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Guaranteed Cost Control for Discrete-Time Uncertain Systems with Saturating Actuators

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Abstract— **This paper considers the robust guaranteed cost control problem for uncertain linear discrete-time systems subject to actuator saturation. By using the linear matrix inequality (LMI) approach, we obtain sufficient conditions for the existence of a state feedback control law, and the problem of designing the optimal guaranteed cost controller is converted to a convex optimization problem with LMI constraints. Numerical examples are given to illustrate the effectiveness of the proposed results.**

I. INTRODUCTION

The problem of designing robust controllers for linear systems with parameter uncertainty has drawn considerable attention in recent control system literature. Much work has been done in order to find a controller which guarantees the robust stability (see, for example [1], [9] and reference therein). However, it is also desirable to design a controller which guarantees not only robust stability but also an adequate level of performance as well. One approach to this problem called guaranteed cost control was firstly presented in [4]. This approach provides an upper bound on a given performance index to all allowed uncertainties. Based on this work, many important results have been proposed in the past decades. In particular, [12] presented finite-horizon and infinite-horizon guaranteed cost controllers for uncertain discrete-time systems with a quadratic performance using difference Riccati equation approach. In [10] and [11], Petersen and Mcfarlane introduced an algebraic Riccati equation approach for designing an optimal guaranteed cost control for continuous-time and discrete-time with a quadratic performance. Although the Riccati equation is a celebrated and powerful tool, it has some drawbacks in solving control problems with some constraints. Moreover, it has recently been emphasized that many problems arising in system and control theory can be converted to the form of linear matrix inequalities (LMI), which belong to the group of convex problems, and thus one can not only find feasible and optimal solutions to them efficiently, but also handle various kinds of additional linear constraints easily (see [2]). For example, Yu presented a linear matrix inequality approach for the design of guaranteed cost controller for linear uncertain systems, and converted the design problem of the optimal guaranteed cost controller to a convex optimization problem with LMI constraints (see [13], [14]).

Another common, but difficult, control problem is to deal with actuator saturation since all control devices are subject to saturation (limited in force, torque, current, flow rate, etc.). This non-linearity causes control systems have to operate under constraints on the magnitude of the control input. These limitations in terms of input constraints must be considered in the controller design. Up to now, the analysis and synthesis of controllers for dynamic systems subject to actuator saturation have been attracting increasingly more attention (see, for example, [3], [7], [6], [5]). And there exist some effective tools to deal with it. However, both actuator saturation and model uncertainty are often encountered in control systems. To deal with these two problems effectively, appropriate design methods are required.

As far as we know, however, little research has been focused on the guaranteed cost control problem for uncertain discrete-time systems subject to actuator saturation. Recently, [15] presented a method to designing the guaranteed cost controller for such systems. However, some inequalities are involved to make the control input do not exceed the saturating level, which may bring considerable conservativeness to minimize the guaranteed cost index. Motivated by the method of [8], we will first transform the saturation non-linearity into a convex polytope of linear systems, and then formulate this problem into a convex optimization problem with constraints given by a set of linear matrix inequalities.

This paper, divided into 5 sections, begins by formulating the problem and giving some preliminary results in Section 2. We will present our main results in Section 3 and two examples are proposed to illustrate the design procedure and its effectiveness in Section 4. The paper is concluded in Section 5.

Notation: The following notations will be used throughout the paper. R denotes the set of real numbers, R^+ denotes the set of non-negative real numbers, R^n denotes the n dimensional Euclidean space and $R^{m\times n}$ denotes all $m \times n$ real matrices. The notation $P \geq Q$ (respectively, $P > Q$), where P and Q are symmetric matrices, means that the matrix $P-Q$ is positive semi-definite (respectively, positive definite). I and 0 denote the identity matrix and zero matrix with compatible dimensions.

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II. Problem Statement and Preliminaries

A. Problem Statement

Let us consider a discrete-time system with uncertainty and actuator saturation.

$$\begin{aligned} x(k+1) &= (A+\Delta A)x(k) + (B+\Delta B)u(k) \\ x(0) &= x_0 \end{aligned} \quad (1)$$

where $x(k) \in R^n$ is the state vector, $u(k) \in R^m$ is the control input vector, A and B are known constant real matrices of appropriate dimensions. The uncertainties are assumed to be of the form

$$[\Delta A \quad \Delta B] = DF(k)[E_1 \quad E_2]$$

where D, E_1, E_2 are constant matrices with compatible dimensions, which represent the structure of uncertainties, and $F(k)$ is an unknown, real, and time-varying matrix with Lebesgue measurable elements satisfying

$$F(k)^T F(k) \leq I$$

The control input u in system (1) is subjected to the following constraints:

$$-\bar{u}_i \leq u_i \leq \bar{u}_i$$

where u_i is the i-th element in the control input u, $\bar{u}_i, i = 1, 2, \cdots, m$ are saturating magnitude. Thus $u(t)$ can be described by $diag\{\bar{u}_i\}\sigma(u)$ without loss of generality. The function $\sigma : R^m \to R^m$, is the standard saturation function defined as follows:

$$\sigma(u) = [\sigma(u_1)\ \sigma(u_2)\ \cdots\ \sigma(u_m)]^T$$

$$\sigma(u_i) = sign(u_i)\min\{1, |u_i|\} \qquad i = 1, 2, \cdots, m$$

For system (1), one performance index we usually used in discrete-time system is the cost function

$$J = \sum_{k=0}^{\infty}[x^T(k)Qx(k) + u^T(k)Rx(k)] \quad (2)$$

where Q and R are given positive-definite symmetric matrices.

The objective of this paper is to develop a procedure to designing the optimal guaranteed cost controller for system (1) which satisfies the performance index (2) subject to actuator saturation.

B. Some Mathematical Tools

Let k_i be the i-th row of the matrix K. We define the symmetric polyhedron,

$$L(K) = \{x \in R^n : |k_i x| \leq 1, i = 1, 2, \cdots, m\}$$

If K is the feedback matrix, then $L(K)$ is the region in the state space where the control is linear in x.

Lemma 1: [15] A state feedback control law $u = Kx$ is said to be a quadratically guaranteed cost controller of system (1) with cost function (2) where there is no actuator saturation in system (1), if there exists a symmetric positive definite matrix $P \in R^{n\times n}$ such that

$$\begin{aligned} &[A+BK+DF(E_1+E_2K)]^T P[A+BK \\ &\quad +DF(E_1+E_2K)] - P + Q + K^T RK < 0 \end{aligned} \quad (3)$$

for all admissible uncertainties.

Lemma 2: [15] If $u = Kx$ is a quadratically guaranteed cost controller of system (1) with cost function (2), where there is no actuator saturation in system (1), then the closed-loop uncertain system

$$x(k+1) = [A+BK+DF(E_1+E_2K)]x(k) \quad (4)$$

is quadratically stable, and the cost function value of the closed-loop system is no more than $J^* = x_0^T P x_0$, which is said to be a guaranteed cost of system (1).

Lemma 3: [8] Let ν be set of $m \times m$ diagonal matrices whose diagonal elements are either 1 or 0. Then there are 2^m elements in ν. Suppose that each element of ν is labeled as $D_i,\ i = 1, 2, \ldots 2^m$ and denote $D_i^- = I - D_i$. Clearly, D_i^- is also an element of ν if $D_i \in \nu$.

Let $K, H \in R^{m\times n}$ be given. For $x(t) \in R^n$, if $\|Hx\|_\infty \leq 1$, then

$$\sigma(Kx) \in co\{D_i Kx + D_i^- Hx : i \in [1, 2, \cdots, 2^m]\}$$

where $co\{\cdot\}$ denotes the convex hull of a set.

Lemma 4: [12] Given matrices $Y = Y^T$, D, E and $R = R^T > 0$ of appropriate dimensions

$$Y + DFE + E^T F^T D^T < 0$$

for all F satisfying $F^T F \leq R$, if and only if there exists some $\varepsilon > 0$ such that

$$Y + \varepsilon DD^T + \varepsilon^{-1} E^T RE < 0$$

.

III. Main Results

In this section, we will give a method for estimating the optimal performance for system (1) with actuator saturation. In order to illustrate the process of designing clearly, we will first give a design approach of control law for system (5) where there is no uncertainty in the state matrix and the input matrix.

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ x(0) &= x_0 \end{aligned} \quad (5)$$

The control input u is also subjected to the constraints: $-\bar{u}_i \leq u_i \leq \bar{u}_i$. We aim to design a state feedback control law $u(t) = Kx(t)$ which make system (5) not only stable but satisfy the performance index (2). Because the system has saturation non-linearity, using Lemma 3, system (5) can be rewritten as follows,

$$\begin{aligned} x(k+1) &= [A + Bdiag\{\bar{u}_i\}(D_i K + D_i^- H)]x(k) \\ x(0) &= x_0 \end{aligned} \quad (6)$$

if $||Hx||_\infty \leq 1$. Similar to Lemma 1, for system (6) if there exist symmetric positive definite matrix $P \in R^{n\times n}$ such that

$$\begin{aligned} &[A + Bdiag\{\bar{u}_i\}(D_i K + D_i^- H)]^T P[A + B \\ &diag\{\bar{u}_i\}(D_i K + D_i^- H)] - P + Q + K^T RK < 0 \end{aligned} \quad (7)$$

then the state feedback control law $u = Kx$ is a quadratically guaranteed cost controller of system (6) with cost function (2). And the cost function value of the closed-loop system is no more than $J^* = x_0^T P x_0$.

We first present the following result.

Theorem 1: For a symmetric positive definite matrix $P \in R^{n\times n}$ and a scalar $\rho \in R^+$, consider the ellipsoid $\mathcal{E}(P,\rho) := \{x \in R^n : \ x^T P x \le \rho\}$. If there exist a symmetric positive definite matrix $G \in R^{n\times n}$ and $V, W \in R^{m\times n}$ such that

$$\begin{bmatrix} -G & G & W^T & X^T \\ G & -\rho Q^{-1} & 0 & 0 \\ W & 0 & -\rho R^{-1} & 0 \\ X & 0 & 0 & -G \end{bmatrix} < 0 \qquad (8)$$

$$\begin{bmatrix} 1 & v_i \\ v_i^T & G \end{bmatrix} \ge 0 \qquad (9)$$

$$\begin{bmatrix} 1 & x_0^T \\ x_0 & G \end{bmatrix} \ge 0 \qquad (10)$$

where

$$X = AG + Bdiag\{\bar{u}_i\}(D_i W + D_i^- V)$$

and v_i is the i-th row of matrix V. Then

$$u(k) = WG^{-1}x(k)$$

is a guaranteed cost control law of system (6) satisfies performance index (2) and $J \le \rho$. Furthermore, $\mathcal{E}(P,\rho)$ is a positive invariant set of system (6).

Proof: Multiplying (7) by $\rho^{\frac{1}{2}} P^{-1}$ on the left and on the right, respectively, we get

$$\begin{aligned} &\rho P^{-1}[A + Bdiag\{\bar{u}_i\}(D_i K + D_i^- H)]^T \\ &\quad \times P[A + Bdiag\{\bar{u}_i\}(D_i K + D_i^- H)]P^{-1} \\ &\quad -\rho P^{-1} + \rho P^{-1} Q P^{-1} + \rho P^{-1} K^T R K P^{-1} < 0 \end{aligned}$$

Let $G = \rho P^{-1}$, $W = \rho K P^{-1}$, $V = \rho H P^{-1}$, and use Schur complement, the above inequality changes into the following form

$$\begin{bmatrix} -G + \rho^{-1} G Q G + \rho^{-1} W^T R W & X^T \\ X & -G \end{bmatrix} < 0 \qquad (11)$$

Applying Schur complement, inequality (11) is equivalent to matrix (8). And saturation non-linearity can be substituted by a convex polytope if

$$\mathcal{E}(P,\rho) \subset L(H)$$

which is equivalent to

$$\rho h_i P^{-1} h_i^T \le 1$$

where h_i is the i-th row of matrix H. Utilizing Schur complement, we get

$$\begin{bmatrix} 1 & h_i(\frac{P}{\rho})^{-1} \\ (\frac{P}{\rho})^{-1} h_i^T & (\frac{P}{\rho})^{-1} \end{bmatrix} \ge 0 \qquad (12)$$

which can be rewritten as (9) by taking $V = \rho H P^{-1}$. From the above discussion, we can see that $u = Kx$ is a quadratically guaranteed cost controller for system (6). Therefore, inequality (10) implies that the closed-loop state trajectory $x(t)$ satisfies $x^T(t) P x(t) \le \rho$. ■

Now we can discuss system (1) using the similar way as in Theorem 1 and derive a condition of designing the optimal guaranteed cost control law in terms of linear matrix inequalities. With the state feedback control $u(k) = Kx(k)$, we can also rewrite system (1) by applying Lemma 3 and convert it to

$$\begin{aligned} x(k+1) &= [A + \Delta A + (B + \Delta B)diag\{\bar{u}_i\} \\ &\qquad \times (D_i K + D_i^- H)]x(k) \qquad (13) \\ x(0) &= x_0 \end{aligned}$$

if $||Hx||_\infty \le 1$, where $[\Delta A \ \ \Delta B] = DF[E_1 \ \ E_2]$. If there exist a symmetric positive define matrix $P \in R^{n\times n}$ such that

$$Q + K^T R K - P + \Psi^T P \Psi < 0 \qquad (14)$$

$$\begin{aligned} \Psi = [&A + Bdiag\{\bar{u}_i\}(D_i K + D_i^- H) \\ &+ DF(E_1 + E_2 diag\{\bar{u}_i\}(D_i K + D_i^- H))] \end{aligned}$$

then the state feedback control law $u = Kx$ is a quadratically guaranteed cost controller of system (13) with the cost function (2). Furthermore, the cost function value of the closed-loop system is no more than $J^* = x_0^T P x_0$.

Theorem 2: For a symmetric positive definite matrix $P \in R^{n\times n}$ and a scalar $\rho \in R^+$, consider the set $\mathcal{E}(P,\rho) := \{x \in R^n : \ x^T P x \le \rho\}$. If there exist a scalar $\varepsilon > 0$, a symmetric positive definite matrix $G \in R^{n\times n}$, and $V, W \in R^{m\times n}$ satisfy

$$\begin{bmatrix} -G & X^T & E^T & G & W^T \\ X & -G + \varepsilon D D^T & 0 & 0 & 0 \\ E & 0 & -\varepsilon I & 0 & 0 \\ G & 0 & 0 & -\rho Q^{-1} & 0 \\ W & 0 & 0 & 0 & -\rho R^{-1} \end{bmatrix} < 0 \qquad (15)$$

$$\begin{bmatrix} 1 & v_i \\ v_i^T & G \end{bmatrix} \ge 0 \qquad (16)$$

$$\begin{bmatrix} 1 & x_0^T \\ x_0 & G \end{bmatrix} \ge 0 \qquad (17)$$

where

$$X = AG + Bdiag\{\bar{u}_i\}(D_i W + D_i^- V)$$

$$E = E_1 G + E_2 diag\{\bar{u}_i\}(D_i W + D_i^- V)$$

and v_i is the i-th row of the matrix V. Then $u(t) = WG^{-1}x(t)$ is a guaranteed cost control law of system (1) satisfies performance index (2) and $J \le \rho$. Furthermore, $\mathcal{E}(P,\rho)$ is a positive invariant set of system (1).

Proof: Multiplying (14) by $\rho^{\frac{1}{2}} P^{-1}$ on the left and on the right, respectively, and let $G = \rho P^{-1}$, $W = \rho K P^{-1}$,

$V = \rho H P^{-1}$. Using Schur complement, we get the following matrix inequality

$$S + \begin{bmatrix} 0 \\ D \end{bmatrix} F \begin{bmatrix} E & 0 \end{bmatrix} + \begin{bmatrix} E & 0 \end{bmatrix}^T F^T \begin{bmatrix} 0 & D^T \end{bmatrix} < 0 \tag{18}$$

where

$$S = \begin{bmatrix} -G + \rho^{-1} GQG + \rho^{-1} W^T RW & X^T \\ X & -G \end{bmatrix}$$

Applying Lemma 4, the above inequality holds for all F satisfying $F^T F \leq I$, if and only if there exist some $\varepsilon > 0$ such that

$$S + \varepsilon \begin{bmatrix} 0 \\ D \end{bmatrix} \begin{bmatrix} 0 & D^T \end{bmatrix} + \varepsilon^{-1} \begin{bmatrix} E^T \\ 0 \end{bmatrix} \begin{bmatrix} E & 0 \end{bmatrix} < 0$$

Using Schur complement, it follows that.

$$\begin{bmatrix} S + \varepsilon \begin{bmatrix} 0 \\ D \end{bmatrix} \begin{bmatrix} 0 & D^T \end{bmatrix} & \begin{bmatrix} E^T \\ 0 \end{bmatrix} \\ \begin{bmatrix} E & 0 \end{bmatrix} & -\varepsilon I \end{bmatrix} < 0$$

Substituting S, E into its original form and applying Schur complement, we finally get inequality (15). The remaining proof is similar to that of Theorem 1, thus omitted. This completes the proof of Theorem 2. ■

Theorem 1 and Theorem 2 give some conditions for the existence of the guaranteed cost controller with the guaranteed cost index $J \leq \rho$. Now we would like to choose from all the $\mathcal{E}(P, \rho)$ that satisfy these conditions such that the guaranteed cost index is minimized. This problem can be formulated as

$$\begin{array}{lll} min\ \rho & s.t. & (8) - (10) \\ & or & (15) - (17) \end{array} \tag{19}$$

If the above optimization problem has an optimal solution $\hat{\rho}, \hat{G}, \hat{W}, \hat{V}$, then $u(k) = WG^{-1}x(k)$ is the optimal guaranteed cost control law of system (5) or (1), which satisfies performance index (2) and $J \leq \hat{\rho}$.

It is clear that (19) is a convex optimization problem with LMI constraints. Therefore, the global minimum of the problem can be reached if it is feasible, and it can be easily solved by using the solver mincx in the LMI Toolbox of MATLAB.

IV. Numerical Examples

In this section, two examples are used to demonstrate that the method presented in this paper is effective and is an improvement over the existing methods.

Example 1: First we consider a discrete-time system without uncertainty.

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ x(0) &= x_0 \end{aligned}$$

with

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1.5 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix} \quad x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$-3 \leq u_i \leq 3,\ i = 1$$

$$Q = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad R = 1$$

Using the method of [15], we obtain the optimal guaranteed cost of the closed-loop system is $J^* = 10.5822$. While by applying Theorem 1 and solving the corresponding optimization problem, we get the optimal guaranteed cost of the closed-loop system is $J^* = 3.9252$. It is obvious that our method gives a lower bound of the guaranteed cost than [15].

Example 2: Now consider an uncertain discrete-time system.

$$\begin{aligned} x(k+1) &= (A + \Delta A)x(k) + (B + \Delta B)u(k) \\ x(0) &= x_0 \end{aligned}$$

with

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1.5 \end{bmatrix}$$

$$B = \begin{bmatrix} 0.3 \\ 0.5 \end{bmatrix} \quad x_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$$

$$E_1 = \begin{bmatrix} 0.2 & 0.3 \\ 0.1 & 0.4 \end{bmatrix} \quad E_2 = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}$$

$$-3 \leq u_i \leq 3,\ i = 1$$

The associated performance index of the system is

$$J = \sum_{k=0}^{\infty} [x^T(k)Qx(k) + u^T(k)Rx(k)]$$

where $Q = diag\{0.5, 0.5\}$, and $R = 1$

Using the method of [15], we obtain the optimal guaranteed cost of the uncertain closed-loop system is $J^* = 12.8988$ and the corresponding control law is

$$K = \begin{bmatrix} -0.4260 & 1.7915 \end{bmatrix}$$

and

$$P = \begin{bmatrix} 3.0716 & 1.0980 \\ 1.0980 & 7.6200 \end{bmatrix}$$

By applying Theorem 2 and solving the corresponding optimization problem, we get the optimal guaranteed cost of the uncertain closed-loop system is $J^* = 4.8066$ and the corresponding control law is

$$K = \begin{bmatrix} -0.2090 & 0.5110 \end{bmatrix}$$

and

$$P = \begin{bmatrix} 1.5261 & 0.5679 \\ 0.5679 & 2.1438 \end{bmatrix}$$

To compare the effect of our method and the method in reference [15] by simulation, we assume that $F = \sin k$. The control law is shown in Fig.1. The state variables of corresponding closed-loop systems are shown in Fig.2. The invariant set is shown in Fig.3. It is obvious with lower control effort that our method brings better performance than that of [15].

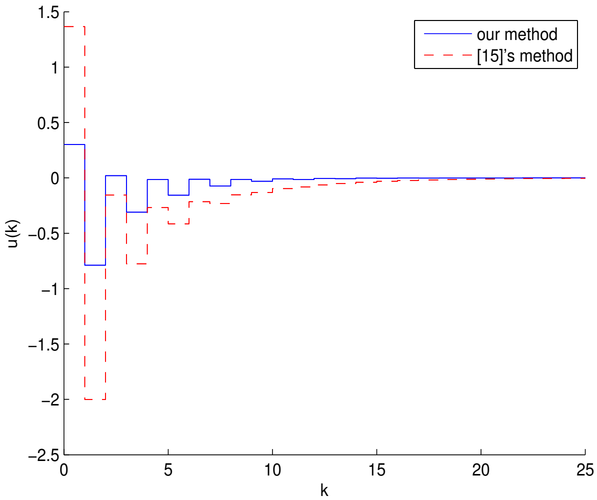

Fig. 1. Control law.

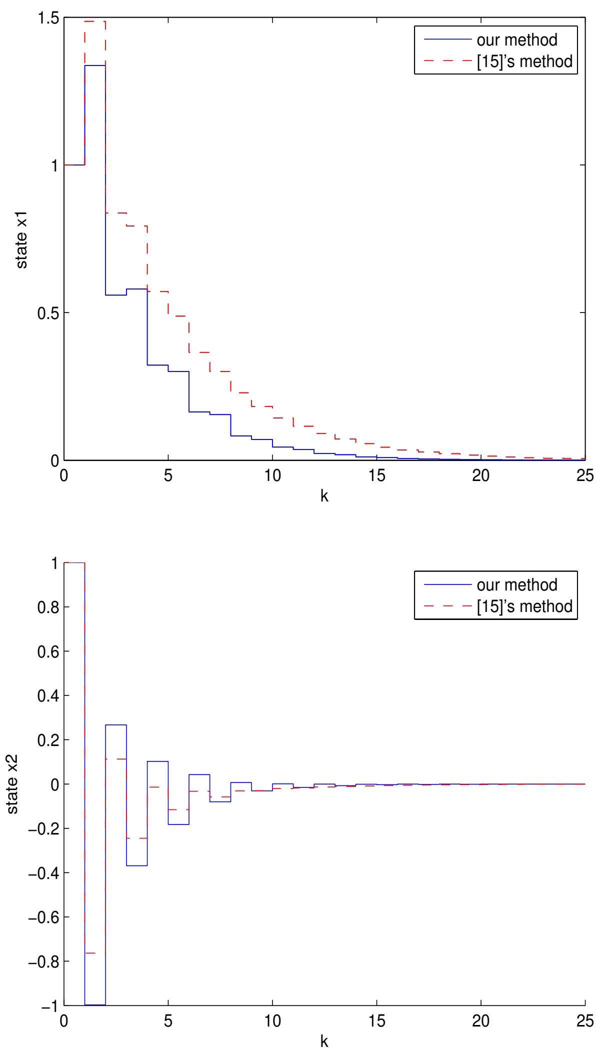

Fig. 2. The closed-loop state variables.

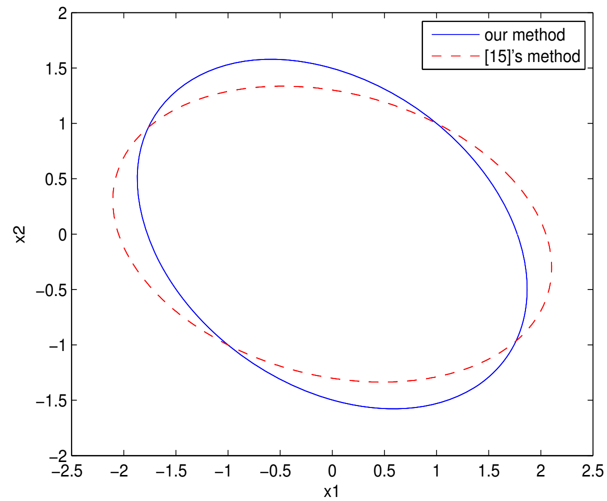

Fig. 3. The invariant set.

V. Conclusion

In this paper, we have presented an LMI based approach to the optimal guaranteed cost control problem via state feedback control laws for a class of uncertain discrete-time systems subject to actuator saturation. By transforming a system with actuator saturation non-linearities into a convex polytope of linear systems, we obtain better results than the existing ones.

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