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A Generalized Rate Dependent Play Operator for Characterizing Asymmetric and Symmetric Hysteresis Nonlinearities

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Abstract— **In this paper, a generalized rate dependent Prandtl-Ishlinskii model is proposed to characterize both the symmetric and asymmetric input-output hysteresis, as a function of the time rate of input. The model is realized upon formulation and integration of a generalized rate dependent play operator capable of generating minor as well as major hysteresis loops with varying slopes of ascending and descending input-output curves. A dynamic threshold function is proposed to enhance the prediction of rate-dependent hysteresis nonlinearities. The validity of the generalized model is demonstrated by comparing its displacement responses with the measured symmetric and asymmetric responses obtained for piezoceramic and magnetostrictive actuators under input frequencies of 50-200 Hz and 10-50 Hz, respectively. The results show the capability of the proposed model to characterize asymmetric and symmetric rate dependent hysteresis nonlinearities in smart actuators.**[1]

I. Introduction

Hysteresis a nonlinear phenomenon that appears in various systems including: ferromagnetic materials, mechanical actuators, and electronic relay circuits. Smart materials based actuators, such as piezoceramic actuators, magnetostrictive actuators, and shape memory alloys, invariably show hysteresis effects [1-4]. The non-differentiable and often unknown hysteresis properties of actuators are known to cause inaccuracies and oscillations in the system responses that may lead to instability of the closed loop system [5]. A number of physical and phenomological models, have been developed to describe hysteresis of actuators and materials [6-9]. The phenomological models such as Preisach model [1], Krasnosel'skii-Pokrovskii operator [10] and Prandtl-Ishlinskii model [2,3] have been widely applied to describe hysteresis. These models have also been used to design controllers for compensating hysteresis effects [10-13]. However, these models have been mostly applied to describe rate independent hysteresis effects, assuming negligible effect of the rate of change of input.

Smart actuators, in general, exhibit rate dependent hysteresis, with either symmetry or asymmetry about the input and/or the output axis [14-16]. As an example, on the basis laboratory measurements, it has been shown that hysteresis loops in magentostrictive actuators is highly asymmetric and the output displacement is strongly rate dependent [15,16]. While the reported data for various piezoceramic actuators under excitation of varying magnitudes and frequencies suggest nearly rate dependent symmetric major and minor hysteresis loops [17,18].

A dynamic density function has been proposed to predict rate dependent hysteresis, when integrated to the classical phenomenological operator-based hysteresis models. Mayergoyz [19] proposed a dynamic Preisach model by adding the time rate of the output in the density function to predict rate dependent hysteresis effects. A dynamical model coupled with the Preisach operator was proposed by Tan and Baras [16] in an attempt to characterize the rate dependent hysteresis effects in a magnetostrictive actuator. However, the application of the Preisach model may be limited for real time inverse controller design, since the model is not analytically invertible.

Prandtl-Ishlinskii model which is a subclass of the Preisach model is analytically invertible. Owing the symmetric nature of the classic play operator, the Prandtl-Ishlinskii model of classical density function can characterize symmetric hysteresis properties. Al Janaideh et al. [20] proposed a rate dependent play operator and dynamic density function to characterize rate dependent hysteresis of a piezoceramic actuator over a wide range of excitation frequencies.

As an extension of the work in [20], in this study, a generalized rate dependent play operator is proposed to describe the symmetry as well as asymmetry hysteresis properties as a function of the time rate of the input. The validity of the proposed model is demonstrated using measured data acquired for piezoceramic and magnetostrictive actuators, which shows symmetric and asymmetric rate dependent hysteresis effects, respectively.

II. Play operator based Prandtl-Ishlinskii model

A. *Classical Prandtl-Ishlinskii model*

Prandtl-Ishlinskii model is a phenomenological hysteresis model that integrates play with a density function to characterize rate independent hysteresis nonlinearities [2]. The play operator is continuous and rate independent hysteresis operator, relating the output and input. The play operator has been described by the motion of a piston within a cylinder of length $2r$ where the instantaneous position of center of the piston is represented by coordinate v, and cylinder position by w [2].

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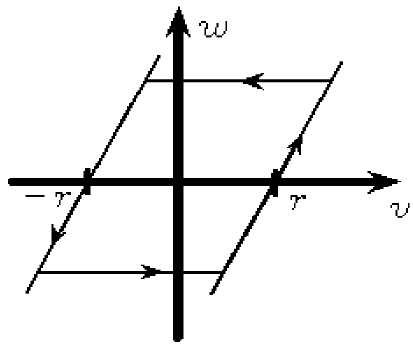

Fig. 1. Play hysteresis operator.

For any input $v(t) \in C_m[0, t_E]$, the play operator is defined by:

$$\begin{aligned} F_r[v](0) &= f_r(v(0), 0) = w(0), \\ F_r[v](t) &= f_r(v(t), F_r[v](t_i)); \\ & \quad t_i < t \leq t_{i+1} \text{ and } 0 \leq i \leq N-1 \\ f_r(v, w) &= \max(v - r, \min(v + r, w)) \end{aligned} \tag{1}$$

where $0 = t_0 < t_1 < ..... < t_N = t_E$ is a partition of $[0, t_E]$ such that the function v is monotone on each of the sub-intervals $[t_i, t_{i+1}]$. The argument of the operator is written in square brackets to indicate the functional dependence, since it maps a function to another function. Due to the nature of the play operator, the above discussions are based on $v(t) \in C_m[0, t_E]$ of continuous and piecewise monotone functions. However, the play operator can be extended to space $C[0, t_E]$ of continuous functions [3].

The Prandtl-Ishlinskii model utilizes the play operator $F_r[v](t)$ to describe relationship between output y_p and input v with a positive density function $p(r)$ and a positive constant q, such that [3]:

$$y_P(t) = qv(t) + \int_0^R p(r) F_r[v, w](t) dr \tag{2}$$

The density function $p(r)$ is identified from experimental data. Prandtl-Ishlinskii model with the density function maps $C[t_o, \infty)$ into $C[t_o, \infty)$. In other words, Lipschitz continuous inputs will yield Lipschitz continuous outputs [4]. However, this hysteresis model produces rate independent and symmetric hysteresis loops. Prandtl-Ishlinskii model can be represented with N play operators as:

$$y_P(t) = qv(t) + \sum_{j=1}^{N} p(r_j) F_{r_j}[v, w](t) \tag{3}$$

B. Rate dependent Prandtl-Ishlinskii model

The fundamental properties of the play operator can be effectively applied to describe the observed rate dependent phenomenon. Particularly, the variations in hysteresis and output amplitude could be accurately described through appropriate selection of the threshold r. The hysteresis of the play is directly related to the threshold r, as evident from the definition of the play operator. An increase in the threshold r yields larger width of the hysteresis operator. Figure 2 illustrates the variations in the play operator output by considering different values of r under a harmonic input $v(t) = 10\sin(2t)$ and initial value $F(v(0), 0) = 0$. The results clearly show that the width of the hysteresis loop increases with an increase in threshold r. Moreover, the peak-to-peak output of the operator decreases with increasing r value. These properties of the play operator conform with those reported for the responses of smart actuators [6], and observed experimentally in this study under inputs at different frequencies. A rate dependent hysteresis play operator may thus be realized using a dynamic threshold $\bar{r} = r(\dot{v})$ as a function of the time rate of input, such that:

$$\begin{aligned} & F_{\bar{r}}(v(t)) = f_{\bar{r}}(v(t), F_{\bar{r}}(v(t_i))) \\ & \quad \text{for } t_i < t \leq t_{i+1} \text{ and } 0 \leq i \leq N-1 \\ & \text{where } f_{\bar{r}}(v, w) = \max(v - \bar{r}, \min(v + \bar{r}, w)); \\ & \text{and } F_{\bar{r}}(v(0)) = f_{\bar{r}}(v(0), 0) = w(0). \end{aligned} \tag{4}$$

The rate dependent Prandtl-Ishlinskii model is subsequently derived by using the rate dependent play hysteresis operator together with density function $p(\bar{r})$ and positive constant q as:

$$y_{\bar{P}}(t) = qv(t) + \sum_{j=1}^{N} p(\bar{r}_j) F_{\bar{r}_j}[v, w](t) \tag{5}$$

The above model can be applied to predict the rate dependent symmetric hysteresis properties of a PZT actuator. Alternatively, the generalized play operator could be utilized to realize asymmetric a dynamic Prandtl-Ishlinskii model with asymmetric input-output relationships.

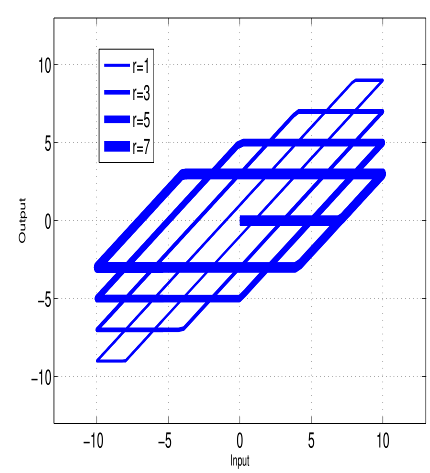

Fig. 2. Influence of threshold r on the output of the play operator, $v(t) = 10\sin(2t)$.

III. Generalized play operator based Prandtl-Ishlinskii hysteresis model

The generalized play operator, as shown in Fig. 3, is a nonlinear play operator. As the input v increases, the output w of the operator increases along the curve γ_l. A decreasing input v causes the output w to decrease along the curve γ_r, as shown in Fig. 3. The minor loops of the input v and the output w are bounded by the limiting curves γ_l and γ_r, which are continuous functions with $\gamma_l \leq \gamma_r$. For a given input $v(t)$, Lipschitz-continuity of the generalized play operator can be ensured if the functions γ_l and γ_r are Lipschitz continuous [2].

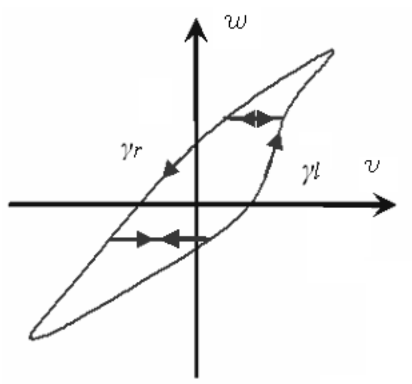

Fig. 3. Generalized play operator.

Analytically, the generalized play operator for any input $v(t) \in C_m[0, t_E]$ is defined by [3]:

$$\begin{aligned} &F_\gamma[v](0) = f_\gamma\left(v(0), 0\right) = w(0); \\ &F_\gamma[v](t) = f_\gamma\left(v(t), F_\gamma[v](t_i)\right); \\ &for \quad t_i < t \leq t_{i+1} \; and \; 0 \leq i \leq N-1 \\ &where, \; f_\gamma(v, w) = \max\left(\gamma_l(v) - r, \min(\gamma_r(v) + r, w)\right) \end{aligned} \tag{6}$$

The generalized Prandtl-Ishlinskii model is subsequently formulated using the generalized play operator $F_\gamma[v](t)$ as:

$$y_{P_\gamma}(t) = qv(t) + \sum_{j=1}^{N} p(r_j) F_{\gamma_j}[v, w](t) \tag{7}$$

In a similar manner, the generalized play hysteresis operator can be modified using the dynamic threshold $\bar{r} = r(\dot{v})$ and integrated in the generalized play operator. By this modification, the symmetry property of the rate dependent play hysteresis operator is relaxed. The generalized rate dependent play operator could be expressed as:

$$\begin{aligned} &F_{\bar{\gamma}}[v](0) = f_{\bar{\gamma}}\left(\gamma(v(0), 0\right) = w(0), \\ &F_{\bar{\gamma}}[v](t) = f_{\bar{\gamma}}\left(\gamma(v(t)), F_{\bar{\gamma}}[v](t_i)\right), \\ &for \; t_i < t \leq t_{i+1} \; and \; 0 \leq i \leq N-1 \\ &f_{\bar{\gamma}}(v, w) = \max\left(\gamma_l(v) - \bar{r}, \min(\gamma_r(v) + \bar{r}, w)\right) \end{aligned} \tag{8}$$

By integrating the generalized rate dependent play operator and the density function together, the generalized rate dependent Prandtl-Ishlinskii model can be expressed as:

$$y_{\bar{P}_\gamma}(t) = qv(t) + \sum_{j=1}^{N} p(\bar{r}_j) F_{\bar{\gamma}_j}[v, w](t) \tag{9}$$

IV. Simulation results

Reported experimental results for different actuators have established that hysteresis is rate dependent symmetric or asymmetric. It has been shown that hysteresis in magnetostrictive actuators is highly asymmetric and the output displacement is strongly rate dependent beyond certain frequencies [12]. Alternatively, the data reported for various piezoceramic actuators under excitation of varying magnitudes and frequencies suggest nearly symmetric major as well as minor hysteresis loops.

As shown in the previous section, the output of the rate dependent Prandtl-Ishlinskii model is directly relates to rate dependent play operator. While the output of the generalized Prandtl-Ishlinskii model depends on the generalized rate dependent play operator. The Prandtl-Ishlinskii model is analyzed using rate dependent play operator and generalized rate dependent play operator. In order to illustrate its influence on the outputs under inputs at varying rates, the simulations are performed under four different input frequencies (1, 2, 3 and 4 Hz) for the two rate dependent play operators to study the influence on the output of the Prandtl-Ishlinskii models.

A dynamic threshold of the following form is chosen:

$$\bar{r} = \varepsilon_1 + \varepsilon_2 \left|\dot{v}\right| + \varepsilon_3 \sqrt{\left|\dot{v}\right|} \tag{10}$$

An input signal of the form: $v(t) = 3 + 10\sin(2\pi f t) + 5\sin(3\pi f t)$ is considered to evaluate minor as well as major hysteresis loops. The chosen simulation parameters are: t_E=6/f, N=301, Δt=0.02/f, $w_0 = 0$, and q=0.457. The parameters for the proposed threshold, described in (12), are taken as: $\varepsilon_1 = 1.4$, $\varepsilon_2 = 0.0545$ and $\varepsilon_3 = 1.0601$. To relax the effect of the density function, a constant density function $p(\bar{r}) = 0.01$ in this simulation is selected for the two rate dependent Prandtl-Ishlinskii models. The generalized rate dependent play operator functions are selected as:$\gamma_r = 3.4v - 1$ and $\gamma_l = 6v + 1.233$.

Figures 4 and 5 illustrate the simulation results at different excitation frequencies of Prandtl-Ishlinskii model of rate dependent play operator and generalized rate dependent PI model of generalized rate dependent play operator, respectively. The results show that as the frequency of the input increases, the hysteresis increases, and the amplitude of the output decreases for major and minor hysteresis loops in both models. The results strongly show that the hysteresis loops of the generalized rate dependent Prandtl-Ishlinskii model of the generalized rate dependent play operator are asymmetric while the hysteresis loops of the rate dependent Prandtl-Ishlinskii model are symmetric. The results clearly demonstrate that the generalized rate dependent play operator relaxes the symmetry of the rate dependent Prandtl-Ishlinskii model output.

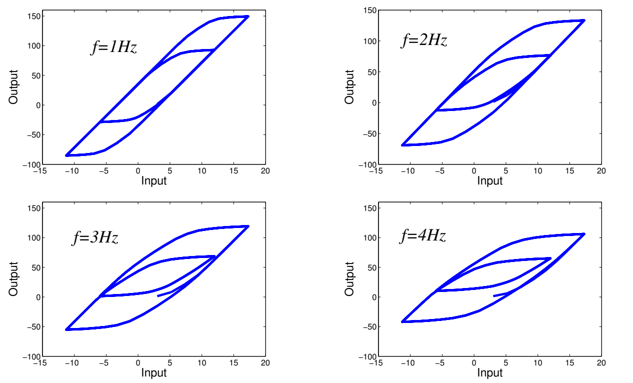

Fig. 4. Simulation results attained from the rate dependent Prandtl-Ishlinskii model of rate dependent play operator at different excitation frequencies.

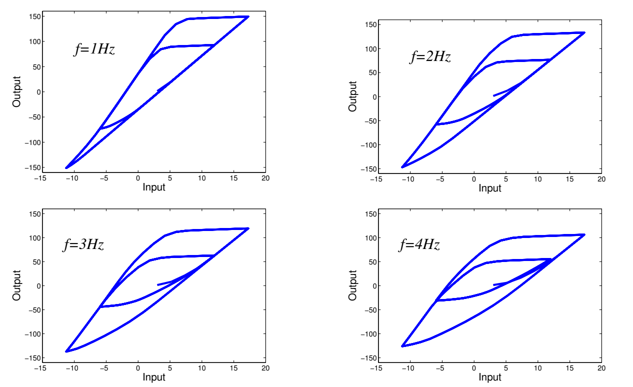

Fig. 5. Simulation results attained from the generalized rate dependent Prandtl-Ishlinskii model of rate dependent play operator at different excitation frequencies.

V. Parameter identification

Experimental results of rate dependent asymmetric and symmetric hysteresis loops are used to identify the parameters of the generalized Prandtl-Ishlinskii model. In this paper, experimental results of magnetostrictive and piezoceramic actuators are used to show the capability of the model to characterize rate dependent symmetric and asymmetric hysteresis loops. On the basis of the observed hysteresis of magnetostrictive actuator, the generalized play operator is proposed as:

$$f_{\bar{\gamma}}(v, w) = \max\left(a_1 v + b_1 - \bar{r}, \min(a_2 v + b_2 + \bar{r}, w)\right) \tag{11}$$

where $a_1 > 0$, $a_2 > 0$, b_1, and b_2 are constants. The dynamic threshold is expressed as:

$$\bar{r} = \alpha \prod_{l=1}^{z} \ln\left(\beta_l + \lambda_l \left|\dot{v}(t)\right|^{\varepsilon_l}\right) \tag{12}$$

where α, λ_l, $\beta_l \geq 1$ and $\varepsilon_l \geq 1$ are positive constants. The order of the rate dependent threshold is determined by the positive integer z. This hysteresis operator can be applied to characterize the rate dependent asymmetric hysteresis effects of smart actuators. In this paper, a second order dynamic threshold is proposed ($z = 2$) to characterize rate dependent hysteresis. The density function of the generalized rate dependent Prandtl-Ishlinskii model is chosen as:

$$p(\bar{r}) = \rho e^{-\tau \bar{r}} \tag{13}$$

where $\rho > 0$ and τ are constants. The density function and the functions $\gamma_l = a_1 v + b_1$ and $\gamma_r = a_2 v + b_2$ of the generalized play operator are not unique, they depend upon the nature of hysteresis of particular material or actuator.

The model parameters are identified through minimization of the error sum function given by:

$$J = \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} \sum_{k=1}^{n_3} \left(y_{\bar{P}_\gamma} - y_m\right)^2 \tag{14}$$

where $y_{\bar{P}_\gamma}$ is the displacement response of the generalized Prandtl-Ishlinskii model corresponding to a particular excitation frequency, and y_m is the measured displacement under the same excitation frequency. The error function is constructed through summation of squared errors over a range of input frequencies and amplitudes, denoted by j ($j = 1 \ldots n_2$) and k ($k = 1 \ldots n_3$), respectively. The index i ($i = 1 \ldots n_1$) refers to the number of data points considered to compute the error function J for one complete hysteresis loop. For the magnetostrictive actuator, one level of input current amplitude of 0.8 A with a bias of 0.1 A ($n_3 = 1$) could be considered, since the data was available only under this excitation. Four different excitation frequencies ($n_2 = 3$), namely, 10, 20, and 50 Hz were considered, while a total of 60 data points ($n_1 = 60$) were available for each hysteresis loop. For the piezoceramic actuator, the input voltage amplitude was limited to 40 V ($n_3 = 1$), while a total of four different frequencies of 50, 100, and 200 Hz($n_2 = 3$) were considered with a total of 50 data points ($n_1 = 50$) for each hysteresis loop. The error minimization is performed using the MATLAB constrained optimization toolbox. The generalized rate dependent Prandtl-Ishlinskii model parameters which identified through the solution of the minimization problem are shown in Table 1.

VI. Model validation

Figure 6 illustrates comparison of results attained from the generalized Prandtl-Ishlinskii model of asymmetric rate dependent play operator with measured data of magnetostrictive actuator at three different frequencies (10, 20, and 50 Hz). The results show the capability of the model to characterize rate dependent asymmetric hysteresis loops of the actuator. The results show that the hysteresis of the measured and model results show increasing hysteresis and decreasing output amplitude with increasing excitation frequency of the input voltage. Figure 7 shows comparison of between the time series of the measured and predicted displacement of the model at different excitation frequencies. The time series of the error under excitation frequencies are shown in Fig. 8. The results suggest peak error of the rate dependent model near 5 μm. In other words, the error of the rate dependent model is approximately unaffected as the excitation frequency of the input current increases.

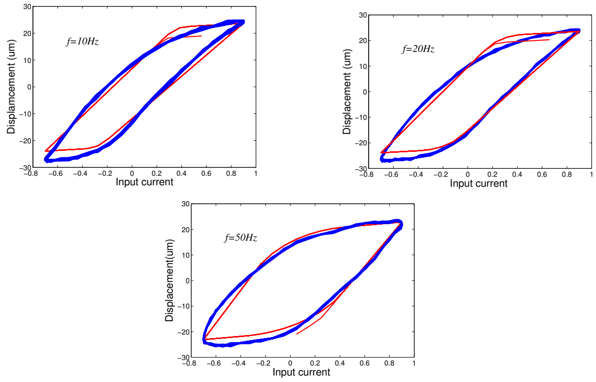

Fig. 6. Comparisons of measured responses of magnetostrictive actuator with the results derived from the generalized rate dependent rate-dependent model under sinusoidal waveform inputs at 10 to 50 Hz different excitation frequencies (—measured ;—model).

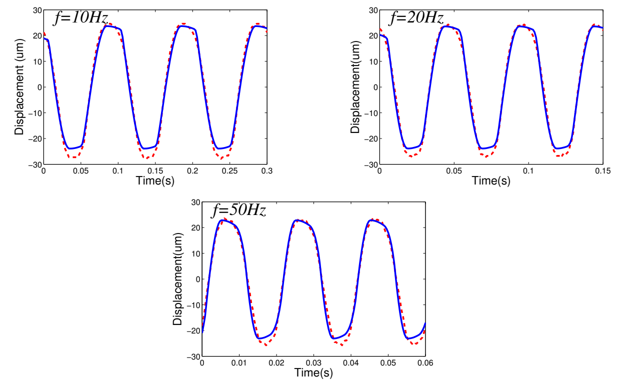

Fig. 7. Time series of measured and predicted displacement of magnetostrictive responses at different excitation frequencies (· · · measured ;— model).

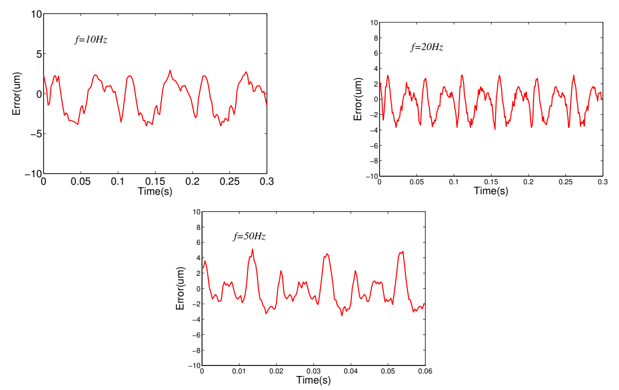

Fig. 8. Time series of error in measured and model displacement of magnetostrictive actuator responses at different excitation frequencies.

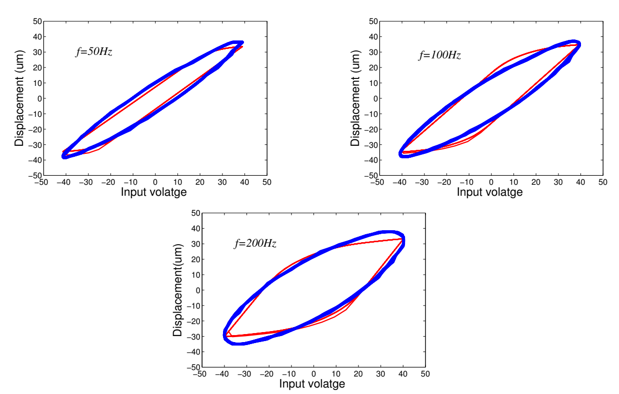

Fig. 9. omparisons of measured responses of piezoceramic actuator with the results derived from the rate dependent Prandtl-Ishlinskii model under sinusoidal waveform inputs at 10 to 100 Hz different excitation frequencies (—measured ;—model).

To show the capability of the model to characterize rate dependent symmetric hysteresis, a comparison between the results of generalized Prandtl-Ishlinskii model and the measured data of piezoceramic actuator at different excitation frequencies of 50, 100, and 200 Hz are carried out in Fig. 9. Time series of displacement responses of the generalized rate dependent Prandtl-Ishlinskii model are further compared with the measured data obtained in Fig. 10. Figure 11 shows the time-series of the model and measured responses. The results suggest very good agreements between the predicted and measured displacement responses irrespective of the excitation frequency, while the peak displacement error, as shown in Fig. 11 is approximately 5 μm.

Parameters	Magnetostrictive actuator	Piezoceramic actuator
α	4.203	2.493
β_1	1.199	1.090
$\beta 2$	1.201	1.020
λ_1	0.0474	21×10^{-4}
λ_2	0.0036	1.0387×10^{-4}
ρ	0.0055	0.0050
τ	- 0.08714	- 0.0173
a_1	20.1916	0.7614
a_2	22.0611	0.7223
b_1	5.3072	1.0882
b_2	2.4484	1.3315
q	2.6251	0.1028

TABLE I

IDENTIFIED PARAMETERS OF THE GENERALIZED RATE DEPENDENT PRANDTL-ISHLINSKII MODEL.

VII. CONCLUSIONS

A generalized rate dependent play operator is proposed for formulating the Prandtl-Ishlinskii model capable of predicting symmetric and asymmetric rate dependent hysteresis nonlinearities. A dynamic threshold function is further proposed and integrated to the model to characterize asymmetric and symmetric rate dependent hysteresis properties. By way of an example, it is shown that the generalized play operator permits for relaxation of the slopes of the major as well as

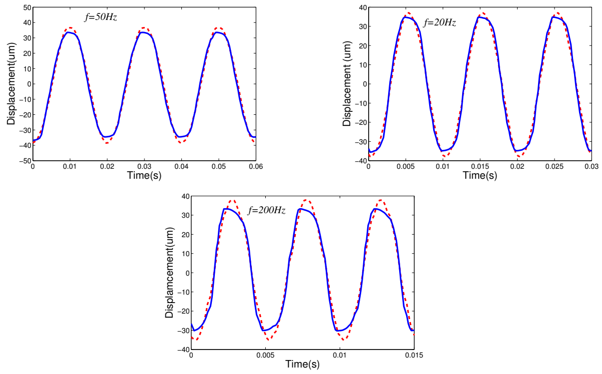

Fig. 10. Time series of measured and model displacement of piezoceramic actuator responses at different excitation frequencies(· · · measured; — model)

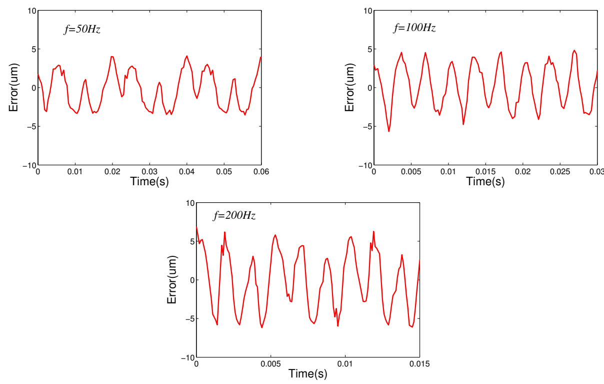

Fig. 11. Time series of error in measured and model displacement of piezoceramic actuator responses at different excitation frequencies.

minor hysteresis loops, and can thus characterize symmetric as well asymmetric input-output relations. The effectiveness of the models is demonstrated by comparing the model responses with the measured symmetric and asymmetric hysteresis loops of the piezoceramic and magnetostrictive actuators, respectively, under different excitation frequencies. From the results, it can be concluded that the Prandtl-Ishlinskii model comprising the generalized rate dependent play operator together with a density function can effectively predict the symmetric as well as asymmetric hysteresis properties of the smart actuators and materials under different rates of inputs.

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References

[1] I. Mayergoyz, *Mathematical Models of Hysteresis and Their Applications*, Elsevier, 2003.
[2] A.Visitian, *Differential Models of Hysteresis*, Springer, 1994.
[3] M. Brokate and J. Sprekels, *Hysteresis and Phase Transitions*, Springer, 1996.
[4] M. Krasnoselskii and A. Pokrovskii, *Systems with hysteresis*, Springer-Verlarg, 1989.
[5] G. Tao and P. V. Kokotovic, "adaptive control of plant with unknown hysteresis," *IEEE Transactions on Automatic control*, vol. 40, pp. 200-212, 1995.
[6] P. Ge and M. Jouaneh, "Modeling hysteresis in piezoceramic actuators," *Precision Engineering*, vol. 17, no. 3, pp. 211–221, 1995.
[7] D. Hughes and J. T. Wen, "Preisach modeling of Piezoceramic and shape memory alloy hysteresis," *Smart Materials and Structures*, vol. 6, pp. 287-300, Feb. 1997.
[8] D.C. Jiles and D.L. Atherton, "Theory of ferromagnetic hysteresis," *Journal of applied physics*, vol. 55, no. 6, pp. 2115-2120, March 1984.
[9] F.T. Calkins , R.C. Smith , A.B. Flatau, "Energy-based hysteresis model for magnetostrictive transducers", *IEEE Transactions on Magnetics*, vol. 36, no. 2, pp. 429-439, 2000 .
[10] W. Galinaities, "Two methods for modeling scalar hysteresis and their using in controlling actuators with hysteresis," Ph.D. dissertation, Dept. Math., Blacksburg, Virginia, USA, 1999.
[11] G. Song, J. Zhao, X. Zhou, and J.A.D. Abreu-Garcia, "Tracking control of a piezoceramic actuator with hysteresis compensation using inverse Preisach model", *IEEE Transactions on Mechatronics*, vol. 10, no. 2, pp. 198-209, 2005.
[12] X. Chen, "Robust control for the systems proceeded by hysteresis." *Proc. of the 25th IASTED international conference on Modeling, identification, and control* , Lanzarote, Spain, pp. 173-178, 2005.
[13] C.Y. Su, Q. Wang, X. Chen, S. Rakheja , "Adaptive variable structure control of a class of nonlinear systems with unknown Prandtl-Ishlinskii hysteresis," *IEEE Trans. on Automatic Control*, vol. 50, no. 12, pp. 2069-2074, 2005.
[14] J. Song, A. D Kiureghian, "Generalized Bouc-Wen model for highly asymmetric hysteresis", *Journal of Engineering Mechanics-ASCE*, vol. 132, no. 6, pp. 610-618, 2006.
[15] R.C. Smith, S. Seelecke, Z. Ounaies, J. Smith, "A free energy model for hysteresis in ferroelectric materials", Journal of Intelligent Material Systems and Structures, vol. 14, no. 11, pp. 719-739, 2003.
[16] X. Tan and J. S. Baras, "Modeling and control of hysteresis in Magnetostrictive actuators," *Automatica*, vol. 40, no. 9, pp. 1469-1480, 2004.
[17] Y. Yu, Z. Xiao, N. G. Naganathan and R.V. Dukkipati, "Dynamic Preisach modelling of hysteresis for the piezoceramic actuator system", *Mechanism and Machine Theory*, vol. 37, no. 1, pp. 75-89, Jan. 2002.
[18] D. Song , C. J. Li, "Modeling of Piezo actuator's nonlinear and frequency dependent dynamics", *Mechatronics*, vol. 9, no. 4, pp. 391-410, June 1999.
[19] I. Mayergoyz, "Dynamic Preisach models of hysteresis", *IEEE Trans. on Magnetics*, vol. 24, pp. 2925-2927, Nov. 1988.
[20] M. Al Janaideh, S. Rakheja ,C.Y. Su, "A generalized Prandtl-Ishlinskii model for characterizing rate dependent hysteresis", *In the proceedings of the 16th IEEE Conference of Control Applications*, pp.343-348, 2007.