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Optimizing Deployment of Multiple Decoys to Enhance Ship Survivability

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Abstract— **In this paper we model a scenario where a ship uses decoys to evade a hostile torpedo. We address the problem of enhancing ship survivability against enemy torpedoes by using single and multiple decoy deployments. We incorporate deterministic ship maneuvers and realistic constraints on turn rates, field of view, etc in the model. We formulate the objective function to quantify and maximize the survivability of the ship in terms of maximizing the intercept time. We introduce the concept of optimal deployment regions, same side deployment, and zig-zag deployment strategies. Finally, we present simulation results.**

I. Introduction

The problem of optimal decoy deployment in naval combats has not attracted much attention in the open literature. The torpedo, a dynamic and lethal underwater weapon [1], comprises of an intelligent homing system which performs search, detects the target, and guides itself to the target ship. The ship uses several protective measures [2], [3], one of which is to launch decoys against the torpedo. In [4], we showed the efficacy of this approach in enhancing ship survivability using a simple model. In this paper, realistic models for the entities have been incorporated in terms of constraint on turn rates, field of view of the torpedo, etc.

II. Problem Formulation

A. The State Equations

The state equations for the torpedo-ship engagement are given below. The variables are explained in Figure 1.

$$\begin{aligned}
\dot{x}^T &= v^T \cos\alpha^T; \;\; \dot{y}^T = v^T \sin\alpha^T \\
\dot{x}^S &= v^S \cos\alpha^S; \;\; \dot{y}^S = v^S \sin\alpha^S \\
\alpha^T_S &= \arctan\{(y^S - y^T)/(x^S - x^T)\} \\
\dot{R}^{TS} &= v^S \cos(\alpha^S - \alpha^T_S) - v^T \cos(\alpha^T - \alpha^T_S) \\
\dot{\alpha}^T_S &= \{v^S \sin(\alpha^S - \alpha^T_S) - v^T \sin(\alpha^T - \alpha^T_S)\}/R^{TS}
\end{aligned} \tag{1}$$

The acceleration command for the torpedo, $\dot{\alpha}^T$, and that for the ship, $\dot{\alpha}^S$, are given by,

$$\dot{\alpha}^T = a^T/v^T; \;\; \dot{\alpha}^S = \pm 1^0/s = \pm 0.0175 \text{ rad/s} \tag{2}$$

$$|\dot{\alpha}^T| \leq 16^0/s = 0.2793 \text{ rad/s} \tag{3}$$

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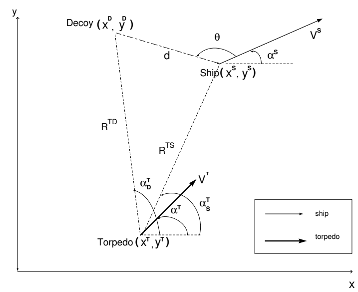

Fig. 1. The engagement geometry

The co-ordinates of the decoy are given by,

$$x^D = x^S + d\cos(\theta + \alpha^S); \;\; y^D = y^S + d\sin(\theta + \alpha^S) \tag{4}$$

The state equations for the torpedo-decoy engagement are,

$$\begin{aligned}
\dot{x}^T &= v^T \cos\alpha^T; \;\; \dot{y}^T = v^T \sin\alpha^T; \;\; \dot{x}^D = 0; \;\; \dot{y}^D = 0 \\
\alpha^T_D &= \arctan\{(y^D - y^T)/(x^D - x^T)\} \\
\dot{R}^{TD} &= -v^T \cos(\alpha^T - \alpha^T_D) \\
\dot{\alpha}^T_D &= -v^T \sin(\alpha^T - \alpha^T_D)/R^{TD}
\end{aligned} \tag{5}$$

The following parameters have been used in the model: Homing range of the torpedo (H); Field of view of the torpedo (FOV); After the target is detected, the torpedo's reduced field of view ($RFOV$); Maximum turn rate of the torpedo ($\dot{\alpha}^T_{max}$); The rate at which it executes the Lost Contact Search (LCS) ($\dot{\alpha}^T_{lcs}$); The torpedo speed during the LCS (v^T_{lcs}); Distance from the ship at which the decoy is deployed (d); Turn rate of the ship ($\dot{\alpha}^S_{max}$); The torpedo blind zone (B); The torpedo guidance cycle time.

B. The Torpedo Dynamics

The torpedo is assumed to guide itself toward the ship, if the ship is inside its field of view, using Proportional Navigation (PN) guidance law till it reaches its homing range. The torpedo acceleration command a^T is,

$$a^T = K_1 v^T \dot{\alpha}^T_S \tag{6}$$

where, $\dot{\alpha}^T_S$ is the rate of change of the LOS angle between the torpedo and the ship. When the torpedo reaches its homing range, it is assumed that the torpedo homes on to the ship using a Pure Pursuit guidance law. The torpedo acceleration command a^T is,

$$a^T = v^T \dot{\alpha}^T_S - K_2(\alpha^T - \alpha^T_S) \tag{7}$$

The torpedo, when lured by the decoy, moves towards the decoy using a PN guidance law and the acceleration

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command is given by (6) with $\dot{\alpha}_S^T$ replaced by $\dot{\alpha}_D^T$. If the decoy is within the torpedo's homing range, the torpedo uses the Pure Pursuit guidance law and its acceleration command is given by (7) with α_S^T replaced by α_D^T and $\dot{\alpha}_S^T$ replaced by $\dot{\alpha}_D^T$. Here, a^T is constrained by (3), which also implies that,

$$\dot{\alpha}_D^T = \text{sgn}(\dot{\alpha}_D^T)\ \max\left\{|a^T/v^T|,\ \dot{\alpha}_{max}^T\right\} \quad (8)$$
$$\dot{\alpha}_S^T = \text{sgn}(\dot{\alpha}_S^T)\ \max\left\{|a^T/v^T|,\ \dot{\alpha}_{max}^T\right\} \quad (9)$$

C. The Torpedo Decision Logic

It is assumed that if the decoy is not deployed inside the $\pm 20^0$ field of view of the torpedo, the decoy fails to divert the torpedo and the torpedo continues to home on to the ship. If the decoy is in the torpedo's field of view, then it moves towards the decoy till it is within 50 meters of the decoy. After which, it is assumed that the torpedo identifies the decoy and heads towards the ship. It is also assumed that the torpedo is not lured by the decoy until the torpedo switches to its autonomous active homing mode.

After identifying the decoy, the torpedo switches back its field of view to $\pm 70^0$ and if the ship is outside its field of view, the torpedo executes a Lost Contact Search (LCS) during which it moves, with a slower turn rate and speed than when compared to the attack mode, in a circular trajectory till it locks on to the ship and starts moving towards the ship. If the ship is beyond the homing range of the torpedo, the torpedo loses the ship and keeps executing the LCS till it reaches its endurance limit.

In the scenario where multiple decoys have been deployed and more than one are active, it is assumed that the torpedo moves towards the nearest decoy which has not been identified as a decoy by the torpedo yet (that is, the range between the torpedo and the decoy is more than 50 m).

In several scenarios where both a decoy and the ship are in the torpedo's field of view, it is assumed that the torpedo is unable to distinguish between the two targets and in order to effectively simulate this scenario the torpedo is assumed to move toward the weighted mean of the positions of the ship and the decoy. A variable w that reflects the probability of the torpedo moving towards the decoy when both the ship and the decoy are in the torpedo's field of view is defined. The trajectories of the engagement for a few values of w are shown in the Figure 2(a). The dotted line represents the trajectory of the torpedo corresponding to the value $w = 1$. We use w=0.9 in our simulation as we assume that the decoy is effective in attracting the torpedo.

D. The Ship Evasive Maneuver

The ship maneuver logic uses ship's relative bearing with the torpedo to compute its evasive maneuver. We omit details. The ship maneuvers till it acquires the required heading given by the logic, and moves along that direction. The time between two evasive maneuver commands is approximately the time taken by the ship to cover about 1 km. Since we assume constant ship speed of 20 knots, the evasive maneuver command cycle is 97.2 seconds.

III. The Single Decoy Case

The values used for the parameters are shown in Table I. The ship was assumed to possess only one decoy. Initial position of the torpedo (shown in Figure 2(b)) was varied, in steps of 15^0, along a circle of radius r = 3 km. The torpedo was always assumed to be released in a direction directly toward the ship.

A. The Objective Function

The objective function to be maximized is the time t_f at which the torpedo intercepts the ship. Thus,

$$\max_{t^D,\ \theta}\ t_f \quad (10)$$

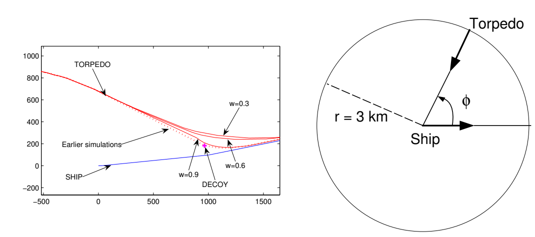
where, t^D and θ are the time of deployment and the angle of deployment, respectively. Also,

$$0 \le t^D \le t^{-D};\quad 0 \le \theta \le 2\pi \quad (11)$$

where, t^{-D} is defined later in this section.

It was observed that carrying out a medium scale constrained nonlinear optimization on this objective function is very difficult. The difficulty can be attributed to the nature of the objective function that has sharp peaks and troughs as a result of which, the gradient based optimization procedure leads us to a the local maxima and almost never to the global maximum. To overcome the above problem, discretization of the decision variables was undertaken, the details of which are given below.

Figures 3(a) and (b) show the objective function plots, the total time of engagement (t_f) for values of (t^D, θ) for different initial positions of the torpedo. We observe that, for every initial position of the torpedo, there always exists a last moment deployment that gives the maximum total time of engagement. This is because the last moment deployment renders the ship outside the torpedo's $\pm 70^0$ field of view



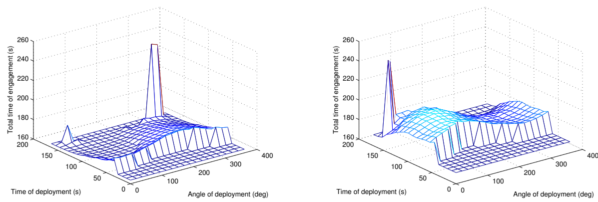
(a) (b)

Fig. 2. (a) Both ship and decoy in torpedo's field of view (b) Initial position of the torpedo

v^S	20 knots	FOV	$\pm 70^0$
v^T	50 knots	$RFOV$	$\pm 20^0$
d	0.1 km	$\dot{\alpha}_{max}^T$	$\pm 16^0/s$
τ	10 min	$\dot{\alpha}_{lcs}^T$	$\pm 8^0/s$
r	3 km	$\dot{\alpha}_{max}^S$	$\pm 1^0/s$
(K_1, K_2)	(10, 3)	v_{lcs}^T	40 knots
H	1.5 km	B	0.1 km

TABLE I

INPUT PARAMETER VALUES



(a) (b)

Fig. 3. (a) $\phi = 75^0$ and (b) $\phi = 180^0$

Strategy	t_f (sec)
Optimum deployment	198.7
Last moment deployment	259
Both optimum and last moment strategy	288.4

TABLE II
THE TOTAL TIME OF ENGAGEMENT (SECONDS) - A COMPARISON

forcing the torpedo to execute its LCS and thus the ship gains a significant amount of extra time. A sample trajectory depicting this scenario is shown in the next subsection.

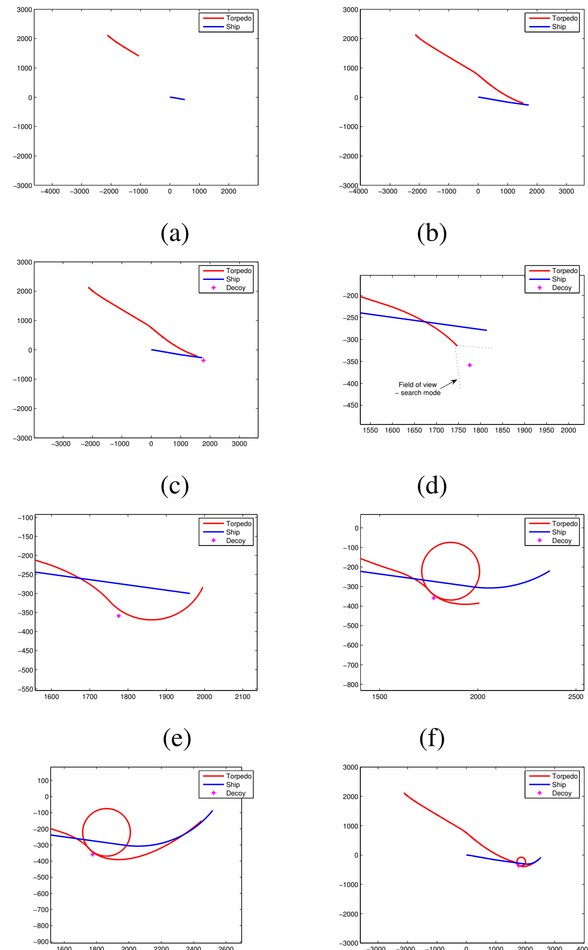
It can also be observed that the last moment deployments occur when the torpedo is very close to the ship. This could be extremely risky. So if we rule out using the last moment deployment option (because of the risk involved), then for most of the initial positions of the torpedo, it can be clearly deduced from the plots that deploying the decoy at the beginning of the engagement proves to be most effective.

Also in some objective function plots certain dips were observed (Figure 3(b)). The deployment in these cases render the decoy outside the torpedo's $\pm 20^0$ field of view, because of which the torpedo does not sense the presence of the decoy and moves towards the ship to intercept it.

B. Last Moment Deployment Strategy

In this section, we will analyze the last moment deployment strategy in detail and present a clear explanation of such an engagement scenario by pausing the engagement at various intervals of time progressively. Successful execution of this strategy requires a carefully chosen angle of deployment. It can also be easily concluded that the torpedo's turn rate during the execution of its LCS has an important role to play in the execution of this strategy.

Figure 4(a) depicts the initial stages of the engagement. The torpedo is allowed to come as close as 200 m of the ship (Figure 4(b)) before the decoy is deployed such that the decoy is inside the torpedo's $\pm 20^0$ field of view (Figure 4(c)). The decoy lures the torpedo towards it till it is within 50 meters from the decoy at which, point the torpedo identifies the decoy (Figure 4(d)) and, since the the ship is outside the torpedo's FOV, it begins to execute LCS. In this process it shoots ahead of the ship (Figure 4(e)) and eventually locks on to the ship (Figure 4(f)). Then, it heads towards the ship and intercepts it (Figures 4(g) and 4(h)).



(a) (b) (c) (d) (e) (f) (g) (h)

Fig. 4. $\phi = 135^0$, $t^D = 170.5s$, $\theta = 303^0$, t=0 - 259 s (a) t=50s: Initial stages (b) t=169s: Before deployment (c) $t = t^D$ (d) t=178s: Decoy identification (e) t=193s: Overshooting of torpedo (f) t=234s: Reacquiring of ship (g) $t = t_f$ (h) $t = t_f$; Original scale

C. Some Special Cases

The last moment deployment strategy, although potentially effective, is risky and is sensitive to the decoy deployment angle. In view of this, it is advisable to keep one decoy reserved till the torpedo comes very close to the ship (about 150 m range from the ship) so that the ship can attempt to enhance its survivability by executing the last moment deployment strategy. Figure 5(a) depicts the scenario where the ship executes the optimum deployment strategy, saves one decoy for the execution of the last moment deployment strategy and executes it successfully. Table II very clearly shows the added advantage in reserving one decoy till the torpedo comes very close to the ship.

Trajectory 5(b) depicts a scenario where the decoy deployment occurs right at the beginning of the engagement and, as mentioned in the previous section, this seems to be optimum in most cases. An exception of this case is shown in Figure 5(c) where the optimum decoy deployment occurs not at the beginning but much later. Figure 5(d) represents

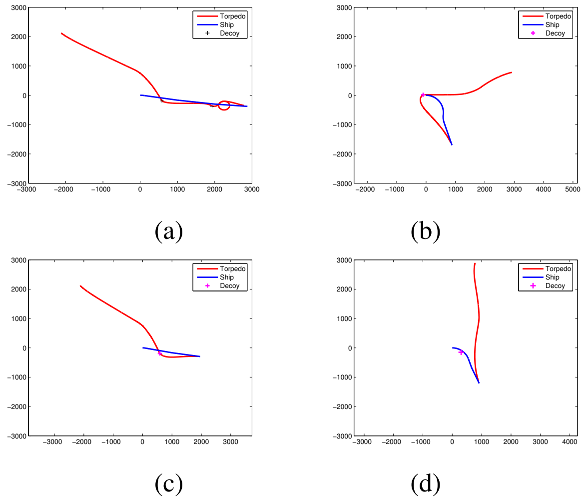


Fig. 5. Some special trajectories (a) Optimum strategy combined with last moment deployment (b) $\phi = 15^0,\ t^D = 0s,\ \theta = 170^0$ (b) $\phi = 135^0,\ t^D = 117.68s,\ \theta = 94.5^0$ (c) $\phi = 75^0,\ t^D = 26s,\ \theta = 132.6^0$ (d) $\phi = 135^0,\ t^D = 170.56s,\ \theta = 303^0$

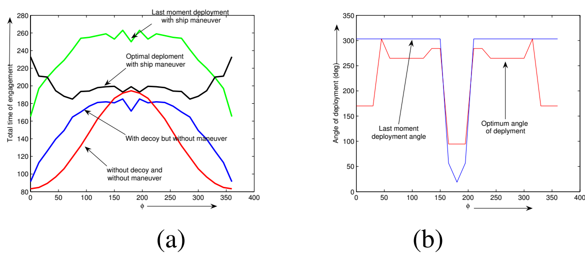


Fig. 6. (a) Total engagement time (b) optimum deployment angle vs ϕ

the case discussed in the previous subsection regarding dips observed in a few plots. The decoy is deployed so that it remains outside the $\pm 20^0$ of the torpedo and is thus rendered ineffective as the torpedo fails to sense the decoy, and intercepts the ship. The trajectory shown in Figure 4(h) depicts this case. The deployment occurs in such a fashion that the torpedo, after recognizing the decoy performs the LCS as the ship is outside its field of view and thus the ship gains a significant amount of time.

D. Analysis of Computational Results and Optimal Regions

Figure 6(a) compares between the total time of engagement for cases when the ship is maneuvering and optimal deployment takes place (t_f^*), when the ship is not maneuvering and has no decoys ($t^{-M,-D}$), and when the ship is maneuvering but has no decoys to deploy (t^{-D}). The optimum angle of decoy deployment is plotted against initial positions of the torpedo in Figure 6(b). It shows that the total time of engagement is higher for all initial positions of the torpedo if a maneuvering ship deploys the decoys optimally (if we ruled out the last moment deployment option).

We introduce the concept of optimal regions by identifying those deployments as a result of which the total time of engagement (t_f) is within 3 seconds and 6 seconds of the optimal total time of engagement. The last moment deployment option is not considered. These regions are computed for different initial positions of the torpedo. Figures 7 show these regions for corresponding initial torpedo positions.

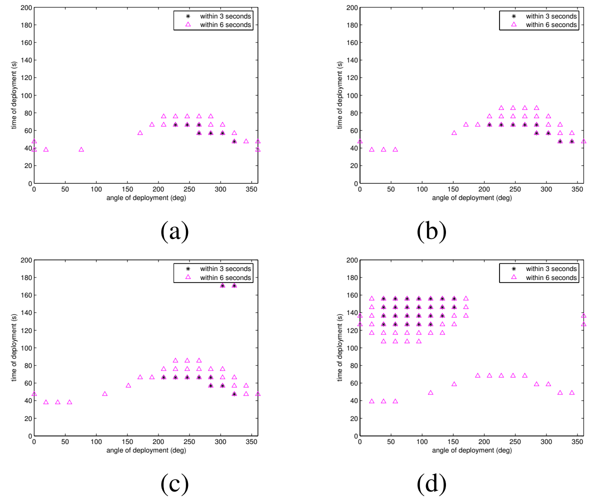


Fig. 7. (a) $\phi = 120^0$ (b) $\phi = 135^0$ (c) $\phi = 150^0$ (d) $\phi = 165^0$

IV. Analysis of Test Cases: Multiple Decoys

Here, we analyze various engagements to study the behavior of the system entities (ship, torpedo, decoys) in some special engagement scenarios. The values used for the input parameters are shown in Table I and $\phi = 135^0$ (Figure 2(b)).

A. Case I: Two decoys; second decoy outside FOV

Consider the case where, when the second decoy deployment occurs, the first decoy is still active and the torpedo has not yet reached within 50 m range of the first decoy (Figure 8(b)). Thus, if the second decoy is outside the torpedo's field of view, the torpedo continues to move towards the first decoy until it reaches within 50 m range (Figure 8(c)), identifies the decoy, and overcomes it to move towards the ship (Figure 8(d)).

B. Case II: Two decoys; both inside FOV

In the case where the second deployment occurs within the field of view of the torpedo and before the torpedo identifies the first decoy, the torpedo is lured by the decoy which is closest to it and heads towards it. After identifying the decoy, it tries to head towards the ship and if the second decoy is active and still inside the torpedo's field of view, the torpedo will head towards this decoy, overcome it, and then move toward the ship. We assume all values to be the same as the previous case. Only the angle of deployment of the second decoy was changed so that the deployment occurs within the torpedo's field of view. Figure 9 shows such an engagement.

C. Case III: Zig-zag deployment; coordinated ship maneuver

When the ship deploys the decoys in a zig-zag fashion with respect to its motion (on the port side and the starboard side, alternatively), the ship is said to execute a *zig-zag* deployment strategy. It was observed that the ship gains a

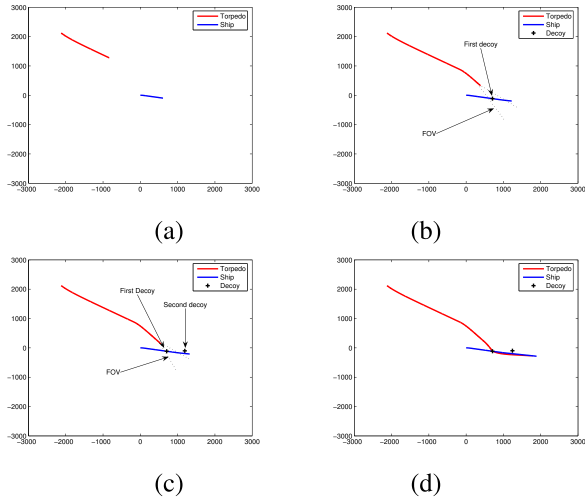


Fig. 8. $\phi = 135^0$, $t \in [60 \quad 120]$, $\theta \in [30^0 \quad 230^0]$ (a) t=60s: first decoy deployment (b) t=120s: second decoy deployment (c) t=130s: torpedo moves towards the first decoy (d) $t = t_f$

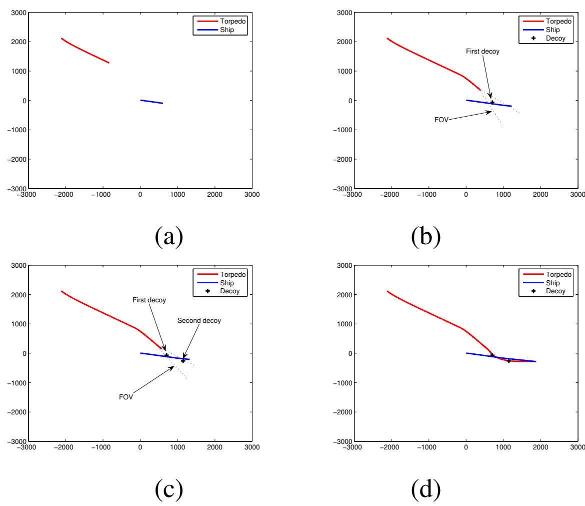


Fig. 9. $\phi = 135^0$, $t \in [60 \quad 120]$, $\theta \in [0^0 \quad 90^0]$ (a) t=60s: first decoy deployment (b) t=120s: second decoy deployment (c) t=130s: torpedo moves towards the closer decoy (d) $t = t_f$

considerable amount of time when compared to deploying decoys on the same side (discussed in the next section). The coordinated ship maneuver plays a key role in the execution of this strategy as will be shown later. The angle of deployment also plays a significant role. All values except the angle of deployment of the third decoy remains the same. Figure 10(b) shows the scenario where this changed value of the deployment angle renders the strategy ineffective as the third decoy fails to attract the torpedo. The deployment times and angles and total engagement time are given in Table III.

D. *Case IV: Zig-zag deployment without ship maneuver*

We consider the *zig-zag* strategy without any ship evasive maneuver. All values of parameters are similar to the values used to demonstrate the zig-zag strategy in the previous section. But it is assumed that the ship does not perform a

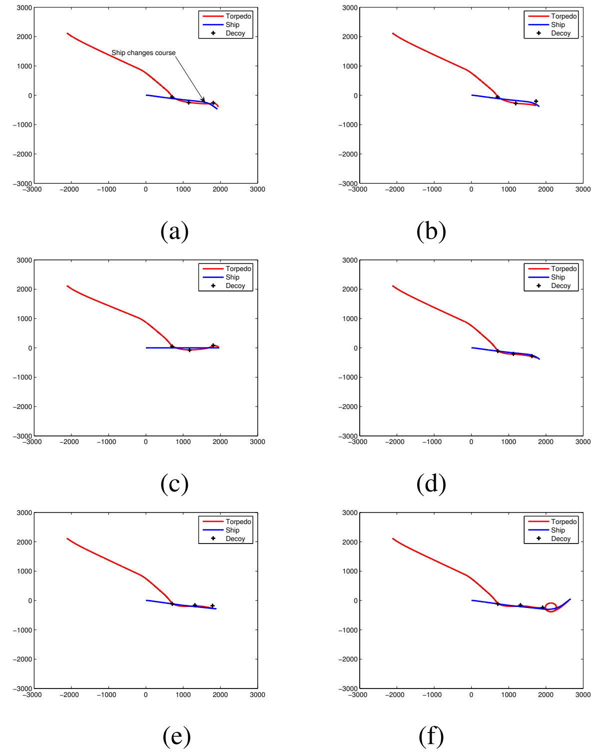


Fig. 10. Zig-zag, same side and last moment deployment (a) Effective zig-zag with coordinated ship maneuver (b) Ineffective zig-zag with coordinated ship maneuver (c) Zig-zag deployment without ship maneuver (d) Same side deployment (e) Ineffective last moment deployment (f) Effective last moment deployment

maneuver. Figure 10(c) represents this scenario and one can observe that a coordinated ship maneuver is instrumental in the ship gaining time. But it can also be seen that even in this case the zig-zag deployment strategy works effectively. The time and angles of deployment, and the total time of engagement are given in Table III.

E. *Case V: Same side deployment*

Consider when all deployments take place on one side of the ship (starboard or port). Deploying the decoys on the same side of the ship actually helps the torpedo move toward the ship as such deployments leave a trail behind the ship. This scenario is shown in Figure 10(d). Thus the *zig-zag* strategy is much more effective than the same side deployment (Table III). The time and angles of deployment, and the total time of engagement are given in Table III.

F. *Case VI: Last moment deployment*

Last moment deployment, discussed in Section 3, advocates the effectiveness of saving one decoy till the end of the engagement to execute the last moment deployment strategy. Figure 10(e) and (f) establish the effectiveness of the strategy and the fact that the angle of deployment plays a vital role in the successful execution of the last moment deployment strategy. Figure 10(e) and (f) depict the scenario where the first two decoys are deployed as in most of the previous cases but the third decoy is saved till the end of the engagement and deployed when the ship is approximately 250 m away from the torpedo. Figure 10(e) shows that an arbitrary choice of angle of deployment results in the failure of this strategy as the decoy deployed at this angle is rendered ineffective as it is outside the torpedo's field of view.

V. Simulation Results: Multiple Decoys

In this section we present the simulation results and establish multiple decoy deployment strategies for the ship. The values used for the input parameters are shown in Table I with reference to Figure 2(b). Initially, the torpedo was always assumed to be released in a direction directly toward the ship. Similar to the single decoy case, simulations are carried out for different initial positions of the torpedo (Figure 2(b)). The ship maneuvers according to the maneuver strategy described in Section 2. For all these computations, the last moment deployment strategy is not considered.

A. Objective function

The objective function t_f to be maximized i is the time at which the torpedo intercepts the ship. Thus,

$$\max_{\delta,\ t_1} \ t_f \tag{12}$$

where, t_1 and δ are the time of deployment of the first decoy and the time lag between consecutive deployments, respectively, and also the decision variables. Also,

$$0 \quad \leq \quad t_1 \ \leq \ t^{-D}; \quad \delta \ \geq \ 0 \tag{13}$$

where, t^{-D} is as defined in Section 2.

The angle of deployment is fixed according to the strategy adopted and is not considered as a decision variable.

B. Same side deployment

With the same side deployment strategy, the angle of deployment of decoys is fixed to 180^0. The optimum values of t_1 and δ were computed for two and three decoys.

It was observed that deploying three decoys instead of two does not give a great advantage. Figure 11(a) compares the total time of engagement when the ship deploys two and three decoys with same side deployment strategy, and a single decoy using the optimal deployment strategy. From the figure one can clearly conclude that deploying three decoys instead of two, while adopting the same side deployment strategy, does not increase the total time of engagement by more than 2 seconds. Figure 11(a) also shows that for certain initial positions of the torpedo deploying a single decoy gives

Deployment strategy	Deployment time (s)	Deployment angle (deg)	t_f (s)
Zig-zag with ship maneuver	[60 120 170]	[30^0 230^0 60^0]	200
Zig-zag without ship maneuver	[60 120 170]	[30^0 230^0 60^0]	193
Same side	[60 120 170]	[30^0 200^0 200^0]	192
Last moment	[60 120 178.5]	[10^0 30^0 30^0]	280

TABLE III

Comparison between cases

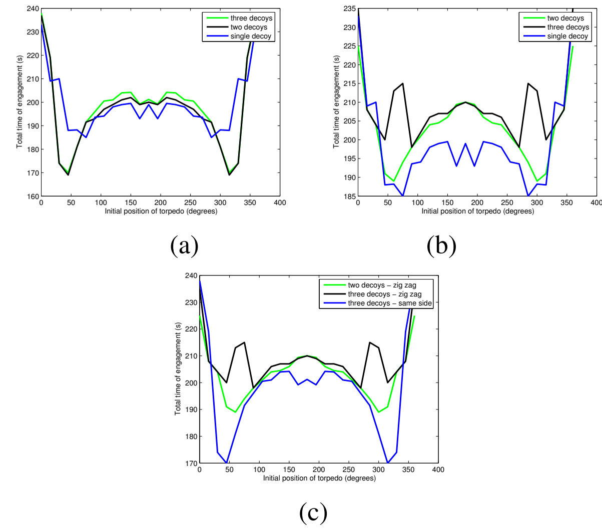


Fig. 11. (a) Multiple same side deployments and single optimum deployment (b) Multiple zig-zag deployments and single optimum deployment (c) Multiple zig-zag deployments and same side deployments

a higher t_f. This is because we no longer use the angle of deployment θ as a decision variable while computing the optimum time of deployment of multiple decoys.

C. Zig-zag deployment

We adopt a zig-zag deployment strategy. The angle of deployment of decoys is 90^0 and 270^0 alternately (port and starboard side). The optimum values of t_1 and δ were computed for two and three decoys.

Figure 11(b) shows that deploying three decoys increases the survivability of the ship when compared to two decoys for certain initial positions of the torpedo ($\phi = 0^0, 15^0, 60^0, 75^0$), when the zig-zag deployment strategy is used. But for most of the initial positions two deployments are as effective as three deployments. Zig-zag deployments clearly prove to be more effective than the same side deployment strategy (Figure 11(c)).

VI. Discussion and Future Work

Single and multiple decoy deployment strategies for enhancing ship survivability was addressed in this paper. The simulation results show the efficacy of the zig-zag deployment strategy and deterministic ship maneuvers coordinated with optimal decoy deployment. Introducing uncertainty in the model and extending the model to deployment of propelled and towed decoys are topics of future research.

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