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Static Output Feedback Control with H_∞ Performance for a Three-Phase Shunt Active Filter

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Abstract— **This paper deals with the synthesis of a static output feedback control law with H_∞ performance for a three-phase shunt active power filter used to compensate current harmonics so that the network current remains quasi-sinusoidal and therefore electric power quality is improved. In a previous work, this system has been successfully driven by a state feedback controller. However, the main goal is to reduce the number of measured outputs. The objective of this work is the design of an output feedback H_∞ controller by non-iterative means. This can be easily achieved by imposing a block diagonal structure on the Lyapunov matrix in order to transform bilinear matrix inequalities into a convex optimization problem. The validity of the synthesized static gain is investigated through numerical simulations.**

I. INTRODUCTION

In many control applications, it is not always possible to have a full access to the state vector, and therefore the control design problem must be formulated as an output feedback one. Both static and fixed-order dynamic output feedback problems attract the attention of the industrial community due to the simplicity of their implementation. They allow one to avoid controller reduction approaches which generally lack performance and stability guarantees. However, unlike state feedback control, there is not one unique general efficient approach that can be used to solve output feedback problems. Each control application has its own algorithm, and many of these algorithms are numerically solved. The lack of tractable approaches to design output feedback controllers is due to the non-linear formulation of the control design problem.

The design of a static or fixed-order dynamic output feedback controller is still an open problem. The reason is that, unlike the state feedback problem, the synthesis of an output feedback controller cannot be formulated in terms of Linear Matrix Inequalities (LMIs). The formulation of an output feedback synthesis problem generally leads to Bilinear Matrix Inequalities (BMIs) which are computationally less tractable than LMIs. Unlike the LMI optimization problems for which many efficient algorithms are available, the BMI optimization problems are non-convex and require a very high computational effort.

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In the control literature [1], different iterative methods have been developed in order to take into consideration the design of output feedback controllers, and particularly to transform BMI problems into LMI ones. These numerical methods involve, at each step, the determination of a solution to a LMI system obtained by fixing some variables. Iterative algorithms have the ability to solve a wide class of control applications. However, they are not guaranteed to find a solution nor to converge to a global minimum. Furthermore, they can be time consuming for high order control problems. Alternatively, different non-iterative design approaches have also been developed [2], [3]. Although they are conservative, they allow one to avoid computational complexity.

In a previous work [4], [5], a shunt active filter system used to depollute a low voltage three-phase electrical network has been successfully driven by a state feedback controller due to the fact that all state variables are accessible. The goal is to reduce the number of measured outputs and therefore an output feedback controller has to be synthesized. In this paper, the main objective is to design a static output feedback control law by non-iterative means using the H_∞ suboptimal control. This can be easily achieved by imposing a block diagonal structure on the Lyapunov matrix in order to convert the corresponding non-linear BMI optimization problem into a convex LMI one. The structured Lyapunov function can introduce some conservatism. On the other hand, it can significantly decrease the complexity of computations. The ability of the synthesized static gain to control a three-phase shunt active filter system will be investigated through numerical simulations.

The paper is organized as follows. The multivariable linear model of a three-phase shunt active filter system is given in Section II. The perturbation rejection problem (H_∞ control problem) is formulated in Section III. The corresponding convex LMI optimization problem is obtained in Section IV. Numerical simulations are provided in Section V. Finally, a conclusion is given in Section VI.

Throughout the paper, the following notations are used:

- The superscripts "T" and "$'$" stand for the transpose and complex conjugate transpose respectively.
- $\mathbb{R}^{m\times n}$ ($\mathbb{C}^{m\times n}$) is the set of all $m\times n$ real (complex) matrices.
- $\mathbb{R}^n$ ($\mathbb{C}^n$) is the set of all vectors with n real (complex) elements.

- I_n is the identity matrix of order n.
- $\mathbf{0}_{m\times n}$ is the $m\times n$ zero matrix.
- For a complex number z, the complex conjugate is denoted as $\bar{z}$.

II. SHUNT ACTIVE FILTER SYSTEM

A. Description of the System

The role of a shunt active power filter is to compensate current harmonics demanded by non-linear loads so that the network current remains quasi-sinusoidal [6]. It is connected in parallel to the electrical network, and composed of three main parts (Fig. 1):

1) A pulse-width modulated voltage source inverter using IGBTs,
2) A switching ripple LC filter used to sink high frequency switching harmonics,
3) A control system used to drive the switching devices.

B. Single Phase Modeling

Fig. 2 shows the electrical circuit of one single phase of a shunt active filter system. The corresponding state model is given by

$$\dot{\mathrm{x}} = A\mathrm{x} + B_u u + B_w w \tag{1}$$

- State vector: $\mathrm{x} = \begin{bmatrix} i_{net} & i_{inv} & v_{C_{af}} \end{bmatrix}^T \in \mathbb{R}^3$
- Control input: $u = u_{inv} \in \mathbb{R}$
- Perturbation vector: $w = \begin{bmatrix} i_{load} & v_{net} \end{bmatrix}^T \in \mathbb{R}^2$

$$A=\begin{pmatrix} -\frac{r_{net}}{L_{net}} & 0 & -\frac{1}{L_{net}} \\ 0 & 0 & -\frac{1}{L_{af}} \\ \frac{1}{C_{af}} & \frac{1}{C_{af}} & 0 \end{pmatrix}, B_w=\begin{pmatrix} 0 & \frac{1}{L_{net}} \\ 0 & 0 \\ \frac{1}{C_{af}} & 0 \end{pmatrix}, B_u=\begin{pmatrix} 0 \\ \frac{1}{L_{af}} \\ 0 \end{pmatrix}$$

C. Three-Phase Modeling

Using (1), one can easily obtain the state model of a three-phase shunt active filter system in the three-phase domain (a,b,c) [4]. Apply the Concordia transformation in order to map the state model from this domain into a two-phase orthogonal stationary frame (α,β),

$$T_{con} = \sqrt{\frac{2}{3}}\begin{pmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix} \tag{2}$$

followed by the Park transformation in order to introduce the constant angular velocity $\dot{\theta}$ of the rotating orthogonal frame,

$$T_{park} = \begin{pmatrix} cos\theta & sin\theta \\ -sin\theta & cos\theta \end{pmatrix} \tag{3}$$

followed by a complex linear transformation in order to obtain a mathematically decoupled model in a complex frame (cd,qi),

$$T_c = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -j \\ -j & 1 \end{pmatrix}; \quad j=\sqrt{-1} \tag{4}$$

The resulting multivariable linear state model of a three-phase shunt active filter system in the frame (cd,qi) is

$$\begin{bmatrix} \dot{\mathrm{x}}_{cd} \\ \dot{\mathrm{x}}_{qi} \end{bmatrix} = \begin{pmatrix} A+j\dot{\theta}I_3 & \mathbf{0} \\ \mathbf{0} & A-j\dot{\theta}I_3 \end{pmatrix}\begin{bmatrix} \mathrm{x}_{cd} \\ \mathrm{x}_{qi} \end{bmatrix} + \begin{pmatrix} B_u & \mathbf{0} \\ \mathbf{0} & B_u \end{pmatrix}\begin{bmatrix} u_{cd} \\ u_{qi} \end{bmatrix} + \begin{pmatrix} B_w & \mathbf{0} \\ \mathbf{0} & B_w \end{pmatrix}\begin{bmatrix} w_{cd} \\ w_{qi} \end{bmatrix} \tag{5}$$

For more details concerning the transition from the frame (a,b,c) to (cd,qi), the reader is referred to [4], [7].

III. PERTURBATION REJECTION PROBLEM

In order to guarantee a suitable harmonic compensation by the active filter system, the three main objectives of the controller to be synthesized are:

1) To decrease the Total Harmonic Distortion (THD) of the network current,
2) To eliminate the fundamental component of the inverter current,
3) To damp resonances caused by the network impedance and the switching ripple filter.

The design of a static output feedback control law with H_∞ performance requires the formulation of the active filter system as a perturbation rejection problem (Fig. 3). According to the defined control objectives, the controlled outputs to be minimized are the fundamental inverter current and the harmonics of the network current. Their extraction requires first order complex-valued low pass filters, and an orthogonal frame rotating at an angular velocity $\dot{\theta}$ equal to

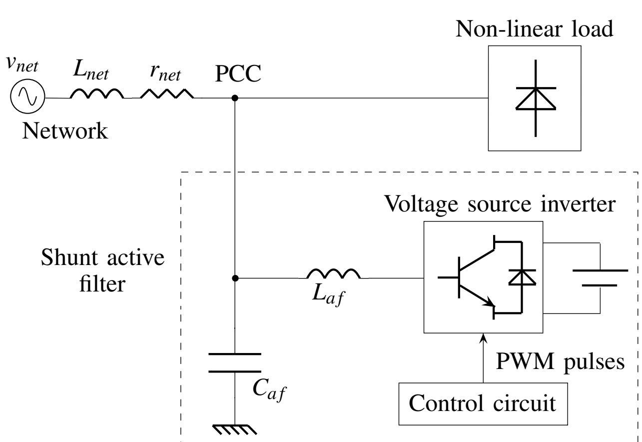

Fig. 1. Shunt active filter configuration

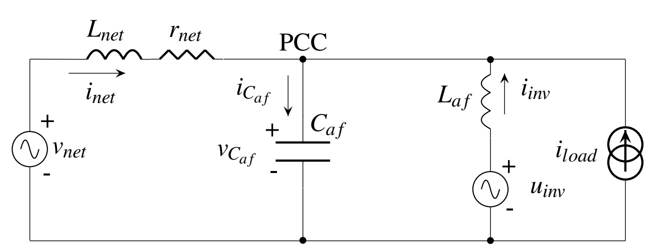

Fig. 2. Single phase equivalent circuit

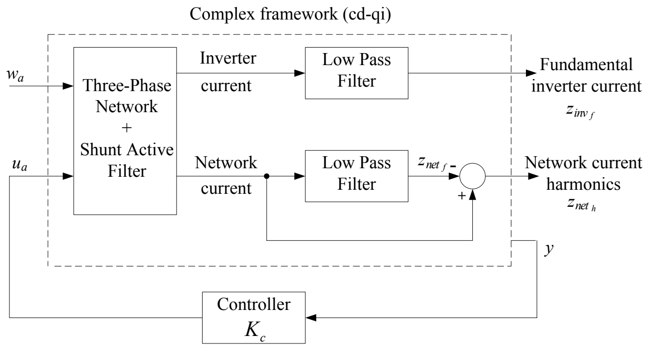

Fig. 3. Perturbation rejection problem

the fundamental system frequency so that the fundamental component of any signal is shifted to the dc component (i.e. zero frequency).

The state model of the low pass filter used to pass the fundamental inverter current in the frame (cd, qi) is

$$\begin{aligned}\begin{bmatrix}\dot{x}_{f_{1cd}}\\ \dot{x}_{f_{1qi}}\end{bmatrix} &= \begin{pmatrix}a_1 & 0\\ 0 & \bar{a}_1\end{pmatrix}\begin{bmatrix}x_{f_{1cd}}\\ x_{f_{1qi}}\end{bmatrix}+\begin{pmatrix}b_1 & 0\\ 0 & \bar{b}_1\end{pmatrix}\begin{bmatrix}i_{inv_{cd}}\\ i_{inv_{qi}}\end{bmatrix}\\ \begin{bmatrix}z_{inv_{f_{cd}}}\\ z_{inv_{f_{qi}}}\end{bmatrix} &= \begin{pmatrix}c_1 & 0\\ 0 & \bar{c}_1\end{pmatrix}\begin{bmatrix}x_{f_{1cd}}\\ x_{f_{1qi}}\end{bmatrix}+\begin{pmatrix}d_1 & 0\\ 0 & \bar{d}_1\end{pmatrix}\begin{bmatrix}i_{inv_{cd}}\\ i_{inv_{qi}}\end{bmatrix}\end{aligned} \quad (6)$$

The state model of the low pass filter used to pass the fundamental network current is

$$\begin{aligned}\begin{bmatrix}\dot{x}_{f_{2cd}}\\ \dot{x}_{f_{2qi}}\end{bmatrix} &= \begin{pmatrix}a_2 & 0\\ 0 & \bar{a}_2\end{pmatrix}\begin{bmatrix}x_{f_{2cd}}\\ x_{f_{2qi}}\end{bmatrix}+\begin{pmatrix}b_2 & 0\\ 0 & \bar{b}_2\end{pmatrix}\begin{bmatrix}i_{net_{cd}}\\ i_{net_{qi}}\end{bmatrix}\\ \begin{bmatrix}z_{net_{f_{cd}}}\\ z_{net_{f_{qi}}}\end{bmatrix} &= \begin{pmatrix}c_2 & 0\\ 0 & \bar{c}_2\end{pmatrix}\begin{bmatrix}x_{f_{2cd}}\\ x_{f_{2qi}}\end{bmatrix}+\begin{pmatrix}d_2 & 0\\ 0 & \bar{d}_2\end{pmatrix}\begin{bmatrix}i_{net_{cd}}\\ i_{net_{qi}}\end{bmatrix}\end{aligned} \quad (7)$$

The variables in models (6) and (7) are selected such that

1) the phases of the filters are equal to 0° at the zero frequency (i.e. dc components are isolated without any harmful phase shift),
2) only dc components pass through the filters.

Fig. 4 shows the frequency responses of the low pass filters in the frame (cd, qi). Combining models (5), (6) and (7) yields the state space model of the open loop system,

$$\begin{aligned}\dot{\mathrm{X}} &= A_c\mathrm{X} + B_{u_c}u_a + B_{w_c}w_a\\ z &= C_{z_c}\mathrm{X}\\ y &= C_c\mathrm{X}\end{aligned} \quad (8)$$

Let $\mathrm{x}_f = \begin{bmatrix}x_{f_1} & x_{f_2}\end{bmatrix}^T \in \mathbb{R}^2$

- State vector (n = 10): $\mathrm{X} = \begin{bmatrix}\mathrm{x}_{cd}^T & \mathrm{x}_{f_{cd}}^T & \mathrm{x}_{qi}^T & \mathrm{x}_{f_{qi}}^T\end{bmatrix}^T \in \mathbb{R}^n$
- Input vector (n_u = 2): $u_a = \begin{bmatrix}u_{cd} & u_{qi}\end{bmatrix}^T \in \mathbb{R}^{n_u}$
- Perturbation vector (n_w = 4): $w_a = \begin{bmatrix}w_{cd}^T & w_{qi}^T\end{bmatrix}^T \in \mathbb{R}^{n_w}$
- Controlled output vector (n_z = 4): $z = \begin{bmatrix}z_{inv_{f_{cd}}} & z_{net_{h_{cd}}} & z_{inv_{f_{qi}}} & z_{net_{h_{qi}}}\end{bmatrix}^T \in \mathbb{R}^{n_z}$
- The measured output vector y only contains the inverter current (i_{inv}), the voltage at the point of common coupling $(v_{C_{af}})$, and the state variable (x_{f_1}) ($y \in \mathbb{R}^{n_y}$; n_y = 6).

$(\star)$ denotes the complex conjugate of the upper non-zero block $(1,1)$.

$$A_c = \left(\begin{array}{ccc:c} A + j\dot{\theta}I_3 & \mathbf{0} & \mathbf{0} & \\ b_1C_{inv} & a_1 & 0 & \mathbf{0}_{5\times5}\\ b_2C_{net} & 0 & a_2 & \\ \cdots & \cdots & \cdots & \cdots\ \cdots\\ & \mathbf{0}_{5\times5} & & \star\end{array}\right), B_{u_c} = \left(\begin{array}{c:c} B_u & \mathbf{0}_{5\times1}\\ \mathbf{0}_{2\times1} & \\ \cdots & \cdots\ \cdots\\ \mathbf{0}_{5\times1} & \star\end{array}\right),$$

$$B_{w_c} = \left(\begin{array}{c:c} B_w & \mathbf{0}_{5\times2}\\ \mathbf{0}_{2\times2} & \\ \cdots & \cdots\ \cdots\\ \mathbf{0}_{5\times2} & \star\end{array}\right), C_{inv} = \begin{pmatrix}0 & 1 & 0\end{pmatrix}, C_{net} = \begin{pmatrix}1 & 0 & 0\end{pmatrix},$$

$$C_{z_c} = \left(\begin{array}{ccc:c} d_1C_{inv} & c_1 & 0 & \mathbf{0}_{2\times5}\\ C_{net} - d_2C_{net} & 0 & -c_2 & \\ \cdots & \cdots & \cdots & \cdots\ \cdots\\ & \mathbf{0}_{2\times5} & & \star\end{array}\right)$$

IV. STATIC OUTPUT FEEDBACK CONTROL WITH H_∞ PERFORMANCE

The static output feedback control law to be synthesized is given by

$$u_a = K_c y \quad (9)$$

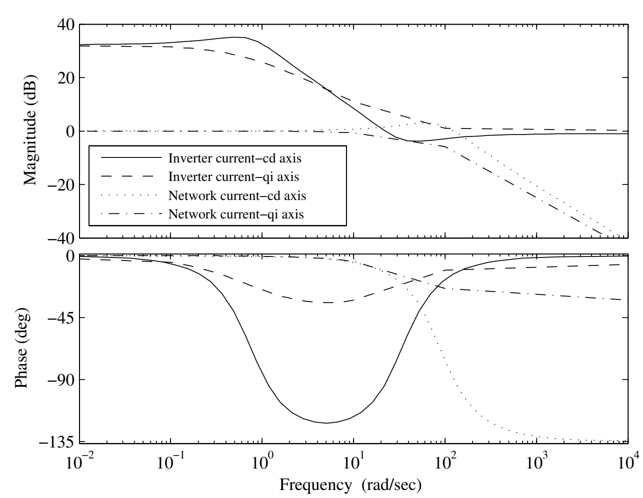

Fig. 4. Low pass filters frequency responses

where K_c is a static complex-valued gain whose structure in the frame (cd, qi) is

$$K_c = \begin{pmatrix} K_{11} & \mathbf{0} \\ \mathbf{0} & \bar{K_{11}} \end{pmatrix} \in \mathbb{C}^{n_u \times n_y}; \quad K_{11} \in \mathbb{C}^{\frac{n_u}{2} \times \frac{n_y}{2}} \tag{10}$$

Its equivalent structure in the frame (α, β) is

$$K_{\alpha\beta} = \begin{pmatrix} Re(K_{11}) & Im(K_{11}) \\ -Im(K_{11}) & Re(K_{11}) \end{pmatrix} \in \mathbb{R}^{n_u \times n_y} \tag{11}$$

Re and *Im* denote the real and imaginary parts respectively.

From (8) and (9), the state space model of the closed loop system is given by

$$\begin{aligned} \dot{\mathrm{X}} &= \underbrace{(A_c + B_{u_c} K_c C_c)}_{A_{cl}} \mathrm{X} + \underbrace{B_{w_c}}_{B_{cl}} w_a \\ z &= \underbrace{C_{z_c}}_{C_{cl}} \mathrm{X} \end{aligned} \tag{12}$$

In this case, the perturbation rejection problem with H_∞ performance consists of finding a controller K_c such that the H_∞ norm of the closed loop system is minimized (or it is less than a prespecified positive bound γ). Using the Bounded Real Lemma [8], this is equivalent to finding a static gain $K_c \in \mathbb{C}^{n_u \times n_y}$ and a symmetric positive definite matrix $Q \in \mathbb{C}^{n \times n}$ such that the following inequality is satisfied

$$\begin{pmatrix} A_{cl}Q + QA_{cl}^{'} & B_{cl} & QC_{cl}^{'} \\ B_{cl}^{'} & -\gamma I_{n_w} & \mathbf{0} \\ C_{cl}Q & \mathbf{0} & -\gamma I_{n_z} \end{pmatrix} < 0 \tag{13}$$

This matrix inequality is bilinear due to the fact that the state variables are not fully accessible (i.e. the output matrix C_c is not an identity one). In order to solve the problem by non-iterative means, the BMI (13) is transformed into a convex LMI by

1) Imposing a block diagonal structure on the Lyapunov matrix Q,

$$Q = \begin{pmatrix} Q_{11} & \mathbf{0} \\ \mathbf{0} & Q_{22} \end{pmatrix} \tag{14}$$

where $Q_{11} \in \mathbb{C}^{n_y \times n_y}$ and $Q_{22} \in \mathbb{C}^{(n-n_y)\times(n-n_y)}$ are two symmetric positive definite matrices.

2) Representing the open loop system (8) by a new state space realization such that the resulting output matrix will be equal to $\begin{pmatrix} I_{n_y} & \mathbf{0}_{n_y \times (n-n_y)} \end{pmatrix}$. Let $\tilde{\mathrm{X}} \in \mathbb{R}^n$ be the new state vector. It is related to the old state vector X via a non-singular state transformation matrix T,

$$\mathrm{X} = T\tilde{\mathrm{X}}; \quad T \in \mathbb{R}^{n \times n} \tag{15}$$

The new state space realization of the closed loop system is

$$\begin{aligned} \dot{\tilde{\mathrm{X}}} &= \underbrace{(\tilde{A} + \tilde{B}_u K_c \tilde{C})}_{A_{cl}} \tilde{\mathrm{X}} + \underbrace{\tilde{B}_w}_{B_{cl}} w_a \\ z &= \underbrace{\tilde{C}_z}_{C_{cl}} \tilde{\mathrm{X}} \end{aligned} \tag{16}$$

where $\tilde{A} = T^{-1}A_cT$, $\tilde{B}_u = T^{-1}B_{u_c}$, $\tilde{B}_w = T^{-1}B_{w_c}$, $\tilde{C} = C_cT$, and $\tilde{C}_z = C_{z_c}T$.

The non-singular matrix T must be selected such that the new output matrix $\tilde{C}$ is equal to

$$C_cT = \begin{pmatrix} I_{n_y} & \mathbf{0}_{n_y \times (n-n_y)} \end{pmatrix} \tag{17}$$

The resulting LMI system is given by

$$\begin{gathered} Q > 0 \\ \begin{pmatrix} \tilde{A}Q + Q\tilde{A}^{'} + \tilde{B}_u Y \tilde{C} + \tilde{C}^{'} Y^{'} \tilde{B}_u^{'} & \tilde{B}_w & Q\tilde{C}_z^{'} \\ \tilde{B}_w^{'} & -\gamma I_{n_w} & \mathbf{0} \\ \tilde{C}_z Q & \mathbf{0} & -\gamma I_{n_z} \end{pmatrix} < 0 \end{gathered} \tag{18}$$

where $Y = K_cQ_{11} \in \mathbb{C}^{n_u \times n_y}$ and Q is a symmetric block diagonal matrix defined in (14).

This system involves complex-valued constant and variable matrices. It should be transformed into a real one by decomposing all matrix variables and coefficients into real and imaginary parts. For example, the decomposition of the constant matrix $\tilde{A}$ is given by $\tilde{A}_1 + j\tilde{A}_2$.

The resulting convex optimization problem subject to two real-valued LMIs can be formulated as

Minimize γ subject to

$$\begin{pmatrix} Q_1 & Q_2 \\ -Q_2 & Q_1 \end{pmatrix} > 0 \tag{19}$$

$$\begin{pmatrix} M & \tilde{B}_{w_1} & E^T & \vdots & N & \tilde{B}_{w_2} & -F^T \\ \tilde{B}_{w_1}^T & -\gamma I_{n_w} & \mathbf{0} & \vdots & -\tilde{B}_{w_2}^T & \mathbf{0} & \mathbf{0} \\ E & \mathbf{0} & -\gamma I_{n_z} & \vdots & F & \mathbf{0} & \mathbf{0} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -N & -\tilde{B}_{w_2} & F^T & \vdots & M & \tilde{B}_{w_1} & E^T \\ \tilde{B}_{w_2}^T & \mathbf{0} & \mathbf{0} & \vdots & \tilde{B}_{w_1}^T & -\gamma I_{n_w} & \mathbf{0} \\ -F & \mathbf{0} & \mathbf{0} & \vdots & E & \mathbf{0} & -\gamma I_{n_z} \end{pmatrix} < 0 \tag{20}$$

where $E = \tilde{C}_{z_1}Q_1 - \tilde{C}_{z_2}Q_2$, $\quad F = \tilde{C}_{z_1}Q_2 + \tilde{C}_{z_2}Q_1$,

$M = \tilde{A}_1Q_1 + Q_1\tilde{A}_1^T - \tilde{A}_2Q_2 + Q_2\tilde{A}_2^T + \tilde{B}_{u_1}Y_1\tilde{C}_1 + \tilde{C}_1^TY_1^T\tilde{B}_{u_1}^T - \tilde{B}_{u_1}Y_2\tilde{C}_2 - \tilde{C}_2^TY_2^T\tilde{B}_{u_1}^T - \tilde{B}_{u_2}Y_1\tilde{C}_2 - \tilde{C}_2^TY_1^T\tilde{B}_{u_2}^T - \tilde{B}_{u_2}Y_2\tilde{C}_1 - \tilde{C}_1^TY_2^T\tilde{B}_{u_2}^T$,

$N = \tilde{A}_1Q_2 + Q_2\tilde{A}_1^T + \tilde{A}_2Q_1 - Q_1\tilde{A}_2^T + \tilde{B}_{u_1}Y_1\tilde{C}_2 - \tilde{C}_2^TW_1^T\tilde{B}_{u_1}^T + \tilde{B}_{u_1}Y_2\tilde{C}_1 - \tilde{C}_1^TY_2^T\tilde{B}_{u_1}^T + \tilde{B}_{u_2}Y_1\tilde{C}_1 - \tilde{C}_1^TY_1^T\tilde{B}_{u_2}^T - \tilde{B}_{u_2}Y_2\tilde{C}_2 + \tilde{C}_2^TY_2^T\tilde{B}_{u_2}^T$.

There are four matrix variables to be determined: Q_1, Q_2, Y_1 and Y_2. Q_1 is a symmetric block diagonal matrix, while Q_2 is a skew symmetric block diagonal one. From the LMI Control Toolbox in MATLAB [9], a LMI solver is used in order to compute the matrix variables Q and Y. Therefore, the resulting complex-valued static output feedback gain is given by

$$K_c = Y Q_{11}^{-1} \tag{21}$$

Fig. 5 shows the magnitude responses of $\left(\frac{I_{net}}{I_{load}}\right)$ with and without control in the complex frame (cd, qi). The closed loop magnitude response is that of a low pass filter which only passes the cd component (i.e. the fundamental component of the load current). Fig. 6 shows the magnitude responses of $\left(\frac{I_{inv}}{I_{load}}\right)$ with and without control. The closed loop magnitude response is that of a band pass filter which only passes the harmonics of the load current. These figures also show the ability of the controller to damp LC resonances.

The conservatism of this non-iterative algorithm can be assessed by comparing the H_∞ performance γ of the output feedback problem to that obtained from a state feedback problem . The upper bound γ of the H_∞ norm of the closed loop active filtering system is equal to 1 in the case of a state feedback controller, and 57 in the case of an output feedback controller. The difference is due to the conservatism introduced by the structured Lyapunov matrix.

V. SIMULATION RESULTS

This section shows the validity and efficiency of the synthesized output feedback control law with H_∞ performance. Consider a power source whose line voltage is 400Vrms and frequency is 50Hz. A load current composed of a fundamental component and harmonics of rank $6k \pm 1$ (k = 1,2,3,4) with decreasing energy is applied to the system in which the inverter is considered as a simple gain. The harmonic spectrum of the generated load current is shown in Fig. 7. Fig. 8 shows the resulting quasi-sinusoidal network current. Its THD has decreased from 29.04% to 4.87% after compensation. The fundamental component of the inverter current is approximately equal to zero as shown in the harmonic spectrum of Fig. 9. The THD of the voltage at PCC has decreased from 14.53% to 1.65% after filtering (Fig. 10). The above results show the ability of the output feedback controller to provide the desired control objectives.

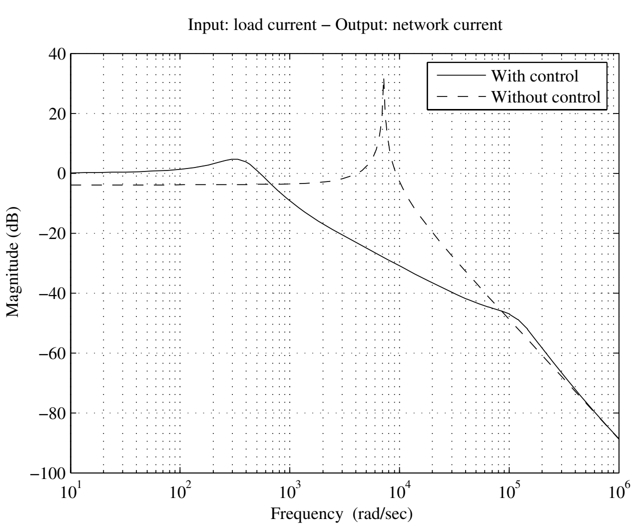

Fig. 5. Open and closed loop behavior of $\left(\frac{I_{net}}{I_{load}}\right)$

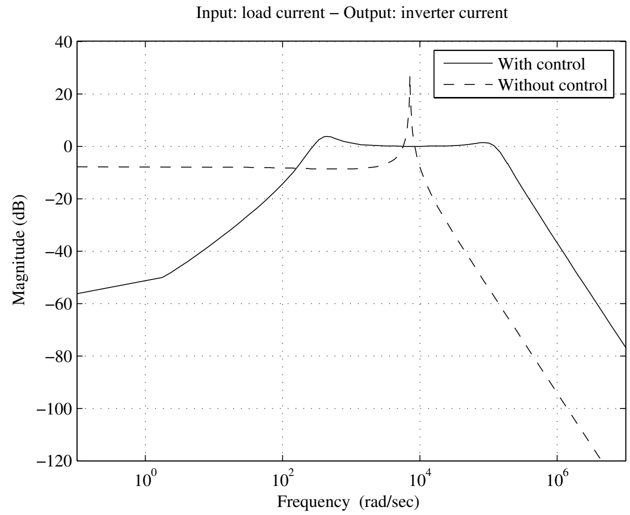

Fig. 6. Open and closed loop behavior of $\left(\frac{I_{inv}}{I_{load}}\right)$

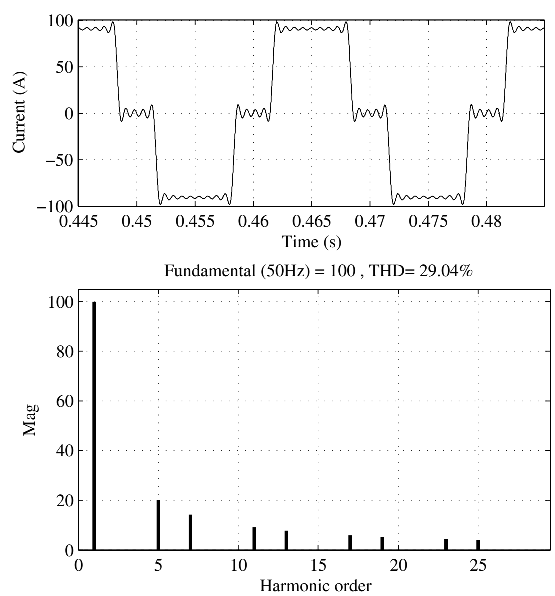

Fig. 7. Harmonic spectrum of the generated load current

VI. CONCLUSION

In this paper, a three-phase shunt active filtering system has been successfully driven by an output feedback controller using only the inverter current and voltage at the point of common coupling as measured outputs. Therefore, there is no need for network current sensors. The design of the static output feedback control law with H_∞ performance has been performed using a non-iterative algorithm, by imposing a block diagonal structure on the Lyapunov matrix. This method can introduce some conservatism, but it decreases the complexity of computations. Simulation results have shown the validity of the output feedback control for a three-phase shunt active filter. Future work will include numerical simulations using a real voltage source inverter, and the real-time implementation of the control algorithm in order to drive a laboratory prototype of an active filtering system.

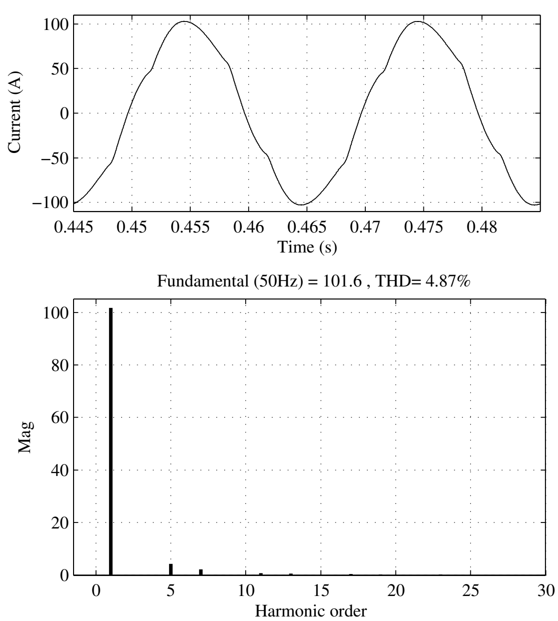

Fig. 8. Harmonic spectrum of the resulting network current

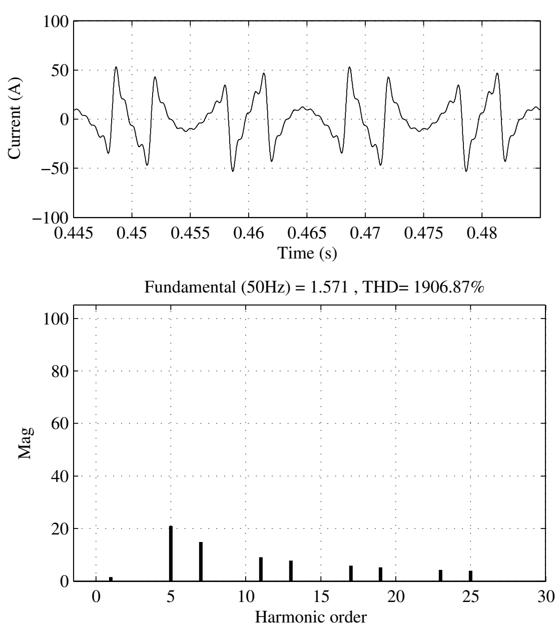

Fig. 9. Harmonic spectrum of the inverter current

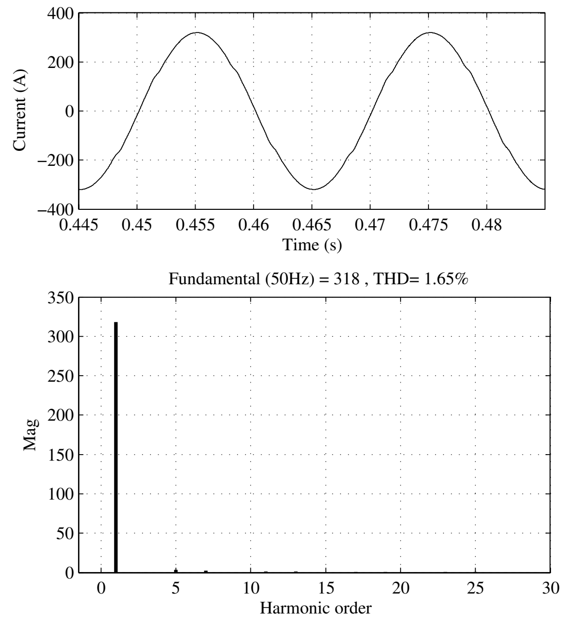

Fig. 10. Harmonic spectrum of the voltage at PCC

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