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Stochastic Controlling Tolerable Fault of Network Control Systems

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***Abstract*— A problem of stabilization about uncertain Networked Control Systems (NCSs) with random but bounded delays is discussed in this paper. By using augmented state-space method, this class of problem can be modeled as discrete-time jump linear system governed by a finite-state Markov chains. A new switched model based on probability is proposed to research problems of reliable control when actuators become ageing or partial disable. Using improved V-K iteration algorithm, a class of reliable controller is designed to make system asymptotically mean square stable under several stochastic disturbance such as random time-delay and stochastic actuator failure and have the maximal redundancy degree.**

I. INTRODUCTION

Feedback control systems wherein the control loops are connected with a real-time network are called networked control systems (NCSs).The main feature of NCSs is that, instead of hardwiring the control devices with point to point connections, sensor, actuators, and controllers are all connected to the network as nodes. The primary advantages of NCSs are low cost, reduced system wiring, simple installation and maintenance, high reliability and ease of system diagnosis and maintenances[1-3].As a result, NCSs have been widely applied to many complicated control systems, such as aviation and aerospace fields, airplane manufacture[4]. The insertion of the communication network in feedback control loop makes the analysis and design of a NCSs complicate because it introduces some problem existing in network into control systems such as limited communication band width , so network-induced delay, wrong order of data packets and data packets dropout often happen inevitably happen during information transmission. Because problems of wrong order and lost of data packets can be converted into problems of time-delay, the basic problem of NCSs is how to treat time-delay.

As to stochastic network-induced delay, many researchers have paid attention on the study of the stability controller design for stabilization and performance achievement in NCSs. A stabilization problem of NCSs was investigated by [5] when the network-induced delay is less than one sampling time. [6] analyzed several important problems specifically that exist in network control systems such as time-delay, dropping network packets and multiple-packet transmission and apply methods of hybrid control system to research stabilization of network control systems when systems has determinate network-induced delay. By using augmented state-space method, [7] converted a stabilization problem of NCSs with random delays into a stabilization problem of jump linear system governed by Markov chains such that the closed-loop system is a jump linear system with one mode, further more, mode-dependent and mode-independent controllers which satisfied mean square stability of system were given.

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In practical application, the required standard of NCSs to security and reliability is very high. Regretfully, both stochastic perturbation and other failures usually exist in NCSs. For example, partial sensors and actuators inactivate because of aging phenomenon. These stochastic faults always lead systems deviate expected dynamic and state characters or even make systems become instable. So, how to deal with the problems of stochastic tolerable fault control of NCSs is becoming more and more important.

Problems of partial sensors inactivation are equal to problems of package dropout which can be solved with common technique, so we focus on problems of reliability when actuators inactivate at certain probability in this paper. Firstly, a switch mode of NCSs is constructed. Secondly, a sufficient condition of tolerable controllers satisfied mean square stability and its design method are given by using LMI method when NCSs have uncertain controlled plant.

II. MODELING OF NCSs

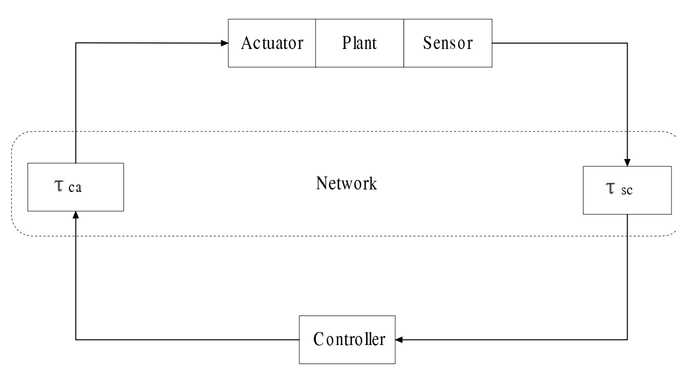

Fig. 1. Network control system with random delay

Without loss general, consider linear uncertain systems with stochastic delayed state described by the following differential equation:

$$x(k+1)=(A+\triangle A(t))x(k)+(B+\triangle B(t))u(k) \qquad (1)$$

where, $x(k)\in\Re^n$ is the state vector, $u(k)\in\Re^m$ is the control input vector. Suppose the uncertain structures of the system (1) are given by

$$\triangle A(t)=DF(t)E,\triangle B(t)=D_1F_1(t)E_1$$

D,D_1,E,E_1 are constant matrices with appropriate dimensions and $F(t),F_1(t)$are Lebesgue measurable satisfied

$$F^\top(t)F(t)\le I,F_1(t)^\top F_1(t)\le I$$

The mode-dependent state feedback control law considered in this paper is:

$$u(k)=K_{r_s(k)}x(k-r_s(k)) \qquad (2)$$

where, $\{r_s(k)\}$ is a bounded random integer sequence with $0\le r_s(k)\le d_s<\infty$ which expresses stochastic delays of network, and d_s is the finite delay bound. If we augment the state variable

$$\tilde{x}(k)=\begin{bmatrix} x^\top(k) & x^\top(k-1) & \dots & x^\top(k-d_s)\end{bmatrix}^\top$$

where $\tilde{x}(k)\in\Re^{(d_s+1)\times n}$,then the closed-loop system is

$$\tilde{x}(k+1)=(\tilde{A}+\hat{D}\hat{F}\hat{E}_{r_s(k)}+\tilde{B}_{r_s(k)}K_{r_s(k)})\tilde{x}(k) \qquad (3)$$

where

$$\tilde{A}=\begin{bmatrix} A & 0 & \cdots & 0 & 0\\ I & 0 & \cdots & 0 & 0\\ 0 & I & \cdots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & \cdots & I & 0\end{bmatrix},\tilde{B}_{r_s(k)}=\begin{bmatrix} B\\ 0\\ 0\\ \vdots\\ 0\end{bmatrix}\tilde{C}_{r_s(k)},$$

$$\tilde{D}=\begin{bmatrix} D\\ 0\\ 0\\ \vdots\\ 0\end{bmatrix},\tilde{D}_1=\begin{bmatrix} D_1\\ 0\\ 0\\ \vdots\\ 0\end{bmatrix},\tilde{E}=\begin{bmatrix} E\\ 0\\ 0\\ \vdots\\ 0\end{bmatrix}^\top,$$

$$\tilde{E}_{r_s(k)}=E_1\tilde{C}_{r_s(k)}K_{r_s(k)},\hat{D}=\begin{bmatrix}\tilde{D} & \tilde{D}_1\end{bmatrix},$$

$$\hat{E}_{r_s(k)}=\begin{bmatrix}\tilde{E}\\ \tilde{E}_{r_s(k)}\end{bmatrix},\hat{F}(k)=\begin{bmatrix}F(k) & \\ & F_1(k)\end{bmatrix},$$

$$\tilde{C}_{r_s(k)}=\begin{bmatrix}0 & \cdots & 0 & I & 0 & \cdots & 0\end{bmatrix}$$

and $\tilde{C}_{r_s(k)}$ has all elements being zero except for the $r_s(k)th$ block being an identity matrix. For simplicity, (3) can be rewritten as the following form

$$\begin{cases}\hat{x}(k)=\hat{A}_{r_s(k)}x(k)\\ x(0)=x_0\in\Re^n\end{cases} \qquad (4)$$

the state of $\hat{A}$ is decided by $r_s(k)$, the jumping rules of random integer sequence $r_s(k)$ can be modeled as a finite state Markov chains, that is

$$Prob\{r_s(k+1)=j|r_s(k)=i\}=P_{ij} \qquad (5)$$

where $0\le i,j\le d_s$. that means, the dynamic characters of bounded random time-delay can be expressed by a step transition probability matrix P. Here, we assume that P is given. More details about the selection of the transition probability matrix P can be seen in [7] and [14].

III. STABILITY ANALYSIS OF NETWORK CONTROL SYSTEM

In this section, a problem of stability of linear jump system is discussed. Some relative definitions and lemmas are as follows

Definition 1 For system (4), let zero point is equilibrium point

1) Asymptotically mean square stable , if for any $x_0\in\Re^n$ and any initial probability distribution $(p_1,\dots,p_s)\in r_s(k)$

$$\lim_{k\to\infty}E\{\|x_k(x_0,\omega)\|^2\}=0$$

2) Stochastic stable, if for any $x_0\in\Re^n$ and any initial probability distribution $(p_1,\dots,p_s)\in r_s(k)$

$$E(\sum_{k=0}^{\infty}\{\|x_k(x_0,\omega)\|^2\})<\infty$$

Lemma 1 Let A,D,E and F be real matrices of appropriate dimensions with $\|F\|<1$. Then we have the following

1) For any scalar $\varepsilon>0$

$$DFE+E^\top F^\top D^\top\le\varepsilon^{-1}D^\top D+\varepsilon E^\top E$$

2) For any matrix $P>0$ and scalar $\varepsilon>0$ satisfying $\varepsilon I-EPE^\top>0$

$$A(t)PA^\top(t)\le APA^\top+APE^\top(\varepsilon I-EPE^\top)^{-1}EPA^\top+\varepsilon DD^\top$$

3) For any matrix $P>0$ and scalar $\varepsilon>0$ satisfying $\varepsilon I-D^\top PD>0$

$$A^\top(t)PA(t)\le A^\top PA+A^\top PD(\varepsilon I-D^\top PD)^{-1}D^\top PA+\varepsilon E^\top E$$

Theorem 1 The closed system (3) is stochastic stability, if there have symmetric positive definite matrixes $Q(i)>0,i\in\{0,\cdots,d_s\}$, satisfying

$$L(i)=\hat{A}_i^\top\bar{Q}(i)\hat{A}_i-Q(i)<0 \qquad (6)$$

where

$$\bar{Q}(i)=\sum_{j=0}^{d_s}p_{ij}Q_j \qquad (7)$$

and $p_{ij}\in P$.

Proof As to closed system (3), choose a Lyapunov function candidate as

$$V(\tilde{x}(k),k)=\tilde{x}(k)^\top Q(\tau_k)\tilde{x}(k)$$

then we have

$$\begin{aligned}
&E\{\Delta(\tilde{x}(k),k)\} \\
&= E\{\tilde{x}(k+1)^\top Q(\tau_{k+1})\tilde{x}(k+1) \mid \tilde{x}(k),\tau_k = i)\} \\
&-\tilde{x}(k)^\top Q(\tau_k)\tilde{x}(k) \\
&= \tilde{x}(k)^\top (\tilde{A}+\hat{D}\hat{F}(k)\hat{E}_i+\tilde{B}_iK_i)^\top \sum_{j=0}^{d_s} p_{ij}Q(j)(\tilde{A}+\hat{D}\hat{F}(k)\hat{E}_i \\
&+\tilde{B}_iK_i)\tilde{x}(k) - \tilde{x}(k)^\top Q(i)\tilde{x}(k) \\
&= \tilde{x}(k)^\top [(\tilde{A}+\hat{D}\hat{F}(k)\hat{E}_i+\tilde{B}_iK_i)^\top \sum_{j=0}^{d_s} p_{ij}Q(j)(\tilde{A}+\hat{D}\hat{F}(k)\hat{E}_i \\
&+\tilde{B}_iK_i) - Q(i)]\tilde{x}(k) < 0
\end{aligned}$$

let

$$L(i) = (\tilde{A}+\hat{D}\hat{F}(k)\hat{E}_i+\tilde{B}_iK_i)^\top \sum_{j=0}^{d_s} p_{ij}Q(j) \times (\tilde{A}+\hat{D}\hat{F}(k)\hat{E}_i+\tilde{B}_iK_i) - Q(i) < 0$$

and $\hat{A}_i = (\tilde{A}+\hat{D}\hat{F}(k)\hat{E}_i+\tilde{B}_iK)$, then compared with definition 1, we can get (8).

This completes the proof of the theorem.

From [10] we know, stochastic stability of the linear jump system of (4) is equivalent to asymptotic mean square stability. The four equivalent conditions which make the linear jump system asymptotic mean square stability are given in [11], so we have Theorem 2 .

Theorem 2 The closed system (4) is asymptotic mean square stability, if there exist positive-definite symmetric matrices $Q_i > 0$ and constants $n_i > 0, i \in \{0,\cdots,d_s\}$, satisfying the following matrix inequality:

$$\begin{bmatrix} Q_j & \Sigma & \Theta & \Lambda \\ * & \Omega & 0 & 0 \\ * & * & \Xi & 0 \\ * & * & * & \Pi \end{bmatrix} > 0 \tag{8}$$

where,

$$\begin{aligned}
&\Sigma = [(\tilde{A}+\tilde{B}_0K_0)^\top Q_0,\cdots,(\tilde{A}+\tilde{B}_{d_s}K_{d_s})^\top Q_{d_s}], \\
&\Theta = [(\tilde{A}+\tilde{B}_0K_0)^\top Q_0\hat{D},\cdots,(\tilde{A}+\tilde{B}_{d_s}K_{d_s})^\top Q_{d_s}\hat{D}], \\
&\Lambda = [n_0\hat{E}_0^\top,\cdots,n_{d_s}\hat{E}_{d_s}^\top], \Omega = diag\{p_{j0}^{-1}Q_0\cdots p_{jd_s}^{-1}Q_{d_s}\}, \\
&\Xi = diag\{p_{j0}^{-1}(n_0I-\hat{D}^\top Q_0\hat{D})\cdots p_{jd_s}^{-1}(n_{d_s}I-\hat{D}^\top Q_{d_s}\hat{D})\}, \\
&\Pi = diag\{p_{j0}^{-1}n_0\cdots p_{jd_s}^{-1}n_{d_s}\}, j \in \{0,1,\cdots,d_s\}
\end{aligned}$$

and $n_iI-\hat{D}^\top Q_i\hat{D} > 0, i \in \{0,1,\cdots,d_s\}$.

Proof From [11], we choose one of four conditions that make system asymptotic mean square stability as follows:

$$\sum_{i=0}^{d_s} p_{ji}\hat{A}_i^\top Q_i\hat{A}_i < Q_j, j \in \{0,\cdots,d_s\} \tag{9}$$

Let $\hat{A}_i^\top = \{(\tilde{A}+B_iK_i)+\hat{D}\hat{F}(k)\hat{E}_i\}$, then from lemma 1 we get

$$\begin{aligned}
\hat{A}_i^\top Q_i\hat{A}_i \leq\ & (\tilde{A}+\tilde{B}_iK_i)^\top Q_i(\tilde{A}+\tilde{B}_iK_i) \\
+\ & (\tilde{A}+\tilde{B}_iK_i)^\top Q_i\hat{D}(n_iI-\hat{D}^\top Q_i\hat{D})\hat{D}^\top Q_i(\tilde{A}+\tilde{B}_iK_i) \\
+\ & n_i\hat{E}_i^\top E_i, i \in \{0,\cdots,d_s\}.
\end{aligned}$$

Together with (9) we obtain

$$\begin{aligned}
&\sum_{i=0}^{d_s} p_{ji}\hat{A}_i^\top Q_i\hat{A}_i - Q_j \leq \sum_{i=0}^{d_s} p_{ji}[(\tilde{A}+\tilde{B}_iK_i)^\top \\
&Q_i\hat{D}(n_iI-\hat{D}^\top Q_i\hat{D})\hat{D}^\top Q_i(\tilde{A}+\tilde{B}_iK_i)+n_i\hat{E}_i^\top E_i] - Q_j < 0
\end{aligned}$$

and then applying Schur complement to the results, we can obtain(8).

Because (8) has several nonlinear items such as $(\tilde{A}+\tilde{B}_0K_0)^\top Q_0$ which can not be resolved by LMI tools directly, the V-K iteration algorithm of [7] is improved to resolve this problem.

For guaranteeing a certain convergent speed, condition (9) becomes

$$\sum_{i=0}^{d_s} p_{ji}\hat{A}_i^\top Q_i\hat{A}_i < \alpha Q_j, j \in \{0,\cdots,d_s\} \tag{10}$$

where, $0 < \alpha < 1$.

1) Request for initial value. Using V-K iteration Design a LQR controller K for the system (1) without considering random delays τ_{sc} in the loop. This controller should satisfy

$$\begin{bmatrix} -X & (AX+BY)^\top & XE^\top & Y^\top E_1^\top & X & Y^\top \\ * & \Upsilon & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon & 0 & 0 & 0 \\ * & * & * & -\varepsilon & 0 & 0 \\ * & * & * & * & -T^{-1} & 0 \\ * & * & * & * & * & -R^{-1} \end{bmatrix} < 0 \tag{11}$$

where $\Upsilon = -(X-\varepsilon(DD^\top + D_1D_1^\top))$, positive-definite symmetric matrix $X > 0$, constant $\varepsilon > 0, K_{ini} = YX^{-1}$. Matrices $T,R > 0$ are corresponding matrices of cost function

$$J = \sum_{k=0}^{\infty}(x_k^\top Tx_k + u_k^\top Ru_k).$$

2) V iteration. Given $K_0 = K_1 = \cdots = K_{d_s} = K_{ini}, \alpha = 1$, and a step transition probability matrix P_0, solve LMI feasibility problem (10) to find $Q_i, i \in \{0,\cdots,d_s\}$.

3) K iteration. Given P_0 and $Q_i, i \in \{0,\cdots,d_s\}$ that have solved in V iteration, solve the eigenvalue problem (10) to find $K_0,K_1,\cdots,K_{d_s}$ guaranteeing variable α be minimum within $0 < \alpha < 1$. That is

$$\min \alpha$$

$$0 < \alpha < 1 \tag{12}$$

$$\begin{bmatrix} \alpha Q_j & \Sigma & \Theta & \Lambda \\ * & \Omega & 0 & 0 \\ * & * & \Xi & 0 \\ * & * & * & \Pi \end{bmatrix} > 0 \tag{13}$$

Others signals of (13) are the same as corresponding signals of (8).

4) Adjust initial matrix P_0. Perturb the transition probability matrix P_0 by adding a small perturbation matrix $\triangle P$ so that $P_0 + \triangle P \to P_0$ .In other words, through adjusting initial matrix P_0, let the new transition probability matrix P_0 tend to expected transition probability matrix P_E .It is notice that

sums of every rows of the perturbation matrix $\triangle P$ must equal zero.

Then back to step 2), repeat the cycle from step 2) to step 4), until the desired transition probability matrix P_E is reached or the V iteration is not feasible.

IV. STABILITY ANALYSIS OF SWITCHED NETWORK CONTROL SYSTEMS WITH UNCERTAINTY

The state equation of this switched control system models as

$$\begin{cases} \hat{x}(k+1) = (A+\triangle A)x(k) + (B+\triangle B)u_{(i_k,r_s(k))}(k) \\ u_{(i_k,r_s(k))}(k) = K_{(i_k,r_s(k))}x(k-r_s(k)) \end{cases} \quad (14)$$

where $i \in \{1,2\}, r_s(k) \in \{0,\cdots,d_s\}$.

From (14), this system has 2 group controllers, i_k indicate the number of controllers groups acting in time k. Because the maximal delay is d_s, every group of time-dependent controllers has d_s+1 controllers. The general structure of switched control system is shown in Fig. 2. where $P \in \Re^{(d_s+1)\times(d_s+1)}$ denotes one step probability matrix

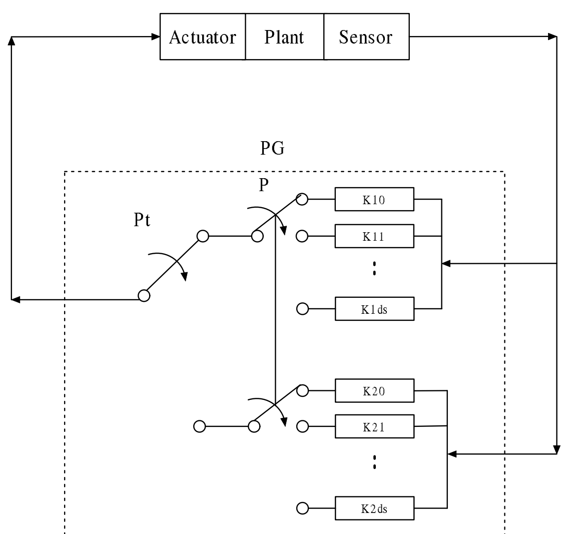

Fig. 2. Stochastic network system under switched control

of stochastic delay of present network, $K_{10} \sim K_{1d_s}$ are mode-dependent controllers satisfied to transition probability matrix P (if controllers $K_{10} = K_{11} = \cdots = K_{1d_s}$, they are mode-dependent controllers and if $K_{10} \neq K_{11} \neq \cdots \neq K_{1d_s}$, they are mode-independent controllers.). $K_{20} \sim K_{2d_s}$ are testing controllers which are used to checkout reliable ability of designed controllers. As to every group of control loop, this system can be seen as an independent jump linear system governed by Markov chains and these two Markov process are irrelative each other. After inserting a switch, these two Markov process becomes relative. Through regulate different switched probability of switch and set suitable gain values of testing controllers, stabilization span and stable performances of system are obtained when actuators of system suffer different fault probability. Firstly, the mean square stability of system theorem is given as follows:

Theorem 3 Assume (14) has switched rules matrix Pt ,the jump rules of jump linear control system governed by a step transition probability matrix P, then the mean square stability of system is equivalent to the existence of symmetric positive definite matrices $Q_0, Q_1, \cdots, Q_{(d_s+1)\times 2}$ and constants $n_i > 0, n_i I - \hat{D}^\top Q_i \hat{D} > 0, i \in \{0,1,\cdots,(2d_s+2)\}$, satisfying

$$\begin{bmatrix} Q_j & \Sigma & \Sigma_1 & \Theta & \Theta_1 & \Lambda & \Lambda_1 \\ * & \Omega & 0 & 0 & 0 & 0 & 0 \\ * & * & \Omega_1 & 0 & 0 & 0 & 0 \\ * & * & * & \Xi & 0 & 0 & 0 \\ * & * & * & * & \Xi_1 & 0 & 0 \\ * & * & * & * & * & \Pi & 0 \\ * & * & * & * & * & * & \Pi_1 \end{bmatrix} > 0 \quad (15)$$

where,

$$\Sigma = [(\tilde{A}+\tilde{B}_0K_{10})^\top Q_0, \cdots, (\tilde{A}+\tilde{B}_{d_s}K_{1d_s})^\top Q_{d_s}],$$
$$\Sigma_1 = [(\tilde{A}+\tilde{B}_0K_{20})^\top Q_{d_s+1}, \cdots, (\tilde{A}+\tilde{B}_{d_s}K_{2d_s})^\top Q_{2d_s+2}],$$
$$\Theta = [(\tilde{A}+\tilde{B}_0K_{10})^\top Q_0\hat{D}, \cdots, (\tilde{A}+\tilde{B}_{d_s}K_{1d_s})^\top Q_{d_s}\hat{D}],$$
$$\Theta_1 = [(\tilde{A}+\tilde{B}_0K_{20})^\top Q_{d_s+1}\hat{D}, \cdots, (\tilde{A}+\tilde{B}_{d_s}K_{2d_s})^\top Q_{2d_s+2}\hat{D}],$$
$$\Lambda = [n_0\hat{E}_0^\top, \cdots, n_{d_s}\hat{E}_{d_s}^\top],$$
$$\Lambda_1 = [n_{d_s+1}\hat{E}_0^\top, \cdots, n_{2d_s+2}\hat{E}_{d_s}^\top],$$
$$\Omega = diag\{p_{j0}^{-1}Q_0 \cdots p_{jd_s}^{-1}Q_{d_s}\},$$
$$\Omega_1 = diag\{p_{j(d_s+1)}^{-1}Q_{(d_s+1)} \cdots p_{j(2d_s+2)}^{-1}Q_{(2d_s+2)}\},$$
$$\Xi = diag\{p_{j0}^{-1}(n_0 I - \hat{D}^\top Q_0\hat{D}) \cdots p_{jd_s}^{-1}(n_{d_s}I - \hat{D}^\top Q_{d_s}\hat{D})\},$$
$$\Xi_1 = diag\{p_{j(d_s+1)}^{-1}(n_{(d_s+1)}I - \hat{D}^\top Q_{(d_s+1)}\hat{D}) \cdots$$
$$p_{j(2d_s+2)}^{-1}(n_{(2d_s+2)}I - \hat{D}^\top Q_{(2d_s+2)}\hat{D})\},$$
$$\Pi = diag\{p_{j0}^{-1}n_0 \cdots p_{jd_s}^{-1}n_{d_s}\},$$
$$\Pi_1 = diag\{p_{j(d_s+1)}^{-1}n_{(d_s+1)} \cdots p_{j(2d_s+2)}^{-1}n_{(2d_s+2)}\},$$
$$j \in \{0,1,\cdots,2d_s+2\}.$$

and $p_{ij} \in P_t \bigotimes P$, signal $\bigotimes$ denotes the matrices Kronecker product.

Proof As to a switched control system which has two groups of controllers, any of these two groups of controllers has d_s+1 jump states. So this switched control system has $2(d_s+1)$ jump states. The switched rules of this system is

$$P_t = \begin{bmatrix} pt_{11} & pt_{12} \\ pt_{21} & pt_{22} \end{bmatrix} = \begin{bmatrix} pt_{11} & 1-pt_{11} \\ pt_{11} & 1-pt_{11} \end{bmatrix}$$

where all elements of matrix P_t are constant within [0,1]. pt_{11} expresses the jump probability from first group jump states to first group jump states at sampling time, pt_{21} expresses the jump probability from second group jump states to second group jump states, pt_{12}, pt_{21} and express the jump probability from first group jump states to second group jump states and the jump probability from second group jump states to first group jump states respectively.

If transition probability matrix of NCSs is denoted by P, the general switched rules of the whole switched control system is described as follows

$$PG = P_t \bigotimes P = \begin{bmatrix} pt_{11}\times P & (1-pt_{11})\times P \\ pt_{11}\times P & (1-pt_{11})\times P \end{bmatrix}$$

where $PG \in \Re^{2(d_s+1)\times 2(d_s+1)}$ and sums of every row equals to 1.

The matrix PG expresses the jump probability between any two states among $2(d_s+1)$ states at sampling time and this theorem can be proved through theorem 2 and Schur complement.

V. RELIABLE ANALYSIS OF NCSs AND DESIGN OF TOLERABLE FAULT CONTROLLERS

A. Reliable analysis of NCSs

Applying the model building in section IV, we obtain steps of reliable analysis of NCSs as follows:

1) Assuming transition probability matrix P is given, request delay-dependent and delay-independent controllers which make system asymptotic mean square stability using theorem 2.

2) Construction the first group controllers using delay-independent controllers solved in step 1) and setting gains K of the second testing controllers equal to zero. we can simulate actuator damage with varying damage degree through selecting different switched probability P_t and then using theorem 3, the maximum redundancy of controllers requested in step 1) which make system asymptotic mean square stability is obtained.

3) Construction the first group controllers using delay-dependent controllers solved in step 1), redundancy degree of these controllers under the condition of part actuators of NCSs are stochastic inactivation is obtain through setting all gains K of testing controllers equal to zero.

B. Design of Tolerable Fault Controllers

1) Let all gains of second group controllers are equation to zero, that is setting gains of testing controllers $K_{20} = K_{21} = \cdots = K_{2d_s} = 0$, and delay-dependent or delay-independent controllers obtained in 1) of V.A subsection as first group controllers. Set initial switched probability value pt_{11} (for simplicity, we substitute pt_{11} with pt). In common, let $pt = 0.9$.

2) Applying theorem 3 and improved V-K iteration algorithm, request controllers $K_{10}, K_{11}, \cdots, K_{1d_s}$ satisfying asymptotic mean square stability of systems when fault probability of actuators is $(1-pt)\%$.

3) Analysis redundant degree of controllers solved in step 2), using reliable analysis method mentioned in section IV .

4) Let controllers solved in step 2) as first group controllers, and adjust switched probability pt, that is $pt = pt - \triangle pt$. Generally, let $\triangle pt = 0.1$; Let all gains of testing controllers equals to zero, and then back to step 2), repeat the cycle from step 2) to step 4), until the redundant degree of system does not increase any more or controllers do not existed. Though this method, a group of controllers with the highest redundant degree is obtained.

Remark 1: If we choose time-dependent controllers as controllers of jump linear system, then we can design a group of tolerable fault controllers which can make system mean square stability when partial controllers of a group of controllers damage entirely through set partial testing controllers' gains and corresponding controllers' gains in first group controllers to 0.

VI. NUMERICAL EXAMPLES

Example 1: Consider a continuous linear model in NCSs as follow:

$$\dot{x}(t) = \begin{bmatrix} -1.4605 & 3.6802 \\ 2.7613 & -6.9299 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \qquad (16)$$

If we set sampling period to $T = 0.1$ second, then discrete model of this model is

$$x(k+1) = \begin{bmatrix} 0.9013 & 0.2491 \\ 0.1869 & 0.5311 \end{bmatrix} x(k) + \begin{bmatrix} 0.0142 \\ 0.0733 \end{bmatrix} u(k) \qquad (17)$$

Let

$$F(k) = F_1(k) = \begin{bmatrix} \sin k & 0 \\ 0 & \cos k \end{bmatrix}, D = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$
$$E = \begin{bmatrix} 0 & 0.1 \\ 0 & 0 \end{bmatrix}, D_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, E_1 = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}.$$

and symmetric positive definite matrices of cost function are $T = I, R = 1$, then we design a LQR controller for the jump system. That is

$$K_{ini} = \begin{bmatrix} -2.9855 & -1.5997 \end{bmatrix}$$

when state transition probability matrix which describe network stochastic delay governed by Markov is

$$P = \begin{bmatrix} 0.5 & 0.5 & 0 \\ 0.3 & 0.6 & 0.1 \\ 0.3 & 0.6 & 0.1 \end{bmatrix},$$

we can get mode-independent and mode-dependent controllers satisfied asymptotic mean square stability of system respectively. They are

$$K = \begin{bmatrix} -2.0039 & 0.9584 \end{bmatrix}$$

and

$$K_{10} = \begin{bmatrix} -18.3289 & -9.7090 \end{bmatrix}$$
$$K_{11} = \begin{bmatrix} 0.0049 & -0.0008 \end{bmatrix}$$
$$K_{12} = \begin{bmatrix} 0.6495 & 0.3295 \end{bmatrix}$$

Consider mode-independent controllers, using reliable analysis method mentioned in subsection V. A we know that system can not satisfy asymptotic mean square stability if $pt = 0.49$, that is failure probability of actuators are 51%. If we stabilize NCSs using mode-dependent controllers, system can not satisfy mean square stability when $pt = 0.34$. That is to say, failure probability of three actuators are $K_{10} = 26.40\%, K_{11} = 36.30\%, K_{13} = 3.30\%$ respectively.

If we set switched probability $pt = 0.32$, we get mode-independent controllers with fault tolerant as follows:

$$K = \begin{bmatrix} -5.5468 & -1.9975 \end{bmatrix}$$

Through reliable analysis in subsection V. A we know, system does not satisfy asymptotic mean square stability if we set switched probability $pt = 0.21$. That is to say, redundancy degree of NCSs is 79%. Under the same switched probability, we obtain mode-dependent controllers with fault tolerant

$$K_{10} = \begin{bmatrix} -11.3736 & -6.2142 \end{bmatrix}$$
$$K_{11} = \begin{bmatrix} -2.5513 & -1.3690 \end{bmatrix}$$
$$K_{12} = \begin{bmatrix} -9.3596 & -5.9244 \end{bmatrix}$$

Through reliable analysis method mentioned we know that NCSs do not satisfy mean square stability if we set switched probability $pt = 0.21$. So redundancy degree satisfied asymptotic mean square stability of NCSs is $K_{10} = 29.63\%, K_{11} = 44.44\%, K_{13} = 4.94\%$. However, if we set switched probability $pt = 0.21$, we can not find mode-dependent or mode-independent controllers which stabilize NCSs respectively. So we think the maximal redundancy degree of NCSs are $K_{10} = 29.63\%, K_{11} = 44.44\%, K_{13} = 4.94\%$ and 79%.

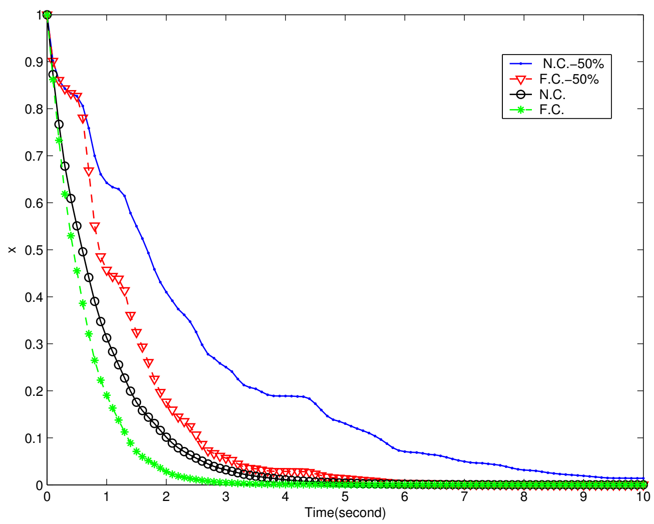

Fig. 3. Results of simulation for mode-independent controller and mode-independent failure tolerant controller

In Fig. 3, N.C. and F.C. denote controlled curves under normal controller and under tolerable fault controller designed at switched probability $pt = 0.50$ respectively, when NCSs have not actuators fault; N.C.-50% and F.C.-50% indicate controlled curves under normal controller and under tolerable fault controller at switched probability $pt = 0.50$ respectively, when there exists 50% actuators fault in NCSs. It is easily shown that constringency speed of NCSs with tolerable fault controller is faster than normal controller in the same condition. When there has 50% actuator fault probability in NCSs, constringency speed of systems with tolerable fault controller is not only faster than that with normal controller but almost the same as the systems with normal controller under the condition that actuators fault probability of NCSs equals to zero. It is shown that tolerant fault controllers has more robust performances compared with non-fault tolerant controllers.

VII. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

This paper presents a problem of stabilization about uncertain Networked Control Systems (NCSs) with random but bounded delays. A new switched model switching based on probability is proposed to research problems of reliable control. Using improved V-K iteration algorithm, a class of reliable controllers is successfully designed to make system asymptotically mean square stable under a serials of stochastic disturbance such as random time-delay and stochastic actuator failures and have the maximal redundancy degree. An example is included to demonstrate the effectiveness of the approach.

B. Future Works

Future work focus on how to make this method more effective to find the maximal redundancy degree and simpler to use.

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