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Flight Control with Amplitude and Rate Constraints: a Command Governor Approach

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Abstract— **In this paper we present an application of a novel constrained control technique known as Command Governor (CG) to deal with the problem of actuator amplitude and rate saturation on aircrafts. The aim of the CG strategy consists of modifying, whenever necessary, the reference signal supplied to a primal compensated system so as to guarantee the fulfillment of existing constraints. CG action is obtained solving on line an optimization procedure which embodies the future plant-evolution according to the receding horizon philosophy. To show the effectiveness of the proposed constrained control algorithm numerical simulations are carried out on a small commercial aircraft of the General Aviation class.**

I. INTRODUCTION

Actuator amplitude and rate saturation is an inherent property common to almost every physical system. It is well recognized that such a type of limitation can lead to performance degradation and even instability.

On complex systems as aircrafts, the actuator saturation represents a problem of paramount importance: the destabilizing effects of actuator saturation have been cited as contributing factors in several mishaps involving aircrafts both of general than military aviation. [1]

In flight control applications many different solutions have been investigated in order to cope with the problems arising from these limitations: anti-windup techniques (AW) [2], software rate-limiters [3], [4], low-high gain (LHG) methods [5] have been successfully applied both to stable and unstable aircrafts models.

Recently, the interest towards methods based on model predictive control (MPC) ideas [6]–[9] is growing due to the availability of powerful and fast computing units [10]. Nonetheless, it must be recalled that, on the theoretical side, such techniques are so popular because of their capability to directly embody the existing constraints in the design phase. In this case, the control action is the result of an optimization procedure which takes into account two objectives: maximize the tracking performance and to guarantee that all prescribed constraints are fulfilled.

Of interest in this paper is a novel constrained control methodology based on conceptual tools of MPC and known as "Command Governor" strategy.

The first theoretical results appeared in [11]. These were based on the preliminary studies reported in [12]. Many mature assessments of the related state of the art can be found in [13]–[18].

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The objective of this constrained control paradigm is to design the so-called CG device. This is a nonlinear predictive unit which is added to a primal compensated plant that is designed so as to perform satisfactorily in the absence of constraints violation. Whenever necessary, CG device computes a modified version of the current reference signal in order to enforce all prescribed constraints with acceptable tracking performance.

The computed reference command is selected according to an on-line quadratic programming (QP) problem where, the objective function is its distance from the actual reference signal and where the constraints take into account the future plant evolution according to receding horizon philosophy. Any standard QP solver can be used for such a type of task and the CG algorithm can be implemented on any off-the-shelf digital-signal-processing board. The computational power required from CG device can be modulated through an appropriate tuning of all design parameters and the current microprocessor technology allows to obtain typical sampling rates used in aeronautical applications [17].

In this respect, the main objective of this paper is to investigate the possibility to apply the CG framework on aircrafts in order to improve the performance of flight control systems which typically operate in the presence of actuators subjected to amplitude and rate saturations. To this end a model of a small commercial aircraft [19], [20] of General Aviation class has been considered as test-case.

The paper is organized as follows: in Section II the aircraft mathematical model and the primal control structure are introduced. In Section III the CG strategy is described together with the main theoretical results. In Section IV extensive numerical results are presented and the benefits of the CG strategy are detailed. Some conclusions end the paper.

II. AIRCRAFT MATHEMATICAL MODEL AND PRIMAL CONTROLLER STRUCTURE

As discussed in the Introduction, this constrained control problem can be viewed as a two-steps procedure. First a primal internal control loop is designed so as to achieve good tracking capabilities w.r.t. the prescribed reference signals. Then the CG strategy is superimposed to guarantee the fulfillment of existing limitations.

In this section the mathematical model of the aircraft is derived and the primal control structure adopted in this flight control application is outlined.

In the so-called polar form [21], the six degree of freedom mathematical model of the aircraft has the following

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structure (See Table I for the meaning of all symbols which follow):

$$W\dot{V} = T\cos(\alpha+\mu_T)\cos\beta - \frac{1}{2}\rho V^2 S C_D + W g_1 \quad (1)$$

$$WV\cos\beta\,\dot{\alpha} = -T\sin(\alpha+\mu_T) - \frac{1}{2}\rho V^2 S C_L + WVq + Wg_2 \quad (2)$$

$$VW\dot{\beta} = -T\cos(\alpha+\mu_T)\sin\beta + \frac{1}{2}\rho V^2 S C_Y - WVr + Wg_3 \quad (3)$$

$$\dot{p}I_x - \dot{r}I_{xz} + qr(I_z - I_y) - pqI_{xz} = \frac{1}{2}\rho V^2 SbC_r \quad (4)$$

$$\dot{q}I_y + rp(I_x - I_z) + (p^2 - r^2)I_{xz} = \frac{1}{2}\rho V^2 S\underline{c}C_m \quad (5)$$

$$-\dot{p}I_{xz} + \dot{r}I_z + pq(I_y - I_x) + qrI_{xz} = \frac{1}{2}\rho V^2 SbC_n \quad (6)$$

$$\dot{\phi} = p + q\tan\theta\sin\phi + r\tan\theta\cos\phi \quad (7)$$

$$\dot{\theta} = q\cos\phi - r\sin\phi \quad (8)$$

$$\dot{\psi} = r\cos\phi\sec\theta + q\sin\phi\sec\theta \quad (9)$$

with

$$g_1 := g(-\cos\alpha\cos\beta\sin\theta + \sin\beta\sin\phi\cos\theta + \sin\alpha\cos\beta\cos\phi\cos\theta)$$

$$g_2 := g(\sin\alpha\sin\theta + \cos\alpha\cos\phi\cos\theta)$$

$$g_3 := g(\cos\alpha\sin\beta\sin\theta + \cos\beta\sin\phi\cos\theta - \sin\alpha\sin\beta\cos\phi\cos\theta)$$

(1) - (6) are the Newton's laws, whereas (7) - (9) the Euler's equations. In presence of external wind the relative wind speed, angle of attack and sideslip are nonlinear function of the wind velocity vector $[u_{\text{wind}}, v_{\text{wind}}, w_{\text{wind}}]^T$. Turbulence is modelled with Dryden's model [22].

Further, the aerodynamics coefficients C_D, C_L, C_Y, C_r, C_m, C_n can be expressed as a function of the state variables $V,\alpha,\beta,\phi,\theta,\psi,p,q,r$, and input δ_e, δ_a, δ_r. T is assumed to be a known function of δ_{th}.

As it will be cleared in next section in order to apply the Command Governor strategy, it is necessary to have a discrete-time pre-compensated linear model of the plant.

To this end, starting from nonlinear model (1) - (9) where we chosen:

1) $\hat{x}_c := [V \;\; \alpha \;\; \beta \;\; \phi \;\; \theta \;\; \psi \;\; p \;\; q \;\; r]^T$ as states vector;
2) $\hat{u}_c := [\delta_e \;\; \delta_a \;\; \delta_r \;\; \delta_{th}]^T$ as inputs vector;
3) $\hat{z}_c := \hat{x}_c$ as outputs vector;
4) $\hat{d}_c := [u_{\text{wind}} \;\; v_{\text{wind}} \;\; w_{\text{wind}}]^T$ as disturbances vector;

a continuous-time linearized model in the neighborhood of a wing leveled straight flight condition (see Table III for major details), is derived with the following structure:

$$\begin{cases} \dot{\hat{x}}_c = \hat{A}_c\;\hat{x}_c + \hat{B}_{d_c}\;\hat{d}_c + \hat{B}_{u_c}\;\hat{u}_c \\ \hat{z}_c = \hat{x}_c + \hat{D}_c\;\hat{d}_c \end{cases} \quad (10)$$

The direct coupling between $\hat{d}_c$ and $\hat{z}_c$ is removed by defying a new state x_c and disturbances vector d_c as it follows:

$$x_c := \hat{x}_c + \hat{D}_c\;\hat{d}_c \qquad d_c := [\hat{d}_c \quad \dot{\hat{d}}_c] \quad (11)$$

As a consequence model (10) is rewritten as:

$$\begin{cases} \dot{x}_c = A_c\;x_c + B_{d_c}\;d_c + B_{u_c}\;u_c \\ z_c = x_c \end{cases} \quad (12)$$

with

$$A_c = \hat{A}_c; \qquad B_{u_c} = \hat{B}_{u_c};$$

$$B_{d_c} = [(\hat{B}_{d_c} - \hat{A}_c\hat{D}_c) \qquad \hat{D}_c] \qquad u_c \equiv \hat{u}_c. \quad (13)$$

Next, starting from model (12), a multivariable Proportional-plus-Integral (PI) controller has been designed using the techniques reported in [23], [24] so as to satisfy stability and good tracking performances w.r.t. reference signals. During the design phase, the controlled variables have been chosen as the pitch, roll and sideslip angles θ, ϕ, β and the throttle command has been kept constant. Further, for the aircraft under consideration, δ_e mainly drives the pitch angle, δ_a drives the roll angle and finally, δ_r mainly controls the excursions of the angle of sideslip. In Fig. 1 the primal control structure adopted in this flight control is shown.

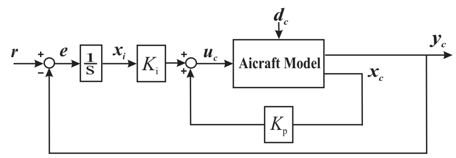


Fig. 1. Primal Control Structure: The PI multivariable action is expressed as $u_c := K_p x_c + K_i x_i$, where x_c are the aircraft model states, $x_i \in \mathbf{R}^3$ are the states of integrators and K_p, K_i gains matrices computed via the standard LQ technique. Note that y_c is the vector of the controlled variables θ,ϕ,β. Finally, r and e are respectively the reference signal and the error signal between r and y_c. For major details concerning all design guidelines of such a control technique see [23], [24]

III. Command Governor Strategy: Mathematical Formulation

The Command Governor is a methodology mainly devoted to enforce all prescribed constraints by properly modifying the reference signal supplied to a pre-compensated plant (Fig. 2). Here we will briefly outline the CG strategy essentials together with the main theoretical results.

In its most common formulation [13]–[18], the CG approach takes into consideration a discrete time closed-loop regulated linear time-invariant plant model on the form:

$$\begin{cases} x(t+1) = \Phi x(t) + G\vartheta(t) + G_d d(t) \\ y(t) = H_y x(t) \\ c(t) = H_c x(t) + L\vartheta(t) + L_d d(t) \end{cases} \quad (14)$$

where

- $t \in \mathscr{Z}_{0+} := \{0,1,\ldots,\}$ is the plant time-step;
- $x(t) \in \mathbf{R}^{n_x}$ is the state vector including plant and compensator states;
- $\vartheta(t) \in \mathbf{R}^{n_r}$ is the manipulable command input vector which, if no constraints were present, would coincide with the reference signal $r(t) \in \mathbf{R}^{n_r}$;
- $d(t) \in \mathscr{D} \subset \mathbf{R}^{n_d}$, is an exogenous disturbance vector and $\mathscr{D}$ a closed convex and compact set having the following structure
$$\mathscr{D} := \{d \in \mathbf{R}^{n_d} : Ud \le h\}, \quad 0_{n_d} \in \mathscr{D} \quad (15)$$
with $U \in \mathbf{R}^{n_u \times n_d}$ full column rank matrix ($\text{rank}(U) = n_d$, $n_u \ge n_d$) and $h := [h_1 \; h_2 \; \ldots h_{n_u}]^T \in \mathbf{R}^{n_u}$ a vector such that $h_p \ge 0$, $p = 1,\ldots,n_u$.
- $y(t) \in \mathbf{R}^{n_r}$ is the output vector which is required to track $r(t)$;

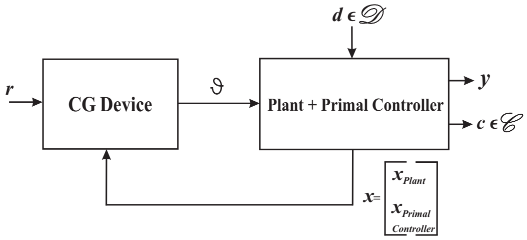


Fig. 2. Reference Governor Structure

- $c(t) \in \mathbf{R}^{n_c}$ is the vector to be constrained, viz. $c(t) \in \mathscr{C} \subset \mathbf{R}^{n_c}$, with $\mathscr{C}$ denoting a closed and convex set having the following structure

$$\mathscr{C} := \{c \in \mathbf{R}^{n_c} : Tc \leq f\} \tag{16}$$

In (16) $T \in \mathbf{R}^{n_t \times n_c}$, $n_t \geq n_c$, $\mathrm{rank}(T) = n_c$ is a matrix which allows to select the constrained outputs, whereas $f \in \mathbf{R}^{n_t}$ is the vector of prescribed constraints.

Under the assumptions that the system (14) must be asymptotically stable and *offset-free* (i.e, $H_y(I-\Phi)^{-1}G = I_{n_r}$) the problem is to design, at each time t, a command $\vartheta(t)$ as a static function of the current state $x(t)$ and the reference signal $r(t)$ (viz. $\vartheta(t) := \underline{\vartheta}(x(t), r(t))$), in such a way that, under all possible disturbance sequences $d(t) \in \mathscr{D}$ and under the constraints $c(t) \in \mathscr{C}$, the CG output $\vartheta(t)$ is the best approximation of $r(t)$ at time t.

The disturbance effect is taken into consideration by means of the following sets recursion:

$$\begin{array}{c} \mathscr{C}_0 := \mathscr{C} \sim L_d \mathscr{D} \\ \mathscr{C}_k := \mathscr{C}_{k-1} \sim H_c \Phi^{k-1} G_d \mathscr{D} \\ \mathscr{C}_\infty := \bigcap_{k=0}^{\infty} \mathscr{C}_k \end{array} \tag{17}$$

where the symbols $\sim$ indicates the *P-difference* [25] between sets, i.e, given two sets $\mathscr{A}$ and $\mathscr{B}$,

$$\mathscr{A} \sim \mathscr{B} := \{\boldsymbol{a} \in \mathbf{R}^n : \boldsymbol{a} + \boldsymbol{b} \in \mathscr{A}, \ \forall \ \boldsymbol{b} \in \mathscr{B}\}$$

It is shown in [26] that the sets $\mathscr{C}_k$ are suitable restrictions of $\mathscr{C}$ such that if the "disturbance free" component of $c(t)$ in (14), depending only on initial state and input, belongs to $\mathscr{C}_\infty$, then $c(t) \in \mathscr{C}$, $\forall \ t \in \mathscr{Z}_{0+}$; further, if $\mathscr{C}_\infty$ is nonempty, all $\mathscr{C}_k$'s are too. In this case they are compact and convex and satisfy the property $\mathscr{C}_k \subset \mathscr{C}_{k-1}$.

By denoting with $\mathscr{W}^{\xi}$ the convex and closed set (assumed nonempty) of all commands $\omega \in \mathbf{R}^{n_r}$ whose corresponding disturbance-free steady-state solutions $\overline{c}_\omega$ of (14) , (i.e, $\overline{c}_\omega := H_c(I_n - \Phi)^{-1}G\omega + L\omega)$ satisfies the constraints with a prescribed tolerance ξ, the CG strategy consists to choose at each time-step, a constant virtual command $v(\cdot) \equiv \omega$, with $\omega \in \mathscr{W}^{\xi}$, such that the corresponding disturbance-free evolution $\overline{c}(k, x(t), \omega)$ of (14), along a virtual time k from the initial condition $x(t)$, (i.e, $\overline{c}(k, x(t), \omega) := H_c(\Phi^k x(t) + \sum_{i=0}^{k-1} \Phi^{k-i-1} G\omega) + L\omega)$, fulfills the constraints $\forall k \in \mathscr{Z}_{0+}$ and its distance from the constant reference of value $r(t)$ is minimal. Such a command ω is applied to the plant, the procedure is repeated at next time $t+1$ on the basis of a new measured state $x(t+1)$.

Consequently, by calling with $\mathscr{V}(x) \subset \mathscr{W}^{\xi}$ the set of all constant commands $\omega \in \mathscr{W}^{\xi}$, whose c-evolutions starting from initial condition x satisfies the constraints also during transients (i.e $\mathscr{V}(x) := \{\omega \in \mathscr{W}^{\xi} : \overline{c}(k, x, \omega) \in \mathscr{C}_k, \forall k \in \mathscr{Z}_{0+}\}$) and provided that $\mathscr{V}(x)$ is nonempty, closed and convex for all $t \in \mathscr{Z}_{0+}$, the CG command is the solution of the following constrained optimization problem:

$$\vartheta(t) := \arg \min_{\omega \in \mathscr{V}(x(t))} J(r(t), \omega) \tag{18}$$

where $J(r(t), \omega)$ is the following quadratic cost index

$$J(r(t), \omega) := \|\omega - r(t)\|_{\Psi}^2 \tag{19}$$

with $\Psi = \Psi' > 0_p$ and $\|x\|_{\Psi}^2 := x'\Psi x$.

In other words, the minimizer (18) represents the best approximation of the reference signal $r(t)$ which, if constantly applied from t onwards to the system (14), would never produce constraints violation. Practical implementation of CG strategy requires a finite-time computable way to solve the optimization problem (18) and there might be no algorithms capable of efficiently computing the minimizer due to the presence of an infinite number of constraints.

To overcome this limitation, it has been shown in [13] and [27] that $\mathscr{V}(x(t))$ is finitely determined, viz. there exists an integer $\overline{k}$ such that if $\overline{c}(k, x, \omega) \in \mathscr{C}_k$, $k \in \{0, 1, ..., \underline{k}\}$, then $\overline{c}(k, x, \omega) \in \mathscr{C}_k \ \forall \ k \in \mathscr{Z}_{0+}$. The constraint horizon $\overline{k}$ can be determined *off-line* by using an algorithm proposed for the first time by Gilbert and Tin Tan [27] which can be translated into the following optimization problem:

$$G_k(j) := \max_{x \in \mathbf{R}^n, \ \omega \in \mathscr{W}^{\xi}} T_j^T \overline{c}(k, x, \omega) - f_j^k$$

$$\text{subject to} \tag{20}$$

$$T_j^T(i, x, \omega) \leq f_j^i, i = 0, ..., k-1$$

where T_j^T, $j = 1, ..., n_t$ denotes the j-th row of T and the terms f_j^i, $i = 0, ..., k-1$ have the following expression:

$$f_j^0 := f_j - \sup_{d \in \mathscr{D}} T_j^T L_d d$$

$$f_j^1 := f_j^0 - \sup_{d \in \mathscr{D}} T_j^T H_c G_d d \tag{21}$$

$$f_j^k := f_j^{k-1} - \sup_{d \in \mathscr{D}} T_j^T H_c \Phi^{k-j} G_d d$$

An algorithm to derive the constraint horizon $\overline{k}$, by means of procedure (20), is as follows:

Step 1. $k \leftarrow 1$;
Step 2. Solve $G_k(j) \ \forall \ j = 1, ..., n_t$;
Step_3. If $G_k(j) \leq 0, \forall j = 1, ..., n_t$:
Set $\overline{k} = k$ Stop;
Step 4. Otherwise $k \leftarrow k+1$, Go To Step 2.

For further details, see [14]. With this scheme, the initial optimization problem (18), which exhibits an infinite number of constraints, is equivalent to a convex quadratic program (QP) problem with a finite number of constraints.

As a consequence, the CG action can be obtained by solving the following QP problem:

$$\vartheta(t) := \min_{\omega}(\omega - r(t))'\psi(\omega - r(t))$$

subject to

$$TH_c\Phi^k x(t) + T\sum_{i=0}^{k-1}\Phi^i G\omega + TL\omega \leq f^k,\ k = 0,...,\overline{k} \quad (22)$$

$$TH_c(I_n - \Phi)^{-1}G\omega + TL\omega \leq f^{k_\varepsilon} - (\xi + \varepsilon)\sqrt{T_j^T T_j}\ j = 1,...,n_t.$$

The symbols ε and k_ε are a fixed positive tolerance and an integer respectively, that allow to approximate the set $\mathscr{C}_\infty$ of (17) with the following set $\mathscr{C}_\infty(\varepsilon)$:

$$\mathscr{C}_\infty(\varepsilon) := \mathscr{C}_{k_\varepsilon} \sim \mathscr{B}_\varepsilon \quad (23)$$

where $\mathscr{B}_\varepsilon$ is the ball of radius ε centered at the origin. Without loss of generality, it is possible affirm that, (see [14]), the integer k_ε can be computed as follows:

$$k_\varepsilon := \frac{\ln(\varepsilon) + \ln(1-\lambda) - \ln(\overline{\sigma}(H_c)\overline{\sigma}(G_d)Md_{max})}{\ln(\lambda)} \quad (24)$$

with $\overline{\sigma}(H_c)$ and $\overline{\sigma}(G_d)$ the max singular values of H_c and G_d matrix respectively, $d_{max} = \max_{d\in\mathscr{D}}\|d\|_2$, and $M > 0$ and $\lambda \in (0,1)$ two constant such that $\|\Phi^k\| \leq M\lambda^k$. For all theoretical aspects, see [14], [15], [26].

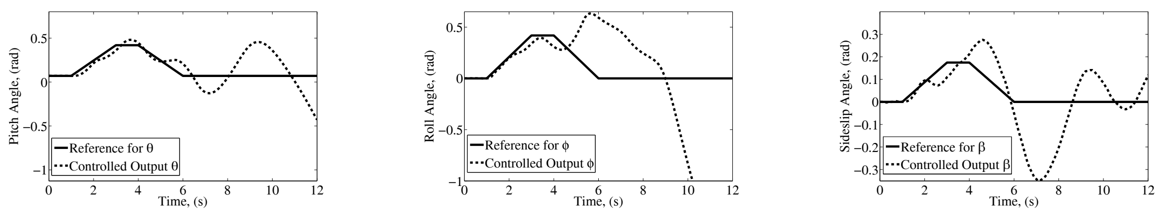
IV. Application of the CG Strategy to civil aircraft

The main objective of this section is to show the recent CG strategy can be considered as an effective methodology to be investigated in field of the general aviation in order to improve the performance of control systems operating in presence of actuators subjected to amplitude and rate saturations. To this end all numerical results obtained with and without the application of the CG strategy will be analyzed in depth.

The PI regulator obtained according to the design guidelines provided in [23], [24] and outlined in Section II has been integrated in the simulation environment [19] of the aircraft nonlinear model. Next, in presence of severe turbulence (maximum wind speed 12 m/s, and standard deviation of about 3 m/s) and by keeping constant the throttle command the civil aircraft equipped with the PI controller has been driven according to the following manoeuvre:

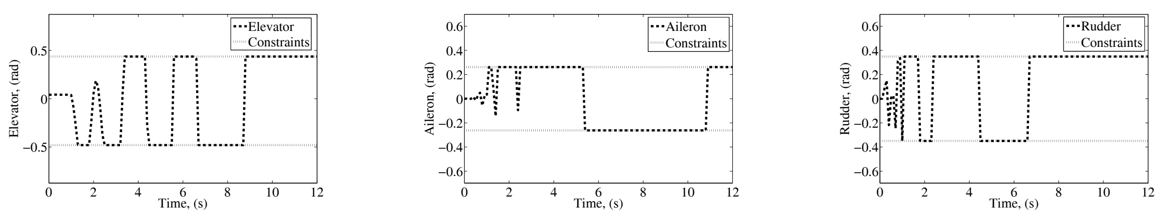
- θ variation w.r.t. the equilibrium value: 0.35 (rad)
- ϕ variation w.r.t. the equilibrium value: 0.42 (rad) (25)
- β variation w.r.t. the equilibrium value: 0.17 (rad)

Figs. 3 show a comparison between the reference trajectories (solid-line) and the related time-behavior of the controlled outputs (dashed-line). It is well remarkable as a fatal loss of



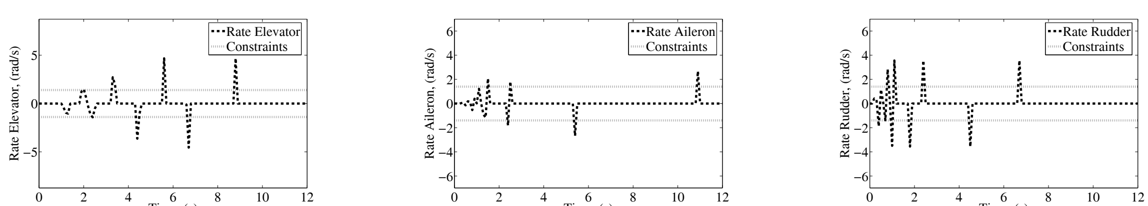
(a) (b) (c)

Fig. 3. Comparison between the pilot-demands (25) (solid-line) and the related controlled outputs (dashed-line) in presence only of the multivariable PI controller.



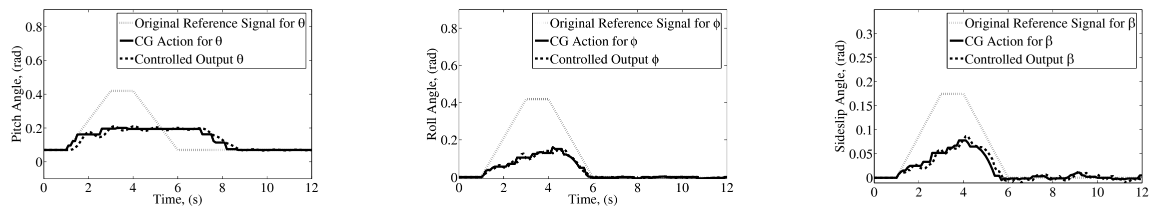
(a) (b) (c)

Fig. 4. Dashed-line: Time-behavior of $\delta_e, \delta_a, \delta_r$ in presence only of the multivariable PI controller. Dotted-line: prescribed constraints.



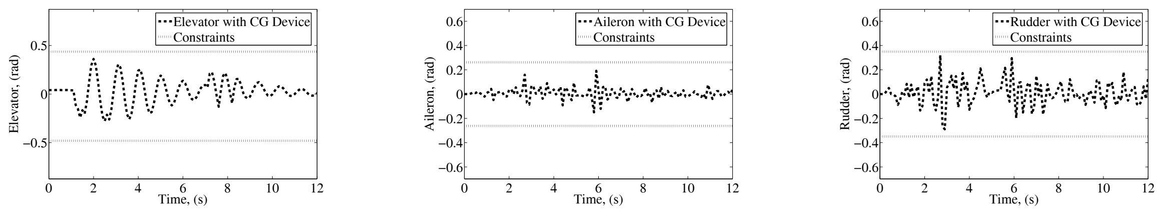
(a) (b) (c)

Fig. 5. Dashed-line: Time-behavior of $\dot{\delta}_e, \dot{\delta}_a, \dot{\delta}_r$ in presence only of the multivariable PI controller. Dotted-line: prescribed constraints.



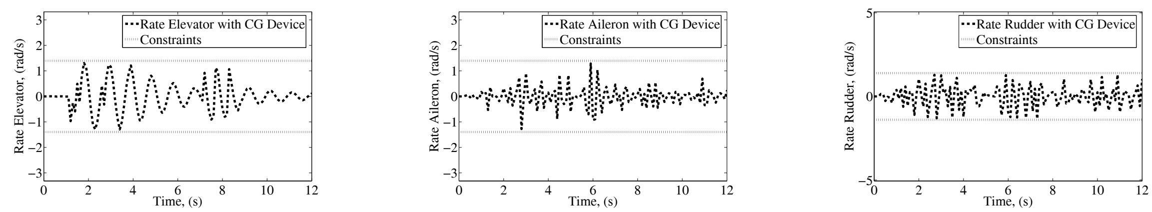
(a) (b) (c)

Fig. 6. Comparison between the pilot-demand (25) (dotted-line), the version modified by CG device (solid-line) and the controlled outputs (dashed-line)



(a) (b) (c)

Fig. 7. Dashed-line: Time-behavior of $\delta_e, \delta_a, \delta_r$, Dotted-line: Prescribed Constraints.



(a) (b) (c)

Fig. 8. Dashed-line: Time-behavior of $\dot{\delta}_e, \dot{\delta}_a, \dot{\delta}_r$, Dotted-line: Prescribed Constraints.

performance is exhibited by the aircraft in correspondence of the abrupt pilot-demand (25). The reasons which lead to this undesirable aircraft behavior can be found by analyzing the following Figs. 4(a)-4(c): here the time-histories (dashed-line) of the excursions of the control surfaces and the related prescribed limitations (dotted-line) are shown. It can be noted as the saturation of all actuators takes place. In particular, after a short chattering among their prescribed lower and upper limits, the deflections of the elevator, aileron and rudder remain locked to their related maximum admissible saturation value. Furthermore, as Figs. 5(a) - 5(c) show, the aggressive pilot manoeuvre (25) also induces a strong violation of the limitations prescribed for the rate (dashed-line) of all the control surfaces. It is worth to stress that if the aircraft is steered with manoeuvres minus aggressive and abrupt than (25) (do not plotted here for brevity), no closed loop instability occurs and satisfactory tracking performance are achieved through the multivariable PI control action. Therefore, in order to recovery stability with acceptable flight performance and to jointly avoid any actuators saturation we equipped the small commercial aircraft under consideration with CG device. The latter has been designed by following all design guidelines provided in the previous section. The resulting CG device has been integrated on the nonlinear model of the aircraft regulated with the PI multivariable controller. In Figs. 6(a) - 6(c) the original reference signal (dotted-line), the outputs (solid-line) of the CG device and the respective time-histories (dashed-line) of the controlled variables θ, ϕ, β are shown. It is quite evident as the time evolution of the new reference signals significantly differs from the original. In particular, note in Fig. 6(a) as the modified reference signal for θ exhibits a more flat shape. Further as it can be observed in Figs. 6(b), 6(c) the new reference signals for ϕ and β have a more sluggish time-behavior. Due to CG action now the controlled outputs exhibit good and tight tracking properties w.r.t. the new reference signals. As a consequence, the aircraft maintains stability with acceptable flight performance. The control surfaces action in presence of the CG device is illustrated in Figs. 7(a)- 7(c): differences w.r.t. the time-behaviors previously shown in Fig. 4(a)-4(c) are quite evident. In particular, note how now, due to the above-discussed corrections performed on the original pilot manoeuvre (25), any deflection of the control surfaces remains locked to the upper or lower limit value. As a consequence, no actuators saturation occurs and an overall enhanced reliability results.

Finally, (see Figs. 8(a)-8(c)) it can be noted as CG device insertion gives rise to strong benefits also on rate of the excursions of the control surfaces: differently from what previously shown in Fig. 5(a)-5(c) now the time-behavior of the rate of all the control surfaces never exceeds its prescribed maximum and minimum limit.

V. CONCLUSIONS

The paper focused on constrained control problem based on Command Governor approach. The main objective was to investigate the possibility to improve the flight control performance on aircrafts in presence of typical limitations acting on amplitude and rate of the actuators. The numerical experiences carried out on model of a small commercial aircraft shown that CG strategy is able to successfully cope with the pernicious effects arising from the presence of these limitations. The considered CG approach can be considered as an appealing methodology to be explored on flight control problems concerning aircrafts of general aviation class.

TABLE I

NOMENCLATURE

Symbol	Meaning
p, q, r	Roll, Pitch, Yaw rate
ϕ, θ, ψ	Roll Pitch, Yaw angle
u_{wind}, v_{wind}, w_{wind}	Long., lateral, vertical wind speed (body-frame)
C_r, C_m, C_n	Roll, Pitch, Yaw moment coefficient
C_D, C_Y, C_L	Drag, Side, Lift force coefficient
I_x, I_y, I_z	Momentum of inertia about X-, Y-, Z- body axis
δ_e, δ_a, δ_r	Elevator, Aileron, Rudder, deflections
δ_{th}, α, β	Throttle command, Angle of attack, Sideslip Angle
W, T, V	Aircraft weight, Thrust, True air speed
S, b, $\underline{c}$	Wing area, Wing span, Mean aerodynamic chord
h, ρ,	Altitude, Air density (function of h)
g, I_{xz}	Gravity acceleration constant, Cross product of inertia
μ_T	Angle between thrust direction and X-body axis

TABLE II

CONSTRAINTS NUMERICAL VALUES

Constrained Variable	Lower Bound	Upper Bound
Elevator	-0.48 (rad)	0.44 (rad)
Aileron	-0.26 (rad)	0.26 (rad)
Rudder	-0.35 (rad)	0.35 (rad)
Rate Elevator	- 1.4 (rad/s)	1.4 (rad/s)
Rate Aileron	- 1.4 (rad/s)	1.4 (rad/s)
Rate Rudder	- 1.4 (rad/s)	1.4 (rad/s)

TABLE III

EQUILIBRIUM VALUES (H=1145 M AND MACH NUMBER=0.28)

Variable	Value	Variable	Value
Elevator	0.04 (rad)	Roll Angle	0 (rad)
Aileron	0 (rad)	Pitch Angle	0.07 (rad)
Rudder	0 (rad)	Yaw Angle	0 (rad)
Throttle	57 (%)	Roll Rate	0 (rad/s)
True Airspeed	124 (m/s)	Pitch Rate	0 (rad/s)
Angle of Attack	0.07 (rad)	Yaw Rate	0 (rad/s)
Angle of Sideslip	0 (rad)		

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