2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

Road Slope and Vehicle Dynamics Estimation

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Abstract— **Driving safely can be acheived by the prevention of risky situations which may require the knowledge of vehicle dynamic state as well as road geometry. It is thus essential to have in real-time a good estimation of the related variables and parameters. Among the parameters of the road that are influencing vehicle longitudinal motion one can find the slope which can not be measured with reduced cost sensor. Vehicle lateral motion is mainly affected by the value of the lateral speed which can not simply measured too. In this paper, an observer based method for the estimation of the vehicle dynamics using a nonlinear vehicle model is proposed. It uses a combination of Extended Kalman filter (EKF) and Luenberger Observer (LO). The observers use several standard measurements such as : the yaw rate, the steering angle and the rotational velocity of the four tires. Experimental tests conducted with a prototype vehicle prove the effectiveness of the proposed approach.**

I. INTRODUCTION

Analysis of road accidents statistics in France [11] show that the number of people killed decreases for the last four years. This is the consequence of several factors : the new driving regulations introduced by the government, the improvement of the infrastructure and the generalization of driver assistance systems (ABS, ESP,...). These systems need the knowledge of the vehicle dynamics and can make advantage of knowing road characteristics such as curvature, slope and bank. Due to technical infeasibility and high sensor cost, these characteristics are sometimes non measurable in real time. It is then of primary importance to develop virtual sensors by the mean of observers which are able to estimate them using only vehicle sensors available in standard.

Several work have been already made to reconstruct the vehicle dynamic state as well as other road characteristics [1], [5], [7], [13]. Different approaches have been developed using Kalman Filter [3], [10], Luenberger observer [13], proportional integral observer [14] and sliding mode observer [13]. In this paper an approach which combines an extended Kalman filter for the estimation of the vehicle dynamics and a Luenberger observer for the estimation of the road slope is proposed. Tire forces are modeled using a Dugoff model.

The paper is organized as follows : Before observer development, it is necessary to choose a representative mathematical model of the vehicle which achieves a good compromise between complexity and accuracy of model results. This model is presented in section 2 where a nonlinear four tires is considered [5], [9]. Section 3 is dedicated to the vehicle dynamics estimation scheme using an Extended Kalman filter (EKF), while the road slope Luenberger Observer (LO) is developed in section 4. Section 5 provides some testing results using a prototype vehicle.

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II. VEHICLE MODEL

Model based estimation methods make use of dynamic model of the system. It is thus essential to establish a dynamic model of the vehicle which both simple and realistic in agreement with the projected study. Among the various existing models in the literature [5], [14], and [9], a nonlinear model [3] with four wheels (coupled model very largely used in simulation) is selected. This model is only used to estimate the longitudinal and lateral vehicle dynamic. Thereafter a simple longitudinal model is developed, it will be used for the reconstruction of the road slope. Vehicle behavior is affected by external forces acting on the vehicle body and road-tire interface. These forces are presented in Fig. 1 and 2, in a vertical and lateral views on road with non zero slope angle.

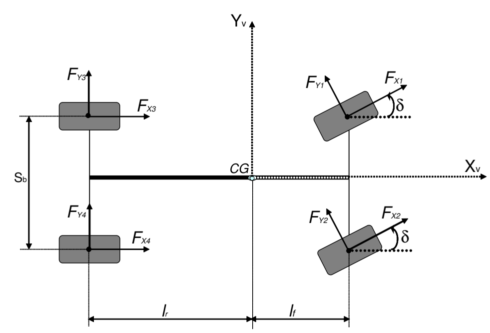

Fig. 1. Vehicle and external forces

The equations of the planar motion of the nonlinear four tires vehicle model are given by the system of three differential equations (1) :

$$\begin{cases} M\dot{V}_x = \sum_{i=1}^{4} F_{X_i} + M\dot{\psi}V_y - C_x V_x^2 - Mg\sin\theta_r \\ M\dot{V}_y = \sum_{i=1}^{4} F_{Y_i} - M\dot{\psi}V_x - C_y V_y^2 \\ I_z\ddot{\psi} = \sum_{i=1}^{4} M_{Z_i} \end{cases} \quad (1)$$

The first differential equation represents the longitudinal motion, the second represents the lateral motion while the

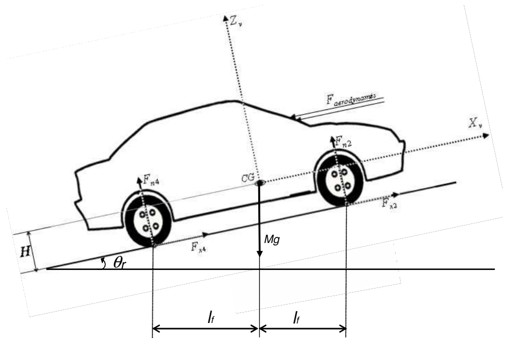

Fig. 2. Vehicle and external forces in slope

third one corresponds to the yaw motion. M is the mass vehicle, θ_r is the road slope, g is the gravitational acceleration and C_x is the aerodynamic resistances coefficient. F_{x_i}, F_{y_i} and M_{z_i} represent respectively the longitudinal forces, lateral forces and the moment around the vertical Z axes at each tire. These quantities are given by the following formulas :

$$\sum_{i=1}^{4} F_{X_i} = (F_{x1}+F_{x2})\cos\delta-(F_{y1}+F_{y2})\sin\delta + (F_{x3}+F_{x4})\cos\beta+(F_{y3}+F_{y4})\sin\beta \quad (2)$$

$$\sum_{i=1}^{4} F_{Y_i} = (F_{x1}+F_{x2})\sin\delta+(F_{y1}+F_{y2})\cos\delta - (F_{x3}+F_{x4})\sin\beta+(F_{y3}+F_{y4})\cos\beta \quad (3)$$

$$\sum_{i=1}^{4} M_{Z_i} = l_f(F_{x1}+F_{x2})\sin\delta+l_f(F_{y1}+F_{y2})\cos\delta + \frac{S_b}{2}(F_{x2}-F_{x1})\cos\delta+\frac{S_b}{2}(F_{x4}-F_{x3}) - \frac{S_b}{2}(F_{y2}-F_{y1})\sin\delta-l_r(F_{y3}+F_{y4}) \quad (4)$$

The nonlinear model can be transformed into standard state-space form with state vector composed of longitudinal speed, lateral speed and yaw rate.

$$x = [V_x \quad V_y \quad \dot{\psi}]$$

The model input is constituted by the steering angle and the four tires rotational velocities.

$$u = [\delta \quad \omega_1 \quad \omega_2 \quad \omega_3 \quad \omega_4]$$

System measurements are actually the longitudinal speed and the yaw rate. the vehicle velocity is considered as the mean of the two translational velocity of the two rear wheels which are the less subjected to the slips.

Longitudinal and lateral forces acting at each tire-road interface are computed using the Dugoff model [6]. This model offers a simple and effective coupling between the longitudinal and lateral forces. The forces are given by :

$$\begin{cases} F_{xi} = C_{xxi}\frac{\lambda_i}{1-\lambda_i}k_i \\ F_{yi} = C_{yyi}\frac{\tan\alpha_i}{1-\lambda_i}k_i \end{cases} \quad (5)$$

with

$$k_i = \begin{cases} (2-\sigma_i)\sigma_i & \text{if} \quad \sigma_i < 1 \\ 1 & \text{if} \quad \sigma_i \geq 1 \end{cases} \quad (6)$$

and

$$\sigma_i = \frac{(1-\lambda_i)\mu_i F_{ni}}{2\sqrt{C_{x0}^2\lambda_i^2+C_{y0}^2\tan^2\alpha_i}} \quad (7)$$

C_{x_0} and C_{y_0} are respectively the longitudinal and lateral stiffness, and μ_i is the friction coefficient for each tire. it is recommended to see ref.[1] to well understand the estimation method of these adhesion variables. λ_i and α_i are respectively longitudinal slip and the sideslip angle of each tire [3] :

$$\begin{cases} \lambda_i = \frac{R\omega_i - V_{xi}}{max(R\omega_i, V_{xi})} \\ \alpha_i = \delta_i - \arctan(\frac{V_{yi}}{V_{xi}}) \end{cases} \quad (8)$$

R is the wheel radius.

The presented model takes also into account load transfer between tires by updating normal forces acting on each of them as a function of road slop, longitudinal and lateral acceleration. The following formulas are used in the sequel :

$$\begin{cases} F_{n1} = \frac{l_r Mg\cos\theta_r}{2(l_r+l_f)} - \frac{HMa_x}{2(l_r+l_f)} - \frac{l_r HMa_y}{S_b(l_r+l_f)} \\ F_{n2} = \frac{l_r Mg\cos\theta_r}{2(l_r+l_f)} - \frac{HMa_x}{2(l_r+l_f)} + \frac{l_r HMa_y}{S_b(l_r+l_f)} \\ F_{n3} = \frac{l_f Mg\cos\theta_r}{2(l_r+l_f)} + \frac{HMa_x}{2(l_r+l_f)} - \frac{l_f HMa_y}{S_b(l_r+l_f)} \\ F_{n4} = \frac{l_f Mg\cos\theta_r}{2(l_r+l_f)} + \frac{HMa_x}{2(l_r+l_f)} + \frac{l_f HMa_y}{S_b(l_r+l_f)} \end{cases} \quad (9)$$

a_x and a_y are respectively longitudinal and lateral vehicle acceleration.

III. EXTENDED KALMAN FILTER

Extended Kalman Filter [8] is an extension of the standard Kalman filter for linear systems. It is also designed as an optimal filter which minimizes the estimation error variance for a linearized model of the nonlinear system around state estimate $\hat{x}$.

Let us consider the following nonlinear discrete-time system :

$$\begin{cases} x_{k+1} = f(x_k, u_k, w_k) \\ Z_k = h(x_k, v_k) \end{cases} \quad (10)$$

where w_k and v_k represent the state and measurement noise vectors respectively.

The EKF algorithm is a recursive process which operates in three steps : a linearization step, a prediction step and an update step. First, one have to proceed to the linearization of the output equation around the forecast. The prediction stage consists in the propagation of both the state estimate and the state estimation error covariance between two sampling instants. Denoting x_{k+1}^- and P_{k+1}^- the state estimation and state error covariance prediction, one can achieve :

$$
\left\{
\begin{array}{lcl}
x_{k+1}^{-} & = & f(\hat{x}_k, u_k, 0) \\
P_{k+1}^{-} & = & A_k P_k A_k^T + Q_k
\end{array}
\right. \quad (11)
$$

where Q_k is the covariance of state noise, A_k is the Jacobian of $f(x_k, u_k, w_k)$.
The update stage occurs at each sampling time, and consists in a corrective stage when the new measurement becomes available. correcting against the measurement, This estimation state make use also of the predicted state and estimation error variance

$$
\left\{
\begin{array}{lcl}
x_{k+1} & = & x_{k+1}^{-} + K_{k+1}(Z_{k+1} - h(\hat{x}_{k+1}^{-})) \\
P_{k+1} & = & (I - K_{k+1} H_k) P_{k+1}^{-}
\end{array}
\right.
$$

where K_k represents the Kalman gain that minimizes the estimation error :

$$
K_{k+1} = P_{k+1} H_k^T (H_k P -_{k+1} H_k^T + R_k)^{-1}
$$

The matrix R_k represents the covariance of the measurement noise, H_k is the Jacobians of measurements.

IV. ROAD SLOPE AND VEHICLE STATE ESTIMATION

A. Vehicle state estimation

The schematic simulation block diagram is represented in Fig. 3. Necessary measurements are the steering angle obtained by an optical coder, rotational speeds of the four tires taken from the ABS system, lateral and longitudinal accelerations, as well as the yaw rate measured by an inertial unit.

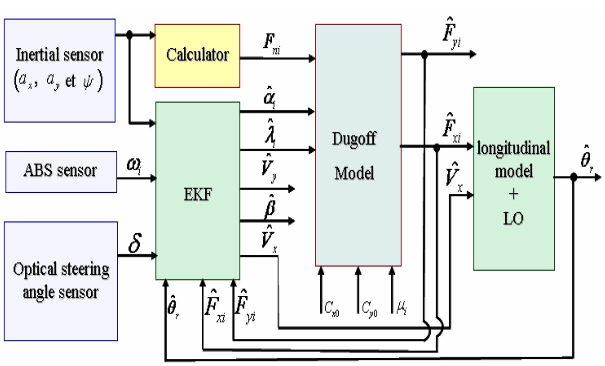

Fig. 3. Simulation block diagram

The EKF in combination with the nonlinear vehicle model represented by system (12) are used for the estimation of the vehicle state. The sampling time is ΔT.

$$
\left\{
\begin{array}{lcl}
\hat{V}_x(k+1) & = & \hat{V}_x(k) + \frac{\Delta T}{M}\Big(\sum_{i=1}^{4} \hat{F}_{X_i}(k) + M\hat{\dot{\psi}}(k)\hat{V}_y(k) \\
 & & -C_x \hat{V}_x^2(k) - Mg\sin\hat{\theta}_r(k)\Big) \\
\hat{V}_y(k+1) & = & \hat{V}_y(k) + \frac{\Delta T}{M}\Big(\sum_{i=1}^{4} \hat{F}_{Y_i}(k) - M\hat{\dot{\psi}}(k)\hat{V}_x(k) \\
 & & -C_y \hat{V}_y^2(k)\Big) \\
\hat{\dot{\psi}}(k+1) & = & \hat{\dot{\psi}}(k) + \frac{\Delta T}{M}\Big(\sum_{i=1}^{4} \hat{M}_{Z_i}(k)\Big)
\end{array}
\right. \quad (12)
$$

B. Road slope estimation

The estimation of the road slope is achieved on the basis a longitudinal model given by equation (13). The vehicle state estimate obtained with the EKF allows the calculation of the longitudinal force. This force can thus be used as an input for the longitudinal model.

$$
\left\{
\begin{array}{lcl}
\dot{V}_x & = & \frac{\hat{F}_{res}}{M} - g\sin\theta_r \\
\dot{\theta}_r & = & 0
\end{array}
\right. \quad (13)
$$

With

$$
\hat{F}_{res} = \sum_{i=1}^{4} \hat{F}_{X_i} + M\dot{\psi}\hat{V}_y - C_x \hat{V}_x^{\,2}
$$

We considered that the slope angle variation is zero, ie the road does not present bumps. Generally, the road slope is less than ten degrees, so one can approximate $\sin\theta_r$ by θ_r. Thus, the system (13) can be written in the following linear form :

$$
\dot{\underline{x}} = \underline{A}\ \underline{x} + \underline{B}\ \underline{u}
$$

where, $\underline{x} = [V_x\ \theta_r]^T$ is the state vector, $\underline{u} = \hat{F}_{res}$ the control input , $\underline{C} = [1 \quad 0]$ the matrix output, while matrices $\underline{A}$ and $\underline{B}$ have the following forms :

$$
\underline{A} = \begin{bmatrix} 0 & -g \\ 0 & 0 \end{bmatrix}, \quad \underline{B} = \begin{bmatrix} 1/M \\ 0 \end{bmatrix}
$$

The system is obviously observable, one can easily verify that the rank of the observability matrix is two. In order to estimate the road slope angle, a simple Luenberger observer [4] is chosen, it is given by :

$$
\dot{\hat{x}} = \underline{A}\ x + \underline{B}\ u + L(y - C\ \hat{x})
$$

with x is the state estimate and L is the observer gain chosen such that the dynamics of the observation error given by $\dot{\tilde{x}} = (A - L\ C)\tilde{x}$, with $\tilde{x} = x - \hat{x}$ are stable. This gain is calculated in such a way that the eigenvalues of $(A - L\ C)$ have negative real parts while being faster that the system model.

V. ESTIMATION AND VALIDATION RESULTS

A. Estimation results

In this section, simulation test results obtained for the estimation of the vehicle state, the sideslip angle and the road slope are presented. The vehicle steering angle δ input is shown in Fig. 4-a, the slope appears in Fig. 9-a, the four wheel rotational speed in Fig. 4-b to Fig. 4-e. The available friction coefficient μ_i is equal to 0.8. The vehicle speed is considered as the mean of the two translational speed of the two rear wheels, this speed is fixed at 25 m/s equal to wR, for the simulation part.

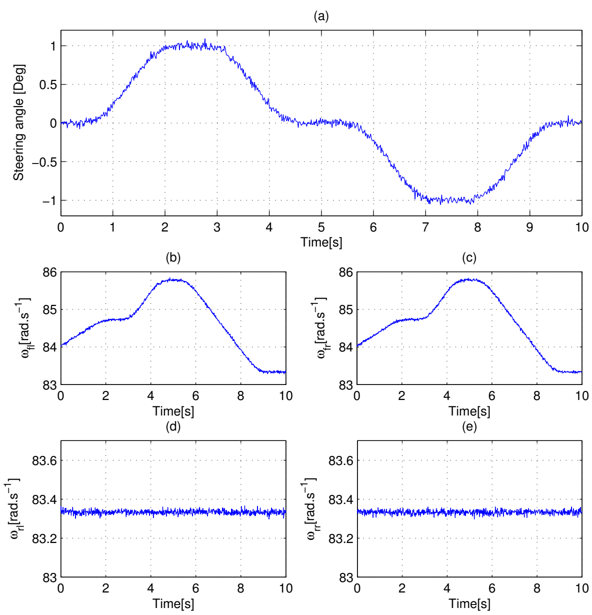

Fig. 4. Steering angle and four wheels rotational speed

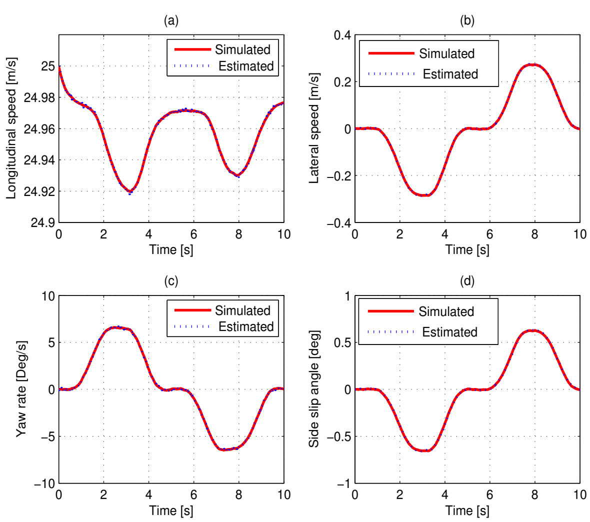

Fig. 5. Estimated and simulated states

Figure (5) shows the estimation results of the longitudinal speed (a), lateral speed (b), yaw rate (c) and the sideslip angle (d). One can conclude that these parameters are well estimated. The estimation errors are presented in figure (6). The state vector estimation error is very small, this enables us to conclude that the estimation results are good.

The estimation results for the longitudinal and the lateral forces applied on each wheel are shown in figures (7) and (8). We can find that the variation of the longitudinal forces

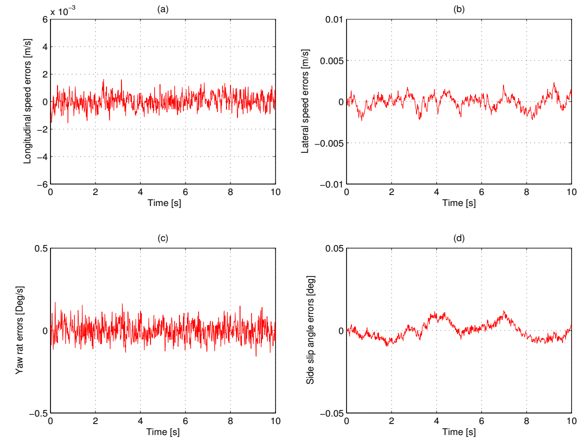

Fig. 6. States estimation errors

for a constant speed is proportional to the road slope, which is logic.

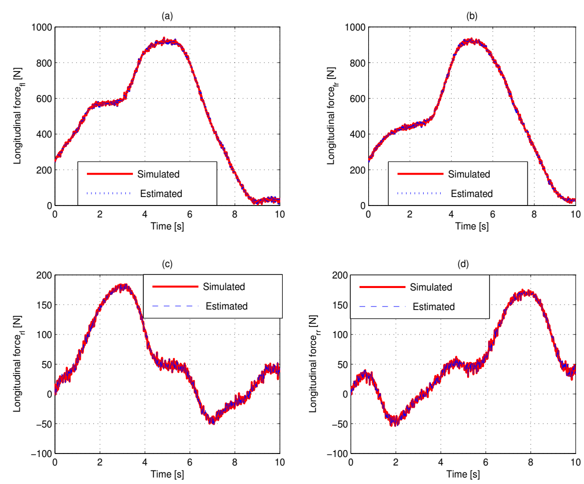

Fig. 7. Estimated and simulated longitudinal forces

The estimation results of the road slope obtained by the use of the Luenberger observer are presented in Fig. 9-a. The estimation error are given in Fig. 9-b, these errors confirm that the road slope is well reconstructed. All the figure confirm the adequate behavior of the model and the observers.

B. Tests with real vehicle measurements

Vehicle and road slope model, as well as the EKF and OL are validated by a set of measurements collected from the Satory test track located in Versailles (France). The used prototype vehicle is equipped with an optical coder sensor for steering angle measurement, an inertial sensor and an ABS system which used to measure the wheels rotational speeds necessary for the estimation. The vehicle is also equipped with a high cost lateral speed sensor "Correvit" which is used a reference sensor for the lateral speed estimation validation.

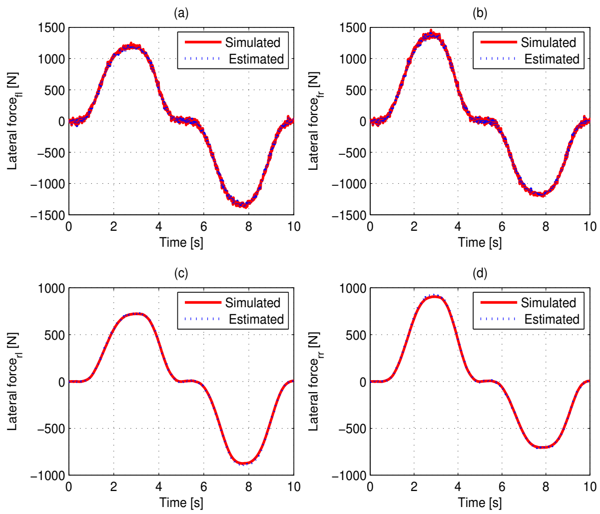

Fig. 8. Estimated and simulated lateral forces

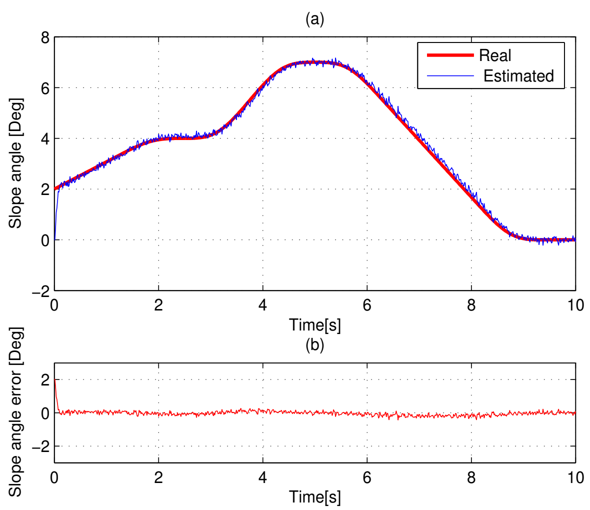

Fig. 9. Estimated and simulated road slope angle and slope error

Figure (10) represents the steering angle (a), and measured and estimated longitudinal speed (b), lateral speed (c) and yaw rate (d). These plots show a good estimation.
The estimation of longitudinal tire/road forces F_{xi} are represented in figure (11).

To validate the model as well as the EKF a comparison between the measured, simulated and estimated longitudinal speed, lateral speed and yaw rate. The results of this comparison are given in Fig. 12 and Fig. 13. To validate the Luenberger observer, we compared the estimated longitudinal speed to the measured one, this appears in Fig. 14.

Finally, we present the curve of the road slope of the test track in figure (15). This slope has been measured by a geometer. All the figure confirm the adequate behavior of the model and the observers. The different plots are very close to each others.

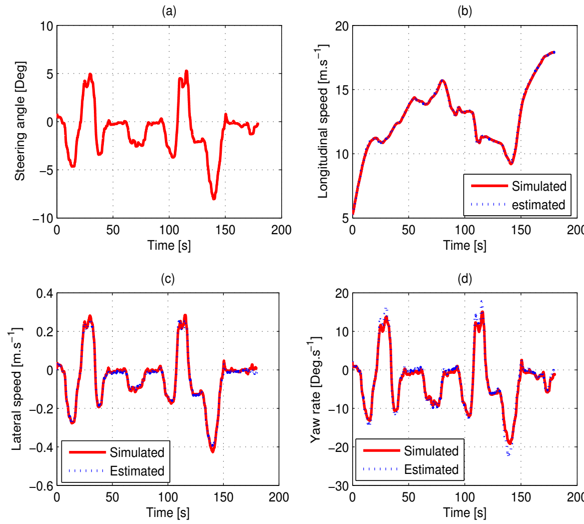

Fig. 10. Steering angle and estimated states

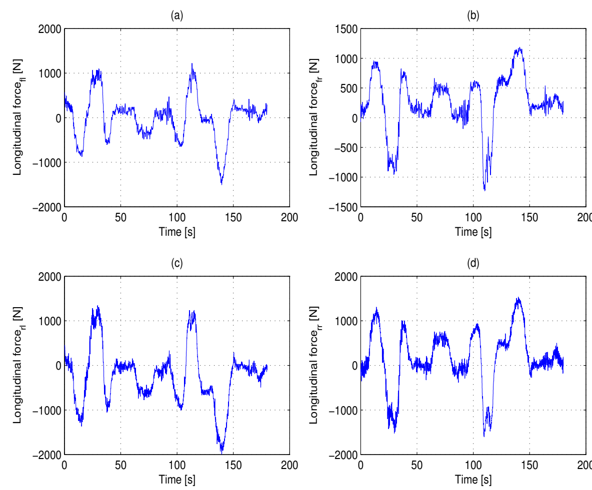

Fig. 11. Longitudinal forces estimation

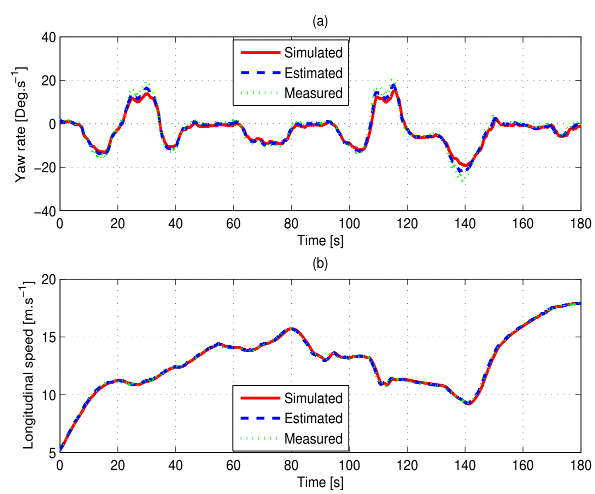

Fig. 12. Simulated, estimated and measured longitudinal speed and yaw rate using EKF

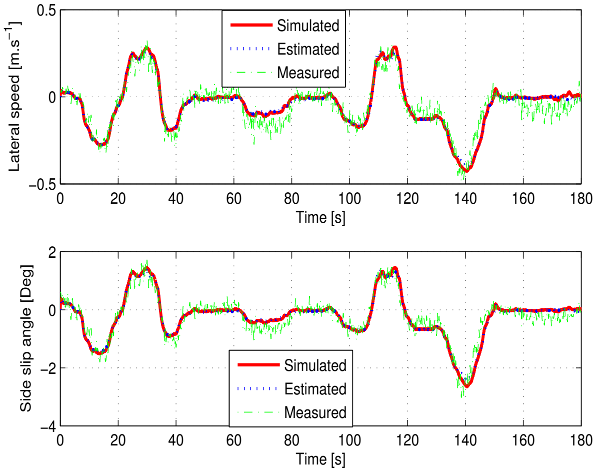

Fig. 13. Simulated, estimated and measured lateral speed and sideslip angle

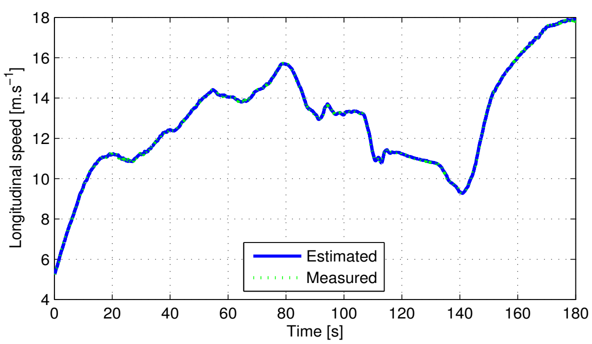

Fig. 14. Estimated and measured longitudinal speed using LO

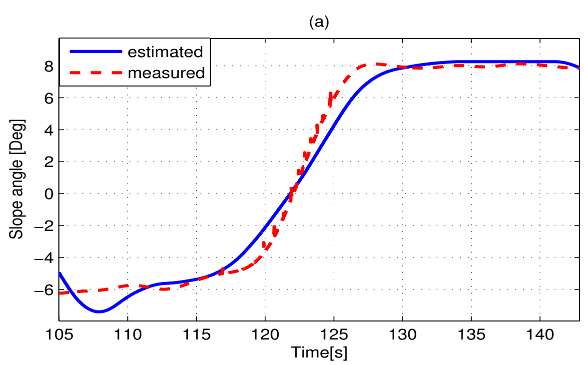

Fig. 15. Road slope estimation

VI. Conclusion

In this paper, a method for the estimation of vehicle state by an EKF, and to reconstitute the road slope using a Luenberger observer is presented. The estimation results have been compared to real measurements collected with a prototype vehicle runing on a test track. The comparisons of measures and estimation results show that the estimated state follows very well the measured one, and the road slope is well reconstructed. The convergence time of the two observers is very short.

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