \tilde{z}_{21}^2 \\
\tilde{T}_{DSet} &= \frac{c\theta_D}{\beta d}\left(-\tilde{a}_1 \tilde{z}_{11} + \tilde{a}_1 r_1\right) + \frac{c}{\beta}\left(1 - \frac{c\theta_D}{d^2}\right)\tilde{z}_{11} \\
&\quad -\frac{\theta_E \theta_D}{d}\left(-\tilde{a}_2 \tilde{z}_{21} - \tilde{a}_3 \tilde{z}_{22} + \tilde{a}_2 r_2\right) \\
&\quad -\left(\theta_E + \theta_D - \frac{c\theta_E \theta_D}{d^2}\right)\tilde{z}_{22} + \frac{c^2 \theta_D}{d^2 \beta}\tilde{\eta}
\end{aligned}$$

The reference state trajectories are computed for the system in normal form. Since the robustifying state feeback controller is designed for the system (3) the reference state has to be transformed into the state space of system (3).

C. Robust stabilizing controller

Recalling the error model as shown in Figure 2 before searching for a robust stabilizing controller it is necessary to fix the required H_∞ attenuation level from w to z. From the uncertainties it is possible to estimate the L_2 gain from the disturbance input w to the controller output u (note that this defines one performance output (4)) which in worst case is 1.85. This value is calculated by assuming a worst case phase shift and a sufficient phase margin for the model uncertainties. The resulting worst case phase shift is then interpreted as worst case L_2 gain. Thus, the H_∞ attenuation level from w to z is 1.85.

Note that with (4) the maps $h(x)$ and $d(x)$ are orthogonal and $d(x)' d(x) = I_{2\times 2}$. Note also that for this first consideration we assume an input disturbance which has no offset. For designing the robustifying feedback controller we apply the γ recalibration strategy recently presented in [4]. Referring to Theorem 2 in [4] it is possible to recalibrate the L_2 gain according to

$$\gamma = \max_x \|d_\infty(x)\| \sqrt{\frac{\gamma^o}{2\max_x \|d(x)\| - \gamma^o}} \qquad (15)$$

The recalibrated H_∞ attenuation level γ used for the controller design and which still guarantees an attenuation level of $\gamma^o = 1.85$ is 3.51 which corresponds to a robust controller much closer to the optimal one. Hence, following *e.g.* [10] we search for a positive definite solution $V(x)$ of

$$\begin{aligned}&\tfrac{\partial V}{\partial x} f(x) + \tfrac{1}{2} h(x)' h(x) \\ &+ \tfrac{1}{2}\tfrac{\partial V}{\partial x} g(x) \left(\tfrac{1}{\gamma^2} I - R(x)^{-1}\right) g(x)' \tfrac{\partial V}{\partial x}' = 0\end{aligned} \qquad (16)$$

where the robust state feedback control law then is

$$u = -g(x)' \left(\frac{\partial V(x)}{\partial x}\right)'.$$

To find a solution of (16) we now perform an iterative procedure. In order to simplify the search for positive definite scalar function, we restrict the possible state operating range with $\|x\|_\infty \leq 1$. For the considered application example this restriction is admissible since the state variables are restricted from the physical point of view, too. For solving (16) we first define the structure (but not the parameters) of a candidate Lyapunov function $V(x)$ and a performance matrix $H(x)$ - with $x' H(x) x = h(x)' h(x)$ - such that (16) is solvable. Note that by a proper choice of $V(x)$ and $H(x)$ the performance matrix has some degrees of freedom which can be used for tuning the performance. However, after calculating the parameters of the candidate Lyapunov function and the missing parameters of the performance matrix (some of them are design parameters), the performance of the resulting controller has to be evaluated. If it is not satisfactory, the same procedure has to be repeated by defining a different candidate Lyapunov function. After some iteration steps we found the following candidate Lyapunov function (parameters need still to be found)

$$\begin{aligned}V(x_1, x_2, x_3, x_4) &= k_1 x_1^2 + k_2 x_2^2 + k_3 x_3^2 + k_4 x_4^2 \\ &+ k_5 x_2 x_4 + k_6 x_1^2 x_3^2 + k_7 x_1 x_3^2 \end{aligned} \quad (17)$$

for which the constants k_1 to k_7 are such that $V(x_1, x_2, x_3, x_4) > 0 \ \forall x$ inside the considered range ($\|x\|_\infty \leq 1$). This solution of (16) can be found if we choose a specific structure of the performance matrix such that

$$H(x) = \begin{pmatrix} h_1^2 & 0 & h_{13}(x) & 0 \\ 0 & h_2^2 & h_{23} & 0 \\ h_{13}(x) & h_{23} & h_3^2 + h_{33_1} x_3 + h_{33} x_3^2 & h_{34} \\ 0 & 0 & h_{34} & h_4^2 \end{pmatrix}$$

where

$$\begin{aligned}h_{13}(x) = h_{13\_1} + h_{13\_2} x_1 - h_{13\_3} x_2 + h_{13\_4} x_3 + h_{13\_5} x_4 \\ + h_{13\_6} x_1 x_2 + h_{13\_7} x_1 x_3 + h_{13\_8} x_1 x_4 + h_{13\_9} x_1^2 \\ + h_{13\_10} x_3^2 + h_{13\_11} x_1 x_3^2 + h_{13\_12} x_3^3 + h_{13\_13} x_1 x_3^3\end{aligned}$$

By setting the tuning parameters h_1, h_2, h_3, h_4, h_{33} and h_{13_13} it is then possible to compute the constants k_1 to k_7, h_{13_1} to h_{13_12} and h_{23} by comparing the coefficients. Note that in this case it is absolutely necessary to check whether the performance matrix $H(x)$ is positive definite $\forall x$ inside the considered range ($\|x\|_\infty \leq 1$). With the tuning parameters

$$h_1 = 7, h_2 = 0.25, h_3 = h_4 = 5, h_{33} = -3, h_{13\_13} = 0.8$$

the remaining parameters of the performance matrix turn out to be those given in Table I for $\gamma = 3.51$, the performance matrix is positive definite and the performance is sufficiently.

$k_1 = 1.57$	$k_2 = 0.0014$	$k_3 = 0.56$
$k_4 = 0.70$	$k_5 = -0.009$	$k_6 = 0.12$
$k_7 = -0.32$		

TABLE I

PARAMETERS OF $V(x_1, x_2, x_3, x_4)$

D. Robustifying feedback control law

The robust stabilizing central controller is now extended to be used for robustifying the feedforward control law. The proposed controller, which solves the RFC problem, is given by (13). For the actual problem the input weights are set to $R(x) = I_{2\times 2}$. The presented approach is possible if (17) is strictly convex and the Hessian matrix of it is positive definite. Furthermore, condition (12) in Theorem 1 has to be satisfied.

Using the controller parameters in Table I the Hessian matrix of the scalar function $V(x_1, x_2, x_3, x_4)$ is positive definite for all $\|x\|_\infty < 1$. Hence, it is a candidate to calculate the gradient difference field $\Gamma_x(e)$ which yields

$$\Gamma_{\tilde{x}}(e) = \begin{pmatrix} \gamma_1(e) \\ 2k_2 e_2 + k_5 e_4 \\ \gamma_2(e) \\ 2k_2 e_4 + k_5 e_2 \end{pmatrix}'$$

where

$$\begin{aligned}\gamma_1(e) &= (2k_6(e_1 + \tilde{x}_1) + k_7) e_3^2 + (2k_1 + 2k_6 \tilde{x}_3) e_1 \\ &\quad + (4k_6(e_1 + \tilde{x}_1)\tilde{x}_3 + 2k_7 \tilde{x}_3) e_3 \\ \gamma_2(e) &= \left(2k_6(e_1 + \tilde{x}_1)^2 + 2k_7(e_1 + \tilde{x}_1) + 2k_3\right) e_3 \\ &\quad + 2k_6 \tilde{x}_3 e_1^2 + (4k_6 \tilde{x}_1 \tilde{x}_3 + 2k_7 \tilde{x}_3) e_1\end{aligned}$$

Note that in order to get a robust feedback control law it is necessary to achieve a L_2 gain from w to u that is less than 1.85. Hence, the performance output for the robustness analysis is equal to the controller output. With $h_e(e) = r_{\tilde{x}}(e)$, $\bar{\lambda}_R = 1$ and with $\max_{\forall t} \|\dot{\tilde{x}}(t)\|_2 = 4\frac{1}{s} = m$ it can numerically be shown that (12) is true for $\kappa = 3$, $\alpha = 0.1$ and $k = 15$. In that case Theorem 2 is valid and the guaranteed robustness level is $\gamma = 1.83$.

According to this, the robustifying state feedback control law is

$$\begin{aligned}v_{fb} &= -\left(2k_1 + 4k_6 \tilde{x}_3 e_3 + 2k_6 e_3^2 + 2k_6 \tilde{x}_3^2\right) e_1 \\ &\quad + (2k_6 \tilde{x}_1 + k_7) e_3^2 + (4k_6 \tilde{x}_1 \tilde{x}_3 + 2k_7 \tilde{x}_3) e_3 \\ \bar{T}_{DSet\_fb} &= -\frac{1}{\theta_D}(2k_4 e_4 + k_5 e_5)\end{aligned}$$

V. Simulation results

The presented controller is now applied to the engine test bench system and compared to the standard control approach. The standard control approach consists of two decoupled PI loops whose parameters are tuned by solving a numerical optimization problem. A precise simulation environment is used in order to test the controller's performance. Therefore, the test bench model includes control input delays, measurement noise, the combustion oscillations and not ideal actuators. Furthermore, since the engine torque is not measurable, it has to be observed. To this end a simple engine torque observer as presented in [14] is implemented.

Figure 4 shows the engine speed signal and the simulated real engine torque and the desired engine torque value for both, the robust tracking controller and the standard control approach. Figure 5 shows the inputs to the engine test bench system for both. The main advantage of the robust tracking controller is the feedforward control which allows a very good tracking performance while the control inputs is without measurement noise. This is very important for test bench control because if the accelerator pedal varies more than in a standard application, the engine will also generate more exhausts and hence the tests become useless. In the comparison of the controller we see that for the robust tracking controller the performance is better and the noise in the dynamometer torque is less.

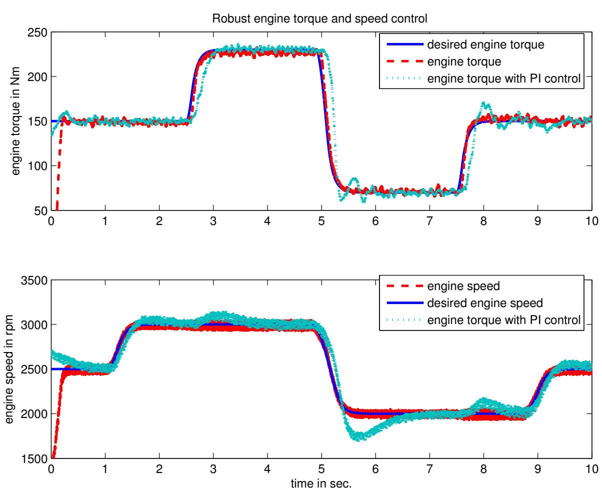

Fig. 4. Tracking results

VI. Conclusions

In this paper a recently presented method for robust tracking of nonlinear systems has been applied to an engine test bench system. For such systems tracking is an important but also difficult task. The main problem thereby is that the model is never known exactly. Nevertheless we showed that an inverse control approach in combination with a robustifying feedback controller solves the problem quite well. The advantage of the presented controller is that once having a central stabilizing controller and a control Lyapunov function it is very easy to calculate the tracking controller. Future work will be concentrated in considering a preview

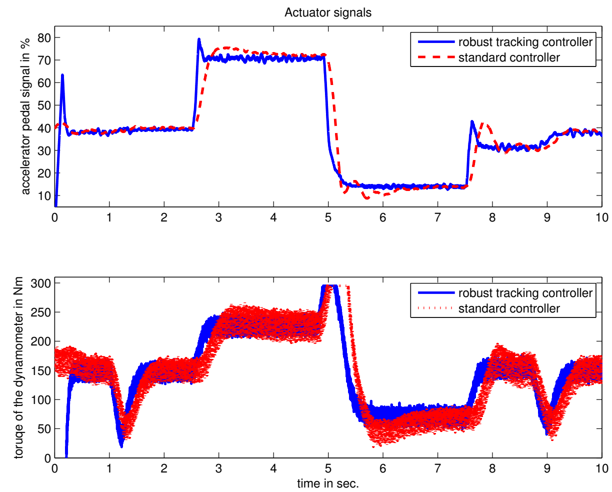

Fig. 5. Control inputs

in the feedback controller structure as well. Furthermore, the controller will be tested in a real environment.

VII. Acknowledgement

The support of LCM - Linz Center of Mechatronics is gratefully acknowledged.

References

[1] B. J. Bunker, M. A. Franchek, B. E. Thomason, "Robust Multivariable Control of an Engine-Dynamometer System", IEEE Transactions on Control Systems Technology, Vol. 5, No. 2, 1997

[2] D. Yanakiev , "Adaptive Control of Diesel Engine-Dynamometer Systems", Proceedings of the 37th IEEE Conference on Decision & Control o Tampa, Florida USA o December 1998

[3] E. Gruenbacher, L. del Re, "Trobust Trajectory Tracking for a Class of Uncertain Nonlinear Systems", NOLCOS, Pretoria, 2007

[4] E. Gruenbacher, P. Colaneri, L. del Re, "Guaranteed robustness bounds for actuator-disturbance nonlinear control",Proceedings of ROCOND,Toulouse 2006

[5] E. Gruenbacher, "Robust Inverse Control of a Class of Nonlinear Systems", VDI Nr. 1120, ISBN 978-3-18-512008-4,2007

[6] U. Kiencke, L. Nielsen, "Automotive Control Systems - For Engine, Driveline and Vehicle", Springer, ISBN 3-540-66922-1, 2000

[7] L. R. Hunt, G. Meyer, "Stable Inversion for Nonlinear Systems", Automatica, Vol. 33(8), pp. 1549-1554, 1997

[8] A. Isidori, "Nonlinear Control Systems - third edition", Springer Verlag, ISBN 3-540-19916-0, 1995

[9] S. Devasia, D. Chen, B. Paden, "Nonlinear Inversion-Based Output Tracking", IEEE Transactions on Automatic Control, Vol. 41(7), 1996

[10] Van der Schaft, A. J., "L_2 -gain analysis of nonlinear systems and nonlinear state feedback H_∞ control", IEEE Trans. Automat. Control, 37, pp. 770-784, 1992.

[11] E. Gruenbacher, L. del Re, "Output Tracking of Non Input Affine Systems using Extended Hammerstein Models", ACC 2005

[12] A. E. Bryson, Y. -C. Ho, "Applied optimal control". Wiley, NY, 1975.

[13] P.Bolzern, P.Colaneri. G.De Nicolao, U.Shaked, "Guaranteed H* bounds for Wiener filtering and prediction", Int. J. Robust and Nonlinear Control, Vol. 12, pp. 41-46, 2002.

[14] E. Gruenbacher, L. del Re, "Adaptive Mean Value Engine Torque Estimation on Engine Test Benches", To be published at the proceedings of 2007 CACS International Automatic Control Conference, National Chung Hsing University, Taichung, Taiwan, 2007

[15] E. Gruenbacher, L. del Re, H. Kokal, M. Schmidt, M. Paulweber, "Online Trajectory Shaping Strategy for Dynamical Engine Test Benches", CCA, Munich, 2006