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Reliable Static Output Control for a Class of Uncertain Systems Via High-order Sampled-data Hold Controller

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Abstract— **The problem of robust reliable controller design against actuator faults for systems with polytopic type uncertainty is addressed. The strategy that using sampled-data control with high-order hold is adopted to tackle the situations that no sampled-data controller with zero-order hold in existence. This problem is formulated in terms of solutions to a set of linear matrix inequalities. The efficiency of the proposed method is demonstrated by a numerical example.**

I. Introduction

More and more advanced technological systems rely on sophisticated control systems to increase their safety and performances. In the event of system component failures, the conventional feedback control designs may result in unsatisfactory performances or even instability, especially for complex systems such as aircrafts, space craft ,nuclear power plants, etc. This has ignited enormous research activities in search for new design methodologies for accommodating the component failures and maintaining the acceptable system stability and performances, so that abrupt degradation and total system failures can be avoided. This types of control is often known as fault-tolerant control (FTC).

The main task of this study is to design a sampled-data *passive* FTC Controller, or the so-called *reliable* controller for continuous-time uncertain linear systems with polytopic uncertainties such that the closed-loop system is reliable and robust stable. Reliable control problems have been extensively studied, and several approaches have been proposed. Such as, for linear systems, there are coprime factorization approach [15], algebraic Riccati equation (ARE) approach [14] [13] [2] [2]; pole region assignment technique [6], Youla parametrization approach [7], linear matrix inequality (LMI) approach [3] [4] [8]; for nonlinear systems, there are Hamilton.Jacobi inequality (HJI) approach [1] [14], variable structure control approach [12], T-S fuzzy model based approach [8][9] [10] [11] and so on.

Form the system modeling point of view, using uncertain models instead of LTI models is more suitable in many real-world safety-critical systems such as aircrafts since there are multiple operating points and the accurate value of parameters is not available. For example, a F-16 aircraft is modeled in polytopic convex polyhedron in [4]. However, only few papers [4] [5] [24] have been investigated on the reliable control design for the uncertain systems, which is a more challenging problem since the controller is needed not only to be robust with respect to the uncertainties but also reliable to accommodate the failures of actuators.

The last decade has witnessed a remarkable improvement in the analysis and the design of systems with polytopic-type uncertainties, ([22] opened a new horizon for linear matrix inequality based approach by virtue of parameter-dependent lyapunov variables. In the following years, many researchers have Struggled for more accurate results with further less conservatism by introducing extra auxiliary variables (see [23] [27] [26] and references there in). However, those improved LMI conditions are almost all of sufficiency, in other words, there may no feasible solutions exist in some cases, and naturally, the probability of infeasibility will be increased while take the actuator failures into consideration. Thus, new control strategy should be resorted to in those situations.

The idea of using generalized sampled-data hold functions instead of the conventional zero-order hold (ZOH) in control systems was first proposed in [16] [21]. Several properties and applications such as decentralized control have been investigated in [17] [18] [19] [20]. However, the problems studied in those papers were formulated as a two-boundary point differential equation whose analytical solution is cumbersome. Here, the high-order sampled data hold function is suggested since the problems can be formulated in finding the coefficients of the HOH function which can transform to a LMI problem.

In this paper, reliable static output feedback with HOH sampled-data control strategy for uncertain continuous-time linear time-invariant systems with convex polytopic uncertainties is presented. The problem is converted to find the generalized gain (coefficient) matrix firstly, and on accounting of using the properties of the null space of output matrices, sufficient conditions for the reliable controller design are given in terms of solutions to a set of linear matrix inequalities.

This paper is organized as follows. SectionII presents

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a introduction about sampled-data control with high-order and gives the problem under consideration and the useful lemmas. In III, a reliable controller design method is proposed. IV presents a illustrative example. Finally, Section V concludes the paper.

Notation: The superscript T stands for matrix transposition and the notation M^{-T} denotes the transpose of the inverse matrix of M. $C^{\perp}$ denotes the matrix such that $CC^{\perp}=0$.

II. Preliminaries and Problem Statements

In this section, we introduce the high-order sampled data control strategy first and give the considered problem in the sequel.

A. High-order Sample-data Control

Consider the following linear time invariant continuous-time systems

$$x(t)=Ax(t)+Bu(t) \tag{1}$$

where $x\in R^n$ is the system state vector, $u(t)\in R^m$ is the control input. Then, a sampled-data controller with H-order hold function can be represented as

$$\begin{aligned}&u_j(t+\kappa T)\\ &=h_j^1(t)x_1[\kappa T]+h_j^2(t)x_2[\kappa T]+,...,h_j^n(t)x_n[\kappa T]\\ &j=1,...,m,\quad 0\le t<T,\quad \kappa=0,1,2,...,\end{aligned} \tag{2}$$

where

$$\begin{cases}h_j^1(t)=(k_{j_0}^1+k_{j_1}^1t+,...,+k_{j_H}^1t^H)\\ h_j^2(t)=(k_{j_0}^2+k_{j_1}^2t+,...,+k_{j_H}^2t^H)\\ \quad\vdots\\ h_j^n(t)=(k_{j_0}^n+k_{j_1}^nt+,...,+k_{j_H}^nt^H)\end{cases} \tag{3}$$

then the (2) can be denoted by the following compact form

$$u_j(t)=[1,t,...,t^H]K_jx[k] \tag{4}$$

where $K_j\in R^{(H+1)\times n}$

$$K_j:=\begin{bmatrix}K_{j_0}\\ K_{j_1}\\ \vdots\\ K_{j_H}\end{bmatrix}=\begin{bmatrix}k_{j_0}^1 & k_{j_0}^2 & \cdots & k_{j_0}^n\\ k_{j_1}^1 & k_{j_1}^2 & \cdots & k_{j_1}^n\\ \vdots & \vdots & \ddots & \vdots\\ k_{j_H}^1 & k_{j_H}^2 & \cdots & k_{j_H}^n\end{bmatrix} \tag{5}$$

It is known that the state of the system (1) under the control law $\sum_{j=1}^{m}u_j(t)$ is given by:

$$\begin{aligned}&x(t)=e^{(t-\lambda T)A}x(\lambda T)+\int_{kT}^{t}e^{(t-\tau)A}Bu(\tau)d\tau,\\ &\qquad\lambda T\le t<(\lambda+1)T,\quad \lambda=0,1,2,...,\end{aligned} \tag{6}$$

By substituting $t=(\lambda+1)T$ in (6), the continuous time system (1) can be represented in the following discretized form

$$x((\lambda+1)T)=A_dx(\lambda T)+B_dKx(\lambda T)\quad \lambda=0,1,2,..., \tag{7}$$

where

$$A_d=\mathscr{D}_A(A,T,H)=e^{AT} \tag{8}$$

$$B_d=\mathscr{D}_B(B,T,H)=\begin{bmatrix}B_{d_0} & B_{d_1} & \cdots & B_{d_H}\end{bmatrix}$$

$$\begin{cases}B_{d_0}:=\int_0^T e^{(T-\tau)A}Bd\tau\\ \qquad=(e^{AT}-I)A^{-1}B\\ B_{d_1}:=\int_0^T e^{(T-\tau)A}tBd\tau\\ \qquad=2(e^{AT}-I)A^{-2}B-TA^{-1}B\\ \vdots\\ B_{d_H}:=\int_0^T e^{(T-\tau)A}t^HBd\tau\\ \qquad=H(e^{AT}-I)A^{-(H+1)}B-\sum_{i=1}^{H}T^iA^{-(H+1-i)}\end{cases} \tag{9}$$

$$\begin{aligned}&K=\begin{bmatrix}K_0^T & K_1^T & \cdots & K_H^T\end{bmatrix}^T\\ &K_i=\begin{bmatrix}K_{1_i}^T & K_{2_i}^T & \cdots & K_{m_i}^T\end{bmatrix}^T \quad \text{for}\quad i=0,...,H\end{aligned} \tag{10}$$

B. Problem Description

Consider the following continuous-time system

$$\begin{aligned}\dot{x}(t)&=A^cx(t)+B_1^cd(t)+B^cu(t)\\ z(t)&=C_1^cx(t)+D_{12}^cu(t)\\ y(t)&=C^cx(t)\end{aligned} \tag{11}$$

where $x\in R^n$ is the system state vector, $u(t)\in R^m$ is the control input, $d(t)\in R^p$ is the disturbance, $y\in R^r$ is the measurement output, $z(t)\in R^q$ is the controlled output. The matrices $A^c,B_1^c,B^c,C_1^c,D_{12}^c$ and C^c are appropriately dimensioned with partially unknown parameters. They belong to the following uncertainty polytope:

$$\begin{aligned}\Omega=\Big\{&(A^c,B_1^c,B^c,C_1^c,D_{12}^c,C^c)\\ &(A^c,B_1^c,B^c,C_1^c,D_{12}^c,C^c)\\ &=\sum_{i=1}^{N}\alpha_i(A_i^c,B_{1i}^c,B_i^c,C_{1i}^c,D_{12i}^c,C_i^c),\alpha_i\ge 0,\sum_{i=1}^{N}\alpha_i=1\Big\}\end{aligned} \tag{12}$$

Without loss of generality, the output matrices $C_i^c, 1\le i\le N$ are assumed to be of full row rank, and let invertible matrices $T_i, 1\le i\le N$, such that

$$C_i^cT_i=\begin{bmatrix}I & 0\end{bmatrix}\quad \text{for}\quad 1\le i\le N \tag{13}$$

Remark 1. For each C_i^c, the corresponding T_i generally is not unique. A special T_i can be obtained by following formula,

$$T_i = \left[\begin{array}{cc} C_i^{cT}(C_i^c C_i^{cT})^{-1} & C_i^{c\perp} \end{array} \right] \quad (14)$$

To investigate the reliable control problem in the event of actuator faults, the fault model must be established first. Let $u_F(t)$ represent the control input vector after failures have occurred. Then the following actuator fault model [4] is adopted for this study:

$$u^F(t) = w_L u(t), \quad L = 1, 2, ..., l_p, \quad l_p \le 2^m - 1 \quad (15)$$

where the scaling factor $w_L (L = 0, 1, ..., l_p, l_p \le 2^m - 1)$ satisfies

$$\begin{array}{c} w_L \in \Omega_f \triangleq \{w_L = diag\,[w_{L1}, w_{L2}, ..., w_{Lm}] \\ w_{Lj} = 0 \text{ or } 1,\ j = 1, 2, ...m\}\,. \end{array} \quad (16)$$

Obviously, when $w_{Lj} = 0$ for $1 \le j \le m$, the fault model (15) corresponds to the case of jth actuator outage. When $w_{Lj} = 1$, it corresponds to the case of no fault in the jth actuator. Without loss of generality, we assume that $w_0 = \boldsymbol{I}$, namely, $L = 0$ corresponds to the normal control input vector $u_F(t) = u(t)$.

The objective of this paper is to design a sampled-data static output feedback controller with H-order hold function for the uncertain systems (11)

$$\begin{array}{c} u(t + \kappa T) = [1, t, ..., t^H]Kx[\kappa T] \\ 0 < t < T, \quad \kappa = 0, 1, 2, ... \end{array} \quad (17)$$

where the gain matrix K is defined by (10), such that the resulting closed-loop system is not only robust stable with respect to the uncertainties but also reliable with respect to the actuator failures.

The following preliminary lemmas will be used in this sequel.

Lemma 1. [25] Let $\xi \in R^n$, $\mathcal{P} = \mathcal{P}^T \in R^{n\times n}$, and $\mathcal{H} \in R^{m\times n}$ such that $rank(\mathcal{H}) = r < n$. The following statements are equivalent:

1) $\xi^T \mathcal{P} \xi < 0$, for all $\xi \ne 0$, $\mathcal{H}\xi = 0$;

2)$\exists \mathcal{X} \in R^{n\times m}$ such that $\mathcal{P} + \mathcal{X}\mathcal{H} + \mathcal{H}^T\mathcal{X}^T < 0$.

This lemma is known as *Finsler's Lemma*.

Lemma 2. [23] If the symmetric matrices $V_i j \in R^{n\times n}$ are such that

$$\begin{array}{ll} V_{ij} + V_{ji} \le 0, & 1 \le j < i \le N \\ \sum\limits_{i=1}^{N} (V_{ij} + V_{ji}) \le 0, & j = 1, ..., N \end{array} \quad (18)$$

then the following inequlity

$$\sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_i \alpha_j V_{ij} \le 0 \qquad \forall \alpha \in \Lambda \quad (19)$$

holds, where Λ is the simplex

$$\Lambda := \left\{ \alpha \in R^N : \alpha_i \ge 0,\ \sum_{i=1}^{N} \alpha_i = 1 \right\} \quad (20)$$

III. Main Results

In this section, we assume the system (11) is disturbance free $(i.e. d(t) = 0)$. Then the discretized uncertain system can be represented in the following form without consideration about the controlled output $z(t)$

$$\begin{array}{l} x(\kappa + 1) = Ax(\kappa) + Bu(\kappa) \\ y(\kappa) = Cx(\kappa) \end{array} \quad (21)$$

The matrices A, B, C are belong to the following discretized uncertainty polytope:

$$\Omega = \left\{ (A, B, C)(A, B, C) = \sum_{i=1}^{N} \alpha_i \right. \\ \left. (A_i, B_i, C_i), \alpha_i \ge 0, \sum_{i=1}^{N} \alpha_i = 1 \right\} \quad (22)$$

where

$$\left\{ \begin{array}{l} A_i = \mathscr{D}_A(A, T, H) \\ B_i = \mathscr{D}_A(B, T, H) \\ C_i = C_i^c \end{array} \right. \quad (23)$$

To make the expression more concise, we denote that

$$B_{Li} = B_i \left[\begin{array}{cccc} w_L & w_L & ... & w_L \end{array} \right]$$

then B_{Li} can be viewed as the generalized input matrix with actuator faults. The following theorem presents a reliable static output feedback controller design for the uncertain system (11).

Theorem 1. *If there exist symmetric matrices $P_{Li} \in R^{n\times n}, V_{Lij} \in R^{2n\times 2n}$ and matrices $G_{Li} \in R^{n\times n}$, $Y \in R^{(H+1)m\times n}$ $1 \le i \le N$, $1 \le L \le l_p$, with the following structure*

$$\begin{array}{l} G_{Li} = \left[\begin{array}{cc} G_{11} & 0 \\ G_{21}^{Li} & G_{22}^{Li} \end{array} \right] \\ V_{Lij} = \left[\begin{array}{cc} V_{11}^{Lij} & V_{12}^{Lij} \\ (V_{12}^{Lij})^T & V_{22}^{Lij} \end{array} \right] \\ Y = \left[\begin{array}{cc} Y_1 & 0 \end{array} \right] \end{array} \quad (24)$$

satisfying the following LMIs,

$$\left[\begin{array}{ccc} -T_i G_{Li} - G_{Li}^T T_i^T & * & * \\ A_i G_{Li} + B_{Lj} Y & -P_{Li} + V_{Lij}^{11} & * \\ P_{Lj} & (V_{ij}^{12})^T & -P_{Lj} + V_{Lij}^{22} \end{array} \right] < 0 \\ 1 \le i, j \le N, \quad (25)$$

$$V_{ij} + V_{ji} \le 0, \qquad 1 \le j < i \le N$$
$$\sum_{i=1}^{N} (V_{ij} + V_{ji}) \le 0, \qquad j = 1, ..., N \tag{26}$$

where $T_i, i = 1, ..., N$, satisfying (34) and are nonsingular, then the system (11) is said to be reliable stable with the H-order sampled-data hold controller (17), where

$$K = Y_1 G_{11}^{-1} \tag{27}$$

Proof. From the structure of L, S_i and (30), (27), we can obtain

$$\begin{aligned} Y &= \begin{bmatrix} KG_{11} & 0 \end{bmatrix} \\ &= \begin{bmatrix} K & 0 \end{bmatrix} \begin{bmatrix} G_{11} & 0 \\ G_{21}^{Li} & G_{22}^{Li} \end{bmatrix} \\ &= KC_iT_iG_{Li} \end{aligned} \tag{28}$$

Substituting Y for $KC_iT_iG_i$ in (25), then the (25) can be rewritten as follows:

$$\begin{bmatrix} -T_iG_{Li} - G_{Li}^TT^T & * & * \\ A_iT_iG_{Li} + B_{Lj}KC_iT_iG_{Li} & -P_{Li} + V_{Lij}^{11} & * \\ P_{Lj} & (V_{Lij}^{12})^T & -P_{Lj} + V_{Lij}^{22} \end{bmatrix} < 0$$
$$1 \le i, j \le N, \tag{29}$$

Let $S_{Li} = (T_iG_{Li})^{-1}$ and pre- and post-multiplying (29) by

$$\begin{bmatrix} S_{Li}^T & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \tag{30}$$

and its transpose, it follows that

$$\begin{bmatrix} -S_{Li} - S_{Li}^T & * & * \\ A_i + B_{Lj}KC_i & -P_{Li} + V_{Lij}^{11} & * \\ P_{Lj}S_{Li} & (V_{Lij}^{12})^T & -P_{Lj} + V_{Lij}^{22} \end{bmatrix} < 0$$
$$1 \le i, j \le N, \tag{31}$$

Multiplying (31) by $\alpha_i\alpha_j$ and summing them, then we have

$$\begin{bmatrix} -\mathscr{S}_L - \mathscr{S}_L^T & * & * \\ \mathscr{A}_d + \mathscr{B}_{dL}K\mathscr{C}_i & -\mathscr{P}_L & * \\ \mathscr{P}_L\mathscr{S}_L & 0 & -\mathscr{P}_L \end{bmatrix} + \begin{bmatrix} 0 & * & * \\ 0 & & \\ 0 & & \mathscr{V}_L \end{bmatrix} < 0 \tag{32}$$

where

$$\begin{aligned} \mathscr{S}_L &= \sum_{i=1}^{N} \alpha_i S_{Li} \\ \mathscr{P}_L &= \sum_{i=1}^{N} \alpha_i P_{Li} \\ \mathscr{V}_L &= \sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_i\alpha_j \begin{bmatrix} V_{Lij}^{11} & V_{Lij}^{12} \\ (V_{Lij}^{12})^T & V_{Lij}^{22} \end{bmatrix} = \sum_{i=1}^{N}\sum_{j=1}^{N} \alpha_i\alpha_j V_{ij} \end{aligned} \tag{33}$$

let $\mathscr{W}_L = \mathscr{S}_L^{-1}$ and pre- and post-multiply (32) by

$$\begin{bmatrix} \mathscr{W}_L^T & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \tag{34}$$

and its transpose, then we can obtain

$$\begin{bmatrix} -\mathscr{W}_L - \mathscr{W}_L^T & * & * \\ (\mathscr{A}_d + \mathscr{B}_{dL}K\mathscr{C}_i)\mathscr{W}_L & -\mathscr{P}_L & * \\ \mathscr{P}_L & 0 & -\mathscr{P}_L \end{bmatrix} + \begin{bmatrix} 0 & * & * \\ 0 & & \\ 0 & & \mathscr{V}_L \end{bmatrix} < 0 \tag{35}$$

Applying Lemma 2 to the inequalities (26), it follows that$\mathscr{V}_L < 0$, from (35), then we have

$$\begin{bmatrix} -\mathscr{W}_L - \mathscr{W}_L^T & * & * \\ (\mathscr{A}_d + \mathscr{B}_{dL}K\mathscr{C}_i)\mathscr{W}_L & -\mathscr{P}_L & * \\ \mathscr{P}_L & 0 & -\mathscr{P}_L \end{bmatrix} < 0 \tag{36}$$

which is equivalent to

$$\begin{bmatrix} \mathscr{P}_L - \mathscr{W}_L - \mathscr{W}_L^T & * \\ (\mathscr{A}_d + \mathscr{B}_{dL}K\mathscr{C}_i)\mathscr{W}_L & -\mathscr{P}_L \end{bmatrix} < 0 \tag{37}$$

Denoting that

$$\begin{aligned} \mathcal{P} &= \begin{bmatrix} \mathscr{P}_L & 0 \\ 0 & -\mathscr{P}_L \end{bmatrix} \\ \mathcal{H} &= \begin{bmatrix} -I & (\mathscr{A} + \mathscr{B}_LK\mathscr{C}) \end{bmatrix} \\ \xi &= \begin{bmatrix} x^T(\kappa+1) & x^T(\kappa) \end{bmatrix}^T \\ \mathcal{X} &= \begin{bmatrix} \mathscr{W}_L^T & 0 \end{bmatrix}^T \end{aligned} \tag{38}$$

then by applying Finsler's Lemma to (37), we can conclude that

$$\xi^T \begin{bmatrix} \mathscr{P}_L & 0 \\ 0 & -\mathscr{P}_L \end{bmatrix} \xi < 0 \tag{39}$$

we can conclude that the system (21) is stable not only with respect to uncertainties but also with respect to the actuator failures from the lyapunov stability theory. Moreover, from (25) , we can deduce that the matrices T_iG_{Li} are positive-definite (not necessarily symmetric) which implies that the matrices G_{Li}, and implicity G_{11}, are invertible because T_i are invertible. Then $Y_1 = KG_{11}$ admits the solution (27). Thus, the proof is complete.

Remark 2. Theorem 1 given a sufficient condition for the design of sampled-data controller with H-order hold function, if $H = 0$, then the controller is of the conventional zero-order hold controller. Compared with HOH, the ZOH controller is suggested if the LMIs (25)(26) is feasible.

IV. **Numerical Example**

To illustrate the effectiveness of our results, a numerical example and its simulation results are given in this section.

Example 1. Consider a uncertain system which belongs to the 2-polytopic convex polyhedron in the form of (11) with $d(t) = 0$ and

$$A_1^c = \begin{bmatrix} 0.4994 & 1.5831 \\ -0.0841 & -0.4500 \end{bmatrix}$$
$$B_1^c = \begin{bmatrix} 1.2608 & 1.2934 \\ 0.9794 & -0.3834 \end{bmatrix} \quad C_1^c = \begin{bmatrix} 0.6603 & 1.6629 \end{bmatrix}$$
$$A_2^c = \begin{bmatrix} 0.9134 & -0.6648 \\ -0.0681 & -1.9108 \end{bmatrix}$$
$$B_2^c = \begin{bmatrix} 0.5673 & 1.7751 \\ -0.8611 & 0.3467 \end{bmatrix} \quad C_2^c = \begin{bmatrix} 1.7925 & 0.8523 \end{bmatrix} \tag{40}$$

With conventional ZOH sampled-data control and sampling period $T = 1s$, the discretized model are:

$$\begin{aligned} A_1^0 &= \begin{bmatrix} 1.5667 & 1.6477 \\ -0.0875 & 0.5785 \end{bmatrix} \quad C_1^0 = C_1^a \\ B_1^0 &= B_{10} = \begin{bmatrix} 2.3971 & 1.3334 \\ 0.7145 & -0.3567 \end{bmatrix} \\ A_2^0 &= \begin{bmatrix} 2.5195 & -0.5557 \\ -0.0569 & 0.1589 \end{bmatrix} \quad C_2^0 = C_2^a \\ B_2^0 &= B_{20} = \begin{bmatrix} 1.1736 & 2.8186 \\ -0.4039 & 0.1050 \end{bmatrix} \end{aligned} \tag{41}$$

It can be checked that there exist a robust controller to stabilize the (40) via LMIs in Theorem 1 while no actuator failures take into consideration. Here, we assume that the 2th actuator is prone to be failed, which means that

$$w_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad w_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \tag{42}$$

Considering the fault case with the above discrete model, the LMIs of Theorem 1 are infeasible. Now we resort to HOH sampled-data control strategy, and letting $H = 1$, then the discretized model can be represented as

$$\begin{matrix} A_1^1 = A_1^0 & & A_2^1 = A_2^0 \\ B_1^1 = \begin{bmatrix} B_{10} & B_{11} \end{bmatrix} & & B_2^1 = \begin{bmatrix} B_{20} & B_{21} \end{bmatrix} \\ C_1^1 = C_1^0 & & C_2^1 = C_2^0 \end{matrix} \tag{43}$$

where

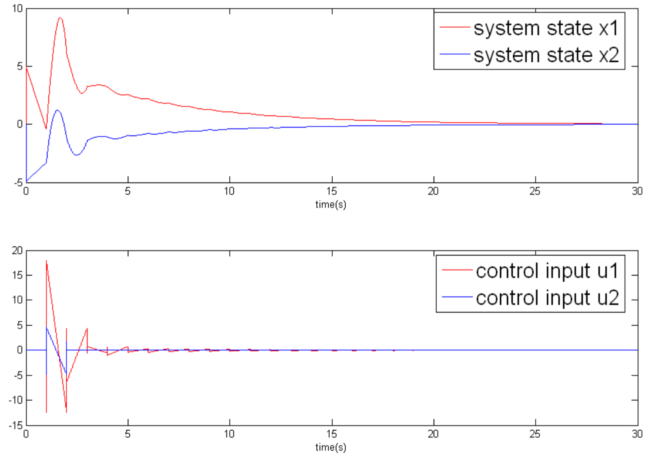

Fig. 1. Evolution curves of the vertex (A_1, B_1, C_1)

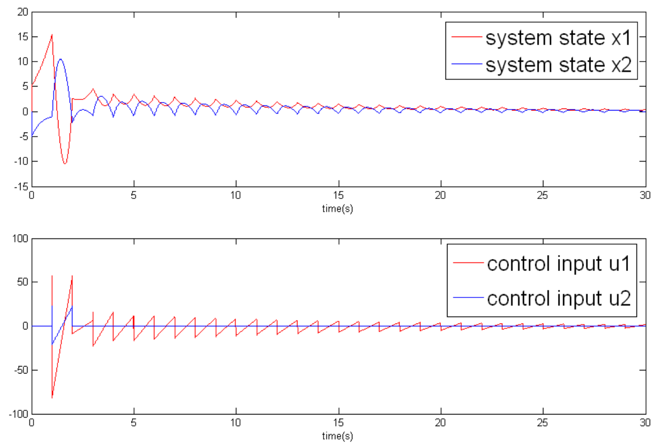

Fig. 2. Evolution curves of the vertex (A_2, B_2, C_2)

$$\begin{aligned} B_{11} &= \begin{bmatrix} 1.0049 & 0.6583 \\ 0.4008 & -0.1824 \end{bmatrix} \\ B_{21} &= \begin{bmatrix} 0.4773 & 1.2032 \\ -0.2563 & 0.0836 \end{bmatrix} \end{aligned} \tag{44}$$

Now, the LMIs of Theorem 1 is feasible, and one of solutions can be obtained as

$$K = \begin{bmatrix} -3.0472 \\ -0.7922 \\ 5.2180 \\ 1.6383 \end{bmatrix} \tag{45}$$

then the sampled data controller can be described as

$$u(t + \kappa T) = \begin{cases} (-3.0472 + 5.2180t)y(\kappa T) \\ (-0.7922 + 1.6383t)y(\kappa T) \end{cases} \\ 0 \le t < T, \quad \kappa = 0, 1, 2, ... \tag{46}$$

To verify the effectiveness of the proposed method, the simulation are given with the following initial condition $x^T(0) = [5, -5]$ and it is assume that the 2th actuator failed at $2s$. Fig.1 and Fig.2 show the simulation results.

Fig.1 is the responses curves of the system' states and control inputs with the vertex (A_1, B_1, C_1). Fig.2 is the responses curves of the uncertain system' states and control inputs with the vertex (A_2, B_2, C_2). It can be observed from the Fig.1 and Fig.2 that the 1-order sampled-data hold controller (46) renders the system (40) not only robust stable with respect to the polytopic uncertainties but also reliable stable with respect to the actuator failures.

V. CONCLUSIONS

In this paper, a reliable static output feedback controller design method for uncertain systems have been proposed based on the high-order sampled-data control strategy. The sampled-data control with HOH was introduced firstly, then, a sufficient condition for the controller design was given in terms of solutions to a set of linear matrix inequalities. A numerical example has shown the effectiveness of the proposed method. The further study will extend the method to H_2 control to take the intersample ripple effect into account.

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