2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

An Adaptive Fuzzy Sliding Mode Control for AQM Systems

Hongwei Wang, Bo Meng, Yuanwei Jing and Xiaoping Liu

Abstract—**In this paper, an active queue management (AQM) algorithm is investigated based on adaptive fuzzy sliding mode control for the problem of congestion control in TCP complex systems with unknown nonlinear disturbance. A sliding surface is constructed based on Lyapunov-Krasovskii method for the particular network model, and a sufficient condition is proposed for robust asymptotic stability of the system in terms of linear matrix inequality (LMI). The corresponding reaching law is designed, which can drive the state trajectory of system onto the sliding surface within limited time. The simulation results show that it can track queue length very quickly under various network conditions, and avoid the congestion of dynamic networks.**

I. Introduction

With the explosive growth and popularity of the Internet in the past years, communication networks have become an essential part of many application in science and engineering. In the current Internet, TCP congestion control mechanisms, while necessary and powerful, are not sufficient to provide good services in all circumstances, especially with the rapid growth in size and the strong requirement for QoS guarantee. So the design of congestion control mechanism is very important.

Active Queue Management (AQM) scheme, a router based congestion control method, has been proposed to improve network utilization. The random early detection (RED) algorithm [1], the earliest well-known AQM scheme, eliminates the flow synchronization problem and attenuates the traffic load by monitoring the average queue length. Unfortunately, RED causes oscillation and instability due to the parameter variations. Therefore, some modified RED schemes, such as FRED [2], ARED [3] and SRED [4], have been proposed in the literature. However, in those studies, both high network utilization and low packet loss can not be guaranteed by only setting control parameters.

Recently, control theory has been widely applied to the analysis and design of TCP networks and congestion control schemes for them. A nonlinear TCP dynamic equation based on fluid-flow model is proposed in [5], which can describe the dynamic behavior of TCP very well. To be more, several congestion control schemes based on the TCP model have been proposed to improve the performance of communication networks. For example, a proportional-integral (PI) controller is developed for a linearized system and implemented using difference equation in [6]. Compared with RED, PI controller is more stable. However, PI controller is sluggish with taking too long time to settle down to the desired queue length, because PI controller improves the steady-state error at the expense of an increase in rise time. In order to overcome the drawbacks of PI controller, [7] developed a proportional-integral-differential (PID) controller scheme for TCP congestion control. In [8], an adaptive proportional-integral (API) controller is developed to eliminate the steady-state error and the sensitivity to variation of the system parameters caused by the inaccuracy. The further investigation shows that the above methods ignore the delay term in some case, which will bring serious consequence. Because the TCP/AQM systems exist time varying round-trip times (RTT) and uncertainties with respect to the number of active TCP sessions through the congestion AQM router, they require more robustness for the designed schemes. [9-12] introduce sliding mode control (SMC) for AQM schemes. SMC is a robust technique well known for its ability to withstand external disturbance and model uncertainties, but they ignore the effect of the time delay in [9, 10]. In [11, 12], the time delay of the control input signal is considered. They analyze the stability only using a state time-delay model without considering the impact of uncertain in [13].

In this paper, a robust adaptive fuzzy sliding mode controller (AFSMC) is designed for the TCP model with unknown nonlinear disturbance, considering state time delay and uncertainties. Adaptive algorithm has online learning ability to deal with the nonlinear systems, and adjust the control rule parameters. As we know that SMC has attractive features such as fast response and good transient response. Therefore, we adopt the adaptive fuzzy sliding mode control method to find appropriate fuzzy rules in fuzzy control implementation and online adaptive rule has the effect of improving the stability property. The problem of designing both a linear sliding surface and reaching motion controller is investigated. A sliding surface is constructed based on Lyapunov-Krasovskill method for the particular network model, and the corresponding reaching law is derived, which can drive the state trajectory of system onto the sliding surface within limited time.

The remainders of this paper are organized as follows. Section II gives TCP dynamics flow control model. Section III presents the adaptive fuzzy sliding mode controller for AQM systems. In Section IV, we compare the performance of the AFSMC and traditional SMC, where we demonstrate the

Manuscript received September 20, 2007. This work is supported by the National Natural Science Foundation of China under grant 60274009 and Specialized Research Fund for the Doctoral Program of Higher Education, under grant 20020145007.

Hongwei Wang, Bo Meng, Yuanwei Jing and Xiaoping Liu are with the school of Information Science and Engineering, Northeastern University, Shenyang, P.R. of China. (Corresponding Hongwei Wang, E-mail: wanghw0819@163.com).

superiority performance of the AFSMC. Finally, we summarize our paper in Section V.

II. THE TCP NETWORK DYNAMICAL MODEL

In [5], a nonlinear dynamic model of a TCP connection through a congested AQM router is developed based on fluid-flow theory and stochastic differential equation analysis. The simplified version that ignores the TCP timeout mechanism is as follows.

$$\begin{cases} \dot{W}(t) = \dfrac{1}{R(t)} - \dfrac{W(t)W(t-R(t))}{2R(t)} p(t-R(t)) \\ \dot{q}(t) = \dfrac{N(t)}{R(t)} W(t) - C(t) \end{cases} \quad (1)$$

where c is the capacity of link, $W(t)$ is the size of TCP congestion window, $q(t)$ is the length of queue in buffer, $P(t)$ is the probability of packet mark/drop $(0 \le p(t) \le 1)$, N is the number of active TCP link, R is the delay of transfer (RTT), including delay of queuing and transmission, which satisfied $R(t) = T_p + q(t)/C(t)$.

To linearize (1), we first assume $R(t) = R_0$, $N(t) = N$ and $C(t) = C$ is normal value of $R(t)$, $N(t)$ and $C(t)$, the equilibrium point (W_0, q_d, p_0) is then defined by $\dot{W} = 0$ and $\dot{q} = 0$. So that

$$\begin{cases} \dot{W} = 0 \Rightarrow W_0^2 p_0 = 2 \\ \dot{q} = 0 \Rightarrow W_0 = \dfrac{R_0 C}{N}, R_0 = \dfrac{q_0}{C} + T_p \end{cases} \quad (2)$$

It is obviously known from (2) that the equilibrium point is unique corresponding to the Internet networks parameters. Let $\delta W(t) = W(t) - W_0$, $\delta q(t) = q(t) - q_d$, $\delta p(t) = p(t) - p_0$. A linearized model is given in [6].

$$\begin{cases} \delta\dot{W}(t) = -\dfrac{N}{R_0^2 C}\left(\delta W(t) + \delta W(t-R_0)\right) - \\ \qquad \dfrac{1}{R_0^2 C}\left(\delta q(t) - \delta q(t-R_0)\right) - \dfrac{R_0 C^2}{2N^2}\delta p(t) \\ \delta\dot{q}(t) = \dfrac{N}{R_0}\delta W(t) - \dfrac{1}{R_0}\delta q(t) \end{cases} \quad (3)$$

Let $x(t) = \left(\delta W(t) \quad \delta q(t)\right)^{\mathrm{T}} = \left(x_1 \quad x_2\right)^{\mathrm{T}}$, $u(t) = \delta p(t)$, $\left(-p_0 \le u(t) \le 1 - p_0\right)$, $\tau = R_0$, the plant (3) can be described as

$$\dot{x}(t) = Ax(t) + A_d x(t-\tau) + Bu(t) \quad (4)$$

where

$$A = \begin{bmatrix} -\dfrac{N}{R_0^2 C} & -\dfrac{1}{R_0^2 C} \\ \dfrac{N}{R_0} & -\dfrac{1}{R_0} \end{bmatrix}, A_d = \begin{bmatrix} -\dfrac{N}{R_0^2 C} & \dfrac{1}{R_0^2 C} \\ 0 & 0 \end{bmatrix},$$

$$B = \begin{bmatrix} -\dfrac{R_0 C^2}{2N^2} & 0 \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} \overline{B} & 0 \end{bmatrix}^{\mathrm{T}}.$$

Note that the plant model (1) is only an approximate model and it ignores the timeout and slow start mechanism. Equation (3) is further made linearization. So we consider a TCP model with uncertain, time-delay and nonlinear.

$$\begin{aligned} \dot{x}(t) = &\left(A + \Delta A(t)\right)x + \left(A_d + \Delta A_d(t)\right)x(t-\tau) + \\ &B\left(u(t) + G\left(x, x(t-\tau), \sigma(t)\right)\right) \end{aligned} \quad (5)$$

where $G\left(x, x(t), \sigma(t)\right)$ denote nonlinear functions, $\sigma(t)$ is a time-varying disturbance parameter. Some matrices can be decomposed into the following

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \Delta A(t) = \begin{bmatrix} \Delta A_{11} & \Delta A_{12} \\ \Delta A_{21} & \Delta A_{22} \end{bmatrix},$$

$$A_d = \begin{bmatrix} A_{d11} & A_{d12} \\ A_{d21} & A_{d22} \end{bmatrix}, \Delta A_d = \begin{bmatrix} \Delta A_{d11} & \Delta A_{d12} \\ \Delta A_{d21} & \Delta A_{d22} \end{bmatrix}.$$

We introduce the following assumptions imposed on system (5).

Assumption 1: The uncertain $\Delta A_{21}, \Delta A_{22}, \Delta A_{d21}$ and ΔA_{d22} satisfy the following form

$$\begin{bmatrix} \Delta A_{21} & \Delta A_{22} \end{bmatrix} = D_1 F_1(t) \begin{bmatrix} E_1 & E_2 \end{bmatrix},$$
$$\begin{bmatrix} \Delta A_{d21}(t) & \Delta A_{d22}(t) \end{bmatrix} = D_2 F_2(t) \begin{bmatrix} E_3 & E_4 \end{bmatrix},$$

where $\|F_i(t)\| \le 1$, and D_1, D_2, E_1, E_2, E_3 and E_4 are some known matrices with appropriate dimensions. In addition, $\|\Delta A_1\| \le \gamma_1$, $\|\Delta A_d\| \le \gamma_2$, where positive scalars γ_1 and γ_2 are not required to be known.

Assumption 2: $G\left(x, x(t-\tau), \sigma(t)\right)$ satisfies the following decomposition

$$\begin{aligned} &G\left(x, x(t-\tau), \sigma(t)\right) \\ &= G_1(x) + G_2\left(x(t-\tau)\right) + \Delta G\left(x(t), x(t-\tau), \sigma(t)\right) \end{aligned} \quad (6)$$

where $G_1(x)$ and $G_2\left(x(t-\tau)\right)$ are unknown continuous functions, $\Delta G\left(x(t), x(t-\tau), \sigma(t)\right)$ is an uncertain function which is also unknown, but bounded

$$\left|\Delta G\left(x(t), x(t-\tau), \sigma(t)\right)\right| \le \alpha_0 + \alpha_1 \|x(t)\| + \alpha_2 \|x(t-\tau)\| \quad (7)$$

where α_0, α_1 and α_2 are unknown positive scalars.

Then (5) is written as follows.

$$\begin{aligned} \dot{x}_1(t) = &\left(A_{11} + \Delta A_{11}\right)x_1(t) + \left(A_{d11} + \Delta A_{d11}(t)\right)x_1(t-\tau) + \\ &\left(A_{12} + \Delta A_{12}\right)x_2(t) + \left(A_{a12} + \Delta A_{d12}\right)x_2(t-\tau) + \\ &\overline{B}\left(u(t) + G\left(x, x(t-\tau), \sigma(t)\right)\right) \end{aligned} \quad (8)$$

$$\dot{x}_2(t) = (A_{21} + \Delta A_{21}) x_1(t) + (A_{d21} + \Delta A_{d21}(t)) x_1(t-\tau) + (A_{22} + \Delta A_{22}) x_2(t) + (A_{d22} + \Delta A_{d22}) x_2(t-\tau) \quad (9)$$

The network system model exists strong uncertainties, nonlinearity and is subject to additive noise. Taking the nonlinearity and uncertainties into consideration, the sliding mode controller of AQM would be an ideal methodology.

III. Design of Sliding Mode Controller

A. Designing Sliding Mode Surface

Without loss of generality, we suppose that the sliding surface is

$$S = \hat{K}x(t) = \begin{bmatrix} -K & I \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = -Kx_1 + x_2 = 0 \quad (10)$$

Substituting (10) into (7) gives the sliding motion

$$\dot{x}_2 = \left[(A_{21} + \Delta A_{21}) \overline{K} + A_{22} + \Delta A_{22} \right] x_2 + \left[(A_{d21} + \Delta A_{d21}(t)) \overline{K} + A_{d22} + \Delta A_{d22}(t) \right] x_2(t-\tau) \quad (11)$$

where $\hat{K} \in R^{1\times 2}, \overline{K} = K^{-1}$.

Lemma 1: For any appropriate dimensions matrix F satisfying $F(t)^{\mathrm{T}} F(t) \le I$, we have

$$2x^{\mathrm{T}} DFEy \le \varepsilon x^{\mathrm{T}} DD^{\mathrm{T}} x + \varepsilon^{-1} y^{\mathrm{T}} E^{\mathrm{T}} Ey$$

for any vector $x \in R^P$, $y \in R^q$ and constant $\varepsilon > 0$, D and E are constant matrices with appropriate dimensions.

Lemma 2: For any appropriate dimensions vectors $x \in R^P$, $y \in R^q$ and constant $\varepsilon > 0$, we have

$$2x^{\mathrm{T}} y \le \varepsilon x^{\mathrm{T}} x + \varepsilon^{-1} y^{\mathrm{T}} y \quad .$$

Theorem 1: Consider system (11). If there exists a symmetric and positive definite matrix P, some matrix W and some positive ε, the following LMI is satisfied

$$\begin{bmatrix} 2A_{21}X + 2A_{22}W + \psi & (E_1X + E_2W)^{\mathrm{T}} & (E_3X + E_4W)^{\mathrm{T}} & W^{\mathrm{T}} & X^{\mathrm{T}} \\ E_1X + E_2W & Y & 0 & 0 & 0 \\ E_3X + E_4W & 0 & Y & 0 & 0 \\ W & 0 & 0 & Y & 0 \\ X & 0 & 0 & 0 & Y \end{bmatrix} < 0 \quad (12)$$

then the designed sliding surface will render the sliding motion asymptotic stability. Where

$$\psi = -Y\left(D_1 D_1^{\mathrm{T}} + D_2 D_2^{\mathrm{T}} + A_{d21} A_{d21}^{\mathrm{T}} + A_{d22} A_{d22}^{\mathrm{T}} \right)$$

$Y = -\varepsilon^{-1}$, $X = \overline{K}P^{-1}$, $P^{-1} = W$.

Proof: For system (11), we choose the following Lyapunov-Krasovskii function

$$V(x) = x_2^{\mathrm{T}} P x_2 + \int_{t-\tau}^{t} \varepsilon x_2^{\mathrm{T}}(v) \left[(E_3\overline{K} + E_4)^{\mathrm{T}} (E_3\overline{K} + E_4) + I + \overline{K}^{\mathrm{T}} \overline{K} \right] x_2(v) dv \quad (13)$$

The time derivative of this function along the trajectory of the system in (13) is given by

$$\begin{aligned} \dot{V} &= 2x_2^{\mathrm{T}} P\dot{x}_2 + \varepsilon x_2^{\mathrm{T}} \left[(E_3\overline{K} + E_4)^{\mathrm{T}} (E_3\overline{K} + E_4) + I + \overline{K}^{\mathrm{T}} \overline{K} \right] x_2 - \varepsilon x_2^{\mathrm{T}}(t-\tau) \left[(E_3\overline{K} + E_4)^{\mathrm{T}} (E_3\overline{K} + E_4) + I + \overline{K}^{\mathrm{T}} \overline{K} \right] x_2(t-\tau) \\ &= 2x_2^{\mathrm{T}} P\Big[(A_{21}\overline{K} + A_{22}) x_2 + (\Delta A_{21}\overline{K} + \Delta A_{22}) x_2 + (A_{d21}\overline{K} + A_{d22}) x_2(t-\tau) + (\Delta A_{d21}\overline{K} + \Delta A_{d22}) x_2(t-\tau) \Big] + \\ &\quad \varepsilon x_2^{\mathrm{T}} \left[(E_3\overline{K} + E_4)^{\mathrm{T}} (E_3\overline{K} + E_4) + I + \overline{K}^{\mathrm{T}} \overline{K} \right] x_2 - \\ &\quad \varepsilon x_2^{\mathrm{T}}(t-\tau) \left[(E_3\overline{K} + E_4)^{\mathrm{T}} (E_3\overline{K} + E_4) + I + \overline{K}^{\mathrm{T}} \overline{K} \right] x_2(t-\tau) \end{aligned} \quad (14)$$

By Lemma 1 and Lemma 2, we can get the following inequality

$$\begin{aligned} & 2x_2^{\mathrm{T}} P(\Delta A_{21}\overline{K} + \Delta A_{22}) x_2 \\ &= 2x_2^{\mathrm{T}} P\left(D_1 F_1(t) (E_1\overline{K} + E_2) \right) x_2 \\ &\le \varepsilon_1^{-1} x_2^{\mathrm{T}} P D_1 D_1^{\mathrm{T}} P x_2 + \varepsilon_1 x_2^{\mathrm{T}} (E_1\overline{K} + E_2)^{\mathrm{T}} (E_1\overline{K} + E_2) x_2 \end{aligned} \quad (15)$$

$$\begin{aligned} & 2x_2^{\mathrm{T}} P(\Delta A_{d21}\overline{K} + \Delta A_{d22}) x_2(t-\tau) \\ &= 2x_2^{\mathrm{T}} P D_2 F_2(t) (E_3\overline{K} + E_4) x_2(t-\tau) \\ &\le \varepsilon_2^{-1} x_2^{\mathrm{T}} P D_2 D_2^{\mathrm{T}} P x_2 + \\ &\quad \varepsilon_2 x_2(t-\tau)^{\mathrm{T}} (E_3\overline{K} + E_4)^{\mathrm{T}} (E_3\overline{K} + E_4) x_2(t-\tau) \end{aligned} \quad (16)$$

$$\begin{aligned} & 2x_2^{\mathrm{T}} P A_{d21} \overline{K} x_2(t-\tau) \\ &\le \varepsilon_3^{-1} x_2^{\mathrm{T}} P A_{d21} A_{d21}^{\mathrm{T}} P x_2 + \varepsilon_3 x_2(t-\tau)^{\mathrm{T}} \overline{K}^{\mathrm{T}} \overline{K} x_2(t-\tau) \end{aligned} \quad (17)$$

$$\begin{aligned} & 2x_2^{\mathrm{T}} P A_{d22} x_2(t-\tau) \\ &\le \varepsilon_4^{-1} x_2^{\mathrm{T}} P A_{d22} A_{d22}^{\mathrm{T}} P x_2 + \varepsilon_4 x_2(t-\tau)^{\mathrm{T}} x_2(t-\tau). \end{aligned} \quad (18)$$

where $\varepsilon_1, \varepsilon_2, \varepsilon_3$ and ε_4 are positive scalars. For simplicity, we choose $\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4 = \varepsilon$, then by substituting (15)-(18) into (14), we get

$$\begin{aligned} \dot{V} &\le x_2^T \Big[2P(A_{21}\overline{K} + A_{22}) + \varepsilon^{-1} P D_1 D_1^T P + \\ &\quad \varepsilon (E_1\overline{K} + E_2)^T (E_1\overline{K} + E_2) + \varepsilon^{-1} P D_2 D_2^T P + \\ &\quad \varepsilon^{-1} P A_{d21} A_{d21}^T P + \varepsilon^{-1} P A_{d22} A_{d22}^T P + \\ &\quad \varepsilon (E_3\overline{K} + E_4)^T (E_3\overline{K} + E_4) + \varepsilon I + \varepsilon \overline{K}^T \overline{K} \Big] x_2 \\ &= x_2^T \varphi x_2 \end{aligned} \quad (19)$$

We find that if $\varphi < 0$, we can obtain $\dot{V} < 0$ by applying the

Schur complement. and using the Lyapunov-Krasovskii function, we can conclude that system (11) is asymptotically stability. Then we make a toolbox to solve $\bar{K}$ in the matlab, and the sliding surface is designed. So we complete the proof of Theorem 1.

B. Designing adaptive fuzzy sliding mode control law

In this study, we will design a control input $u(t)$ to maintain the system states on the sliding surface $S(t)$ for all $t>0$. considering the unknown nonlinear disturbance, an adaptive fuzzy control law is designed such that the reaching condition is satisfied.

First, a fuzzy system is collection of fuzzy IF-THEN rules of the form

R^j : IF x_1 is A_1^j and $\cdots$ and x_n is A_n^j THEN y is B^j .

By using the strategy of singleton fuzzification, product inference and center-average defuzzification, the output of the fuzzy system is

$$y(x)=\frac{\sum_{j=1}^{m} y^j\left(\prod_{i=1}^{n}\mu_{A_i^j}(x_i)\right)}{\sum_{j=1}^{m}\left(\prod_{i=1}^{n}\mu_{A_i^j}(x_i)\right)} \tag{20}$$

where $\mu_{A_i^j}(x_i)$ is the membership function of linguistic variable x_i, and y^j is the point in R at which μ_B^j achieves its maximum value (assume that $\mu_{B^i}(y^i)=1$).

By introducing the concept of the fuzzy basis function vector $\xi(x)$, (20) can be written as

$$y(x)=\theta^{\mathrm{T}}\xi(x) \tag{21}$$

where $\theta=[\theta_1,...,\theta_m]^{\mathrm{T}}$, $\xi(x)=[\xi_1(x),...\xi_2(x)]^{\mathrm{T}}$, and $\xi_j(x)$ is defined as

$$\xi_j(x)=\frac{\prod_{i=1}^{n}\mu_{A_i^j}(x_i)}{\sum_{j=1}^{m}\left(\prod_{i=1}^{n}\mu_{A_i^j}(x_i)\right)}$$

In this paper, we will take two universal fuzzy systems $\widehat{G}_1(x,\theta_1)$ and $\widehat{G}_2(x(t-\tau),\theta_2)$ to approximate the uncertain terms, where θ_1 and θ_2 contain tunable parameters.

Then, by using the above fuzzy rule, we construct the following fuzzy systems

$$\widehat{G}_1(x,\theta_1)=\theta_1^{\mathrm{T}}\xi_1(x)$$

and

$$\widehat{G}_2(x(t-\tau),\theta_2)=\theta_2^{\mathrm{T}}\xi_2(x(t-\tau))$$

where $\xi_1(x)$ and $\xi_2(t-\tau)$ are fuzzy basis functions.

According to the universal approximation theorem there exist optimal approximation parameters θ_1^* and θ_2^* [14], such that $\xi_1(x)^{\mathrm{T}}\theta_1^*$ and $\xi_2(x(t-\tau))^{\mathrm{T}}\theta_2^*$ can approximate $\bar{G}_1(x(t))$ and $\bar{G}_2(x(t-\tau))$ to any desired degree. The parameters θ_1^* and θ_2^* are defined as follows

$$\theta_1^*=\arg\min_{\theta_1\in\Omega_{G1}}\left(\sup_{x\in\Omega_x}\left|\widehat{G}_1(x,\theta_1)-\bar{G}_1(x)\right|\right) \tag{22}$$

$$\theta_2^*=\arg\min_{\theta_2\in\Omega_{G2}}\left(\sup_{x(t-\tau)\in\Omega_x}\left|\widehat{G}_2(x(t-\tau),\theta_2)-\bar{G}_2(x)\right|\right) \tag{23}$$

where Ω_{G1},Ω_{G2} and Ω_x denote the sets of suitable bounds on θ_1,θ_2 and x, respectively. We assume that θ_1,θ_2 and x never reach the boundaries of Ω_{G1},Ω_{G2} and Ω_x . And the minimum approximation error satisfies the following assumption.

Assumption3: The approximation error between $\bar{G}_1(x(t))$ $\bar{G}_2(x(t-\tau))$ and $\xi_1(x)^{\mathrm{T}}\theta_1^*$, $\xi_2(x(t-\tau))^{\mathrm{T}}\theta_2^*$ satisfied the following inequality

$$\begin{aligned}&\left|\bar{G}_1(x)-\theta_1^{*T}\xi_1(x)+\bar{G}_2(x(t-\tau))-\theta_2^{*T}\xi_2(x(t-\tau))\right|\\&\le m_0+m_1\|x\|+m_2\|x(t-\tau)\|\end{aligned} \tag{24}$$

where m_0,m_1 and m_2 are unknown scalars. These two parameters θ_1^* and θ_2^* will be learned by using the adaptive algorithms.

Theorem 2: For system (5), the following control law is chosen

$$u=u_1+u_2+u_3 \tag{25}$$

where

$$u_1=-(\hat{K}B)^{-1}\left(\theta_1^{\mathrm{T}}\xi_1(x)+\theta_2^{\mathrm{T}}\xi_2(x(t-\tau))\right) \tag{26}$$

$$u_2=-(\hat{K}B)^{-1}\left(\vartheta_0+\vartheta_1\|x\|+\vartheta_2\|x(t-\tau)\|\right)\mathrm{sgn}(S) \tag{27}$$

$$u_3=-(\hat{K}B)^{-1}\beta S \tag{28}$$

Then, the sliding surface is reachable in a finite time.

In (26)-(28), β is a positive scalar, $\theta_1,\theta_2,\vartheta_0,\vartheta_1$ and ϑ_2 are adaptive parameters, whose adaptive laws are as follows.

$$\dot{\theta}_1=r_1S\xi_1(x) \tag{29}$$

$$\dot{\theta}_2=r_2S\xi_2(x(t-\tau)) \tag{30}$$

$$\dot{\vartheta}_0=r_3|S| \tag{31}$$

$$\dot{\vartheta}_1=r_4|S|\|x\| \tag{32}$$

$$\dot{\vartheta}_2 = r_5 |S| \|x(t-\tau)\| \tag{33}$$

where r_1, r_2, r_3, r_4 and r_5 are positive scalars.

In the above controller, u_1 contains two fuzzy logic systems used to approximate the unknown nonlinear functions, u_2 is an adaptive controller used to compension for the time-varying uncertainties, and u_3 is used to further guarantee the sliding mode function $S(t)$ asymptotically converge to zero.

Proof: For system (5), defining the following Lyapunov function

$$V = \frac{1}{2}\left[S^2 + \frac{1}{r_1}\bar{\theta}_1^{\mathrm{T}}\bar{\theta}_1 + \frac{1}{r_2}\bar{\theta}_2^{\mathrm{T}}\bar{\theta}_2 + \frac{1}{r_3}\bar{\vartheta}_0^2 + \frac{1}{r_4}\bar{\vartheta}_1^2 + \frac{1}{r_5}\bar{\vartheta}_2^2\right] \tag{34}$$

where $\bar{\theta}_1 = \theta_1^* - \theta_1, \bar{\theta}_2 = \theta_2^* - \theta_2$, $\bar{\vartheta}_0 = \vartheta_0^* - \vartheta_0$, $\bar{\vartheta}_1 = \vartheta_1^* - \vartheta_1$, $\bar{\vartheta}_2 = \vartheta_2^* - \vartheta_2$.

The time derivative along the state trajectory of system (5) is

$$\begin{aligned}\dot{V} = S\dot{S} + \frac{1}{r_1}\bar{\theta}_1^{\mathrm{T}}\dot{\bar{\theta}}_1 + + \frac{1}{r_2}\bar{\theta}_2^{\mathrm{T}}\dot{\bar{\theta}}_2 + \frac{1}{r_3}\bar{\vartheta}_0^{\mathrm{T}}\dot{\bar{\vartheta}}_0 + \\ \frac{1}{r_4}\bar{\vartheta}_1^{\mathrm{T}}\dot{\bar{\vartheta}}_1 + \frac{1}{r_5}\bar{\vartheta}_2^{\mathrm{T}}\dot{\bar{\vartheta}}_2\end{aligned} \tag{35}$$

As we known

$$\begin{aligned}S\dot{S} &= \hat{K}x\hat{K}\dot{x} \\ &= \hat{K}x\hat{K}\left[\left(A + \Delta A(t)\right)x + \left(A_d + \Delta A_d(t)\right)x(t-\tau) + \right. \\ &\quad \left. B\left(u(t) + G(x, x(t-\tau), \sigma(t))\right)\right]\end{aligned} \tag{36}$$

Let

$$\begin{cases}\bar{G}_1(x) = \hat{K}Ax + G_1(x) \\ \bar{G}_2(x(t-\tau)) = \hat{K}A_d x(t-\tau) + G_2(x(t-\tau))\end{cases} \tag{37}$$

In the following, we use two fuzzy logic systems $\hat{G}_1(x, \theta_1) = \theta_1^{\mathrm{T}}\xi_1(x)$ and $\hat{G}_2(x(t-\tau), \theta_2) = \theta_2^{\mathrm{T}}\xi_2(x(t-\tau))$ to approximate $\bar{G}_1(x(t))$ and $\bar{G}_2(x(t-\tau))$. By Assumption 2 and 3, substituting (25)-(33) and (37) into (36), then the following inequality is obtained

$$\begin{aligned}S\dot{S} \le & \left|\hat{K}x\right|\left[(\vartheta_0^* - \vartheta_0) + (\vartheta_1^* - \vartheta_1)\|x\| + (\vartheta_2^* - \vartheta_2)\|x(t-\tau)\|\right] \\ & + \left[(\theta_1^* - \theta_1)^{\mathrm{T}}\xi_1(x) + (\theta_2^* - \theta_2)^{\mathrm{T}}\xi_2(x(t-\tau))\right]\hat{K}x - \beta S^2\end{aligned} \tag{38}$$

Then we can get the following inequality by substituting (29)-(33) and (38) into (35), we can get

$$\dot{V} \le -\beta S^2 \tag{39}$$

From (39), it is easy to see that the reaching condition is satisfied.

IV. SIMULATION RESULTS

In this section, we validate the effectives and performance of the scheme of this paper by simulation. The choosing of the parameters is based on [15]. $N = 50$, $C = 300$ packets/s, $R_0 = 0.533\text{s}$, $q_d = 100$ packets. Thus, the parameters are achieved.

$$A = \begin{bmatrix}-0.6 & -0.01 \\ 94 & -2\end{bmatrix},\ A_d = \begin{bmatrix}-0.6 & 0.01 \\ 0 & 0\end{bmatrix},\ B = \begin{bmatrix}10 \\ 0\end{bmatrix}.$$

For simulation, we chose the following uncertain parameter as follows.

$\Delta A_{21} = \Delta A_{d21} = 0.1\sin t$, $\Delta A_{22} = \Delta A_{d22} = 0.02\cos t$, $\beta = 2$, $G_1(x) = \frac{1}{2}\|x\|^2$, $G_2(x(t-\tau)) = \|x(t-\tau)\|^2$, $\Delta G = 0.1\sin t$.

Using LMI toolbox in the matlab, we can get $\bar{K}$, thus sliding surface is obtained as follows

$$S(x) = 3x_1 + x_2$$

From the above section, we can also get the reaching law, choosing adaptive parameters as $r_1 = 5$, $r_2 = 1$, $r_3 = 8$, $r_4 = 3$, $r_5 = 10$.

The membership functions are chosen as

$$\mu_1(x_i) = \exp\left[-\left((x_i + \pi/6)/(\pi/24)\right)^2\right]$$

$$\mu_2(x_i) = \exp\left[-\left((x_i + \pi/12)/(\pi/24)^2\right)\right]$$

$$\mu_3(x_i) = \exp\left[-\left(x_i/(\pi/24)\right)^2\right]$$

$$\mu_4(x_i) = \exp\left[-\left((x_i - \pi/12)/(\pi/24)\right)^2\right]$$

$$\mu_5(x_i) = \exp\left[-\left((x_i - \pi/6)/(\pi/24)\right)^2\right]$$

The performance and effectiveness of the proposed AFSMC are verified in a series of numerical simulation via matlab/simlink for the network.

In Fig.1 we choose the network parameters as above. We

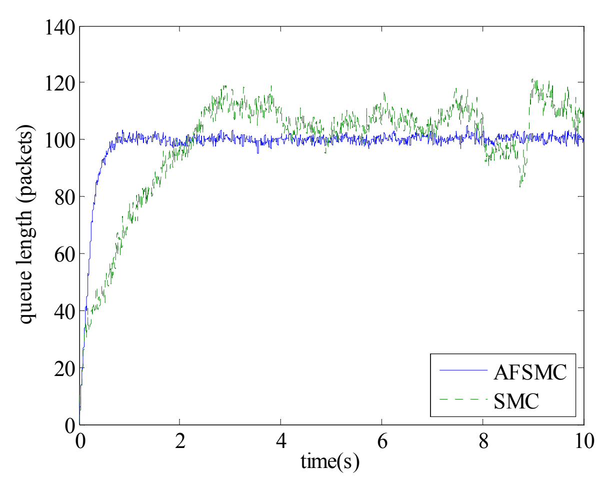

Fig.1. Queue length responses with fixed parameters

can see that AFSMC obtain fast and stability responses. However, SMC has big chattering and slow responses.

In order to test the robust performance of AFSMC, we vary N from 50 to 130, C from 300 to 250. The simulation results are given in Fig.2 and Fig.3. SMC have strong instability by the improper parameters. However, AFSMC maintains the instantaneous queue length closed to the desired queue. Therefore, we can conclude that AFSMC scheme performs well characteristic under varied network parameters, and it is very adapt to the network congestion control.

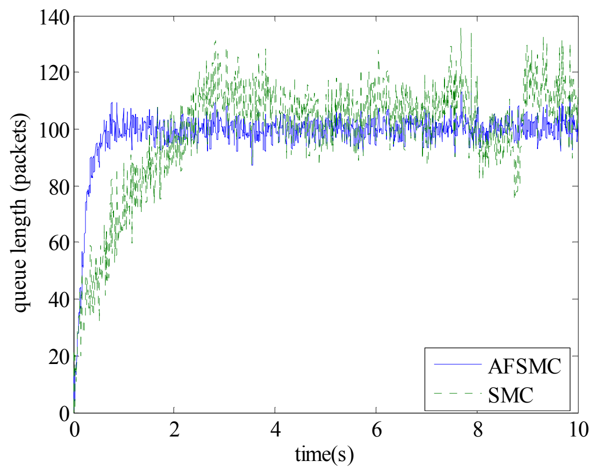

Fig.2. Queue length responses with varied network parameters and disturbance

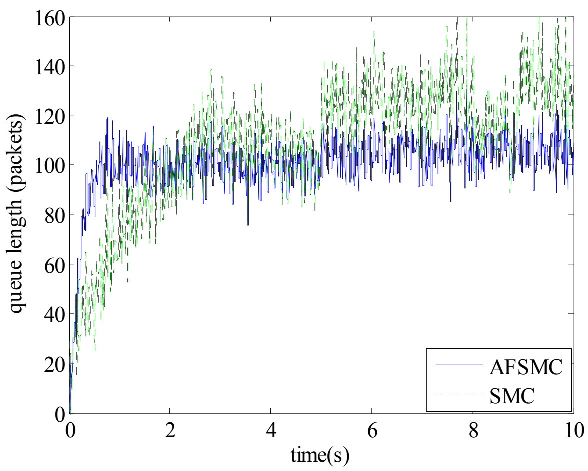

Fig.3. Queue length responses with varied network parameters disturbance and bursting flows

V. Conclusion

In this paper, we have proposed an adaptive fuzzy sliding mode controller for AQM systems with time varying, uncertainties and unknown nonlinear disturbance. The sliding surface and the corresponding reaching law are designed. Different from the existing literature, a Lyapunov-Krasovskii function is constructed for the practice TCP network, and the sliding motion is asymptotic stability. The designed adaptive fuzzy sliding mode control reaching law can drive the state trajectory of the system onto the sliding surface in limited time. The simulation results verify the validity of our main results, and the presented method can obtain faster transients and avoid network congestion.

References

[1] S.Flogd and V.Jacobson, "Random early detection gateways for congestion avoidance," *IEEE/ACM Transactions on Networking*, vol. 1, no 4, pp. 397–413, 1993.

[2] D. Lin, R. Morris, "Dynamics of random early detection," *In Proceedings of the ACM SiGCom'97,* Cannes, 1997, pp. 127–137.

[3] W Feng, D Kandlur and D Saha. "A self-configuring RED gateway," *In Proceedings of the IEEE INFOCOM,* New York, 1999, pp. 1320–1328.

[4] T J Ott, T V Lakshman and L H Wong. "SRED: stabilized RED," *In Proceedings of the IEEE INFOCOM,* New York, 1999, pp. 1346–1355.

[5] V Misra, W B Gong and D Towsley. "Fluid-based Analysis of a Network of AQM Routers Supporting TCP Flows with an Application to RED," *In Proceedings of ACM/S IGCOMM,* Stockholm, 2000, pp. 152–160.

[6] C. Hollot, V. Misra, D. Towsley and W. B. Gong, "On designing improved controllers for AQM routers supporting TCP flows," *In Proceedings of the IEEE INFOCOM,* Alaska, USA, 2001, pp. 1726–1734.

[7] S Ryu, C Rump, "Design of load-adaptive queue management for Internet congestion control," *In Proceedings of ICOIN,* Cheju Island, Korea, 2003, pp. 24–34.

[8] C Wang, "API: adaptive proportional-integral algorithm for active queue management under dynamic," *In Proceedings of High Performance Switching and Routing,* Phoenix, Arizona, 2004, pp.51–55.

[9] P Yan, Y Gao, and Ö Hitay. "A variable structure control approach to active queue management for TCP with ECN," *IEEE Transaction on Control Systems Technology*, vol.13, no.2, pp.203–215, 2005.

[10] F Y Ren, C Lin, and X H Yin. "Design a congestion controller based on sliding mode variable structure control," *Computer communications,* vol. 28, no. 9, pp. 1050–1061, 2005.

[11] F J Yin, M D Georigi and Y W Jing. "Robust stabilization of uncertain input delay for internet congestion control," *In Proceedings of the 2006 American Control Conference Minneapolis*, Minnesota, USA, 2006, pp. 5576–5580.

[12] S. Manfredi, M di Bernardo and F Garofalo, "Reduction-based robust active queue management control," *Control Engineering Practice*, vol.15, pp. 177–186, 2007.

[13] F. M. Yu, H. Y. Chung and S. Y. Chen, "Fuzzy sliding mode controller design for uncertain time-delay systems with nonlinear input," *Fuzzy Sets and System*, vol. 140, no. 2, pp. 359–374, 2003.

[14] L.X.Wang, "Adaptive Fuzzy control: Design and stability analysis," *Prentice-Hall, Englewood Cliffs*, NJ, 1994, pp.140–154.

[15] P F Quet, H Özbay. "On the design of AQM supporting TCP flows using robust control theory," *IEEE Transactions on Automatic control*, vol. 4, pp. 1031–1036, 2004.