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Adaptive NN Control for a Class of Strict-feedback Nonlinear Systems

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***Abstract*— An adaptive neural network control (ANNC) is proposed for a class of strict-feedback uncertain nonlinear systems with both unknown system nonlinearities and unknown virtual control gain nonlinearities. The continuous function separation technique and RBF neural network are introduced to model system nonlinearities. A systematic procedure for synthesis of ANNC is developed by combining the backstepping technique and Lyapunov stability theory. An important feature of the proposed algorithm is that the order of dynamic compensator of ANNC is only identical to the order n of controlled system, such that it can reduce the computation load and makes particularly suitable for parallel processing in actual implementation. In addition, the resulted closed-loop system is proven to be semi-global uniform ultimate bound and the possible controller singularity problem can be removed. Finally, numerical simulation example are presented to illustrate the tracking performance of the proposed algorithm.**

***Index Terms*— Uncertain nonlinear systems, neural networks, adaptive control, backstepping technique.**

I. INTRODUCTION

IN the past decades, the adaptive control of nonlinear systems with linearly parameterized uncertainty has achieved significant progress (see [1]∼[3] and references therein). For systems with high uncertainty, which cannot be modelled or repeatable, adaptive control approach obtained further development by means of neural network (NN) control schemes(e.g., [4]∼[7]) or fuzzy control schemes(e.g., [8]∼[10]) based on the idea of backstepping.

However, there is a substantial "dimension curse" restriction in the aforementioned works, that is, the number of hidden units becomes prohibitively large as we move to high dimensional systems, which imposes that there are many parameters need to be tuned in the approximator-based adaptive control schemes, such that the learning times tend to become unacceptably large for the systems of higher order and time-consuming process is unavoidable when the approximator-based adaptive controllers are implemented. This drawback restricts the applicability of the methods. This problem has been first pointed out and researched in [11] and [12], and further discussed in [13]∼[17] when using adaptive fuzzy control schemes or NN control schemes.

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In this paper, motivated by the pioneering works proposed by Professor Yang in [11]∼[17], with respect to a class of strict-feedback nonlinear systems in the presence of unstructured nonlinearities and unknown virtual control gain nonlinearities, a systematic procedure is developed for the synthesis of the stable adaptive NN tracking controller.

II. NOTATION AND PRELIMINARIES

A. RBF Neural Network

In control engineering, RBF neural networks are usually used as a tool for modelling nonlinear functions because of their good capabilities in function approximation. They belong to a class of linearly parameterized networks. For comprehensive treatment of neural networks approximation, see [4]. RBF neural networks can be described as $w^T S(z)$ with input vector $z \in R^n$, weight vector $w \in R^l$, node number l, and basis function vector $S(z) \in R^l$. Universal approximation resluts indicate that, if l is chosen sufficiently large, then $w^T S(z)$ can approximate any continuous function to any desired accuracy over a compact set. In this paper, we use the following RBF neural networks to approximate a smooth function $h(z): R^q \to R$

$$h_{nn}(z) = w^T S(z) \tag{1}$$

where the input vector $z \in \Omega \subset R^n$, weight vector $w = [w_1, w_2, \ldots, w_l]^T \in R^l$, the neural network node number $l > 1$, and $S(z) = [s_1(z), s_2(z), \ldots, s_l(z)]^T$, with $s_i(z)$ being chosen as the commonly used Gaussian functions, which have the form

$$s_i(z) = \exp\left[\frac{-(z-\mu_i)^T(z-\mu_i)}{\eta_i^2}\right], i = 1, 2, \ldots, l$$

where $\mu_i = [\mu_{i1}, \mu_{i2}, \ldots, \mu_{in}]^T$ is the center of the receptive field and η_i is the width of the Gaussian function.

For the unknown nonlinear function $f(x)$, we have the following approximation over the compact sets Ω

$$f(x) = w^{*T} S(x) + \varepsilon. \quad \forall x \in \Omega \subseteq R^n \tag{2}$$

where $S(x)$ is the basis function vector, ε is the approximation error, and w^* is an unknown ideal constant weight vector.

The ideal weight vector w^* in (2) is an "artificial" quantity required only for analytical purposes. Typically, w^* is chosen as the value of w that minimizes $|\ \varepsilon\ |$ for all $x \in \Omega$, where $\Omega \subseteq R^n$ is a compact set, i.e.,

$$w^* := \arg\min_{w \in R^n} \left\{ \sup_{x \in \Omega} |\ f(x) - w^T S(x)\ | \right\}.$$

We make the following assumption on the approximation error.

Assumption 1: Over a compact region $\Omega \in R^n$

$$|\varepsilon| \leq \varepsilon^*$$

where $\varepsilon^* > 0$ is an unknown bound.

The following lemma provides a new description for the continuous function by use of combination of continuous function separation technique and RBF NN approximation, which enables one to deal with nonlinear parameterization and will result in a solution to the robust adaptive NN control problem of nonlinear parameterized systems.

Lemma 1: [17] For any given real continuous function $f(x,\theta)$ with $f(0,\theta) = 0$, when the continuous function separation technique and RBF NN approximation technique are used, then $f(x,\theta)$ can be denoted as follows

$$f(x,\theta) = \bar{S}(x)Ax \tag{3}$$

where $\bar{S}(x) = [1, S(x)] = [1, s_1(x), s_2(x), \cdots, s_l(x)]$, $s_i(x), i = 1,2,\cdots,l$ are the RBF basis functions which are known and l is the node number. $A^T = [\varepsilon, W^T]$, $\varepsilon^T = [\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n]$ is a vector of the approximation error and $W = \begin{bmatrix} w_{11}^* & w_{12}^* & \cdots & w_{1n}^* \\ w_{21}^* & w_{22}^* & \cdots & w_{2n}^* \\ \vdots & \vdots & \cdots & \vdots \\ w_{l1}^* & w_{l2}^* & \cdots & w_{ln}^* \end{bmatrix}$ is a weight matrix.

III. PROBLEM FORMULATION

Consider an uncertain nonlinear dynamic system in the following form

$$\begin{cases} \dot{x}_i = g_i(\bar{x}_i,\theta)x_{i+1} + f_i(\bar{x}_i,\theta), & 1 \leq i \leq n-1 \\ \dot{x}_n = f_n(x,\theta) + g_n(x,\theta)u \\ y = x_1 \end{cases} \tag{4}$$

where $x = [x_1, x_2, \cdots, x_n]^T \in R^n$ is the system state, $u \in R$ is the control input, $y \in R$ is the output of system and $\theta \in \Theta \subset R^q$ is an q-dimension of parameter uncertain vector, where Θ is a compact set. Let $\bar{x}_i = [x_1, x_2, \cdots, x_i]^T$. $f_i(\bar{x}_i,\theta)$'s are unknown smooth system functions with $f_i(0,\theta) = 0$ and $g_i(\bar{x}_i,\theta)$'s are unknown smooth functions which are referred to as the virtual control gain ones, all of which are continuous functions depending on the state x

For system (4), the following assumptions are introduced.

Assumption 2: The uncertain virtual control gain functions $g_i(\bar{x}_i,\theta)$, $i = 1,2,\ldots,n$ are confined within a certain range such that

$$0 < b_{\min} \leq |g_i(\bar{x}_i,\theta)| \leq b_{\max} \tag{5}$$

where $b_{\min}$ and $b_{\max}$ are the lower and upper bound parameters respectively.

The above assumption implies that the smooth virtual control gain functions $g_i(\bar{x}_i,\theta), i = 1,2,\ldots,n$ are strictly either positive or negative. From now on, without loss of generality, we shall assume $g_i(\bar{x}_i,\theta) \geq b_{\min} > 0, i = 1,2,\ldots,n, \forall (x,\theta) \in R^n \times \Theta$. Assumption 2 is reasonable because $g_i(\bar{x}_i,\theta)$ being away from zero is the controllable conditions of system (4).

The primary goal of this paper is to track a given reference signal $y_d(t)$ while keeping the states and control bounded. That is, the output tracking error $z_1 = y(t) - y_d(t)$ should be small. The given reference signal $y_d(t)$ is assumed to be available together with its n time derivatives, and that $y_d^{(n)}(t)$ is piecewise continuous. Moreover, the vector $\bar{x}_{d(i+1)} = [y_d, y_d^{(1)}, \ldots, y_d^{(i)}]^T$ is bounded, i.e., for some $\kappa > 0$, $\| \bar{x}_{d(i+1)} \| < \kappa, i = 1,2,\cdots,n$.

IV. DESIGN OF ROBUST ADAPTIVE NN CONTROL

A. Control Design Procedure

We give the proceeding of the backstepping design as follows.

Step 1. Define the error variable $z_1 = x_1 - y_d$, then

$$\dot{z}_1 = g_1(\bar{x}_1,\theta)x_2 + f_1(x_1,\theta) - \dot{y}_d \tag{6}$$

Since $f_1(x_1,\theta)$ is an unknown continuous function with $f_1(0,\theta) = 0$, according to Lemma 1, $f_1(x_1,w)$ can be expressed as

$$\begin{aligned} f_1(x_1,\theta) =& \bar{S}_1(x_1)A_1x_1 \\ =& \bar{S}_1(x_1)A_1z_1 - \bar{S}_1(x_1)A_1y_d \end{aligned} \tag{7}$$

Letting $c_{\theta 1} = \| A_1 \|$, $A_1^m = c_{\theta 1}^{-1}A_1$(we obtain $\| A_1^m \| \leq 1$), $\omega_1 = A_1^m z_1$. Then, defining a error variable $z_2 = x_2 - \alpha_1$ where α_1 is an intermediate stabilizing function and substituting (7) into (6), we get

$$\dot{z}_1 = g_1(\bar{x}_1,\theta)(z_2 + \alpha_1) + c_{\theta 1}\bar{S}_1(x_1)\omega_1 + \nu_1 \tag{8}$$

where $c_{\theta 1}$ is an unknown constant and $\nu_1 = \xi_1(x_1)A_1y_d - \dot{y}_d$ is a bounded function.

Consider the stabilization of the subsystem (8) and the Lyapunov function candidate is given as follows

$$V_1(z_1,\hat{\lambda}_1) = \frac{1}{2}z_1^2 + \frac{1}{2}b_{\min}\Gamma_1^{-1}\tilde{\lambda}_1^2 \tag{9}$$

where Γ_1 is a positive constant. λ_1 and $\tilde{\lambda}_1$ will be designed later. The time derivative of V_1 is

$$\begin{aligned} \dot{V}_1(z_1,\hat{\lambda}_1) =& z_1(g_1(\bar{x}_1,\theta)(z_2 + \alpha_1) + c_{\theta 1}\bar{S}_1(x_1)\omega_1 + \nu_1) \\ & - b_{\min}\Gamma_1^{-1}\tilde{\lambda}_1\dot{\hat{\lambda}}_1 \end{aligned} \tag{10}$$

We calculate some items in (10) first. Let $\gamma_1 > 0$, we can get

$$\begin{aligned} c_{\theta 1}\bar{S}_1(x_1)\omega_1 z_1 =& c_{\theta 1}\bar{S}_1(x_1)\omega_1 z_1 - \gamma_1^2\omega_1^T\omega_1 \\ & + \gamma_1^2\omega_1^T\omega_1 \\ =& -\gamma_1^2\left(\omega_1 - \frac{c_{\theta 1}}{2\gamma_1^2}\bar{S}_1 z_1\right)^2 \\ & + \frac{c_{\theta 1}^2}{4\gamma_1^2}\bar{S}_1\bar{S}_1^T z_1^2 + \gamma_1^2\omega_1^T\omega_1 \\ \leq & \frac{c_{\theta 1}^2}{4\gamma_1^2}\bar{S}_1\bar{S}_1^T z_1^2 + \gamma_1^2\omega_1^T\omega_1 \end{aligned} \tag{11}$$

and according to Yang's inequation, given any positive constant $\rho > 0$, we have

$$\nu_1 z_1 \leq \eta_1 \psi_1(x_1) \parallel z_1 \parallel \leq \frac{\eta_1^2}{4\rho^2}\psi_1^2(x_1)z_1^2 + \rho^2 \tag{12}$$

where $\eta_1 = \max(\parallel A_1 y_d \parallel, \parallel \dot{y}_d \parallel)$ and $\psi_1(x_1) = 1 + \parallel \bar{S}_1 \parallel$. Noting (11) and (12), we can get

$$\begin{aligned} &c_{\theta 1}\bar{S}_1(x_1)\omega_1 z_1 + \nu_1 z_1 \\ &\leq \frac{c_{\theta 1}^2}{4\gamma_1^2}\bar{S}_1\bar{S}_1^T z_1^2 + \gamma_1^2\omega_1^T\omega_1 + \frac{\eta_1^2}{4\rho^2}\psi_1^2(x_1)z_1^2 \\ &\quad + \rho^2 \\ &\leq b_{\min}\lambda_1\Phi_1(x_1)z_1^2 + \gamma_1^2\omega_1^T\omega_1 + \rho^2 \\ &\leq b_{\min}\hat{\lambda}_1\Phi_1(x_1)z_1^2 + b_{\min}\widetilde{\lambda}_1\Phi_1(x_1)z_1^2 \\ &\quad + \gamma_1^2\omega_1^T\omega_1 + \rho^2 \end{aligned} \tag{13}$$

where $\Phi_1(x_1) = \frac{1}{4\gamma_1^2}\bar{S}_1\bar{S}_1^T + \frac{1}{4\rho^2}\psi_1^2$, $\lambda_1 = \max(b_{\min}^{-1}c_{\theta 1}^2, b_{\min}^{-1}\eta_1^2)$, $\widetilde{\lambda}_1 = (\lambda_1 - \hat{\lambda}_1)$ and $\hat{\lambda}_1$ is the estimate of λ_1.

Therefore, substituting (13) into (10), we can get

$$\begin{aligned} \dot{V}_1(z_1, \hat{\lambda}_1) \leq & g_1(\bar{x}_1, \theta)z_1 z_2 + g_1(\bar{x}_1, \theta)\alpha_1 z_1 \\ & + b_{\min}\hat{\lambda}_1\Phi_1(x_1)z_1^2 \\ & + b_{\min}\Gamma_1^{-1}\widetilde{\lambda}_1\left(\Gamma_1\Phi_1(x_1)z_1^2 - \dot{\hat{\lambda}}_1\right) \\ & + \gamma_1^2\omega_1^T\omega_1 + \rho^2 \end{aligned} \tag{14}$$

Given a design constant $k_1 > 0$, we choose the intermediate stabilizing function α_1 and the adaptive law for $\hat{\lambda}_1$ as

$$\alpha_1 = -\left(k_1 + \hat{\lambda}_1\Phi_1(x_1)\right)z_1 \tag{15}$$

$$\dot{\hat{\lambda}}_1 = \Gamma_1[\Phi_1(x_1)z_1^2 - \sigma_1(\hat{\lambda}_1 - \lambda_1^0)] \tag{16}$$

where λ_1^0 and σ_1 are design parameters. In light of Assumption 2, we can get

$$\begin{aligned} g_1\alpha_1 z_1 = & g_1[-(k_1 + \hat{\lambda}_1\Phi_1(x_1))z_1^2] \\ \leq & b_{\min}[-(k_1 + \hat{\lambda}_1\Phi_1(x_1))z_1^2] \end{aligned} \tag{17}$$

and by use of Yang's inequation again, yields

$$g_1 z_1 z_2 \leq \frac{1}{4}z_1^2 + g_1^2 z_2^2 \tag{18}$$

Using (15), (16), (17) and (18), $\dot{V}_1$ is converted into

$$\begin{aligned} \dot{V}_1(z_1, \hat{\lambda}_1) \leq & -\left(b_{\min}k_1 - \frac{1}{4}\right)z_1^2 - \frac{1}{2}b_{\min}\sigma_1\widetilde{\lambda}_1^2 \\ & + g_1^2 z_2^2 + \gamma_1^2\omega_1^T\omega_1 + \delta_1 \end{aligned} \tag{19}$$

where $\delta_1 = \rho^2 + \frac{1}{2}b_{\min}\sigma_1 \mid \lambda_1 - \lambda_1^0 \mid^2$.

Step 2.

$$\dot{z}_2 = g_2(\bar{x}_2, \theta)x_3 + f_2(\bar{x}_2, \theta) - \dot{\alpha}_1 \tag{20}$$

Then the time derivative of α_1 is

$$\begin{aligned} \dot{\alpha}_1 = & \frac{\partial\alpha_1}{\partial x_1}\dot{x}_1 + \frac{\partial\alpha_1}{\partial\hat{\lambda}_1}\dot{\hat{\lambda}}_1 + \frac{\partial\alpha_1}{\partial y_d}\dot{y}_d \\ = & \frac{\partial\alpha_1}{\partial x_1}(g_1(\bar{x}_1, \theta)x_2 + f_1(\bar{x}_1, \theta)) \\ & + \frac{\partial\alpha_1}{\partial\hat{\lambda}_1}\dot{\hat{\lambda}}_1 + \frac{\partial\alpha_1}{\partial y_d}\dot{y}_d \\ = & f_{12}(z_1, \bar{x}_2, \theta) + \frac{\partial\alpha_1}{\partial y_d}\dot{y}_d \end{aligned} \tag{21}$$

Substituting (21) into (20), we get

$$\begin{aligned} \dot{z}_2 = & -g_1^2(\bar{x}_1, \theta)z_2 + g_2(\bar{x}_2, \theta)x_3 + f_2'(\bar{z}_2, y_d, w) \\ & - \frac{\partial\alpha_1}{\partial y_d}\dot{y}_d \end{aligned} \tag{22}$$

where $f_2'(\bar{z}_2, y_d, \theta) = g_1^2(\bar{x}_1, \theta)z_2 + f_2(\bar{x}_2, \theta) - f_{12}(z_1, \bar{x}_2, \theta)$.

We also use Lemma 1 to treat the unknown function $f_2'(\bar{z}_2, y_d, \theta)$ and obtain

$$\begin{aligned} f_2'(\bar{z}_2, y_d, \theta) = & \bar{S}_2(\bar{z}_2, y_d)A_2[\bar{z}_2, y_d]^T \\ = & \bar{S}_2 A_2^1 \bar{z}_2^T + \bar{S}_2 A_2^2 y_d \\ = & c_{\theta 2}\bar{S}_2\omega_2 + \bar{S}_2 A_2^2 y_d \end{aligned}$$

where $\omega_2 = A_2^m \bar{z}_2^T$ and $c_{\theta 2} = \parallel A_2^1 \parallel = \lambda_{\max}^{1/2}(A_2^{1T}A_2^1)$, such that $A_2^1 = c_{\theta 2}A_2^m$ and $\parallel A_2^m \parallel \leq 1$. Defining the error variable $z_3 = x_3 - \alpha_2$, a direct substitution of above equation gives

$$\dot{z}_2 = -g_1^2(\bar{x}_2, \theta)z_2 + g_2(\bar{x}_2, \theta)(z_3 + \alpha_2) + c_{\theta 2}\bar{S}_2\omega_2 + \nu_2 \tag{23}$$

where $\nu_2 = \bar{S}_2 A_2^2 y_d - \frac{\partial\alpha_1}{\partial y_d}\dot{y}_d$.

Choosing Lyapunov function candidate

$$V_2 = V_1 + \frac{1}{2}z_2^2 + \frac{1}{2}b_{\min}\Gamma_2^{-1}\widetilde{\lambda}_2^2$$

where $\widetilde{\lambda}_2 = (\lambda_2 - \hat{\lambda}_2)$ and $\lambda_2 = \max(b_{\min}^{-1}c_{\theta 2}^2, b_{\min}^{-1}\eta_2^2)$.

A similar procedure with (11), (12) and (13) is used and the time derivative of V_2 becomes

$$\begin{aligned} \dot{V}_2 \leq & -\left(b_{\min}k_1 - \frac{1}{4}\right)z_1^2 + \gamma_1^2\omega_1^T\omega_1 + \delta_1 \\ & + z_2\left(g_2(z_3 + \alpha_2) + b_{\min}\hat{\lambda}_2\Phi_2 z_2\right) + \gamma_2^2\omega_2^T\omega_2 + \rho^2 \\ & + b_{\min}\Gamma_2^{-1}\widetilde{\lambda}_2(\Gamma_2\Phi_2 z_2^2 - \dot{\hat{\lambda}}_2) \end{aligned}$$

where $\Phi_2 = \frac{1}{4\gamma_2^2}\bar{S}_2\bar{S}_2^T + \frac{1}{4\rho^2}\psi_2^2$, $\parallel \nu_2 \parallel \leq \eta_2\psi_2$ and $\psi_2 = 1 + \parallel \bar{S}_2 \parallel + \parallel \frac{\partial\alpha_1}{\partial y_d} \parallel$.

Now, choose the intermediate stabilizing function α_2 and adaptive law as

$$\alpha_2 = -\left(k_2 + \hat{\lambda}_2\Phi_2\right)z_2 \tag{24}$$

$$\dot{\hat{\lambda}}_2 = \Gamma_2\left[\Phi_2 z_2^2 - \sigma_2(\hat{\lambda}_2 - \lambda_2^0)\right] \tag{25}$$

where $k_2 > 0$, λ_2^0 and σ_2 are design constants. Using same procedure as (17) and 18), then $\dot{V}_2$ is converted to

$$\dot{V}_2 \leq -\sum_{i=1}^{2}\left(b_{\min}k_i - \frac{1}{4}\right)z_i^2 - \frac{1}{2}b_{\min}\sum_{i=1}^{2}\sigma_i\widetilde{\lambda}_i^2 + g_2^2 z_3^2 + \sum_{i=1}^{2}\gamma_i^2\omega_i^T\omega_i + \delta_2 \quad (26)$$

where $\delta_2 = 2\rho^2 + \frac{1}{2}b_{\min}\sum_{i=1}^{2}\sigma_i \mid \lambda_i - \lambda_i^0 \mid^2$.

A similar procedure is employed recursively for each step $k(3 \leq k \leq n-1)$. By considering the equation of system (4) for $i = k, \dot{x}_k = g_k(\bar{x}_k, \theta)x_{k+1} + f_k(\bar{x}_k, \theta)$, and the Lyapunov function candidate

$$V_k = V_{k-1} + \frac{1}{2}z_k^2 + \frac{1}{2}b_{\min}\Gamma_k^{-1}\widetilde{\lambda}_k^2$$

where $\widetilde{\lambda}_k = (\lambda_k - \hat{\lambda}_k)$.

We may design the intermediate stabilizing function α_k, and learning law for $\hat{\lambda}_k$, which take similar forms of (24) and (25), respectively. The controller u for the system (4) shall be constructed in step n.

Step n: Define the error variable as $z_n = x_n - \alpha_{n-1}$, we have

$$\dot{z}_n = f_n(x,\theta) + g_n(x,\theta)u - \dot{\alpha}_{n-1}$$

Using the similar way to (21) in Step 2, we have

$$\begin{aligned}\dot{\alpha}_{n-1} =& \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}\left\{x_{j+1} + f_j(\bar{x}_j,\theta)\right\} \\ &+ \frac{\partial\alpha_{n-1}}{\partial\hat{\lambda}_{n-1}}\dot{\hat{\lambda}}_{n-1} + \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(j-1)}}y_d^{(j)} \\ =& f_{(n-1)n}(\bar{z}_{n-1},\bar{x}_n,\bar{x}_{d(n-1)},\theta) \\ &+ \sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial y_d^{(j-1)}}y_d^{(j)}\end{aligned}$$

Then

$$\begin{aligned}\dot{z}_n =& -g_{n-1}^2 z_n + f_n'(\bar{z}_n,\bar{x}_{d(n-1)},\theta) + g_n(x,\theta)u \\ &- \sum_{j=1}^{n-1}\left(\frac{\partial\alpha_{n-1}}{\partial y_d^{(j-1)}}y_d^{(j)}\right)\end{aligned}$$

where $f_n' = f_n(x,\theta) - f_{(n-1)n}(\bar{z}_{n-1},\bar{x}_n,\bar{x}_{d(n-1)},\theta) + g_{n-1}^2(\bar{x}_{n-1},\theta)z_n$.

We also use Lemma 1 to deal with the unknown function $f_n'(\bar{z}_n,\bar{x}_{d(n-1)},w)$ and obtain

$$\begin{aligned}f_n' =& \bar{S}_n A_n[\bar{z}_n,\bar{x}_{d(n-1)}]^T \\ =& \bar{S}_n A_n^1[\bar{z}_n,x_n]^T + \bar{S}_n A_n^2\bar{x}_{d(n-1)}^T \\ =& \bar{S}_n A_n^1\bar{z}_n^T + \bar{S}_n A_n^2\bar{x}_{d(n-1)}^T\end{aligned}$$

Let $\omega_n = A_n^{m1}\bar{z}_n^T$, $c_{\theta n} = \parallel A_n^{m1} \parallel$ and $A_n^1 = c_{\theta n}A_n^{m1}$.

$$\dot{z}_n = -g_{n-1}^2 z_n + c_{\theta n}\xi_n w_n + g_n(x,\theta)\alpha_n + v_n$$

where $u = \alpha_n$ and $v_n = -\bar{S}_n A_n^2\bar{x}_{d(n-1)}^T - \sum_{j=1}^{n-1}(\frac{\partial\alpha_{n-1}}{\partial y_d^{(j-1)}}y_d^{(j)})$.

Taking the following Lyapunov function candidate

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{1}{2}b_{\min}\Gamma_n^{-1}\widetilde{\lambda}_n^2$$

where $\widetilde{\lambda}_n = (\lambda_n - \hat{\lambda}_n)$ and $\lambda_n = b_{\min}\max(b_{\min}^{-1}c_{\theta n}^2, b_{\min}^{-1}\eta_n^2)$. Then its time derivative is

$$\begin{aligned}\dot{V}_n =& \dot{V}_{n-1} + z_n(-g_{n-1}^2 z_n + g_n(x,\theta)\alpha_n + c_{\theta n}\xi_n w_n + v_n) \\ &- b_{\min}\Gamma_{n1}^{-1}\widetilde{\lambda}_n\dot{\hat{\lambda}}_n \\ \leq& -\sum_{i=1}^{n-1}\left(b_{\min}k_i - \frac{1}{4}\right)z_i^2 - \frac{1}{2}b_{\min}\sum_{i=1}^{n-1}\sigma_i\widetilde{\lambda}_i^2 \\ &+ \sum_{i=1}^{n}\gamma_i^2\omega_i^T\omega_i + \delta_{n-1} + \rho^2 \\ &+ z_n\left(g_n(x,w)\alpha_n + b_{\min}\hat{\lambda}_n\Phi_n z_n\right) \\ &+ b_{\min}\Gamma_n^{-1}\widetilde{\lambda}_n\left(\Gamma_n\Phi_n z_n^2 - \dot{\hat{\lambda}}_n\right)\end{aligned} \quad (27)$$

where $\Phi_n = \frac{1}{4\gamma_n^2}\bar{S}_n\bar{S}_n^T + \frac{1}{4\rho^2}\psi_n^2$, $\parallel v_n \parallel \leq \eta_n\psi_n$, $\psi_n = 1 + \parallel \bar{S}_n \parallel + \sum_{j=1}^{n-1}\left(\parallel \frac{\partial\alpha_{n-1}}{\partial y_d^{(j-1)}} \parallel\right)$.

Now, we get $k_n > 0$ as a design constant and are ready to choose the controller with adaptive law in step n as

$$u = \alpha_n = -\left(k_n + \hat{\lambda}_n\Phi_n\right)z_n \quad (28)$$

$$\dot{\hat{\lambda}}_n = \Gamma_n\left[\Phi_n z_n^2 - \sigma_n(\hat{\lambda}_n - \lambda_n^0)\right] \quad (29)$$

By means of Assumption 2, we have

$$g_n(x,\theta)\alpha_n z_n \leq -b_{\min}(k_n + \hat{\lambda}_n\Phi_n)z_n^2$$

It follows from the recursive control design procedure above, so that

$$\begin{aligned}\dot{V}_n \leq& -\sum_{i=1}^{n-1}\left(b_{\min}k_i - \frac{1}{4}\right)z_i^2 \\ &- \frac{1}{2}b_{\min}\sum_{i=1}^{n}\sigma_i\widetilde{\lambda}_i^2 + \sum_{i=1}^{n}\gamma_i^2\omega_i^T\omega_i + \delta_n \\ \leq& -\sum_{i=1}^{n}\left(-b_{\min}k_i - \frac{1}{4}\right)z_i^2 \\ &- \frac{1}{2}b_{\min}\sum_{i=1}^{n}\sigma_i\widetilde{\lambda}_i^2 + \gamma^2 \parallel \omega \parallel^2 + \delta_n\end{aligned} \quad (30)$$

where $\delta_n = n\rho^2 + \frac{1}{2}b_{\min}\sum_{i=1}^{n}\sigma_i \mid \lambda_i - \lambda_i^0 \mid^2$, $\omega = [\omega_1, \omega_2, \ldots, \omega_n]^T$ and $\gamma = (\gamma_1^2 + \gamma_2^2 + \ldots + \gamma_n^2)^{1/2}$.

We are now in a position to state our main result on semi-global robust adaptive NN controller.

Theorem 1: Consider the overall closed-loop system (4), (28) and (29) and suppose that the packaged uncertain functions $f_i'(\bar{z}_i,\bar{x}_{d(i)},\theta), i = 1,2,\cdots,n$ can be dealt with by Lemma 1. If we pick $\gamma < 1$, $k_i > \frac{5}{4}b_{\min}^{-1}, i = 1,2,\ldots,n$ in (30), then the robust adaptive NN tracking control $u = \alpha_n$, the intermediate stabilizing functions α_i and adaptive laws for

$\hat{\lambda}_i$ can make all the solutions $(z(t), \hat{\lambda})$ of the derived closed loop system uniformly ultimately bounded. Furthermore, given any $\mu_1 > 0$, we can tune our design constants such that the output error $z_1 = y(t) - y_d(t)$ satisfies $\lim_{t\to\infty} | z_1(t) | \le \mu_1$.

Proof: Note that $\omega_i = A_i^m \bar{z}_i^T$ and $\|A_i^m\| \le 1$, $i = 1, ..., n$, so we obtain

$$\omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{bmatrix} = \begin{bmatrix} A_1^m & 0 & \cdots & 0 \\ A_2^{m1} & A_2^{m2} & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ A_n^{m1} & A_n^{m2} & \cdots & A_n^{mn} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_n \end{bmatrix} = Az$$

and

$$\| \omega \| \le \| A \| \| z \| \le \| z \| \tag{31}$$

Now, if choosing $\gamma < 1$, then (30) becomes

$$\dot{V}_n \le -\sum_{i=1}^{n} \left(-b_{\min} k_i - \frac{5}{4} \right) z_i^2 - \frac{1}{2} b_{\min} \sum_{i=1}^{n} \sigma_i \widetilde{\lambda}_i^2 + \delta_n \tag{32}$$

If picking $k_i > \frac{5}{4} b_{\min}, i = 1, 2, \ldots, n$ from (32), we obtain

$$\begin{aligned} \dot{V}_n & \le -z^T Q z - \tfrac{1}{2} b_{\min} \widetilde{\lambda}^T Q_1 \widetilde{\lambda} + \delta_n \\ & \le -z^T Q z - \tfrac{1}{2} b_{\min} \widetilde{\lambda}^T Q_1 \widetilde{\lambda} + \delta_n \\ & \le -2 c_1 V_n + \delta_n \end{aligned} \tag{33}$$

where $Q = \mathrm{diag}[b_{\min} k_1 - \frac{5}{4}, b_{\min} k_2 - \frac{5}{4}, \ldots, b_{\min} k_{n-1} - \frac{5}{4}, b_{\min} k_n - \frac{5}{4}]$, $Q_1 = \mathrm{diag}[\sigma_1, \sigma_2, \ldots, \sigma_n]$, $c_1 = \min\{(\lambda_{\min}(Q) - 1)/2, \lambda_{\min}(Q_1)/\lambda_{\max}(\Gamma)\}$, $\Gamma = [\Gamma_1, \Gamma_2, \ldots, \Gamma_n]^T$. From (33), we obtain

$$V_n(t) \le \frac{\delta_n}{2 c_1} + V_n(t_0) e^{-2 c_1 (t - t_0)}. \quad \forall t \ge t_0 \ge 0.$$

It results that the solutions of composite closed-loop system are uniformly ultimately bounded, and implies that, for any $\mu_1 > (\delta_n / c_1)^{1/2}$, there exists a constant $T > 0$ such that $| z_1(t) | \le \mu_1$ for all $t \ge t_0 + T$. The last statement holds readily since $(\delta_n / c_1)^{1/2}$ can be made arbitrarily small if the design parameters $\hat{\lambda}^0$ and σ are chosen appropriately. Finally, we have proved Theorem 1. ∎

V. Simulation Examples

In this section, we will discuss the following second-order plant in the simulation as

$$\begin{cases} \dot{x}_1 = (3 + x_1) x_2 + x_1 + x_1^2 \\ \dot{x}_2 = (2 + x_1) u \end{cases} \tag{34}$$

with the output $y = x_1$. Clearly, system (34) is of strict-feedback form and have the uncertain virtual control gain functions g_1 and g_2 with satisfying Assumption 2, we can use Theorem 1 to design the robust adaptive NN tracking controller.

For system (34), we can design the following controller by use of Theorem 1.

The first stabilizing function α_1 is

$$\alpha_1 = -(5 + \hat{\lambda}_1 \Phi_1) z_1 \tag{35}$$

where $z_1 = y - y_d$, $\Phi_1 = \frac{1}{4\gamma_1^2} \bar{S}_1 \bar{S}_1^T + \frac{1}{4\rho^2} \psi_1^2$ and adaptive law for $\hat{\lambda}_1$ as

$$\dot{\hat{\lambda}}_1 = 1000 \left[\Phi_1 z_1^2 - 0.00002 (\hat{\lambda}_1 - 0.1) \right] \tag{36}$$

and we obtain the controller law as

$$u = -(5 + \hat{\lambda}_2 \Phi_2) z_2 \tag{37}$$

where $z_2 = x_2 - \alpha_1$ and $\Phi_2 = \frac{1}{4\gamma_2^2} \bar{S}_2 \bar{S}_2^T + \frac{1}{4\rho^2} \psi_2^2$.

Then adaptive law is

$$\dot{\hat{\lambda}}_2 = 30 \left[\Phi_2 z_2^2 - 0.00017 (\hat{\lambda}_2 - 0.3) \right] \tag{38}$$

It is well known that the selection of the centers and widths of RBF has a great influence on the performance of the adaptive NN controller, and that Gaussian RBF NNs arranged on a regular lattic on R^n can uniformly approximate sufficiently smooth functions on closed bounded subsets. Accordingly, in the following simulation studies, we select the centers and widths as: Neural network $W_1^{*T} S_1(x_1)$ contains five nodes, with centers $\mu_l (l = 1, 2, \cdots, 5)$ evenly spaced in $[-1, 1]$ and widths $\eta_l = 1 (l = 1, 2, \cdots, 5)$. Neural networks $W_2^{*T} S_2(z_1, z_2, y_d)$ contains 125 nodes, with centers $\mu_l (l = 1, 2, \cdots, 125)$ evenly spaces in $[-1, 1] \times [-1, 1] \times [-1, 1]$, and widths $\eta_l (l = 1, 2, \cdots, 125)$. The following initial conditions and controller design parameters are adopted in the simulation: $x(0) = [0, 0]^T$, $\hat{\lambda}(0) = [0, 0]^T$, and $\gamma_1 = \gamma_2 = 0.5$, $\rho = 0.5$.

Figs. 1, 2 and 3 show the simulation results for system (34) with the reference signal $y_d = \sin(0.5t)$.

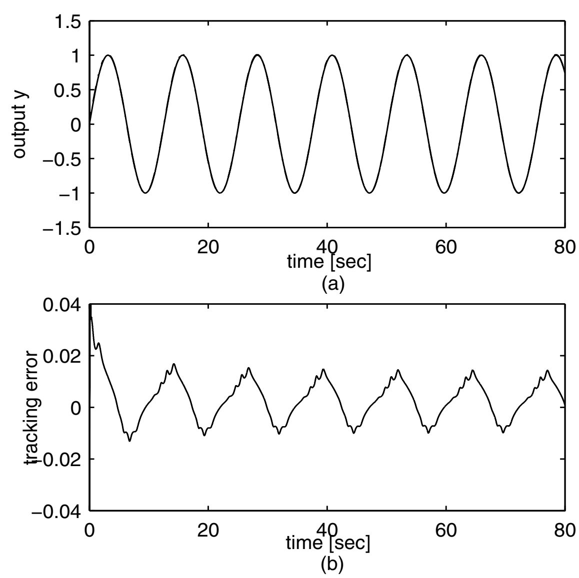

Fig. 1. Simulation results for Plant Σ_1 with $y_d = \sin(0.5t)$ (a) System output y and reference signal y_d (solid line: y and dashed line: y_d). (b) Tracking error z_1.

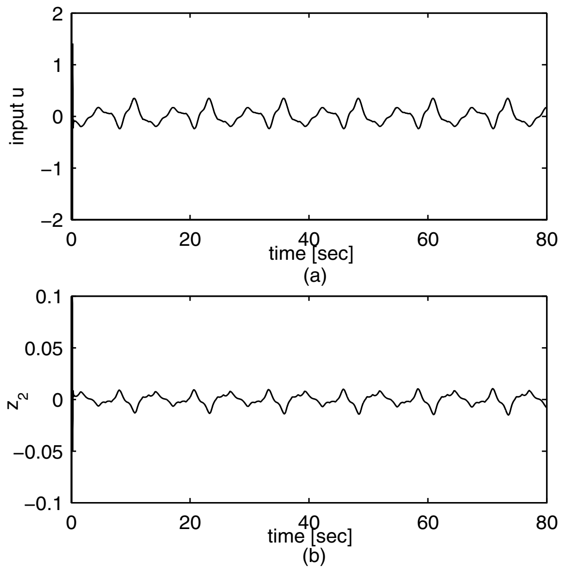

Fig. 2. Simulation results with $y_d = \sin(0.5t)$ (a) Control input u. (b) Intermediate error variable z_2.

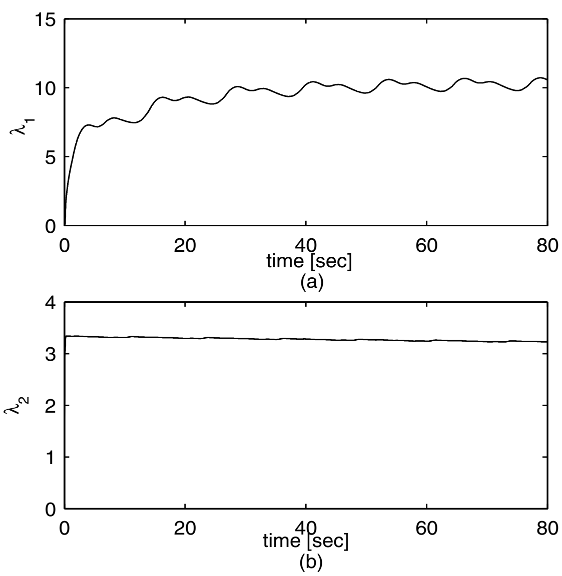

Fig. 3. Simulation results with $y_d = \sin(0.5t)$ (a) Adaptive parameter $\hat{\lambda}_1$. (b) Adaptive parameter $\hat{\lambda}_2$.

VI. Conclusion

We have considered the tracking control problem for a class of strict-feedback uncertain nonlinear systems. The systems may possess a wide class of uncertainties referred to as unstructured uncertainties, which are not linearly parameterized and have no prior knowledge of the bounding functions. We have incorporated the continuous function separation technique with RBF NN to model the unstructured uncertain functions in the systems and proposed an adaptive NN tracking control algorithm by combining backstepping technique with Lyapunov stability theory. The proposed algorithm can guarantee that the closed-loop system is SGUUB. The main feature of the algorithm proposed is that the order of dynamic compensator of ANNC is only identical to the order n of controlled system. Additionally, the computation load of the algorithm can be reduced, and it is a convenience to realize this algorithm for applications.

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This paper is dedicated to the memory of Professor Yansheng Yang, who died on 30th September 2005, for his pioneering research and distinguished contribution in the field of adaptive fuzzy control or NN control.

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