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Variable-structure PI Controller for Tank Level Process

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Abstract—**A variable structure (VS) PI controller for the level process is proposed. It is shown via the theory and simulations presented that the VS PI controller has higher performance than the conventional PI controller for the process considered. Tuning rules for the VS PI controller are given.**

I. INTRODUCTION

LEVEL control in various tanks and vessels is one of the most common control problems in the process industry. It can usually be categorized into (a) the process control in which maintaining the level to a certain set point is the primary objective, and (b) the control in which large level fluctuations are allowed and even assumed – the case of so called surge vessels. In the latter case the primary control objective is the outflow stabilization. In the present paper, only the first category of level control objectives will be considered.

Normally level is controlled by a PI or PID controller, which can be implemented as a part of a distributed control system (DCS) or locally. The controllers are tuned in accordance with established methods [1]-[4]. However, very often satisfactory performance can hardly be achieved. This happens due to the integrating character of the process, which in combination with the integral term of the PI/PID controller results in a double integrator in the loop. The presence of the integral term is absolutely necessary (to ensure zero error in a steady state) and the use of a PI controller with an integrating process usually results in oscillatory transients. In summary, level process is not as easy to control in terms of providing a good performance as it might seem.

PID controllers are used more seldom for the considered process than PI controllers, as the performance improvement due to introduction of the derivative term is marginal while the derivative term would amplify the measurement noise. Therefore, we limit our analysis and design to the case of PI controllers. However, the use of a PID controller would differ from the presented analysis only by the tuning rules.

The variable-structure (VS) control was proposed a few decades ago and was mainly developed as a sliding mode control [5]. However, practically the same ideas were used as a foundation of such recent developments in the automatic control as hybrid systems and switched systems. It was shown in [5] using examples of second-order systems that via switching feedback gains dependent on the state variables, the closed-loop system could be made stable even if the plant was unstable or marginally stable. The same idea is used in this work and being further developed for application to the tank level control. The objective of this paper is, therefore, to develop a VS PI controller for the level process and a methodology of its tuning.

The paper is organized as follows. At first the model of a PI-controlled level process disturbed by flow change is considered. Performance of the loop is analyzed in the state space. Then the dynamics of the loop having valve dynamics is considered. After that, tuning rules for a VS PI controller are derived. And finally, a simulation example is given.

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II. SIMPLIFIED MODEL OF LEVEL PROCESS AND VS PRINCIPLE

The model of the level process can be schematically represented by a tank, which has a controlled inflow and uncontrolled outflow (Fig. 1). In many cases the actual arrangement is the opposite: the inflow is uncontrolled, and level control is done via manipulating the outflow. Yet, the second situation can be transformed into the first one via changing the flow signs.

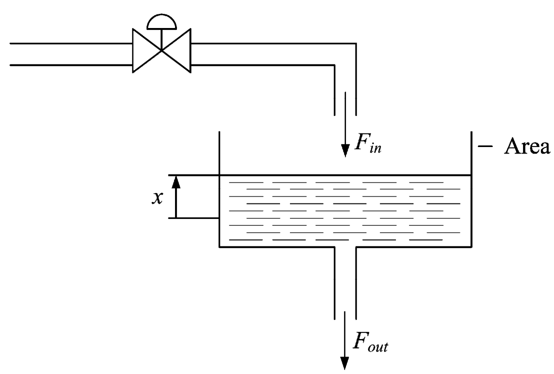

Fig.1. The tank level process.

Let us consider that we can manipulate the inflow through some linear dynamics, so that in a steady state the inflow is proportional to the controller command. (Note: In practice, the valve opening is usually proportional to the controller command, but the flow is not necessarily proportional to the controller command and also depends on the upstream pressure. However, this dependence can be linearized by the use of the flow controller cascaded with the level controller.)

Write the equation of the process.

$$\dot{x}_1 = \frac{1}{a}(F_{in} - F_{out}) \qquad (1)$$

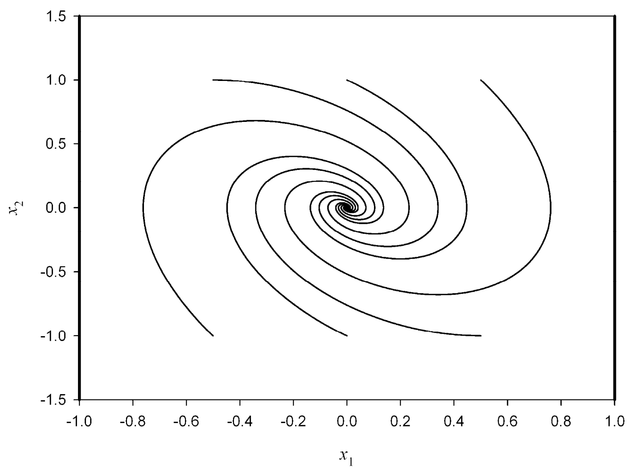

Fig.2. Phase portrait of PI controlled level process (underdamped).

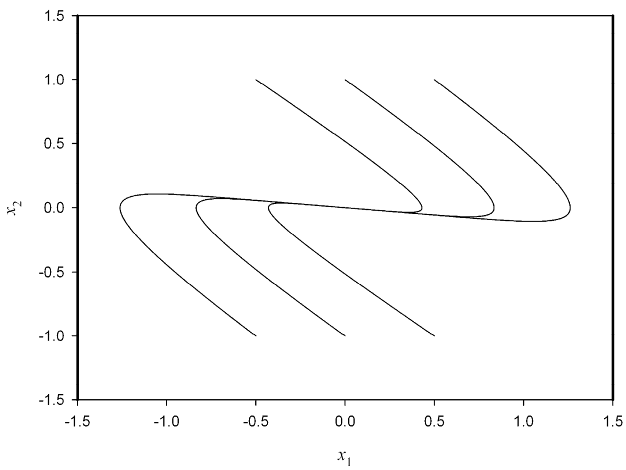

Fig.3. Phase portrait of PI controlled level process (overdamped).

where x_1 is the level value, F_{in} is the controlled flow to the tank, F_{out} is the uncontrolled flow from the tank, a is the cross-sectional area of the tank (it is assumed the tank has such geometry that a is constant).

Let the process be controlled by a PI controller given by the following equation in the Laplace domain.

$$u(s) = K\left(1+\frac{1}{Ts}\right)e(s) \tag{2}$$

where u is control, K is the controller proportional gain, T is the controller integral time constant, s is the Laplace variable, e is the error (the difference between the level set point and the actual level value).

At this point, we will consider that the control u produced by the controller is equal to the inflow (no actuator-valve dynamics): $F_{in}=u$, that the outflow is zero, and that the set point value is zero, so that $e=-x_1$. Rewrite equations (1) and (2) in the normal form:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\dfrac{K}{Ta}x_1 - \dfrac{K}{a}x_2 \end{cases} \tag{3}$$

Get rid of time in (3) via dividing the second equation by the first one and obtain the equations of the state trajectories.

$$\frac{dx_2}{dx_1} = -\frac{K}{a}\left(\frac{1}{T}\frac{x_1}{x_2}+1\right) \tag{4}$$

Depending on the parameters K and T of the controller, equation (4) can represent either an underdamped (oscillatory) process (Fig. 2) or an overdamped process (Fig. 3).

To analyze advantages and drawbacks of each of the presented controllers with respect to the level control process, we should give the control objectives for this process.

Firstly, level controller is a regulator: the set point is usually constant, and the servo properties (set point tracking) are not specified. The main objective of the controller is to attenuate (reject) possible disturbances. Secondly, the only possible disturbance is the change of outflow. This change is often an abrupt change due to connection or disconnection of consumers. If, for example, the initial state is the equilibrium point (inflow is equal to outflow) then a decrease of outflow would cause an instantaneous change of the state from (0,0) to $(0,x_{02})$. The same would happen if the outflow increased. Therefore, the typical situation that needs to be analyzed is the motion from the point $(0,x_{02})$. The respective system trajectories for an overdamped and underdamped processes are presented in Fig. 4. Thirdly, the control objective is to minimize the effect of this disturbance, which is manifested as level increase (decrease) from the set point. The maximum level deviation corresponds to the distance between the point of intersection of the horizontal axis by the trajectory and the origin (Fig. 4). Fourthly, another control objective is to ensure a smooth and possibly non-oscillatory (overdamped) transient.

Let us analyze how those objectives (criteria) are related to the controller and process parameters. The closed-loop system is a linear second order system with the following characteristic polynomial:

$$P(\lambda) = \lambda^2 + \frac{K}{a}\lambda + \frac{K}{Ta},$$

which has the following roots: $\lambda_1 = -\frac{K}{2a}\left(1+\sqrt{1-\frac{4a}{KT}}\right)$, and $\lambda_2 = -\frac{K}{2a}\left(1-\sqrt{1-\frac{4a}{KT}}\right)$. The damping ratio of the closed-loop system is, therefore, $\xi = \sqrt{\frac{KT}{4a}}$. Let us also analyze how each point $(0,x_{02})$ is mapped to the horizontal axis in terms of the distance of the point of intersection from the origin. This requires the solution of equations (3).

Consider the case of $\lambda_1 \neq \lambda_2$. The analytical solution can be given as follows:

$$x = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t} \tag{5}$$

where $c_1 = -c_2 = -\frac{a}{K}\frac{x_{02}}{\sqrt{1-\frac{4a}{TK}}}$, $x_{02} = \dot{x}_1(0)$, and λ_1 and

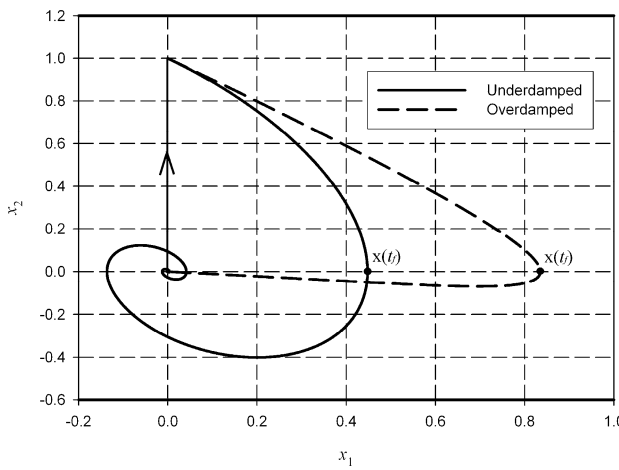

Fig.4. Phase trajectories for instantaneous outflow decrease.

λ_2 are given above. The powers of the exponents are complex in the case of the underdamped process and real in the case of the overdamped process. Let us denote the time corresponding to the intersection of the horizontal axis by the trajectory t_f. This time can be found as a solution of the equation

$$\dot{x}_1(t_f) = c_1\left(\lambda_1 e^{\lambda_1 t_f} - \lambda_2 e^{\lambda_{12} t_f}\right) = 0\ , \qquad (6)$$

from which time t_f can be derived as

$$t_f = \frac{1}{\lambda_2 - \lambda_1} \ln\left(\frac{\lambda_1}{\lambda_2}\right), \qquad (7)$$

and consequently

$$x_1(t_f) = c_1\left(e^{\lambda_1 t_f} - e^{\lambda_2 t_f}\right), \qquad (8)$$

A number of calculations of the values of $x_1(t_f)$ for different controller integral time constant values, subject to x_{02}=1, and the unity controller gain ($K/a = 1$), that are depicted in Fig. 4 are presented in Table 1. Table 1 presents the comparison of the transient process characteristics for different values of T and the same values of K/a, which ensures the same initial trajectory slopes – as per (4). By changing the controller gain K one can vary the transient characteristics of the process, including the possibility of reducing time t_f and the maximum level deviation $x_1(t_f)$. However, firstly, that would be an additional way of enhancement of the transient dynamics, and secondly, the model being considered is a simplified one, which does not account for the existence of the actuator-valve dynamics, and in practice the possibility of the controller gain increase is limited.

One can see from Table 1 and the phase portraits Fig. 2, 3 and 4 that each of those two different transients has some properties that are valuable to the level control process. The underdamped response has the points of intersection of the state trajectories with the horizontal axis that are located closer to the origin than the ones for the underdamped response. Therefore, the maximum deviation of level from the set point due to a disturbance in the form of the instantaneous outflow change will be smaller for this type of control. On the other hand, the transient that corresponds to the overdamped response is much more suitable for the level process than the oscillatory response. Therefore, a tradeoff between the disturbance attenuation and the quality of the transient takes place.

The use of the variable structure principle [5] would, in our opinion, resolve the noted tradeoff allowing one to utilize the advantages of both controllers: the better disturbance attenuation provided by the underdamped control and the quality of the transient of the overdamped controller. If we create a variable-structure controller such that takes the underdamped control if $x_1x_2>0$ and overdamped control if $x_1x_2\leq 0$ then both useful properties of the two controllers would be utilized, and the phase portrait of the system would look as given in Fig. 5.

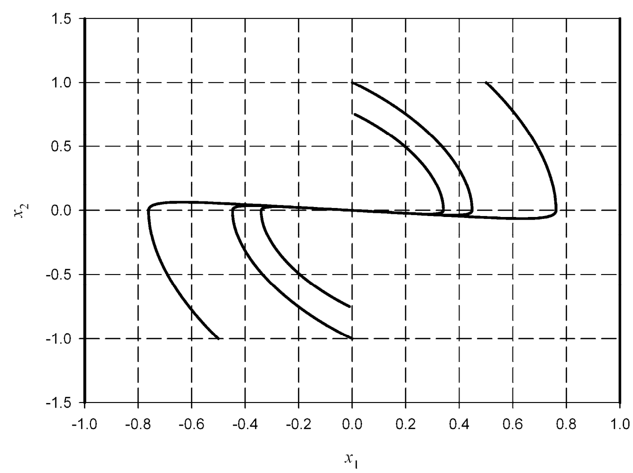

Fig.5. Phase portrait of level process (variable-structure).

In Fig. 5, the trajectories of the 1st and 3rd quadrants are from Fig. 2, and of the 2nd and 4th quadrants are from Fig. 3. One can see that the points of intersection with the horizontal axis are the same as in Fig. 2, and after that intersection the process becomes overdamped. Therefore, the variable-structure controller given in Fig. 5 utilizes the valuable properties of both controllers considered above.

III. Detail Process Model and Controller Tuning

A model of the variable structure PI controller for the level process was presented above. Yet, this model addresses an ideal situation. Namely, the output of the controller is considered to be the inflow to the tank. In fact, this model does not account for the dynamics of the actuator and of the valve and the flow build-up due to the valve position change. The existence of these dynamics results in some lag in the inflow with respect to the controller command. This lag can be relatively precisely modeled by the first-order plus dead time (FOPDT) dynamics [3], [4] given by the following transfer function:

$$W_a(s) = \frac{e^{-\tau s}}{T_a s + 1} \qquad (9)$$

Therefore, the detail model of the level process can be presented as the block diagram in Fig. 6. In Fig. 6, the controller parameters K and T are assumed to take two different sets of values depending on the states of the system.

TABLE I
SETTLING TIME AND MAXIMUM LEVEL DEVIATION

T	0.1	0.3	0.7	1.0	5.0	10.0
t_f	0.452	0.739	1.05	1.21	2.15	2.66
$x_1(t_f)$	0.252	0.379	0.495	0.546	0.762	0.835

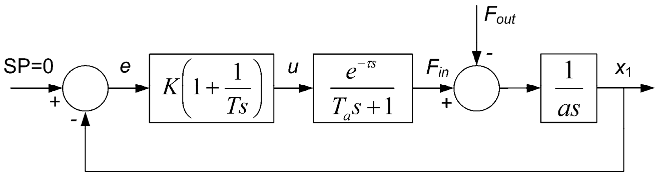

Fig.6. Level process dynamics.

Therefore, we have demonstrated that the controller design should ensure a relatively "fast" response when the trajectory is in quadrants 1 and 3 of the phase plane x_1Ox_2, and a relatively "slow" response when the trajectory is in quadrants 2 and 4. Despite the fact that now the order of the systems is higher than two, we are still able to partition the state space of that system as "flat" considering only the variables x_1 and x_2 that represent the principal dynamics. The other variables should, however, be accounted for in selecting the optimal controller parameters.

If the dynamics of the actuator-valve were neglected the following criteria for the optimal controller design in the two partitions of the state space could be used. *In quadrants 1 and 3, the PI controller is to provide a minimal level deviation as a response to the step change of the outflow*, subject to the constraint of the stability of the closed-loop system with this controller. (Note: the stability constraint is not necessary but in practical terms it is better to limit the controller gain, which would also be beneficial to the overall performance considering the issue of back tracking discussed below). *In quadrants 2 and 4, the PI controller is to provide minimum settling time of the transient process from the initial condition* (x_{01},0), subject to the constraint on the non-oscillatory character of the transient process. However, due to the presence of the dynamics of the actuator-valve the inflow always lags with respect to the controller command. This results in the distortion of the ideal partitioning of the state space. In fact, when the command to decrease the valve opening comes to the actuator-valve (which corresponds to the point (x_{01},0)) the valve still continues to increase the opening due to the lag. This effect distorts the optimal settings of the controller if those are found separately for each partition. In that respect the solution of the controller parameters optimization problem, when all four parameters are optimized simultaneously, and the switching conditions are determined by $x_1x_2>0$ and $x_1x_2\leq 0$ would be more efficient. In this case we will not be able to use two different objective functions any more, but need to formulate a certain one criterion that would characterize the overall dynamics of the system. There are a number of criteria that meet that requirement. The most popular ones are: the minimal integral absolute error (IAE), and the minimal integral time by absolute error (ITAE), as the response to the unit step, which are given as follows.

$$Q_{IAE} = \int_0^\infty |x_1(t)| dt \to \min , \tag{10}$$

$$Q_{ITAE} = \int_0^\infty |x_1(t)| t dt \to \min , \tag{11}$$

where the system is as given in Fig. 6, the external input is the step change of the outflow $F_{out}(t)$=-1(t), the error is equal to the negative output – as the set point is zero.

Simulations show that the use of criteria (10) or (11) for a conventional PI controller for the level process allows for achieving a compromise between the minimum level deviation and the character of the transient process. However, the use of the ITAE criterion provides a less oscillatory transient, which is more suitable for the level process (yet the level deviation is slightly higher). Therefore, let us select the ITAE criterion for the optimization of the controller parameters for the level process.

Let us find the optimal controller settings for a certain normalized set of process parameters, after which the actual settings for each particular process can be computed by scaling of this normalized solution. Let the normalized transfer function of the open-loop system be as follows:

$$W_{ol}(s) = K_0\left(1+\frac{1}{T_0 s}\right)\frac{e^{-\tau_0 s}}{s(s+1)} , \tag{12}$$

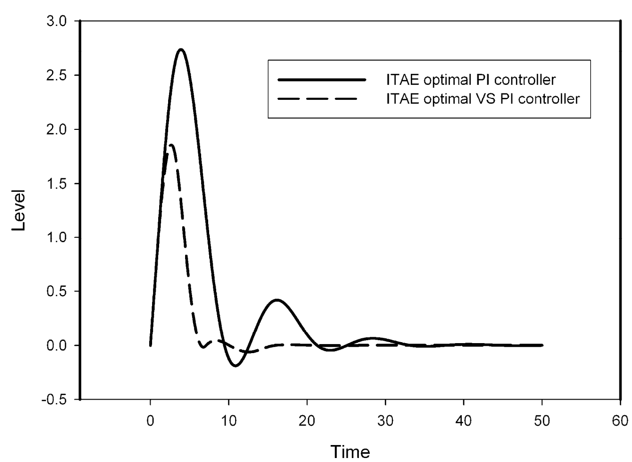

Fig.7. Response to step change in outflow for ITAE optimized system.

Formulate the optimization problem for the conventional PI controller as the problem of finding such values of K_0 and T_0 that minimize the objective function (11). The results of the optimization are given in Table 2, where the value of the objective function for the optimal set of the controller parameters is given too. The optimal settings are computed for a number of different values of the time delay τ_0.

TABLE II
ITAE OPTIMAL PI CONTROLLER SETTINGS

τ_0	0.1	0.2	0.5	0.7	1.0	1.5
K_0	2.764	1.584	0.7925	0.6201	0.4803	0.3562
T_0	4.744	5.053	6.153	6.898	8.045	9.922
$ITAE$	8.45	15.44	42.62	67.43	116.55	234.87

TABLE III
ITAE OPTIMAL VS PI CONTROLLER SETTINGS

τ_0	0.1	0.2	0.5	0.7	1.0	1.5
K_{01}	2.509	1.760	0.5744	0.4841	0.3563	0.3242
T_{01}	1.221	1.417	1.75	2.059	2.777	3.819
K_{02}	3.944	2.892	0.7560	0.6193	0.4227	0.4193
T_{02}	91.92	>100.0	>100.0	>100.0	36.44	31.77
$ITAE$	1.48	2.41	18.41	29.76	65.11	111.02

Now let us solve the same optimization problem for the VS PI controller. The VS PI controller has four parameters: K_{01}, T_{01} for the controller in quadrants 1 and 3, and K_{02}, T_{02} for the controller in quadrants 2 and 4. The results of the optimization are given in Table 3, along with the value of the objective function for the optimal set of the controller parameters.

The comparison of the ITAE value of Table 2 and 3 shows that the VS PI controller provides a significant improvement of the system performance if assessed in terms of the objective function values. Fig. 7 provides the plots of the transient response of the two controllers for time delay τ_0=0.5. One can see that VS PI controller is superior in comparison with the conventional PI controller in both the maximum level deviation and the quality of the transient process.

It should be noted that Table 3 gives the optimal controller settings subject to the use of the so-called "back-initialization" of the controller, when in the VS PI controller consisting of two PI controllers, the output of the inactive controller is initialized to the output of the active controller (using the integral term constant). This mode is used to ensure a continuous control action. If the discontinuous controller action is allowed, the VS PI controller will provide even higher performance improvement.

IV. VS CONTROLLER DESIGN METHODOLOGY

The VS PI level controller design should, therefore, include the following steps.

A. *State-space partitioning.* Despite the fact that the account of the actuator-valve dynamics might affect the partitioning of the state space with respect to the switching between the two controllers, the effect of possible augmentation would be small, and also it would not be expedient to make the partition boundary a tuning parameter, which depends on the actuator-valve dynamics. Therefore, we shall consider the switching from the "underdamped" controller to the "overdamped" controller the same as in the ideal case (Fig. 8). However, it is expedient to provide some "dead-zone" in the partitioning around the point x_1=0. This "dead-zone" would ensure more sluggish operation of the process at level values around the set point, when the control switching may be otherwise caused by the measurement noise and some small outflow fluctuations. This "dead-zone" is denoted in Fig. 8 as of width $2h$. The value of h should be determined from the level process observations, so that h must be slightly higher than the amplitude of the observed noise. It can also be noted that the initial parts of the trajectories in this dead zone for the overdamped controller and for the underdamped controller are very close (see Fig. 4), as the motion in this mode is mostly determined by the reaction of the process to the disturbance rather than by the controller action.

B. *Parameter selection for the VS PI controller.* Controller parameters selection is based on the data of Table 3 and recalculation formulas. It is assumed that the model of the process is given by the FOPDT plus integrator (parameters a, T_a, and τ) is known. Considering a, T_a, and τ to be known values, the ITAE optimal controller parameters can be computed as follows. At first the value of relative dead time computed as $\tau_0 = \tau / T_a$. After that the values of K_{01}, T_{01}, K_{02},,T_{02} are selected from Table 3 for the calculated value of τ_0. The in-between values are computed via interpolation. To obtain the formulas for the scaled optimal solution use the substitution $s'=T_a s$ in the formula for the transfer function of the open-loop system, where s' is a scaled Laplace variable.

$$W_{ol}(s) = K\left(1 + \frac{1}{Ts}\right)\frac{e^{-\tau s}}{s(T_a s + 1)}$$

$$W_{ol}(s') = \frac{KT_a}{a}\left(1 + \frac{T_a}{Ts'}\right)\frac{e^{-\tau s'/T_a}}{s'(s' + 1)}$$

Therefore, $\tau_0 = \tau / T_a$, $K_0 = \frac{KT_a}{a}$, $T_0 = T / T_a$ and K_1, T_1, K_2, T_2 are computed as follows:

$$K_{1(2)} = K_{01(02)}\frac{a}{T_a}, \quad T_{1(2)} = T_{01(02)} T_a$$

C. *Account for the effect of controllers back initialization.* Controller back initialization involves setting the initial value of the integral term of the inactive controller to a certain value at the time of the controller switching, so that the initial output of the controller should be equal to the output of the active controller. Through this mechanism, a continuous control is provided – even with the use of a switching control strategy. It should be noted that the control continuity is ensured at the expense of certain performance deterioration. In fact, the switching controller strategy without back initialization would provide a better performance in terms of the objective function (ITAE) value. Yet, the industry requirement is that the switching controller must provide a continuous control. For the level process, a discontinuous control might create abrupt fluctuations in the inflow, which would affect the upstream process (the source). The provision for back initialization is accounted for in the optimal settings of Table 3.

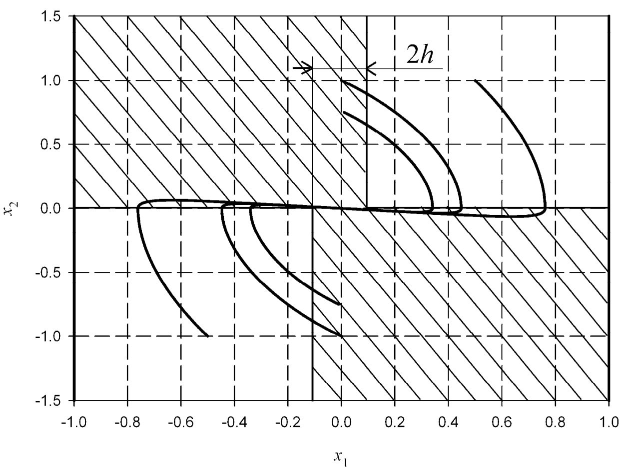

Fig.8. State space partitioning.

V. SIMULATION EXAMPLE

The example shows the system response to the situation imitating random connections and disconnections of consumers, so that the outflow from the tank changes by steps that have random amplitude and occur at random times (Fig. 9, dashed line). Consider two types of control of the process given by the transfer function $W_{ol}(s) = \frac{e^{-0.5s}}{s(s+1)}$. In the first case, the process is controlled by a conventional PI controller optimized per integral absolute error (IAE) criterion, so that the controller transfer function is $W_{PI}(s) = 1.060\left(1+\frac{1}{7.233s}\right)$. In the second option, the process is controlled by the VS PI controller also optimized as per IAE criterion, so that the controller transfer functions are $W_{VSPI1}(s) = 1.070\left(1+\frac{1}{1.884s}\right)$ and $W_{VSPI2}(s) = 1.824\left(1+\frac{1}{21.307s}\right)$ (for the two regions of the state space). The choice of the IAE criterion (versus ITAE criterion used above, for example) is due to the character of the situation analyzed. Time that acts as a weight in the ITAE criterion should not be involved in the criterion used for the performance assessment in the simulated situation, as the process is running continuously, and errors in the beginning should have the same weight as the errors in the end of the test. Fig. 9 demonstrates two different performances of the two controllers assessed as level deviations. The conventional PI controller produces the value of the criterion IAE_{PI}=255.2, while the VS PI controller gives the criterion value IAE_{VSPI}=154.8, which is significantly lower than the former.

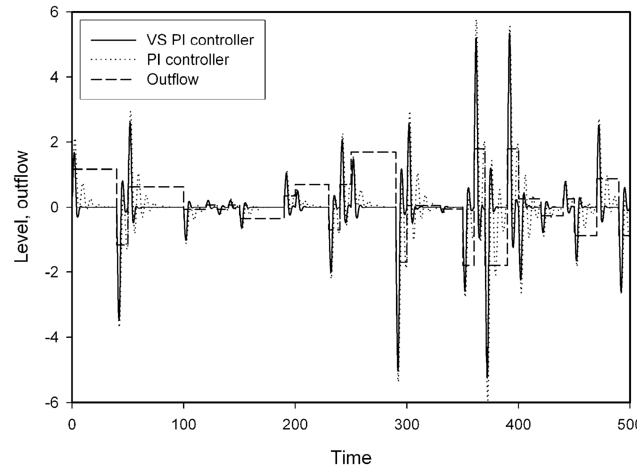

Fig.9. Level response to random step changes in outflow for IAE optimized PI control and IAE optimized VS PI control.

VI. CONCLUSION

A variable-structure PI controller is proposed. The VS principle utilized has a very simple physical interpretation, which is the application of a relatively aggressive control if the disturbance comes to the process - aimed at minimizing its effect, and the application of a relatively sluggish control after this disturbance is matched by the controller action – to ensure a damped transient process of coming to the set point. It is shown that the proposed controller is superior to the conventional PI controller in terms of providing a better disturbance attenuation and the quality of the transient process. This superiority is demonstrated via finding the optimal settings (and the optimization criterion values) for the conventional PI and VS PI controllers, and the analysis of the performance of those optimized controllers. The simulation example demonstrates the design methodology. An industrial application has been implemented and testing of the controller performance is in progress.

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