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Multivariable Decoupling Internal Model Control for Grinding Circuit

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Abstract—**Grinding circuit (GC) of mineral processing industry is characterized by its multivariable, severe coupling and multiple time delay nature. The product particle size and the mill throughput of GC are the important performance indexes directly related to the performance of the subsequent process and the production rate of the overall mineral processing plant respectively. However, they are hard to control effectively with conventional control strategies due to the above complex characteristics of GC. In this paper, a multivariable decoupling internal model control (MDIMC) scheme is adopted to handle such intricate process. Control studies have been performed by simulation tests for servo, regulatory, disturbance rejection and robustness problems.**

I. INTRODUCTION

GRINDING circuit (GC) is the most important operation unit in mineral processing industry. The function of it is to liberate the valuable minerals from the discardable gangue so as to help the subsequent beneficiation process [1]. The product fineness from GC affects the performance of the subsequent beneficiation process in terms of product concentrate grade and metal recovery rate, and the grinding yield decides the production rate of the overall mineral processing production. Therefore, a close control of GC is extremely important in order to provide fine product for downstream operations, improve the grinding yield and save energy losses.

Controlling the performance of a closed GC is a challenging problem due to its complex dynamic characteristics, such as multivariable, severe coupling, and multiple time delays. Traditionally, the GC is controlled by multi-loop PI/PID controllers which are heavily detuned to avoid multivariable interactions [2]. However, this strategy cannot eliminate the interactions exist in system well. Moreover, it results in highly sluggish closed loop responses. Therefore, some advance control strategies, such as adaptive control and artificial neural network (ANN) based control [3,4], have been applied in control of GC. Especially, in recent years, model predictive control (MPC) scheme, a widely used multivariable control algorithm in chemical process, has been applied in GC successfully [5-8].

Up to present, multivariable internal model control scheme has been a new and extensive concerned multivariable control algorithm in chemical process industries and other areas [9-13]. Compared with other multivariable control strategies, especially the widely used MPC scheme, the multivariable internal model control is found to be of many advantages, such as insensitive to the plant-model mismatches, higher robustness and better performance of disturbance rejection, expeditiousness and flexibility. Especially, it can be easy to design the stability and robustness by tuning the adjustable filter time parameters. Therefore, for GC characterized by multivariable, severe coupling and multiple time delays, the multivariable internal model control is more suitable in this case.

This paper presents an application of multivariable decoupling internal model control (MDIMC) in GC. The rest of the paper is organized as follows: the description of a typical GC, its control problem and process model are given in Section 2. After a brief description of MDIMC scheme in Section 3, the detailed process of the controller design for GC is illustrated in Section 4, and the simulation tests are conducted in Section 5. Section 6 concludes the paper.

II. GRINDING CIRCUIT DESCRIPTION

A. Process Description

The grinding circuit under study operates in a closed-circuit as shown in Fig.1. It consists of a ball mill, a hydrocyclone, a sump and other associated conveying sets.

Fresh ore from an ore bin is fed onto the conveyer belt by a vibratory feeder at a certain speed, and then is conveyed into the ball mill inlet, together with certain amount of water flow (called mill water). The knocking and tumbling action of iron balls within the revolving mill crush the ore inside to fine particles. The slurry containing the fine product is discharged from the mill to the sump, and pumped to the hydrocyclone for classification. The slurry is separated into two streams by the centrifugal force of hydrocyclone: an overflow stream containing the finer particles and an underflow stream containing the coarser particles as circulating load. The

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underflow is recycled back to the mill for regrinding, while the overflow is the final desired product and then transported to the subsequent beneficiation procedure.

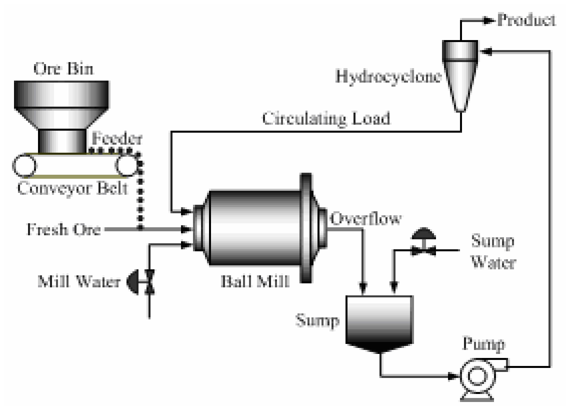

Fig. 1. Schematic diagram of grinding circuit

B. Control Problem Description

There are two important performance indexes existing in the GC shown above: product particle size (PPS) and mill throughput (MT). The PPS directly affects the performance of the subsequent beneficiation process. The MT directly relates to production rate of the overall mineral processing plant. Therefore, the control objective of GC can be stated as a fixed PPS setpoint at a MT setpoint corresponding to a value just below the maximum tonnage constraint. The manipulated variables available to achieve the above control objective are the fresh ore feed rate to the mill and the sump water addition rate to the sump. In addition, the sump level and the percent solids in the mill are controlled by manipulating the pump rate and the mill water addition rate to the mill in proportion to the fresh feed rate respectively, by using local controllers.

C. Process Model

Traditionally, process mode for automatic control can be established on the basis of physical laws governing the behavior of the true system and often be referred to as a first-principle model. Alternatively the model is derived from measurements of input-output data (open loop responses) from the real plant. This method relies heavily on system identification and the resulting model is called an empirical model or input-output model [8]. The input-output model provides necessary information required for the selection of a proper control strategy, design and tuning of appropriate controllers [7].

For the above shown GC, an input-output model developed by Ref. [7] at a certain operating condition is shown in Eq. (1).

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \dfrac{-0.425e^{-1.52s}}{11.7s+1} & \dfrac{0.1052(47.1s+1)}{11.5s+1} \\ \dfrac{2.977}{5.5s+1} & \dfrac{1.063e^{-2.26s}}{2.5s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (1)$$

where y_1 stands for product particle size (PPS), defined as the fraction of particles in cyclone overflow passing a sieve of 104 μm. aperture; y_2 is the mill throughput (MT), defined as the flow rate of slurry through the mill (kg/min); u_1, u_2 denote manipulated variables, namely the fresh feed rate (kg/min) and water addition rate to the sump (kg/min) respectively.

It is evident that severe interactions exist between the manipulated and controlled variables. Moreover, these input-output loops in the GC system have different time delays, and for a particular loop its output could be affected by all the inputs trough different time. As for conventional multi-loop PI/PID controllers, the generally become sluggish, leading to poor control performances and precise control of the process is impossible in real practice. Multivariable decoupling internal mode control (MDIMC) scheme is much suitable for this case.

III. Multivariable Decoupling Internal Mode Control Algorithms

MDIMC is proposed to effectively control multivariable processes with multiple time delay [9-13]. The underlying algorithms of MDIMC scheme for a two inputs two outputs (TITO) process can be simply described as following.

Consider the IMC system in Fig.2, where $G(s)$ and $\tilde{G}(s)$ respectively represent the transfer functions of the plant and its model, and $K(s)$ is the IMC controller. Assume the decoupling control of the system is solvable, which also means that $G(s)$ is stable and non-singular [9,13]. Moreover, assume

$$G(s) = \tilde{G}(s) = \begin{bmatrix} g_{11}(s) & g_{12}(s) \\ g_{21}(s) & g_{22}(s) \end{bmatrix}, \text{ and } K(s) = \begin{bmatrix} k_{11}(s) & k_{12}(s) \\ k_{21}(s) & k_{22}(s) \end{bmatrix}$$

where $g_{ij}(s) = g_{ij0}(s)e^{-\tau_{ij}s}$ and $g_{ij0}(s)$ are strictly proper and stable transfer functions, and $\tau_{ij} \geq 0$.

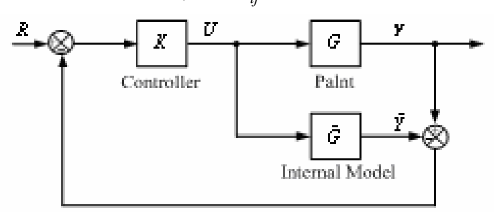

Fig. 1. Internal model control

The closed-loop transfer matrix H can be derived from Fig. 2 as $H = GK[I + (G - \tilde{G})K]^{-1}$, which reduces to GK under the condition $G = \tilde{G}$. Thus, the closed-loop is decoupled if and only if GK is decoupled (diagonal and non-singular) and the IMC system is internally stable if and only if K is stable. Therefore, the task is to characterize all stable and realizable controllers K and the resulting $H = \text{diag}(h_{11}, h_{22})$ such that $GK = H$ is decoupled.

It can be conducted that in order for $GK = H = \text{diag}(h_{11}, h_{22})$, the elements of K should satisfy the condition:

$$\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = \begin{bmatrix} \frac{g_{22}}{|G|}h_{11} & \frac{-g_{12}}{g_{11}}k_{22} \\ \frac{-g_{21}}{g_{22}}k_{11} & \frac{g_{11}}{|G|}h_{22} \end{bmatrix} = \begin{bmatrix} \frac{g_{22}}{|G|}h_{11} & \frac{-g_{12}}{|G|}h_{22} \\ \frac{-g_{21}}{|G|}h_{11} & \frac{g_{11}}{|G|}h_{22} \end{bmatrix} \quad (2)$$

$H = \mathrm{diag}(h_{11}, h_{22})$ usually being take as follows [9]

$$h_{11} = f_1(s)e^{-\tau(h_{11})s} \prod_{z \in Z^+_{|G|}} \left(\frac{z-s}{z+s}\right)^{\eta_z(h_{11})} \quad (3)$$

$$h_{22} = f_2(s)e^{-\tau(h_{22})s} \prod_{z \in Z^+_{|G|}} \left(\frac{z-s}{z+s}\right)^{\eta_z(h_{22})} \quad (4)$$

where $f_i(s), i = 1,2$ is the ith loop IMC filter, which is usually chosen as $f_i(s) = 1/(\alpha_i s+1)^{r_i}$, where α_i and r_i are the filter time constants and order respectively, $\tau(h_{ii})$ denotes time delay of h_{ii}, $\eta_z(h_{ii}), z \in Z^+_{G^{ii}}$ denotes an integer v such that $\lim_{s \to z} h_{ii}/(s-z)^v$ exists and is non-zero, $Z^+_{|G|}$ is the set of unstable zeros. The values of $\tau(h_{ii})$ and $\eta_z(h_{ii})$ need to meet the following conditions [9]

$$\tau(h_{ii}) \ge \tau(|G|) - \tau_i \quad (5)$$

$$\eta_z(h_{ii}) \ge \eta_z(|G|) - \eta_i(z) \quad (6)$$

Eq. (5) is a characterization of the decoupled ith loop transfer function in terms of their time delays and unstable zeros which indicates the minimum amount of time delay and unstable zeros that the ith decoupled loop transfer function must contain. In Eq. (5) and Eq. (6), τ_i and $\eta_i(z)$ are determined as follows [9]

$$\tau_1 \triangleq \min\{\tau(g_{22}), \tau(g_{21})\} \quad (7\text{-}1)$$

$$\tau_2 \triangleq \min\{\tau(g_{11}), \tau(g_{12})\} \quad (7\text{-}2)$$

$$\eta_1(z) \triangleq \min\{\eta_z(g_{21}), \eta_z(g_{22})\} \quad (8\text{-}1)$$

$$\eta_2(z) \triangleq \min\{\eta_z(g_{12}), \eta_z(g_{11})\} \quad (8\text{-}2)$$

In addition, the time delays and unstable zeros of controller K diagonal elements k_{ii} need to meet the following conditions [9]

$$\tau(k_{11}) \ge \tau(g_{22}) - \tau_1 \quad (9\text{-}1)$$

$$\tau(k_{22}) \ge \tau(g_{11}) - \tau_2 \quad (9\text{-}2)$$

$$\eta_z(k_{11}) \ge \eta_z(g_{22}) - \eta_1(z), z \in Z^+_{g_{22}} \quad (10\text{-}1)$$

$$\eta_z(k_{22}) \ge \eta_z(g_{11}) - \eta_2(z), z \in Z^+_{g_{11}} \quad (10\text{-}2)$$

IV. Design of MDIMC for GC

From the input-output model (described as Eq. (1))of GC can be obtained

$$G(s) = \begin{bmatrix} \frac{-0.425e^{-1.52s}}{11.7s+1} & \frac{0.1052(47.1s+1)}{11.5s+1} \\ \frac{2.977}{5.5s+1} & \frac{1.063e^{-2.26s}}{2.5s+1} \end{bmatrix} \quad (11)$$

and

$$|G| = -\frac{0.425 \times 1.063}{(11.7s+1)(2.5s+1)}e^{-3.78s} - \frac{0.1052 \times 2.977 \times (47.1s+1)}{(11.5s+1)(5.5s+1)} \quad (12)$$

Notice that $G(s)$ described as Eq. 11 is stable and non-singular, therefore, the decoupling problem with stability via the IMC structure is solvable. However, $\phi(s) = |G|$ is a complicated form, which is difficult to acquire time delay and unstable zeros by the above MDIMC algorithm. Therefore, it is need to employ a model reduction based method to find a much simpler form yet good approximation to $\phi(s) = |G|$.

A. Model Reduction Based on Recursive Least Squares (RLS) in Frequency Domain

Because that two-order model can cover a wide range of dynamics and yet to be low enough for its economic implementation [9], the complicated Eq. (12) is translated into a two-order rational function plus dead time model described as Eq. (13) by the improved employing frequency domain RLS identification algorithm in this paper.

$$\hat{\phi}(s) = \frac{b_2 s^2 + b_1 s^1 + b_0}{a_2 s^2 + a_1 s^1 + 1} e^{-Ls} \quad (13)$$

where a_1, a_2, b_0, b_1, b_2 are the parameters to be estimated, L is the time delay to be identified.

The identification process based on improved frequency domain RLS algorithm is given following

Step 1: Choose $N = 100$, set $\Delta L = 0.02$, and obtain $L_i = L_0 + (i-1)\Delta L, i = 1, \cdots, N$.

Step 2: For each L_i, find a rational approximation solution $\hat{\phi}_0(s)$ to modified model $\phi_0(s) = \phi(s)e^{L_i s} = |G|e^{L_i s}$ with following algorithms

$$\begin{cases} \theta_r = \theta_{r-1} + P(r)\{\phi_0(j\omega_r)\overline{\mho(j\omega_r)} + \overline{\phi_0(j\omega_r)}\mho(j\omega_r) \\ \qquad - \left[\mho(j\omega_r)\overline{\mho^{\mathrm{T}}(j\omega_r)} + \overline{\mho(j\omega_r)}\mho^{\mathrm{T}}(j\omega_r)\right]\} \\ Q(r-1) = \left[I - \frac{2P(r-1)\mathrm{Re}(\mho(j\omega_r))\mathrm{Re}(\mho^{\mathrm{T}}(j\omega_r))}{1 + 2\mathrm{Re}(\mho^{\mathrm{T}}(j\omega_r))P(r-1)\mathrm{Re}(\mho(j\omega_r))}\right]P(r-1) \\ P(r) = \left[I - \frac{2Q(r-1)\mathrm{Im}(\mho(j\omega_r))\mathrm{Im}(\mho^{\mathrm{T}}(j\omega_r))}{1 + 2\mathrm{Im}(\mho^{\mathrm{T}}(j\omega_r))Q(r-1)\mathrm{Im}(\mho(j\omega_r))}\right]Q(r-1) \end{cases} \quad (14)$$

where

$\theta = [a_1 \quad a_2 \quad b_0 \quad b_1 \quad b_2]^{\mathrm{T}}, \phi_0(j\omega_r) = u(j\omega_r)\phi(j\omega_r)e^{-L_i j\omega_r}$

$\mho(j\omega_r) = [-\phi_0(j\omega_r)j\omega_r \quad -\phi_0(j\omega_r)(j\omega_r)^2 \quad (j\omega_r)^{-1} \quad 1 \quad j\omega_r]$,

$u(j\omega_r) = (j\omega_r)^{-1}$ denotes the step input signal.

Step 3: For each rational approximation solution $\hat{\phi}_0(s)$ obtained in step 2, calculate $\phi_0(s) = \phi(s)e^{L_i s}$, and then evaluate the corresponding approximation error e in Eq. (15)

$$e = N^{-1}\sum_{r=1}^{N}\left|(j\omega_r)^{-1}\hat{\phi}(j\omega_r) - (j\omega_r)^{-1}\phi(j\omega_r)\right|^2 \quad (15)$$

Step 4: Take as the solution $\hat{\phi}(s)$ that yields the minimum error e in Eq. (15).

The above identification procedure is implemented by Matlab language, the initial parameters are evaluated as following

$$\begin{cases} \theta_0 = [0.00001 \quad 0.00001 \quad 0.00001 \quad 0.00001 \quad 0.00001]^{\mathrm{T}} \\ P(0) = 10^6 I_{5\times 5} \\ \omega_0 = 0.1, \Delta\omega = 0.001, \omega_i = \omega_0 + \Delta\omega \times i, i = 0, \cdots, 1000 \end{cases}$$

Finally, the best approximation reduced-order model of $\phi(s) = |G|$ is obtained as shown in Eq. (16)

$$\widehat{\phi}(s) = \frac{-22.7358s^2 - 14.2767s^1 - 0.765}{70.8148s^2 + 16.0724s^1 + 1} e^{-1.24s} \qquad (16)$$

It is easy to find $\widehat{\phi}(0) = \phi(0) = |G(0)| = -0.765$ .Therefore, there is no steady error between $\widehat{\phi}(s)$ and $|G|$. The excellent model approximation effect of $\widehat{\phi}(s)$ can be seen in Fig. 3.

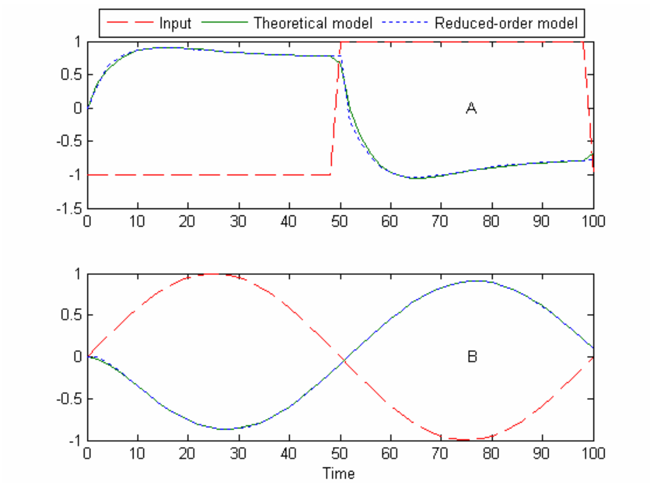

Fig. 3. Effect curves of model approximation.
A: with unit square wave input; B: with unit sine input

B. Multivariable Decoupling Internal Model Controller Design

According to the best reduced-order model $\widehat{\phi}$ of $|G|$, it is clear that $\tau(\widehat{\phi}) = 1.24$, and

$$\tau_1 \triangleq \min\{\tau(g_{22}), \tau(g_{21})\} = \min\{\tau(\frac{1.063e^{-2.26s}}{2.5s+1}), \tau(\frac{2.977}{5.5s+1})\} = 0 ,$$

hence, $\tau(\widehat{\phi}) - \tau_1 = 1.24$ and $\tau(k_{11}) \geq \tau(g_{22}) - \tau_1 = 2.26$.

Since $\widehat{\phi}$ is of minimum phase, $Z^{+}_{|\widehat{G}|} = \{\phi\}$ and there is no need to calculate $\eta_1(z)$.

According to Eq. (3), we have $h_{11}(s) = e^{-1.24s} / (\alpha_1 s + 1)$ with the filter chosen as $f_1(s) = 1/(\alpha_1 s + 1)$. From Eq. (2), k_{11} and k_{21} are obtained as

$$k_{11} = -\frac{1.063(70.8148s^2 + 16.0724s^1 + 1)}{(22.7358s^2 + 14.2767s^1 + 0.765)(2.5s+1)(\alpha_1 s + 1)} e^{-2.26s}$$

$$k_{21} = \frac{2.977(70.8148s^2 + 16.0724s^1 + 1)}{(22.7358s^2 + 14.2767s^1 + 0.765)(\alpha_1 s + 1)(5.5s+1)}$$

By the similarly calculation, with $h_{22}(s) = e^{-1.24s} / (\alpha_2 s + 1)$, k_{22} and k_{12} can be obtained as

$$k_{22} = \frac{0.425(70.8148s^2 + 16.0724s^1 + 1)}{(22.7358s^2 + 14.2767s^1 + 0.765)(11.7s+1)(\alpha_2 s + 1)} e^{-1.52s}$$

$$k_{12} = \frac{0.1052(70.8148s^2 + 16.0724s^1 + 1)(47.1s+1)}{(22.7358s^2 + 14.2767s^1 + 0.765)(\alpha_2 s + 1)(11.5s+1)}$$

The open-loop response of the developed controller K with $\alpha_1 = \alpha_2 = 0.1$ is shown in Fig. 4. Obviously, controller K is stable. Because that the G also is stable, the designed nominal MDIMC system is closed –loop stable.

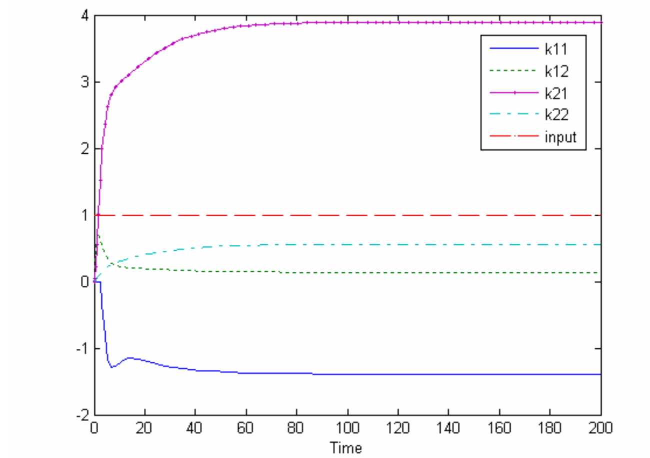

Fig. 4. Open-loop response for the developed controller

V. Simulation Tests

Firstly, the closed-loop responses of the grinding circuit with the developed nominal MDIMC scheme are simulated for step change setpoints. For comparison, two groups of filter time parameters $\alpha_1 = \alpha_2 = 0.5$ and $\alpha_1 = \alpha_2 = 5$ are tuned. By adding step change at $t = 18\,\text{min}$. and $t = 100\,\text{min}$. to the twofold setpoint inputs respectively, the nominal MDIMC system responses according to the two groups of filter time parameters are obtained as shown in Fig. 5.

From Fig. 5, it is clearly seen that no matter the setpoint changes in PPS or MT, the setpoints are reached smoothly with no overshoot by using the proposed method, and the twofold process output responses are accurately decoupled from each other. Moreover, it can be observed that lesser filter time parameters $\alpha_1 = \alpha_2 = 0.5$ can quicken the response speed, however, the magnitudes of manipulated variables response, especially the fresh feed rate, are increased, and vice versa.

To demonstrate effectiveness of the proposed MDIMC scheme for GC further more, square pulse setpoint inputs are added to the MDIMC system. To simulate the influence on system caused by the fluctuations in size distribution and hardness of the feed material in actual grinding process, a step change of load disturbance with a magnitude of 0.02 to each of the twofold process inputs at $t = 140\,\text{min}$. simultaneously. Moreover, two-way white noises with the maximal magnitude of 0.0005 and 0.005 are merged into the feedback channel of the grinding process, which can imitate the 5% stochastic errors arose from measuring of the PPS and the MT in actual operation. Simulation results according to two groups of filter time parameters $\alpha_1 = \alpha_2 = 0.5$ and $\alpha_1 = \alpha_2 = 5$ are shown in Fig.6.

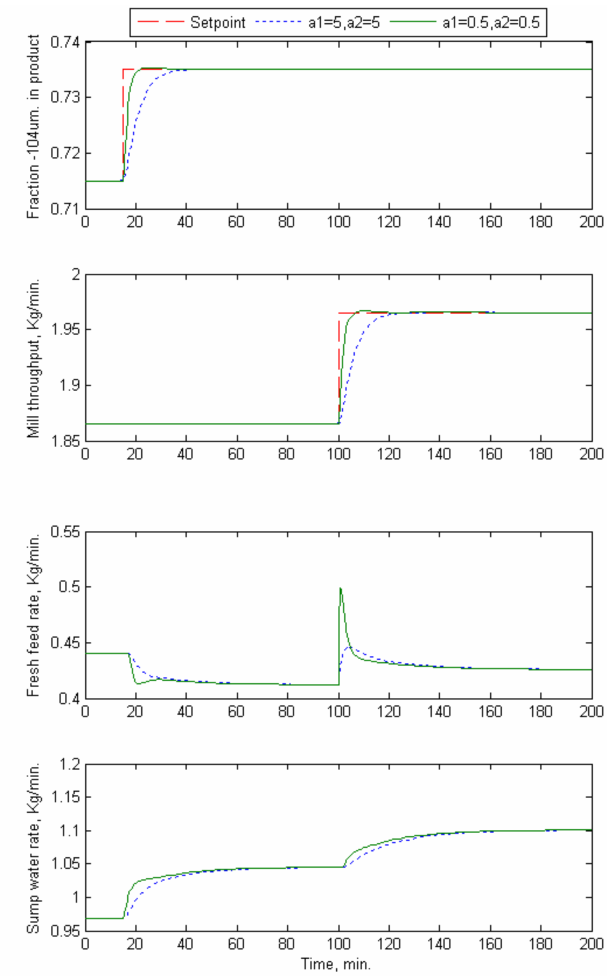

Fig. 5. Closed-loop response of the nominal MDIMC system

It is clearly seen that even if load disturbances and white noises are existed, the twofold process output responses are almost decoupled from each other, and able to reach the setpoints smoothly with no overshoot for both the positive and negative setpoint changes.

While taking the less filter time constants $\alpha_1 = \alpha_2 = 0.5$, the manipulated variable of fresh feed rate trembles with a certain extent because of white noise existing, however, the sump water rate is influenced less. Under the load disturbances, the PPS and the MT departure their setpoints and produce peak jumps, however, they will come back to track their respective setpoints quickly. Such results show the developed MDIMC system for GC with better performance of load disturbance rejection and fault tolerance.

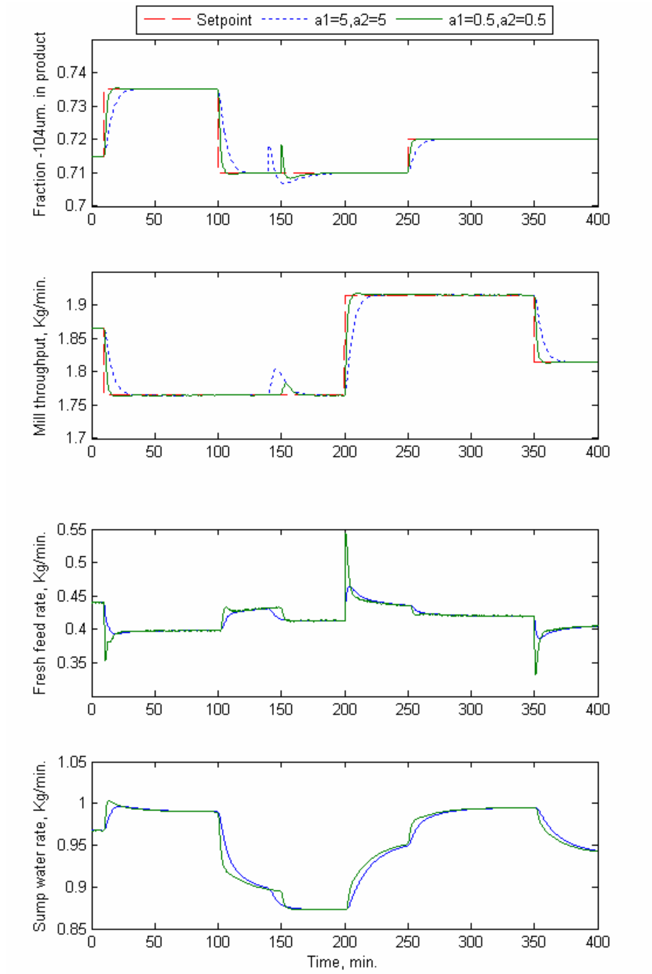

Fig. 6. Closed-loop response of the nominal MDIMC system with load disturbance and white noise

To demonstrate the robustness of the proposed scheme, another input-output model developed by Ref. [7] at a different operating condition, which is shown in Eq.(17), is employed to denote the grinding process.

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \dfrac{-0.495e^{-0.21s}}{16.55s+1} & \dfrac{0.1125(51.6s+1)}{15.5s+1} \\ \dfrac{3.425}{6.6s+1} & \dfrac{1.115e^{-2.13s}}{2.55s+1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \quad (17)$$

Obviously, the parameters of the Eq. (17) have much changes compared with the Eq. (1). For comparison, the same simulation tests as above doing are taken for this perturbed system, and the simulation results are shown in Fig. 7.

It is seen from Fig.7 that the closed-loop responses of the perturbed system are still very smooth with no overshoot or offset, and the performance of decoupling, load disturbance rejection and fault tolerance are uniformly satisfactory. Such simulation results prove the perfect robustness performance of the developed MDIMC system. This is because that the controller contains filters, thus insensitive to the plant-model mismatches.

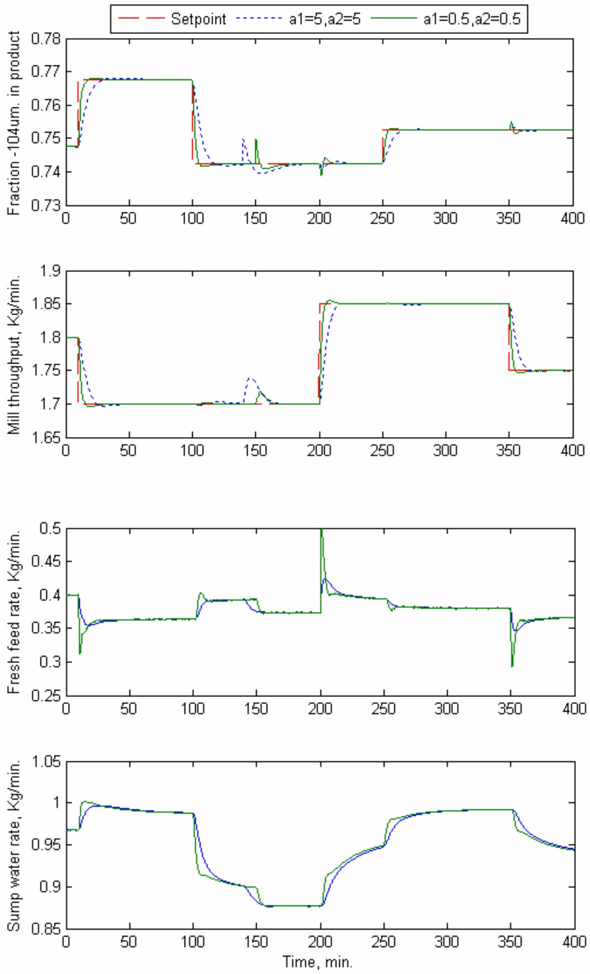

Fig. 7. Closed-loop response of the perturbed MDIMC system with load disturbance and white noise

VI. CONCLUSION

Multivariable decoupling internal mode control (MDIMC) scheme has been employed to handle the strong coupling multivariable system of GC. A TITO model of GC has been utilized for process control. Model reduction based on improved RLS in frequency domain was proposed to simplify controller design. At last, several simulation tests show the better performance of decoupling, setpoint response, load disturbance rejection, fault tolerance and robustness of the developed MDIMC system for GC.

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