

# Dynamic Output Feedback Linearization Based Adaptive Control of Nonlinear MIMO Systems

Eduard Petlenkov, Juri Belikov, Sven Nõmm and Małgorzata Wyrwas

**Abstract**—This paper discusses application of dynamic output feedback linearization algorithm for adaptive control of nonlinear MIMO systems. Neural Network based Simplified Additive Nonlinear AutoRegressive eXogenous (NN-SANARX) structure is used for identification of nonlinear MIMO systems. This structure imposes a restriction on model adaptation. The model is divided into adaptable and nonadaptable parts. After that history-stack adaptation with dynamic output feedback linearization is used for adaptive control of nonlinear MIMO systems. The effectiveness of the adaptive control technique proposed in the paper is demonstrated on numerical example.

## I. INTRODUCTION

DYNAMIC output feedback linearization algorithm proposed in [9] was specified for the case of NN-based ANARX structure in [3] and [6]. In [1] and [2] it was applied to control of nonlinear SISO systems. Two methods for application of this algorithm to control of nonlinear MIMO systems were shown in [4] and [5]. The technique proposed in [4] introduces a simplified NN-ANARX model. It imposes restrictions on the model, but calculation of a vector of control signals comes down to a solution of a system of linear equations. The technique proposed in [5] is based on introducing an additional neural network into the closed loop control system without simplification of the model. Our experiments have shown higher robustness of the first approach. That is why adaptive controller considered in this paper is based on this method.

While there are many sophisticated training methods for neural networks based nonlinear autoregressive exogenous models (NN-NARX), the NARX structure itself suffers from several drawbacks. First is that in general NARX structure is not always linearizable. Second drawback is that in general it is not realizable in the classical state-space form [6]-[8]. Since the majority techniques for system analysis and control design are based on classical state-space description, such structure is highly undesirable for the further analysis

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and control design. Additive NARX (ANARX) and Neural Network based Additive NARX (NN-ANARX) structures (the structures with reduced coupling) with dynamic output feedback linearization algorithm based on these models [3], [9] were proposed to bridge the gap.

This paper gives a brief overview of previous researches on NN ANARX model based control of nonlinear MIMO systems and designs an adaptive controller for a wide class of nonlinear MIMO systems. All the calculations and simulations shown in this paper are performed in MATLAB/Simulink environment.

The main contribution of this paper is devoted to identification and adaptive control of nonlinear MIMO systems. An adaptive control technique based on dynamic output feedback linearization of Neural Network based Simplified ANARX model with history-stack adaptation is proposed in the paper.

### A. Outline of the Paper

The paper is organized as follows. An overview of previous research of NN-ANARX structure and dynamic output feedback linearization based control is given in section II. Methods for application of this algorithm to control of nonlinear MIMO systems are shown in section III. Adaptive control of nonlinear MIMO systems is discussed in section IV and the effectiveness of the proposed technique is demonstrated on numerical example in section V. Conclusions are drawn in the last section.

## II. NN-ANARX STRUCTURE BASED DYNAMIC OUTPUT FEEDBACK LINEARIZATION OF NONLINEAR SYSTEMS

Discrete time Nonlinear AutoRegressive eXogenous (NARX) models are represented by high order difference equation.

$$y(t) = f(y(t-1), \dots, y(t-n), u(t-1), \dots, u(t-n)). \quad (1)$$

This model can be obtained by training a multilayer perceptron and is capable of modeling a wide class of nonlinear systems, but from the control system point of view, this structure has several serious drawbacks. This structure can not be represented in classical state-space form, there is no possibility to separate different time steps and it is not always linearizable by a dynamic feedback.

The idea of separating time-instances was proposed in [10]. ANARX model, presented in [8] and [3] for nonlinear SISO systems and in [4] and [5] for nonlinear MIMO

systems, is a subclass of NARX models having all time instances separated

$$y(t) = f_1(Z(t-1)) + \dots + f_n(Z(t-n)) = \sum_{i=1}^n f_i(Z(t-i)), \quad (2)$$

where for SISO systems  $Z(t) = [y(t), u(t)]^T$  and for MIMO systems  $Z(t) = [y_1(t), \dots, y_m(t), u_1(t), \dots, u_r(t)]^T$ . Here  $r$  is the number of inputs and  $m$  is the number of outputs of the model.

It is very convenient to obtain ANARX model (2) by training a neural network of the structure depicted in figure 1.

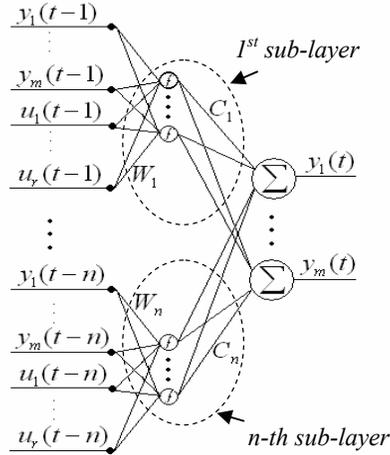


Fig. 1. Structure of neural network representing MIMO ANARX model

It can be seen from the figure that neural network representing ANARX model is a restricted connectivity neural network. Its hidden layer consists of  $n$  sub-layers corresponding to the  $n$ -th order of the model. This model is called NN-based ANARX or NN-ANARX and can be represented by the following high order difference equation.

$$y(t) = \sum_{i=1}^n C_i \varphi_i(W_i \cdot Z(t-i)), \quad (3)$$

where  $\varphi_i(\cdot)$  is an activation function of neurons of the corresponding sub-layer,  $W_i$  and  $C_i$  are matrixes of synaptic weights of inputs and outputs of  $i$ -th sub-layer.

ANARX model (2) can always be linearized by using the following dynamic output feedback [9]

$$u(t) = F^{-1}(y(t), \eta_1(t)), \quad (4)$$

where

$$F = f_1(Z(t)) = \eta_1(t) \quad (5)$$

and the dynamics of the feedback linearization is represented by the following system of  $n-1$  first order difference equations.

$$\begin{aligned} \eta_1(t+1) &= \eta_2(t) - f_2(Z(t)) \\ &\vdots \\ \eta_{n-2}(t+1) &= \eta_{n-1}(t) - f_{n-1}(Z(t)) \\ \eta_{n-1}(t+1) &= v(t) - f_n(Z(t)) \end{aligned} \quad (6)$$

Here  $v(t)$  is the desired output of the system (reference).

If ANARX model is obtained in the form of neural network (3), equations (5)-(6) can be rewritten by using parameters of the neural network as follows [3]

$$F = C_1 \varphi_1(W_1 \cdot Z(t)) = \eta_1(t) \quad (7)$$

$$\eta_1(t+1) = \eta_2(t) - C_2 \varphi_2(W_2 \cdot Z(t))$$

$$\vdots \quad (8)$$

$$\eta_{n-2}(t+1) = \eta_{n-1}(t) - C_{n-1} \varphi_{n-1}(W_{n-1} \cdot Z(t))$$

$$\eta_{n-1}(t+1) = v(t) - C_n \varphi_n(W_n \cdot Z(t))$$

Dynamic linearization (7)-(8) can be used as an output feedback controller for nonlinear identified by NN-ANARX model (3). The problem of practical application of this control technique is calculation of values of control signal  $u(t)$  or, in case of MIMO systems, vector of controls  $[u_1(t), \dots, u_r(t)]$  from (7).

Control technique (7)-(8) was successfully applied to control of nonlinear SISO systems by using Newton's method for numerical calculation of (4) in [1] and [2]. Our experiments have shown that 3 iterations guarantee sufficient precision [1], [2], but when we have to calculate numerous controls this task becomes extremely complex. Inverse of the function of several arguments can not be calculated numerically fast enough to satisfy the needs of the control system. Practically it means that numerical algorithms do not converge or the result is not precise and can not be used as a control signal. Because of the problem described above the algorithm can not be applied to systems with multiply inputs. Also, numerical calculation of control signal slows down the speed of the control system, which may be critical in real time applications. Two methods were proposed to overcome these problems and to apply dynamic linearization (7)-(8) to control of nonlinear MIMO systems in [4] and [5]. To show the restrictions imposed by these methods on adaptive control, NN-ANARX based control of nonlinear MIMO systems will be briefly discussed in the next section.

### III. CONTROL OF NONLINEAR MIMO SYSTEMS

In [5], dynamic output feedback linearization algorithm (7)-(8) was applied to control of nonlinear MIMO systems by an additional static neural network approximating function (4). According to Stone-Weirstrass theorem [11], [12], a two-layer perceptron with nonlinear sigmoid activation functions of its hidden layer neurons is capable of approximating any arbitrary continuous map to within a

desired accuracy.

First sub-network of neural network depicted in figure 1 can be simulated as a separate system with random inputs

$$[\eta_{11}(t), \dots, \eta_{1m}(t)]^T = C_1 \phi_1(W_1 \cdot Z(t)). \quad (9)$$

By using this training data set, an additional neural network (10) approximating (4) can be trained

$$[u_1(t), \dots, u_r(t)]^T = C_0 \phi_0(W_0 \cdot [y_1(t), \dots, y_m(t), \eta_{11}(t), \dots, \eta_{1m}(t)]^T). \quad (10)$$

System (8), (10) is then used as an output feedback controller for nonlinear systems identified by NN-ANARX model (3).

An alternative approach to application of dynamic output feedback linearization algorithm (7)-(8) to control of nonlinear MIMO systems was proposed in [4]. This method is based on Simplified NN-ANARX model and allows exact calculation of the vector of control signals (not approximation).

NN-based Simplified ANARX model (NN-SANARX) has linear activation functions on the first sub-layer of the hidden layer of the network (3) depicted in figure 1 and can be formalized by the following equation

$$y(t) = C_1 \cdot W_1 \cdot Z(t-1) + \sum_{i=2}^n C_i \phi_i(W_i \cdot Z(t-i)). \quad (11)$$

It can be seen from (11) that in order to identify a nonlinear system, neural network representing NN-SANARX model (11) has to have at least two sub-layers of the hidden layer. It means that the order of the nonlinear model is two or greater ( $n \geq 2$ ).

NN-SANARX model (11) is a sub-class of ANARX models (2) and NN-ANARX models (3). Consequently, it also can always be linearized by dynamic output feedback (7)-(8). This algorithm can be used model based dynamic output feedback control of nonlinear systems identified by (11). It has to be mentioned that for MIMO systems  $Z(t)$ ,  $u(t)$  and  $y(t)$  are the following vectors:

$$u(t) = [u_1(t), \dots, u_r(t)], \quad (12)$$

$$y(t) = [y_1(t), \dots, y_m(t)], \quad (13)$$

$$Z(t) = [y_1(t), \dots, y_m(t), u_1(t), \dots, u_r(t)]. \quad (14)$$

$W_i \in \mathfrak{R}^{l_i \times (m+r)}$  and  $C_i \in \mathfrak{R}^{m \times l_i}$  are input and output matrixes of synaptic weights of the corresponding  $i$ -th sub-layer of the hidden layer of the network depicted in figure 1 and representing ANARX structure. Here  $l_i$  is the number of neurons in the  $i$ -th sub-layer.

Lets define matrix  $T$  as

$$T = C_1 \cdot W_1. \quad (15)$$

It follows from definition (15) that  $T \in \mathfrak{R}^{m \times (m+r)}$ . It has  $m+r$  columns and can easily be separated into two matrixes  $T_1$  and  $T_2$  such that

$$T \cdot Z(t) = T_1 \cdot y(t) + T_2 \cdot u(t), \quad (16)$$

where  $T_1 \in \mathfrak{R}^{m \times m}$  is a square matrix and  $T_2 \in \mathfrak{R}^{m \times r}$ . Equation (7) takes now the following form

$$F = T_1 \cdot y(t) + T_2 \cdot u(t) = \eta_1(t). \quad (17)$$

In case of MIMO systems,  $u(t)$  and  $y(t)$  are vectors and vector of control signals  $u(t)$  has to be calculated on each sample by solving system of linear equations (17). It has a solution if and only if  $\text{rank}(T_2) \geq r$ . It is possible only if  $r \geq m$ . Systems with  $r = m$  were considered in [4]. For this class of nonlinear MIMO systems identified by NN-SANARX model,  $T_2 \in \mathfrak{R}^{m \times m}$  is a square matrix. If matrix  $T_2$  is a nonsingular matrix, system (17) has a unique solution, which can be found as

$$u(t) = T_2^{-1}(\eta_1(t) - T_1 \cdot y(t)). \quad (18)$$

System (8), (18) can now be used as a dynamic output feedback controller for systems identified by NN-based Simplified ANARX structure (11). Nonsingularity of matrix  $T_2$  is the criterion of applicability of the model for model based control [4] and can be very easily checked after identification of NN-SANARX model (11) by using obtained matrixes of synaptic weights (15)-(16) of the model.

Neural Networks based structure of the model makes possible to use on-line training as the controller's adaptation technique, but both control algorithms (8), (10) and (8), (18)

discussed in this section impose an important restriction on model adaptation.

In case of control based on dynamic output feedback linearization of NN-ANARX model with additional neural network (8), (10), parameters  $W_1$  and  $C_1$  of the first sub-layer of model (3) are used for off-line training of network (11). Matrixes of parameters  $W_0$  and  $C_0$  of network (11) depend on  $W_1$  and  $C_1$  and can not be retrained on-line.

In case of control based on dynamic output feedback linearization of NN-SANARX model (8), (18), parameters  $W_1$  and  $C_1$  of the first sub-layer of model (11) should guarantee nonsingularity of matrix  $T_2$  and therefore can not be changed when control system is running.

Our experiments have also shown much higher robustness of NN-SANARX model based control (8), (18) in nonadaptive case. That is why this approach was chosen for designing of a dynamic output feedback linearization based adaptive controller for nonlinear MIMO systems, which will be discussed in detail in the next section of the paper.

#### IV. NN-SANARX MODEL BASED ADAPTIVE CONTROL OF NONLINEAR MIMO SYSTEMS

The aim of the adaptive controller is to modify its behavior in response to changes in the dynamics of the process and disturbances. A real-time estimator is a central part in most adaptive controllers [13]. In case of model based neurocontrollers, parameters of neural network based model have to be adjusted. Thus, classical dynamic output linearization can be combined with neural networks based adaptation.

The requirement to learn from incidents or samples from the environment has parallels with human learning and memory. There are two types of memories: short-term memory and long-term memory. The theory says that all new information to be memorized must first be processed through the short-term memory [14].

History-stack adaptation (HSA) algorithm [14] was used to adapt the model. It retains a short history of process patterns (short-time memory) that can represent an approximation to the nonlinear process dynamics. This information containing in the History Stack (HS) is then transferred into the long-time memory by means of training algorithm.

The control system consists of unknown nonlinear plant preceded by the linearizing feedback (8), (18). Unknown plant is modeled by the NN-SANARX structure (11). At each time-step the plant model is adjusted by HSA algorithm. The HS operates as a First-In-First-Out (FIFO) stack containing  $n_p$  patterns. At each time-step  $k$  HS accepts (memorizes) a net pattern  $Z(t)$  from the process and discards (forgets) the oldest pattern  $Z(t - n_p)$ . These elements  $Z(t), Z(t-1), \dots, Z(t - n_p + 1)$  constitute the training set and are used in  $n_c$  cycles to update the weights [14], [15]. To achieve a good process performance the proper choice of the parameters  $n_p$  and  $n_c$  is essential [15].

It was shown in the previous section that if the neural network is adjusted on-line by an adaptation algorithm, matrix  $T_2$  may become singular or close to singular that according to (16)-(18) leads to extreme growth of control signal  $u(t)$ . That is why first sub-layer of the network shown in figure 1 and representing SANARX model of a nonlinear MIMO system has to be excluded from the adaptation procedure. Two matrixes of synaptic weights of the model  $W_1$  and  $C_1$  should remain constant after checking nonsingularity of matrix  $T_2$  when the model is trained off-line.

It has to be mentioned that following from (3) and (7), NN-ANARX model can be represented as

$$y(t) = \eta_1(t-1) + \sum_{i=2}^n C_i \cdot \varphi_i(W_i \cdot Z(t-i)), \quad (19)$$

where, from the control point of view, vector  $\eta_1(t)$  comes at each time step from the dynamics (8) of the controller.

From the other hand, for NN-based SANARX model (11)

$$\eta_1(t) = C_1 \cdot W_1 \cdot Z(t). \quad (20)$$

Now model (11) can be separated into adaptive  $y^a(t)$  and nonadaptive  $y^n(t)$  parts so that

$$[y_1(t), \dots, y_m(t)] = [y_1^n(t), \dots, y_m^n(t)] + [y_1^a(t), \dots, y_m^a(t)], \quad (21)$$

where

$$[y_1^n(t), \dots, y_m^n(t)] = C_1 \cdot W_1 \cdot Z(t-1) \quad (22)$$

and

$$[y_1^a(t), \dots, y_m^a(t)] = \sum_{i=2}^n C_i \cdot \varphi_i(W_i \cdot Z(t-i)). \quad (23)$$

According to (19), etalon vectors for training of network (23) representing adaptable part of the model can always be very easily found as

$$[y_1^a(t), \dots, y_m^a(t)] = [y_1(t), \dots, y_m(t)] - [\eta_{1,1}(t), \dots, \eta_{1,m}(t)]. \quad (24)$$

Here vector  $[y_1(t), \dots, y_m(t)]$  is obtained from the output of the controlled system and vector  $[\eta_{1,1}(t), \dots, \eta_{1,m}(t)]$  is computed by the dynamic feedback controller (8).

History-stack adaptation technique is proposed for adaptation of (23). Set of vectors  $\{y(t-i), u(t-i), y^a(t-i), i=1, \dots, n_p\}$  is used as the training set for model adaptation by on-line training of (23). Vector  $y^a(t)$  is defined by (24) and  $n_p$  is the number of patterns in the stack (size of the history-stack).

The structure of the control scheme for adaptive control of nonlinear MIMO systems based on dynamic output feedback linearization of NN-SANARX model with history-stack adaptation is presented in figure 2.

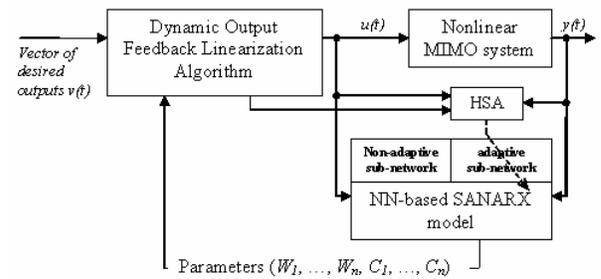


Fig. 2. Adaptive control of nonlinear MIMO systems

It can be seen from the figure that the adaptation algorithm (HSA) utilizes current inputs and outputs from the controlled system together with the vector of inner states of the controller. Thus, adaptation is based on the following vectors

$$HSA \begin{cases} [u_1(t), \dots, u_r(t)] \\ [y_1(t), \dots, y_m(t)] \\ [\eta_{1,1}(t), \dots, \eta_{1,m}(t)] \end{cases} \quad (25)$$

and adaptation is applied to only a part of the model - to adaptive sub-network (23).

Closed loop control system with adaptation was simulated on a nonlinear MIMO test system with input/output disturbances. Our experiments have shown high effectiveness and robustness of the proposed technique. Consider the following numerical example.

#### V. NUMERICAL EXAMPLE

A nonlinear MIMO discrete-time system [16], [17], [4] is represented by the following input-output equation

$$\begin{aligned} y_1(t+1) &= 0.4y_1(t) + \frac{u_1(t)}{1+u_1^2(t)} + 0.2u_1^3(t) + 0.5u_2(t) \\ y_2(t+1) &= 0.2y_2(t) + \frac{u_2(t)}{1+u_2^2(t)} + 0.4u_2^3(t) + 0.2u_1(t) \end{aligned} \quad (26)$$

System (21) was simulated and the obtained set of input-output data was used for training of MIMO NN-based SANARX structure (27) with two sub-layers corresponding to the second order of model (11).

$$\begin{aligned} [y_1(t), y_2(t)]^T &= C_1 \cdot W_1 \cdot [y_1(t-1), y_2(t-1), u_1(t-1), u_2(t-1)]^T + \\ &+ C_2 \cdot \varphi_2(W_2 \cdot [y_1(t-2), y_2(t-2), u_1(t-2), u_2(t-2)]^T) \end{aligned} \quad (27)$$

As it was shown in [4], minimal order of NN-SANARX model representing a nonlinear dynamic system is 2 ( $n=2$ ), because of linearity of the first sub-layer. The first layer consisted of 2 linear neurons ( $I_1 = 2$ ) and the second layer consisted of 5 nonlinear neurons ( $I_2 = 5$ ) with hyperbolic tangent sigmoid activation functions.

Levenberg-Marquardt (LM) training algorithm was chosen to perform off-line training of the model since neural network representing ANARX model is a restricted connectivity network and LM algorithm is much more efficient compared to other techniques when the network contains no more than a few hundred weights [18]. Also the training speed of LM algorithm is much higher and the feed forward neural network trained with it can better model the nonlinearity [19]. Identified parameters of model (27) can be found in [4] where nonadaptive control of system (26) by dynamic output feedback linearization of NN-SANARX model is demonstrated.

By using these parameters, NN-SANARX model based

dynamic output feedback controller for system (26) can be represented by the following equations

$$\begin{aligned} [u_1(t), u_2(t)]^T &= T_2^{-1}([\eta_{11}(t), \eta_{12}(t)]^T - T_1 \cdot [y_1(t), y_2(t)]^T) \\ [\eta_{11}(t+1), \eta_{12}(t+1)]^T &= [v_1(t), v_2(t)]^T - C_2 \phi(W_2 \cdot [y_1(t), y_2(t), u_1(t), u_2(t)]^T) \end{aligned} \quad (28)$$

where  $T_1$  and  $T_2$  are defined by (16) and  $[v_1(t), v_2(t)]^T$  is the vector of reference signals corresponding to two controllable outputs of the system. See [4] for more details.

In [4] feedback controller (28) was successfully applied to nonadaptive control of nonlinear MIMO system (26). Our experiments have also shown that because of high robustness of the considered control technique, it is capable of compensating high input and output disturbances as well as flowing of parameters of the controlled system (up to 100%) when these disturbances and changes in the dynamics of the system do not occur in the same time. Problems (high oscillations and big static errors) arise when they present in the control loop simultaneously. The aim of the following experiment is to show the effectiveness of the propose adaptive control technique in case of presence of high disturbances on inputs and outputs of the controlled system together with changing of its parameters.

For this experiment, (26) takes the following form

$$\begin{aligned} y_1(t+1) &= (0.4 + \Delta_1(t))y_1(t) + (1 + \Delta_2(t))\frac{u_1(t)}{1+u_1^2(t)} + 0.2u_1^3(t) + 0.5u_2(t) \\ y_2(t+1) &= 0.2y_2(t) + (1 + \Delta_3(t))\frac{u_2(t)}{1+u_2^2(t)} + (0.4 + \Delta_4(t))u_2^3(t) + 0.2u_1(t) \end{aligned} \quad (29)$$

where

$$\Delta_1(t) = \begin{cases} -0.15e^{5 \cdot 10^{-4} \cdot t}, & t < 1400 \\ 0.2 \cos(0.0027 \cdot t) - 0.2, & t \geq 1400 \end{cases}, \quad (30)$$

$$\Delta_2(t) = \begin{cases} 6.5 \cdot 10^{-4} \cdot t, & t < 610 \\ -3.5 \cdot 10^{-4} \cdot t + 0.15, & t \geq 610 \end{cases}, \quad (31)$$

$$\Delta_3(t) = 2.5 \cdot 10^{-4} \cdot t, \quad (32)$$

$$\Delta_4(t) = \begin{cases} 0.2 \sin(6.5 \cdot 10^{-3} \cdot t) - 0.2, & t < 1700 \\ 0.12 \cos(3 \cdot 10^{-3} \cdot t), & t \geq 1700 \end{cases}. \quad (33)$$

In (30)-(33),  $t \in [0; 2500]$ . Changes of parameters in time corresponding to (30)-(33) are also illustrated in figure 3. Disturbances given to two inputs and two outputs of system (29)-(33) are depicted in figure 4.

NN-SANARX model based dynamic output feedback linearization with history-stack adaptation corresponding to the structure of the closed loop system depicted in figure 2, was used for adaptive control of system (29)-(33).

We suggest to implement the gradient descent training

algorithm in adaptive control scheme. The main advantage of latter is that it consists of a higher number of shorter iterations. On each iteration updated weights can be calculated very fast that is critical in some real-time applications. The stack length of HSA was set to 200 patterns ( $n_p = 200$ ).

The result of the control system simulation is depicted in figure 5. It can be seen from the figure that outputs  $y_1(t)$  and  $y_2(t)$  of system (29)-(33) are capable of simultaneous tracking the desired reference signals  $v_1(t)$  and  $v_2(t)$  compensating undesirable influences of disturbances and changes in the dynamics of the process thus meeting the aim of an adaptive control system.

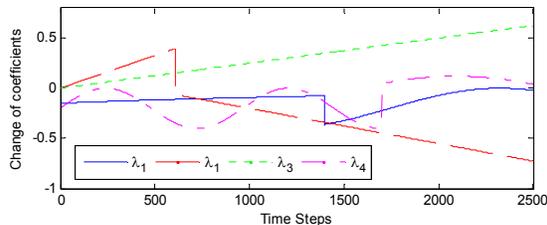


Fig. 3 Changes of parameters of the controlled system in time

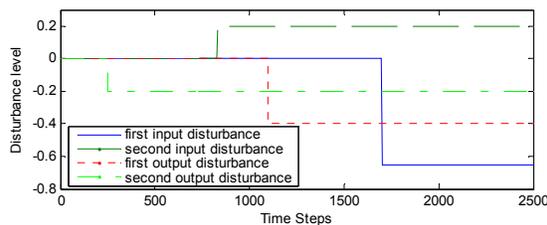


Fig. 4 Input and output disturbances of the controlled system

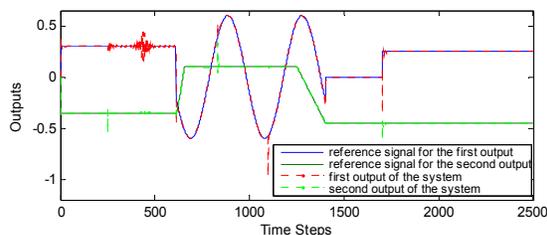


Fig. 5 NN-SANARX model based adaptive control of nonlinear MIMO system

## VI. CONCLUSION

This paper designs an adaptive feedback controller for nonlinear MIMO system modeled by Neural Networks based Simplified ANARX structure. Classical dynamic output feedback linearization is combined with neural network based history-stack adaptation.

Because of specific of NN-SANARX model based control, the model has to be divided into adaptable and nonadaptable parts in order to make the control algorithm adaptive. The proposed adaptation technique is based on the stack of controller's input and output values as well as inner states of the dynamic feedback controller.

Experiments have shown high robustness of the proposed control technique and capability the adaptive control system to modify its behavior in response to changes in the

dynamics of the process and disturbances .. The control technique proposed in the paper can applied to control of a wide class of nonlinear MIMO systems.

## REFERENCES

- [1] E. Petlenkov, S. Nömm, and Ü. Kotta, "NN-based ANARX structure for identification and model-based control," *Proc. 9th International Conference on Control Automation Robotics & Vision (ICARCV 2006)*, Singapore, 2006, pp. 2284-2288.
- [2] E. Petlenkov, S. Nömm, and Ü. Kotta, "Adaptive Output Feedback Linearization for a Class of NN-based ANARX Models," *Proc. 6th IEEE International Conference on Control and Automation*, Guangzhou, China, May 30 – June 1, 2007, pp. 3173-3178.
- [3] Ü. Kotta, S. Nömm, and F. Chowdhury, "On a new type of neural network-based input-output model: The ANARMA structure," *Proc. of the 5th IFAC Symposium on nonlinear control systems NOLCOS*, St. Petersburg, Russia, July 2001.
- [4] E. Petlenkov, "NN-ANARX Structure Based Dynamic Output Feedback Linearization for Control of Nonlinear MIMO Systems," *Proc. Of the 15th Mediterranean Conference on Control and Automation, MED'07*, June 2007, Athens, Greece, pp. 1-6.
- [5] E. Petlenkov, and J. Belikov, "NN-ANARX Structure for Dynamic Output Feedback Linearization of Nonlinear SISO and MIMO Systems: Neural Networks Based Approach," *In Proc. Of the 26th Chinese Control Conference (CCC2007)*, Zhangjiajie, China, July 2007, vol. 4, pp. 138-145.
- [6] Ü. Kotta, F. Chowdhury, and S. Nömm, "On realizability of neural networks-based input-output models in the classical state space form," *Automatica*, 2006, vol. 42, no. 7, pp. 1211-1216.
- [7] Ü. Kotta and N. Sadegh, "Two approaches for state space realization of narma models: bridging the gap," *Mathematical and Computer Modelling of Dynamical Systems*, vol. 8, no. 1, 2002, pp. 21-32.
- [8] F. N. Chowdhury, Ü. Kotta and S. Nömm, "On realizability of neural-networks-based input-output models," *Proc. of the 3rd Int. Conf. on Differential Equations and Applications*, St. Petersburg, Russia, vol. 6, 2000, pp. 47-51.
- [9] R. Pothin, Ü. Kotta, and C. Moog, "Output feedback linearization of nonlinear discrete-time systems," *Proc. of the IFAC Conf. on Control system design*, Bratislava, 2000, pp. 174-179.
- [10] F. N. Chowdhury, "Input-output modeling of nonlinear systems with time-varying linear models. *IEEE Transactions on Automatic Control*, vol. 7, 2000, pp. 1355-1358.
- [11] M. H. Stone, "The generalized Weierstrass approximation theorem," *Mathematics Magazine*, vol. 21, 1948, pp. 167-184, 237-254.
- [12] K. J. Hunt, G. R. Irwin and K. Warwick, *Neural Network Engineering in Dynamic Control Systems: Advances in Industrial Control*, Springer-Verlag, London, 1995.
- [13] K. J. Åström and B. Wittenmark. *Adaptive control*. Addison-Wesley Publishing Company, 1989.
- [14] P. M. Mills, A. Z. Zomaya and M. O. Tade, *Neuro-Adaptive Process Control*, John Wiley & Sons, Inc., England, 1996.
- [15] K. Vassiljeva, and E. Rüstern, "On-Line Identification and Adaptive Control of Nonlinear Systems using Neural Networks," *In Proc. of the 9th Biennial Baltic Electronics Conference: BEC2004*, Tallinn, Estonia, October 3-6, 2004, pp. 149-152.
- [16] X. Li, Y. Bai and L. Yang, "Neural Network Online Decoupling for a Class of Nonlinear System," *In Proc. of the 6th World Congress on Intelligent Control and Automation*, Dalian, China, June 21-23, 2006, pp. 2920-2924.
- [17] H. L. Shu, "PID neural network for analysis of the multivariable systems," *Automation transaction*, vol.25, no.1, 1999, pp. 105-111.
- [18] M. T. Hagan and M. B. Menhaj, "Training feedforward networks with the Marquardt algorithm," *Neural Networks, IEEE Transactions on*, Vol. 5, Issue 6, November 1994, pp. 989-993.
- [19] F. Declercq and R. De Keyser, "Comparative study of neural predictors in model based predictive control" *Proc. of The International Workshop on Neural Networks for Identification, Control, Robotics, and Signal/Image Processing*, Venice, Italy, 21-23 Aug. 1996, pp. 20-28.