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The multimodel approach for a numerical second order sliding mode control of highly non stationary systems

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Abstract— **In this work, the multimodel approach is exploited in order to ameliorate the discrete second order sliding mode control (2-DSMC) performances in the case of highly non stationary systems. Simulation results show a notable improvement relatively to the classical 2-DSMC, especially in the reaching phase.**

I. INTRODUCTION

Variable structure systems and sliding mode control (SMC) theory have, since the 70's, raised up the researchers interest thanks to its robustness with respect to parameter variations and external perturbations [4], [6], [14], [22], [31]. Sliding mode control systems which are particular cases of the variable structure systems are closed loop systems with discontinuous controller gains that switch the system structure in order to maintain its trajectory inside a predetermined subspace called "sliding surface" [31]. The main idea is to react immediately to any deviation from the sliding surface by a powerful enough control input. The system's dynamics, in sliding mode, depend on the sliding surface's parameters. Face of the many advantages of the digital control strategy [1], the discretization of the SMC has become an interesting research field. Unfortunately, discretisized sliding mode control laws are confronted to the dilemma performance-robustness because they need a model of the system [19], [23]. The discontinuous term which guaranties the robustness of sliding mode control laws must not be of a large amplitude in discrete ones, otherwise, it generates oscillations on the sliding function and can even lead to instability. This is due to the fact that the sampling rate is reduced [30]. Many approaches have been suggested in order to overcome this phenomenon [7], [8], [20], [25], [29], [30], However, the reduction of the oscillations amplitude was obtained at the cost of the control law robustness.

In the eighties, Levantovsky [13] and Emelyanov [5] proposed a control technique, called high order sliding mode control, where not only the sliding function is reduced to zero, but also its high order derivatives. In the case of the r-order sliding mode control, the discontinuity is applied on the $(r-1)$ derivative of the control. The effectif control is obtained by $(r-1)$ integrations and can, then, be considered as a continuous signal. In other words, the oscillations generated by the discontinuous control are transferred to the higher derivatives of the sliding function. This approach permits to reduce the oscillations amplitude, the notorious sliding mode systems's robustness remaining intact [12].

This approach can be exploited in order to resolve the chattering problem in discrete time. However, the results are less brilliant than in continuous time implementation. In fact, in case of relatively important parameter variations and external perturbations, the amplitude of the discontinuous term, noted M, which must be large enough to dominate them, can involve a static error on the sliding surface and on the output.

In this work, we propose a solution to this last problem. The idea is based on the multimodel approach. It consists in diminishing the distance between the model and the system parameters, which allow us to use a smaller amplitude of M.

II. SLIDING MODE CONTROL

In continuous time, the control input is updated continuously so that the sliding function sign is opposite to the sign of its derivative. That means:

$$S(t,x)\dot{S}(t,x) < 0 \quad (1)$$

where $S(t,x)$ is the sliding function and x is the state. This inequality is the fondamental condition for sliding mode[2]. One choice of control input that can be used is:

$$u(t) = -Msign(S(t,x)) \quad (2)$$

M is a positive constant whose choice depends on the system's model parameters, the setpoint, the perturbations and the model parameters variations. Because it is impossible to apply such a control law in practice (the imperfection of the actuators and sensors does not allow the SMC application at an infinite frequency), oscillations appear on the sliding surface and on the state. The oscillations amplitude largeness is proportional to that of M. The equivalent control is among the ways to reduce them. It is the average value of the discontinuous control (2), assumed to involve a sliding mode, and it is calculated using a model of the system. The discontinuous term (2) is added to the equivalent control in order to guaranty the robustness of the control law. Its amplitude is then reduced to a value relatively low, but sufficient to compensate external perturbations and parameters variations.

As calculators are more and more used for control algorithms' implementation. So, many researches have aimed to conceive discrete time sliding mode control laws. Unfortunately, the variable structure theory have been formally developed for continuous time implementation and its performances are guaranteed only for a reduced sampling step. A first order approximation of the condition (1) induces the chattering phenomenon and can even lead to instability [21]

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and [29]. Many authors tried to resolve this problem. Among the first contributions, we can cite those of Milosavljevic [20], Sarpturk [29], Furuta [7] and Sira Ramirez [30]. These works are based on the quasi sliding mode concept introduced by Milosavljevic [20]. This concept asserts that the continuous SMC existence conditions do not guaranty its existence in discrete time.

Most of discrete sliding mode control laws use an equivalent control calculated by using a model of the system. In order to ensure robustness with respect to parameter variations and external perturbations, a high frequency term is added to the equivalent control. This last term is generally in the form $-M\ sign(S(k))$. Softened forms are also met : saturation, hyperbolic tangent [9], [21]... This last strategy diminishes the chattering amplitude, but on the other hand, deteriorates the system's robustness. Some authors proposed an on line estimation of the perturbations [21], [24], [32]. In this case, the perturbation variation rate influence the estimation performance.

Gao proposed the reaching law [8]. For a discrete time system of the form:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) \\ y(k) = Hx(k) \end{cases} \tag{3}$$

where $x(k)$ is the state vector and $y(k)$ is the output, the control law is:

$$u(k) = (C^T B)^{-1}[\varphi\, S(k) - C^T Ax(k) - T_e M \text{sign}(S(k))] \tag{4}$$

with T_e is the sampling step, $\varphi \in [0,\ 1[$ and $S(k) = C^T x(k)$ where C^T is the sliding function parameters' vector.

A particular case of high order sliding mode control is the second order sliding mode control and was essentially used for chattering suppression purpose. In discrete time, the performances are acceptable within a certain range of parameter variations, beyond which, the reaching phase can be controlled no more. If we try to increase the discontinuous term amplitude in order to overcome the high parameter variations, a static error appears on the state and on the output. A solution to this last problem is proposed in this work by using the multimodel approach.

III. An asymptotic numerical second order sliding mode control

Let's consider the non linear system defined by:

$$\dot{x} = f(t, x, u) \tag{5}$$

with :

- $x(t) = [x_1(t), ..., x_n(t)]^T \in X$ state vector, $X \subset R^n$.
- $u(t, x)$ is the control.
- $f(t, x, u)$ is a function supposed sufficiently differentiable, but in an uncertain manner.

We denote by $S(t, x)$ the sliding function. It is a differentiable function with its $(r-1)$ first derivatives relatively to the time depending only on the state $x(t)$ (that means they contain no discontinuities) [10].

The objective of first order sliding mode control is to force the state to move on the switching surface $S(t, x) = 0$. In high order sliding mode control, the purpose is to force the state to move on the switching surface $S(t, x) = 0$ and to keep its $(r-1)$ first successive derivatives null [10] :

$$S(t, x) = \dot{S}(t, x) = ... = S^{(r-1)}(t, x) = 0 \tag{6}$$

r is the sliding mode order.

In second order sliding mode control, the following condition must be satisfied:

$$S(t, x) = \dot{S}(t, x) = 0 \tag{7}$$

Salgado [10] considered a new sliding surface $\sigma(t, x)$ defined by:

$$\sigma(t, x) = \dot{S}(t, x) + \alpha S(t, x) \tag{8}$$

with α is a positive constant.

The equivalent control expression is given by:

$$\begin{aligned} \dot{u}_{eq}(t) = -\tfrac{1}{C^T \frac{\partial}{\partial u} f(t,x,u)} \Big(& C^T \tfrac{\partial}{\partial} f(t, x, u) \\ & + C^T \tfrac{\partial}{\partial x} f(t, x, u)\dot{x}(t) + \alpha \dot{S}(t, x) \Big) \end{aligned} \tag{9}$$

The effective control to apply to the system (5) is obtained by integration of the following relation:

$$\dot{u}(t) = \dot{u}_{eq}(t) + u_{dis}(t) \tag{10}$$

with $\quad u_{dis}(t) = -M\, sign\,(\sigma(t, x))$

The convergence conditions are described in [10]. For the discrete-time system defined by (3). The sliding function is taken in this linear form:

$$S(k) = C^T (x(k) - x_d(k)) \tag{11}$$

with $x_d(k)$ is the desired state vector.

We consider the new sliding function $\sigma(k)$ defined by:

$$\sigma(k) = S(k+1) + \beta S(k) \tag{12}$$

with $\beta \in [0, 1[$ and:

$$\begin{aligned} S(k+1) &= C^T (x(k+1) - x_d(k+1)) \\ &= C^T \left(Ax(k) + Bu(k) - x_d(k+1)\right) \end{aligned} \tag{13}$$

The equivalent control that forces the system to evolute on the sliding function is deduced from :

$$\sigma(k+1) = \sigma(k) = 0 \tag{14}$$

The equations (12), (13) and (14) give:

$$S(k+1) + \beta S(k) = 0 \tag{15}$$

with,

$$\begin{aligned} S(k+1) = \sigma(k+1) - \beta S(k) &= C^T (x(k+1) - x_d(k+1)) \\ &= C^T \left(Ax(k) + Bu_{eq}(k) - x_d(k+1)\right) \end{aligned} \tag{16}$$

Then :

$$u_{eq}(k) = (C^T B)^{-1}[-\beta\, S(k) - C^T Ax(k) + C^T (x_d(k+1)] \tag{17}$$

The robustness is ensured by the use of a discontinuous term (sign of the new sliding function $\sigma(k)$). By analogy with the continuous-time case, we apply to the system (3) the integral of the discontinuous term. In this last case, a first order transformation is considered:

$$u_{dis}(k) = u_{dis}(k-1) - T_e M \, sign\,(\sigma(k)) \qquad (18)$$

The control at the instant k is then :

$$u(k) = u_{eq}(k) + u_{dis}(k) \qquad (19)$$

In what follow, this control law is noted 2-DSMC.

IV. Limits of the 2-DSMC

Although the 2-DSMC proved to be better than the 1-DSMC in terms of chattering reduction, an improvement can be made in the case of highly non stationary systems necessitating a relatively large amplitude of the discontinuous term M. In fact, an important value of M induces oscillations of the sliding surface $\sigma(k)$. A non null average of these oscillations involve a static error on the sliding surface $S(k)$ and on the output. This phenomenon is illustrated via simulations. The non stationary model of a muscle engine is considered [14]. A nominal continuous model of the considered system can be written as follow:

$$\begin{cases} \dot{x}(t) = A_c x(t) + B_c u(t) \\ y(t) = H_c x(t) \end{cases} \qquad (20)$$

with :

$$A_c = \begin{bmatrix} 0 & 1 \\ -65,1 & -20,3 \end{bmatrix} \quad B_c = \begin{bmatrix} 0 \\ 73,1 \end{bmatrix}$$
$$H_c = [1 \quad 0]$$

By consideration of the non stationarity, a discrete model is:

$$\begin{cases} x(k+1) = (A + \Delta A)\left(\; x(k) \; \right) + (B + \Delta B)u(k) \\ y(k) = Hx(k) \end{cases} \qquad (21)$$

with :

$$A = \begin{bmatrix} 1 & T_e \\ -65.1T_e & (1-20.3T_e) \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 73.1T_e \end{bmatrix}$$
$$\Delta A = \begin{bmatrix} 0 & 1 \\ -65.1sg(k)\ \Delta v\ T_e & -20.3sg(k)\ \Delta v\ T_e \end{bmatrix}$$
$$\Delta B = \begin{bmatrix} 0 \\ -73.1sg(k)\ \Delta v\ T_e \end{bmatrix} \quad H = [1 \quad 0]$$

$$sg(k) = \begin{cases} 1 \;\; if \quad k < 120 \\ -1 \quad ifnot \end{cases}$$

Δv is the parametric variation amplitude.
The sliding step T_e is chosen, according to the system's dynamics, equal to $0.05s$.
We represent on the figures 1 to 2 the obtained results with high parametric variations $\Delta v = 0.2$ and low parameter variations $\Delta v = 0.02$, M's amplitude being kept constant (equal to 0.03). We observe that if the parametric variation is relatively important, the sliding surface' behavior does no more correspond to the desired one (figure 1). Besides, the output response is affected (figure 3). Finally, we observe

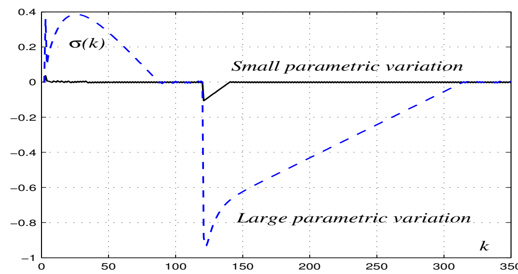

Fig. 1. Evolutions of $\sigma(k)$ for two different levels of parametric variation.

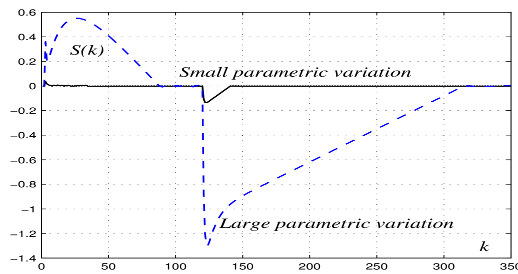

Fig. 2. Evolutions of $S(k)$ for two different levels of parametric variation.

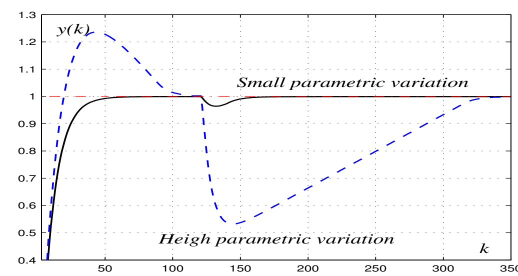

Fig. 3. Output's evolutions for two different levels of parametric variation.

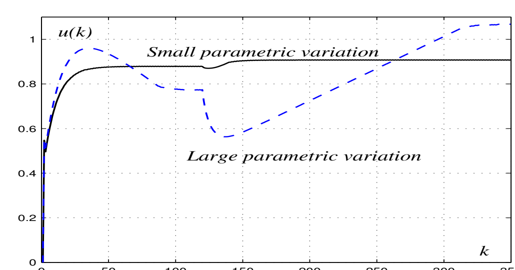

Fig. 4. Control's evolutions for two different levels of parametric variation.

no chattering since M's amplitude remained relatively low (figure 4).

If we increase the amplitude of the discontiuous term M in order to ensure its dominance over the relatively high parametric variation ($\Delta v = 0.2$), we remark that oscillations appear on the control (figure 7) and induce chattering on the sliding function $\sigma(k)$ (figure 5). Consequently, a static error appear on the output signal (figure 6). From the curves corresponding to the two values of M, we can conclude that the gained convergence rapidity was at the cost of a presence of the chattering and of the static error. As a solution to

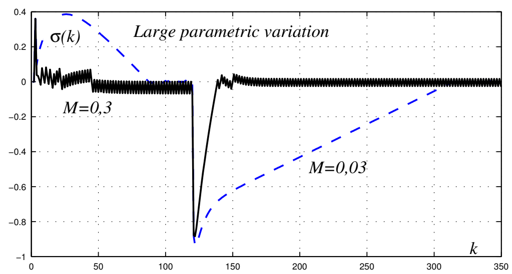

Fig. 5. Evolutions of $\sigma(k)$ for two different values of M.

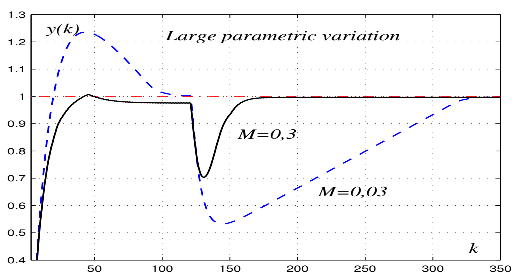

Fig. 6. Outputs's evolutions for two different values of M.

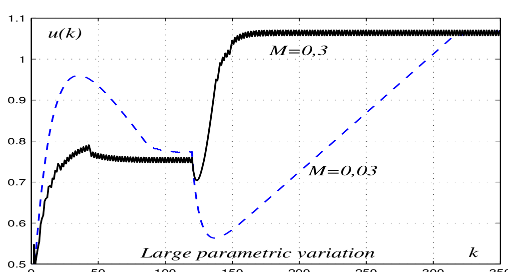

Fig. 7. Control's evolutions for two different values of M.

that problem, we propose to reduce the distance between the model and the system's parameters, which shall allow us to diminish M's amplitude. The multimodel approach, caracterised by good performances in case of relatively important parametric variations, is chosen for the system modeling. In the following paragraph, we introduce the multimodel aproach. Then, we present the discrete multimodel second order sliding mode approach. The improvement offered by this last approach is illustrated via a numerical simulation.

V. Improvement of the 2-DSMC performances by the multimodel approach

Instead of exploiting one global model of the system for the equivalent control calculation, the multimodel approach suggests the use of some partial models that express the process dynamics. Two problems must be resolved: the construction of the partial models and the choice of the right one at the right time [18]. If the final model is build by the fusion technique, we must, of course, compute partial models validities.

A. Construction of the partial models

Some approaches have been proposed for the systematic determination of a generic models base. In [11], Ksouri L. proposed a models' base based on the Kharitonov's algebric approach. Four extreme models and a medium one can be exploited by the multimodel strategy. Ben Abdennour et al. [15], [16], [17], [26], [27], [28] have proposed two contributions for the systematic determination of the models'base. The first is based on the Chiu's approach for fuzzy classification [3] and the second exploites the classification strategy based on the Kohenen card.

B. The validities computing

The validities estimation is insured by the residue approach:

$$v_i(k) = \frac{1 - \frac{r_i(k)}{\sum_{c=1}^{m_d} r_c(k)}}{m_d - 1}, \quad i \in [1, m_d] \tag{22}$$

$$r_i(k) = |y(k) - y_i(k)| \tag{23}$$

with $y(k)$ is the system's output, $y_i(k)$ is the output of the i^{th} model and m_d is the models number.
In order to reduce the perturbation phenomenon due to the inadequate models, we reinforce the validities as follow:

$$v_i^{renf}(k) = v_i(k) \prod_{\substack{c=1 \\ i \neq c}}^{m_d} \left(1 - e^{-\left(\frac{r_c(k)}{g}\right)^2}\right) \tag{24}$$

with g is a positive coefficient. The normalized reinforced validities are given by:

$$v_{in}^{renf}(k) = \frac{v_i^{renf}(k)}{\sum_{c=1}^{m_d} v_c^{renf}(k)} \tag{25}$$

C. The Multimodel 2-DSMC

As already mentioned, the 2-DSMC helps to reduce the chattering phenomenon by the integration of the discontinuous term which is supposed to guaranty the robustness of the control law. The choice of the discontinuous term amplitude is related to the parametric variations and to the external perturbations affecting the system. Consequently, if they are relatively important, M must be large enough to ensure a rapid convergence of the sliding function. Unfortunately, this

induces a static error on the sliding function and on the system's state. As a solution to this problem, we propose to reduce the distance between the model and the system parameters and then keep a relatively low amplitude of the discontinuous term M.
The multimodel discrete second order sliding mode control (MM-2-DSMC) approach structure is shown by the figure 8.

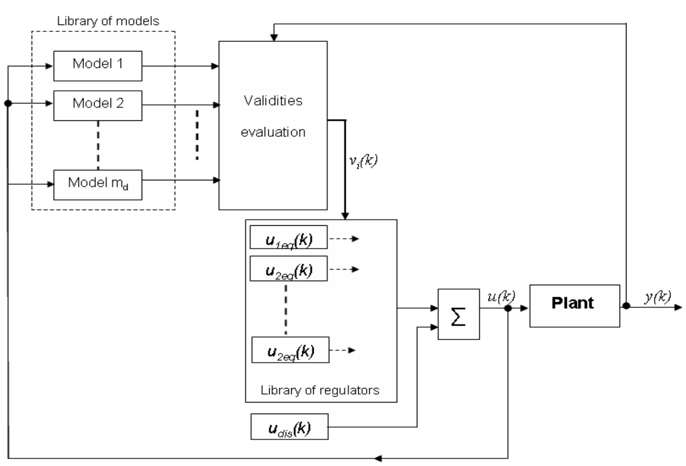

Fig. 8. The structure of a multimodel discrete second order sliding mode control (MM-2-DSMC).

The control applied to the system is given by the following relation:

$$u(k) = v_1(k)u_{1eq}(k)+v_2(k)u_{2eq}(k)+v_3(k)u_{3eq}(k)+u_{dis}(k); \tag{26}$$

with

- $v_i(k)$: validity of the local model M_i,
- $u_{ieq}(k)$: the equivalent 2-DSMC calculated using the local model M_i,
- $u_{dis}(k)$: the discontinuous term of the control.

D. Simulation results

The proposed control law (MM-2-DSMC) is applied on the above considered non stationary system. The 2-DSMC given by the relation (4) is also applied for a comparison study. The considered parametric variation is relatively important ($\Delta v = 0.2$). For the 2-DSMC calculation a medium model is used.
For the MM-2-DSMC calculation, we use a model's base composed of tree partial models: the medium model and the two extremal models. The models are of the following form:

$$M_i : \begin{cases} x(k+1) = A_i x(k) + B_i u(k) \\ y(k) = H_i x(k) \end{cases} \tag{27}$$

with:

- M_1: $A_1 = \begin{bmatrix} 1 & T_e \\ -3.25 & -0.015 \end{bmatrix}$ $B_1 = \begin{bmatrix} 0 \\ 3.65 \end{bmatrix}$
  $H_1 = [1 \quad 0]$
- M_2: $A_2 = \begin{bmatrix} 1 & T_e \\ -3.32 & -0.115 \end{bmatrix}$ $B_2 = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$
  $H_2 = [1 \quad 0]$
- M_3: $A_3 = \begin{bmatrix} 1 & T_e \\ -3.19 & 0.085 \end{bmatrix}$ $B_3 = \begin{bmatrix} 0 \\ 4.3 \end{bmatrix}$
  $H_3 = [1 \quad 0]$

The validities are evaluated by using the residue approach. Tree partial controls are calculated according to the three partial models by the expression:

$$u_i(k) = u_{ieq}(k) + u_{dis}(k) \qquad i = 1, 2, 3 \tag{28}$$

$u_{ieq}(k)$ is calculated by the expression :

$$u_{ieq}(k) = (C^T B_i)^{-1}[-\beta\, S(k) - C^T A_i x(k) + C^T (x_d(k+1)]$$

$u_{dis}(k)$ is approximated by a first order discretisation:

$$u_{dis}(k) = u_{dis}(k-1) - T_e M\, sign\,(\sigma(k))$$

We take, $M = 0.03$, $\beta = 0.3$ and $\phi = 0.3$ for the evaluation of the MM-2-DSMC and the 2-DSMC performances.

The evolutions of the output, the sliding function, the control and the phase plane trajectory are represented respectively on the figures 9, 10, 11 and 12. A notable improvement is observed in the case of the MM-2-DSMC relatively to the 2-DSMC. In fact, the sliding function keeps a null value apart from the parametric variations. On the contrary, with the 2-DSMC, the sliding function leaves the sliding surface in case of relatively important parameter variations. A notable improvement is also noted on the output evolution which observes the desired dynamic and is robust with respect to the parameter variations.

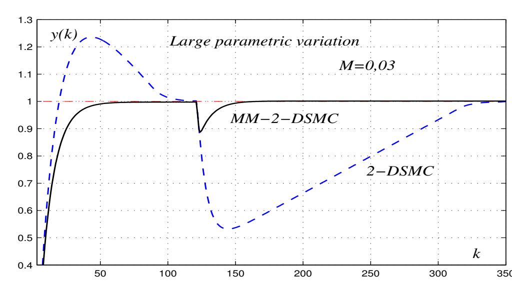

Fig. 9. The output evolutions.

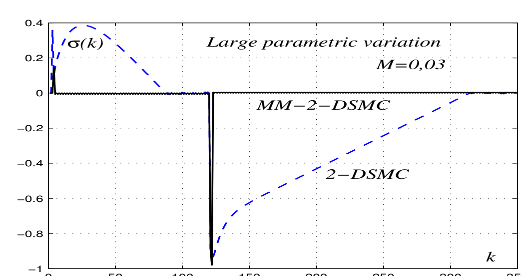

Fig. 10. Evolutions of the sliding function $\sigma(k)$.

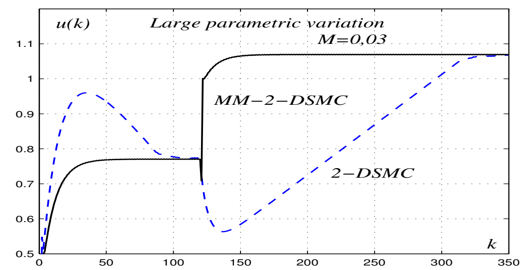

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Fig. 11. The control evolutions.

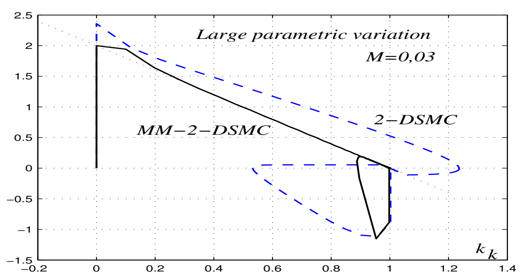

Fig. 12. The phase plane.

VI. CONCLUSION

In this work, we proposed a combination between the multimodel approach and the 2-DSMC in order to insure the robustness of the closed loop system, essentially during the reaching phase and in presence of large parameter variations. The use of a multimodel improved, remarkably, the convergence rapidity without a need to increase the discontinuous term's amplitude and, thus, avoiding the chattering phenomenon. Simulation results show a notable performances improvement offered by the proposed MM-2-DSMC relatively to case where the 2-DSMC is exploited.

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