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A New Class of Modular Adaptive Controllers, Part I: Systems with Linear-in-the-Parameters Uncertainty

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Abstract— **This paper presents a novel adaptive nonlinear control design which achieves modularity between the controller and the adaptive update law. Modularity between the controller/update law design provides flexibility in the selection of different update laws that could potentially be easier to implement or used to obtain faster parameter convergence and/or better tracking performance. For a class of linear-in-the-parameters (LP) uncertain Euler-Lagrange systems subject to additive bounded non-LP disturbances, the result in this (Part I) paper is based on a controller that uses a model-based feedforward adaptive term in conjunction with the recently developed Robust Integral of the Sign of the Error (RISE) feedback term. Modularity in the adaptive feedforward term is made possible by considering a generic form of the adaptive update law and its corresponding parameter estimate. This generic form of the update law is used to develop a new closed-loop error system and stability analysis that does not depend on nonlinear damping to yield the modular adaptive control result.**

I. Introduction

A variety of adaptive control results have been developed to compensate for linear-in-the-parameters (LP) uncertainty in nonlinear systems. Most of this research has exploited Lyapunov-based techniques (i.e., the controller and the adaptive update law are designed based on a Lyapunov analysis); however, Lyapunov-based methods restrict the design of the adaptive update law. For example, many of the previous adaptive controllers are restricted to utilizing gradient update laws to cancel cross terms in a Lyapunov-based stability analysis. Gradient update laws often exhibit slow parameter convergence which leads to a degraded transient performance of the tracking error in comparison to other possible adaptive update laws (e.g., least-squares update law). Several results have been developed in literature that aim to augment the typical position/velocity tracking error based gradient update law including: composite adaptive update laws [1]–[3]; prediction error based update laws [4]–[8]; and various least-squares update laws [9]–[11]. The adaptive update law in these results are all still designed to cancel cross terms in the Lyapunov-based stability analysis. In contrast to these results, researchers have also developed a class of modular adaptive controllers (cf. [4], [6]–[8]) where a feedback mechanism is used to stabilize the error dynamics provided certain conditions are satisfied on the adaptive update law. For example, nonlinear damping [5], [12] is typically used to yield an input-to-state stability (ISS) result with respect to the parameter estimation error where it is assumed a priori that the update law yields bounded parameter estimates. Often the modular adaptive control development exploits a prediction error in the update law (e.g., see [1], [4]–[7]), where the prediction error is often required to be square integrable (e.g., [4], [6], [7]). A brief survey of modular adaptive control results is provided in [4].

Recently, a new high gain feedback control strategy coined the Robust Integral of the Sign of the Error (RISE) in [13] was developed that contains a unique integral signum term which can accommodate for sufficiently smooth bounded disturbances. A significant outcome of this new control structure is that asymptotic stability is obtained despite a fairly general uncertain disturbances. In fact, the early work in [14]–[24] illustrate how different RISE-based controllers/estimation methods can be used to yield an asymptotic result for nonlinear systems with LP or non-LP uncertainty and additive bounded disturbances without an adaptive feedforward component. Since the RISE method exploits high gain feedback, results such as [13], [25], [26] were developed with various modifications to the stability analysis to amalgamate the RISE feedback with model-based adaptive or neural network feedforward components. The results in [13] experimentally demonstrate the well accepted paradigm that the inclusion of an adaptive feedforward term can reduce the control effort, improve the transient performance, and reduce the steady-state error over feedback only methods. However, the results in [13], [25], [26] were developed using the typical Lyapunov-based gradient adaptive update law. Since the RISE feedback mechanism alone can yield an asymptotic result without a feedforward component to cancel cross terms in the stability analysis, the research in this paper is motivated by the following question: *Can the RISE control method be used to yield a new class of modular adaptive controllers?*

The results in this work provide the first investigation of the ability to yield controller/update law modularity using the RISE feedback. Specifically in this (Part I) paper, we consider dynamic systems with structured (i.e., LP) and unstructured uncertainties and develop a controller with modularity between the controller/update law, where a model-based adaptive feedforward term is used in conjunction with

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the RISE feedback term [13], [25]. Part II of this paper [27], presents a neural network extension of the result for non-LP systems [26].

The RISE-based modular adaptive approach is different than previous work (cf. [4], [5], [7]) in the sense that it does not rely on nonlinear damping. The use of the RISE method in lieu of nonlinear damping has several potential advantages that motivate this investigation including: an asymptotic modular adaptive tracking result can be obtained for nonlinear systems with non-LP additive bounded disturbances; the dual objectives of asymptotic tracking and controller/update law modularity are achieved in a single step unlike the two stage analysis required in some results (cf., [4], [7]); the development does not require that the adaptive estimates are a priori bounded; and the development does not require a positive definite estimate of the inertia matrix or a square integrable prediction error as in [4], [7]. Modularity in the adaptive feedforward term is made possible by considering a generic form of the adaptive update law and its corresponding parameter estimate. The general form of the adaptive update law includes examples such as gradient, least-squares, and etc. This generic form of the update law is used to develop a new closed-loop error system, and the typical RISE stability analysis is modified to accommodate the generic update law. New sufficient gain conditions are derived to prove an asymptotic tracking result.

While the current result encompasses a large variety of adaptive update laws, an update law design based on the prediction error is not possible because the formulation of a prediction error requires the system dynamics to be completely LP. Future efforts can be focussed on developing a RISE-based adaptive controller for a completely LP system that could also use a prediction error/torque filtering approach. Also, one of the shortcomings of current work is that only a semi-global asymptotic stability is achieved, and further investigation is needed to achieve a global stability result [28].

II. Dynamic Model and Properties

The class of nonlinear dynamic systems considered in this paper can be described by the following Euler-Lagrange formulation:

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) + F(\dot{q}) + \tau_d(t) = \tau(t). \quad (1)$$

In (1), $M(q) \in \mathbb{R}^{n\times n}$ denotes the inertia matrix, $V_m(q,\dot{q}) \in \mathbb{R}^{n\times n}$ denotes the centripetal-Coriolis matrix, $G(q) \in \mathbb{R}^n$ denotes the gravity vector, $F(\dot{q}) \in \mathbb{R}^n$ denotes friction, $\tau_d(t) \in \mathbb{R}^n$ denotes a general nonlinear disturbance (e.g., unmodeled effects), $\tau(t) \in \mathbb{R}^n$ represents the torque input control vector, and $q(t)$, $\dot{q}(t)$, $\ddot{q}(t) \in \mathbb{R}^n$ denote the link position, velocity, and acceleration vectors, respectively. The subsequent development is based on the assumption that $q(t)$ and $\dot{q}(t)$ are measurable and that $M(q)$, $V_m(q,\dot{q})$, $G(q)$, $F(\dot{q})$ and $\tau_d(t)$ are unknown. Moreover, the following properties and assumptions will be exploited in the subsequent development.

Property 1: The inertia matrix $M(q)$ is symmetric, positive definite, and satisfies the following inequality $\forall\xi(t) \in \mathbb{R}^n$:

$$m_1 \|\xi\|^2 \leq \xi^T M(q)\xi \leq \bar{m}(q)\|\xi\|^2 \quad (2)$$

where $m_1 \in \mathbb{R}$ is a known positive constant, $\bar{m}(q) \in \mathbb{R}$ is a known positive function, and $\|\cdot\|$ denotes the standard Euclidean norm.

Property 2: If $q(t)$, $\dot{q}(t) \in \mathcal{L}_\infty$, then $V_m(q,\dot{q})$, $F(\dot{q})$ and $G(q)$ are bounded. Moreover, if $q(t)$, $\dot{q}(t) \in \mathcal{L}_\infty$, then the first and second partial derivatives of the elements of $M(q)$, $V_m(q,\dot{q})$, $G(q)$ with respect to $q(t)$ exist and are bounded, and the first and second partial derivatives of the elements of $V_m(q,\dot{q})$, $F(\dot{q})$ with respect to $\dot{q}(t)$ exist and are bounded.

Property 3: The nonlinear disturbance term and its first two time derivatives, i.e. $\tau_d(t)$, $\dot{\tau}_d(t)$, $\ddot{\tau}_d(t)$ are bounded by known constants.

Property 4: Part of the dynamics in (1) can be linearly parameterized as

$$Y_d\theta \triangleq M(q_d)\ddot{q}_d + V_m(q_d,\dot{q}_d)\dot{q}_d + G(q_d) + F(\dot{q}_d) \quad (3)$$

where $\theta \in \mathbb{R}^p$ contains the constant unknown system parameters, and $Y_d(q_d,\dot{q}_d,\ddot{q}_d) \in \mathbb{R}^{n\times p}$ is the desired regression matrix that contains known nonlinear functions of the desired link position, velocity, and acceleration, $q_d(t)$, $\dot{q}_d(t)$, $\ddot{q}_d(t) \in \mathbb{R}^n$, respectively.

Property 5: The desired trajectory is assumed to be designed such that $q_d^{(i)}(t) \in \mathbb{R}^n$ $(i = 0, 1, ..., 4)$ exist and are bounded.

III. Control Objective

The objective is to design a continuous modular adaptive controller which ensures that the system tracks a desired time-varying trajectory $q_d(t)$ despite uncertainties and bounded disturbances in the dynamic model. To quantify this objective, a position tracking error, denoted by $e_1(t) \in \mathbb{R}^n$, is defined as

$$e_1 \triangleq q_d - q. \quad (4)$$

To facilitate the subsequent analysis, filtered tracking errors [29], denoted by $e_2(t)$, $r(t) \in \mathbb{R}^n$, are also defined as

$$e_2 \triangleq \dot{e}_1 + \alpha_1 e_1 \quad (5)$$

$$r \triangleq \dot{e}_2 + \alpha_2 e_2 \quad (6)$$

where α_1, $\alpha_2 \in \mathbb{R}$ denote positive constants. The filtered tracking error $r(t)$ is not measurable since the expression in (6) depends on $\ddot{q}(t)$.

IV. Control Development

The open-loop tracking error system can be developed by premultiplying (6) by $M(q)$ and utilizing the expressions in (1), (4), and (5) to obtain the following expression:

$$M(q)r = Y_d\theta + S + \tau_d - \tau \quad (7)$$

where the auxiliary function $Y_d(t)\theta \in \mathbb{R}^n$ was defined in (3), and the auxiliary function $S(q,\dot{q},t) \in \mathbb{R}^n$ is defined as

$$\begin{aligned} S \triangleq\ & M(q)(\alpha_1\dot{e}_1 + \alpha_2 e_2) + M(q)\ddot{q}_d - M(q_d)\ddot{q}_d \\ & + V_m(q,\dot{q})\dot{q} - V_m(q_d,\dot{q}_d)\dot{q}_d \\ & + G(q) - G(q_d) + F(\dot{q}) - F(\dot{q}_d). \end{aligned} \quad (8)$$

Based on the open-loop error system in (7), the control torque input is composed of an adaptive feedforward term plus the RISE feedback term as

$$\tau \triangleq Y_d\hat{\theta} + \mu. \tag{9}$$

In (9), $\mu(t) \in \mathbb{R}^n$ denotes the RISE feedback term defined as [13]–[15], [25]

$$\begin{aligned} \mu(t) \;\triangleq\; & (k_s+1)e_2(t) - (k_s+1)e_2(0) \\ & + \int_0^t [(k_s+1)\alpha_2 e_2(\sigma) + \beta_1 sgn(e_2(\sigma))]d\sigma \end{aligned} \tag{10}$$

where k_s, $\beta_1 \in \mathbb{R}$ are positive constant control gains, $Y_d(t)$ was introduced in (3), and $\hat{\theta}(t) \in \mathbb{R}^p$ denotes a subsequently designed parameter estimate vector. The closed-loop tracking error system can be developed by substituting (9) into (7) as

$$M(q)r = Y_d(\theta - \hat{\theta}) + S + \tau_d - \mu. \tag{11}$$

To facilitate the subsequent modular adaptive control development and stability analysis, the time derivative of (11) is expressed as

$$\begin{aligned} M(q)\dot{r} \;=\; & -\frac{1}{2}\dot{M}(q)r + \tilde{N}(t) + N_B(t) \\ & -(k_s+1)r - \beta_1 sgn(e_2) - e_2 \end{aligned} \tag{12}$$

where the fact that the time derivative of (10) is given as

$$\dot{\mu}(t) = (k_s+1)r + \beta_1 sgn(e_2) \tag{13}$$

was utilized. In (12), the unmeasurable/unknown auxiliary terms $\tilde{N}(e_1, e_2, r, t)$, $N_B(t) \in \mathbb{R}^n$ are defined as

$$\tilde{N}(t) \triangleq -\frac{1}{2}\dot{M}(q)r + \dot{S} + e_2 + \tilde{N}_0 \tag{14}$$

$$N_B(t) \triangleq N_{B_1}(t) + N_{B_2}(t) \tag{15}$$

where $N_{B_1}(t) \in \mathbb{R}^n$ is given by

$$N_{B_1} \triangleq \dot{Y}_d\theta + \dot{\tau}_d, \tag{16}$$

and the sum of the auxiliary terms $\tilde{N}_0(t)$, $N_{B_2}(t) \in \mathbb{R}^n$ is given by

$$N_{B_2}(t) + \tilde{N}_0 = -\dot{Y}_d\hat{\theta} - Y_d\dot{\hat{\theta}}. \tag{17}$$

Specific definitions for $\tilde{N}_0(t)$, $N_{B_2}(t)$ are subsequently defined based on the definition of the adaptive update law for $\hat{\theta}(t)$. The structure of (12) and the introduction of the auxiliary terms in (14)-(17) is motivated by the desire to segregate terms that can be upper bounded by state-dependent terms and terms that can be upper bounded by constants. Specifically, depending on how the adaptive update law is designed, analysis is provided in the next section to upper bound $\tilde{N}(t)$ by state-dependent terms and $N_B(t)$ by a constant. The need to further segregate $N_B(t)$, is that some terms in $N_B(t)$ have time derivatives that are upper bounded by a constant, while other terms have time-derivatives that are upper-bounded by state dependent terms. The segregation of these terms based on the structure of the adaptive update law (see (17)), is key for the development of a stability analysis for the modular RISE-based adaptive update law/controller.

V. Modular Adaptive Update Law Development

A key difference between the traditional modular adaptive controllers that use nonlinear damping (cf., [5], [8], [30]) and the current RISE-based approach is that the RISE-based method does not exploit the ISS property with respect to the parameter estimation error. The current approach does not rely on nonlinear damping, but instead uses the ability of the RISE technique to compensate for smooth bounded disturbances. In general, previous nonlinear damping-based modular adaptive controllers first prove an ISS stability result provided the adaptive update law yields bounded parameter estimates (e.g., $\hat{\theta}(t) \in \mathcal{L}_\infty$ via a projection algorithm), and then use additional analysis along with assumptions (PD estimate of the inertia matrix, and square integrable prediction error, etc.) to conclude asymptotic convergence. In contrast, since the RISE-based modular adaptive control approach in this paper does not exploit an ISS analysis, the assumptions regarding the parameter estimate are modified. The following development requires some general bounds on the structure of the adaptive update law and the corresponding parameter estimate to segregate the components of the auxiliary terms introduced in (14)-(17). Specifically, instead of assuming that $\hat{\theta}(t) \in \mathcal{L}_\infty$, the subsequent development is based on the less restrictive assumption that the parameter estimate $\hat{\theta}(t)$ can be described as

$$\hat{\theta}(t) = f_1(t) + \Phi(q, \dot{q}, e_1, e_2, t). \tag{18}$$

In (18), $f_1(t) \in \mathbb{R}^p$ is a known function such that

$$\|f_1(t)\| \leq \gamma_1 \tag{19}$$

$$\left\|\dot{f}_1(t)\right\| \leq \gamma_2 + \gamma_3\|e_1\| + \gamma_4\|e_2\| + \gamma_5\|r\|$$

where $\gamma_i \in \mathbb{R}$, $(i = 1, 2, ..., 5)$ are known non-negative constants (i.e., the constants can be set to zero for different update laws), and $\Phi(q, \dot{q}, e_1, e_2, t) \in \mathbb{R}^p$ is a known function that satisfies the following bound:

$$\|\Phi(t)\| \leq \rho_1(\left\|\begin{bmatrix} e_1^T & e_2^T \end{bmatrix}^T\right\|)\left\|\begin{bmatrix} e_1^T & e_2^T \end{bmatrix}^T\right\| \tag{20}$$

where the bounding function $\rho_1(\cdot) \in \mathbb{R}$ is a positive, globally invertible, nondecreasing function. The estimate in (18) is assumed to be generated according to an update law of the following general form

$$\dot{\hat{\theta}}(t) = g_1(t) + \Omega(q, \dot{q}, e_1, e_2, r, t). \tag{21}$$

In (21), $g_1(t) \in \mathbb{R}^p$ is a known function such that

$$\|g_1(t)\| \leq \delta_1 \tag{22}$$

$$\|\dot{g}_1(t)\| \leq \delta_2 + \delta_3\|e_1\| + \delta_4\|e_2\| + \delta_5\|r\|$$

where $\delta_i \in \mathbb{R}$, $(i = 1, 2, ..., 5)$ are known non-negative constants, and $\Omega(q, \dot{q}, e_1, e_2, r, t) \in \mathbb{R}^p$ satisfies the following bound:

$$\|\Omega(t)\| \leq \rho_2(\|z\|)\|z\| \tag{23}$$

where the bounding function $\rho_2(\cdot) \in \mathbb{R}$ is a positive, globally invertible, nondecreasing function, and $z(t) \in \mathbb{R}^{3n}$ is defined as

$$z(t) \triangleq \begin{bmatrix} e_1^T & e_2^T & r^T \end{bmatrix}^T. \tag{24}$$

Remark 1: The update law in (21) depends on the unmeasurable signal r. But it is assumed that the update law in (21) is of the form which upon integration yields an estimate $\hat{\theta}(t)$ that is independent of r. Thus the controller needs only the measurable signals for implementation.

The structure of the adaptive estimate and the adaptive update law is flexible in the sense that any of the terms in (18) and (21) can be removed for any specific update law and estimate. For example if all the error-dependent terms in (18) are removed, then the condition on $\hat{\theta}(t)$ is the same as in the standard nonlinear damping-based modular adaptive methods (i.e., $\hat{\theta}(t) \in \mathcal{L}_\infty$). In this sense, the ISS property with respect to the parameter estimation error is automatically proven by considering this special case of $\hat{\theta}(t)$. The results in this paper are not proven for estimates or update laws with additional terms that are not included in the generic structure in (18) and (21). For example, a standard gradient-based update law is of the form (21), but the corresponding estimate (obtained via integration by parts) is not of the form (18) due to the presence of some terms that are bounded by the integral of the error instead of being bounded by the error. However, the same gradient-based update law and its corresponding estimate can be used in (9) if a smooth projection algorithm is used that keeps the estimates bounded. As shown in [13], and [25], the standard gradient-based update law can be used in (9) without a projection algorithm, yet including this structure in the modular adaptive analysis is problematic because the integral of the error could be unbounded (so this update law could not be used in nonlinear damping based-modular adaptive laws without a projection either). Since the goal in this paper is to develop a modular update law, a specific update law cannot be used to inject terms in the stability analysis to cancel the terms containing the parameter mismatch error. Instead, the terms containing the parameter mismatch error are segregated depending on whether they are state-dependent or bounded by constant (see (17)).

Based on the development given in (18)-(22), the terms $\tilde{N}_0(t)$ and $N_{B_2}(t)$ introduced in (14)-(17) are defined as

$$\tilde{N}_0(t) \triangleq -\dot{Y}_d\Phi - Y_d\Omega \tag{25}$$

$$N_{B_2}(t) \triangleq -\dot{Y}_d f_1 - Y_d g_1. \tag{26}$$

In a similar manner as in [14], the Mean Value Theorem can be used along with the inequalities in (20) and (23) to develop the following upper bound for the expression in (14):

$$\left\|\tilde{N}(t)\right\| \leq \rho(\|z\|)\|z\| \tag{27}$$

where the bounding function $\rho(\cdot) \in \mathbb{R}$ is a positive, globally invertible, nondecreasing function. The following inequalities can be developed based on the expressions in (15), (16), (26), their time derivatives, and the inequalities in (19) and (22):

$$\|N_B(t)\| \leq \zeta_1 \qquad \left\|\dot{N}_{B_1}(t)\right\| \leq \zeta_2 \tag{28}$$
$$\left\|\dot{N}_{B_2}(t)\right\| \leq \zeta_3 + \zeta_4\|e_1\| + \zeta_5\|e_2\| + \zeta_6\|r\|$$

where $\zeta_i \in \mathbb{R}$, $(i = 1, 2, ..., 6)$ are known positive constants.

VI. Stability Analysis

Theorem: The controller given in (9), (18) and (21) ensures that all system signals are bounded under closed-loop operation and that the position tracking error is regulated in the sense that

$$\|e_1(t)\| \to 0 \qquad \text{as} \qquad t \to \infty$$

provided the control gain k_s introduced in (10) is selected sufficiently large (see the subsequent proof), α_1 and α_2 are selected according to the following sufficient conditions:

$$\alpha_1 > \frac{\beta_2}{4} + \frac{1}{2} \tag{29}$$
$$\alpha_2 > \frac{\beta_2}{2} + \beta_3 + \frac{\beta_4}{2} + 1$$

and β_i $(i = 1, 2, 3, 4)$ are selected according to the following sufficient conditions:

$$\beta_1 > \zeta_1 + \frac{1}{\alpha_2}\zeta_2 + \frac{1}{\alpha_2}\zeta_3 \tag{30}$$
$$\beta_2 > \zeta_4 \qquad \beta_3 > \zeta_5 \qquad \beta_4 > \zeta_6$$

where β_1 was introduced in (10), and β_2-β_4 are introduced in (33).

Proof: Let $\mathcal{D} \subset \mathbb{R}^{3n+1}$ be a domain containing $y(t) = 0$, where $y(t) \in \mathbb{R}^{3n+1}$ is defined as

$$y(t) \triangleq [z^T(t) \quad \sqrt{P(t)}]^T. \tag{31}$$

In (31), the auxiliary function $P(t) \in \mathbb{R}$ is defined as

$$P(t) \triangleq \beta_1\|e_2(0)\| - e_2(0)^T N_B(0) - \int_0^t L(\tau)d\tau \tag{32}$$

where the auxiliary function $L(t) \in \mathbb{R}$ is defined as

$$\begin{aligned} L(t) \triangleq\ & r^T(N_B(t) - \beta_1 sgn(e_2)) \\ & -\beta_2\|e_1(t)\|\|e_2(t)\| - \beta_3\|e_2(t)\|^2 \\ & -\beta_4\|e_2(t)\|\|r(t)\| \end{aligned} \tag{33}$$

where $\beta_i \in \mathbb{R}$ $(i = 1, 2, 3, 4)$ are positive constants chosen according to the sufficient conditions in (30). Provided the sufficient conditions introduced in (30) are satisfied, the following inequality can be obtained in a similar manner as in [25]:

$$\int_0^t L(\tau)d\tau \leq \beta_1\|e_2(0)\| - e_2(0)^T N_B(0). \tag{34}$$

Hence, (34) can be used to conclude that $P(t) \geq 0$.

Let $V_L(y,t) : \mathcal{D} \times [0,\infty) \to \mathbb{R}$ be a continuously differentiable positive definite function defined as

$$V_L(y,t) \triangleq e_1^T e_1 + \frac{1}{2} e_2^T e_2 + \frac{1}{2} r^T M(q) r + P, \qquad (35)$$

which satisfies the following inequalities:

$$U_1(y) \leq V_L(y,t) \leq U_2(y) \qquad (36)$$

provided the sufficient conditions introduced in (29)-(30) are satisfied. In (36), the continuous positive definite functions $U_1(y)$, and $U_2(y) \in \mathbb{R}$ are defined as $U_1(y) \triangleq \lambda_1 \|y\|^2$, and $U_2(y) \triangleq \lambda_2(q) \|y\|^2$, where λ_1, $\lambda_2(q) \in \mathbb{R}$ are defined as

$$\lambda_1 \triangleq \frac{1}{2} \min\{1, m_1\} \qquad \lambda_2(q) \triangleq \max\left\{\frac{1}{2}\bar{m}(q), 1\right\}$$

where m_1, $\bar{m}(q)$ are introduced in (2). After taking the time derivative of (35), $\dot{V}(y,t)$ can be expressed as

$$\begin{aligned} \dot{V}_L(y,t) &= r^T M(q)\dot{r} + \frac{1}{2} r^T \dot{M}(q) r \\ &\quad + e_2^T \dot{e}_2 + 2 e_1^T \dot{e}_1 + \dot{P}. \end{aligned}$$

The derivative $\dot{P}(t) \in \mathbb{R}$ can be expressed as

$$\begin{aligned} \dot{P}(t) &= -L(t) = -r^T (N_B - \beta_1 sgn(e_2)) \\ &\quad + \beta_2 \|e_1(t)\| \|e_2(t)\| + \beta_3 \|e_2(t)\|^2 \\ &\quad + \beta_4 \|e_2(t)\| \|r(t)\|. \end{aligned} \qquad (37)$$

After utilizing (5), (6), (12), (13), (21), and (37), $\dot{V}(y,t)$ can be simplified as follows:

$$\begin{aligned} \dot{V}_L(y,t) &= r^T \tilde{N} - (k_s + 1) \|r\|^2 - \alpha_2 \|e_2\|^2 \\ &\quad - 2\alpha_1 \|e_1\|^2 + 2 e_2^T e_1 + \beta_2 \|e_1\| \|e_2\| \\ &\quad + \beta_3 \|e_2\|^2 + \beta_4 \|e_2\| \|r\|. \end{aligned} \qquad (38)$$

Based on the fact that

$$2 e_2^T e_1 \leq \|e_1\|^2 + \|e_2\|^2$$

$\dot{V}_L(y,t)$ can be upper bounded using the squares of the components of $z(t)$ as follows:

$$\begin{aligned} \dot{V}_L(y,t) &\leq r^T \tilde{N} - (k_s + 1) \|r\|^2 - \alpha_2 \|e_2\|^2 - 2\alpha_1 \|e_1\|^2 \\ &\quad + \|e_1\|^2 + \|e_2\|^2 + \frac{\beta_2}{2} \|e_1\|^2 + \frac{\beta_2}{2} \|e_2\|^2 \\ &\quad + \beta_3 \|e_2\|^2 + \frac{\beta_4}{2} \|e_2\|^2 + \frac{\beta_4}{2} \|r\|^2. \end{aligned} \qquad (39)$$

By using (27), the expression in (39) can be rewritten as follows:

$$\dot{V}_L(y,t) \leq -\lambda_3 \|z\|^2 - \left[\left(k_s - \frac{\beta_4}{2}\right) \|r\|^2 - \rho(\|z\|) \|r\| \|z\|\right] \qquad (40)$$

where $\lambda_3 \triangleq \min\{2\alpha_1 - \frac{\beta_2}{2} - 1, \alpha_2 - \frac{\beta_2}{2} - \beta_3 - \frac{\beta_4}{2} - 1, 1\}$; hence, α_1, and α_2 must be chosen according to the sufficient condition in (29). After completing the squares for the terms inside the brackets in (40), the following expression can be obtained:

$$\dot{V}_L(y,t) \leq -\lambda_3 \|z\|^2 + \frac{\rho^2(\|z\|) \|z\|^2}{4\left(k_s - \frac{\beta_4}{2}\right)} \leq -U(y) \qquad (41)$$

where $U(y) = c \|z\|^2$, for some positive constant c, is a continuous, positive semi-definite function that is defined on the following domain:

$$\mathcal{D} \triangleq \left\{ y \in \mathbb{R}^{3n+1} \mid \|y\| \leq \rho^{-1}\left(2\sqrt{\lambda_3\left(k_s - \frac{\beta_4}{2}\right)}\right)\right\}.$$

The inequalities in (36) and (41) can be used to show that $V_L(y,t) \in \mathcal{L}_\infty$ in $\mathcal{D}$; hence, $e_1(t)$, $e_2(t)$, and $r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Given that $e_1(t)$, $e_2(t)$, and $r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, standard linear analysis methods can be used to prove that $\dot{e}_1(t)$, $\dot{e}_2(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$ from (5) and (6). Since $e_1(t)$, $e_2(t)$, $r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, the assumption that $q_d(t)$, $\dot{q}_d(t)$, $\ddot{q}_d(t)$ exist and are bounded can be used along with (4)-(6) to conclude that $q(t)$, $\dot{q}(t)$, $\ddot{q}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Since $q(t)$, $\dot{q}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, (18)-(22) can be used to prove that $\hat{\theta}(t), \dot{\hat{\theta}}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Since $q(t)$, $\dot{q}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, Property 2 can be used to conclude that $M(q)$, $V_m(q,\dot{q})$, $G(q)$, and $F(\dot{q}) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Thus from (1) and Property 3, we can show that $\tau(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Given that $r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, (13) can be used to show that $\dot{\mu}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Since $\dot{q}(t)$, $\ddot{q}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, Property 2 can be used to show that $\dot{V}_m(q,\dot{q})$, $\dot{G}(q)$, $\dot{F}(q)$ and $\dot{M}(q) \in \mathcal{L}_\infty$ in $\mathcal{D}$; hence, (12) can be used to show that $\dot{r}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Since $\dot{e}_1(t)$, $\dot{e}_2(t)$, $\dot{r}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, the definitions for $U(y)$ and $z(t)$ can be used to prove that $U(y)$ is uniformly continuous in $\mathcal{D}$.

Let $\mathcal{S} \subset \mathcal{D}$ denote a set defined as follows:

$$\begin{aligned} \mathcal{S} &\triangleq \{y(t) \subset \mathcal{D} \mid U_2(y(t)) \\ &< \lambda_1 \left(\rho^{-1}\left(2\sqrt{\lambda_3\left(k_s - \frac{\beta_4}{2}\right)}\right)\right)^2\}. \end{aligned} \qquad (42)$$

The region of attraction in (42) can be made arbitrarily large to include any initial conditions by increasing the control gain k_s (i.e., a semi-global type of stability result) [14]. Theorem 8.4 of [30] can now be invoked to state that

$$c \|z(t)\|^2 \to 0 \quad \text{as} \quad t \to \infty \quad \forall y(0) \in \mathcal{S}. \qquad (43)$$

Based on the definition of $z(t)$, (43) can be used to show that

$$\|e_1(t)\| \to 0 \quad \text{as} \quad t \to \infty \quad \forall y(0) \in \mathcal{S}. \qquad (44)$$

VII. Conclusion

A RISE-based approach was presented to achieve modularity in the controller/update law for Euler-Lagrange systems. Specifically, for systems with structured and unstructured uncertainties, a controller was employed that uses a model-based feedforward adaptive term in conjunction with the RISE feedback term (see [13], [25]). The adaptive feedforward term was made modular by considering a generic form of the adaptive update law and its corresponding parameter estimate. This generic form of the update law was used to develop a new closed-loop error system, and the typical RISE stability analysis was modified. New sufficient gain conditions were derived to show asymptotic tracking of the desired link position.

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