2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

A new inventory level APIOBPCS-based controller

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***Abstract*— Modern companies have realized that reducing inventory levels as much as possible without losing sales opportunities is an effective way to reduce costs and to have more profitability. This fact is true not only for large companies but also for mid-size and small companies on account of the high maintenance and opportunity costs associated with large inventory stocks. In this paper we want to introduce into the inventory management field advanced methodologies and tools from de industrial automation and modern control theory. In this way, a new approach to the Automatic Pipeline Feedback Order-Based Production Control System (APIOBPCS) is presented. The proposed control system structure add to the APIOBPCS a PID (Proportional, Integrative and Derivative) controller as well as an Extended Kalman filter, acting as demand predictor. The main objective of this controller is to stabilize and to regulate the inventory level around a desired set-point value, in spite of a demand with cyclic and stochastic components. Along this work, the dynamics and delays of the productive process were modeled as a pure delay. The Kalman filter estimates de parameters of a Volterra time-series model to forecast futures values of the demand in order to compensate production delays. A control error analysis for the proposed controller is also presented in order to obtain bounds for the control error and to probe controller stability. This analysis is also useful to make decisions about the desired inventory level for a given demand prediction error. Finally, the inventory control system is tested by simulations showing a good performance and better results than those achieved by using traditional techniques.**

***Index Terms*— Production systems, inventory level control, prediction, Extended Kalman Filter.**

I. INTRODUCTION

Until recently, production and sales managers used to control inventory levels by means of two powerful but limited tools: intuition and experience. However, the highly competitive market, the changing customer preferences and the complexity of modern production and sales operations, even in small markets, have pushed manager for improving their decision processes. Among others, the decision on how much or when to order plays an important role in modern companies and it is not anymore convenient to regulate stock levels without a quantitative assessment of the involved factors.

Inventories are resources needed for production or commercialization processes, that are kept idle, waiting to be used when necessary. These resources can be of any kind: men, machines, raw material, money, graduates from the educational system, dean management, water tank-based irrigation system, etc. Inventories are used to compensate or regulate the imbalances of the normal sequence of activities in production and sales processes. In other words, inventories should have a stabilizing effect on material flow patterns [1]. An important problem in production planning and sales projections is the demand, which is usually unknown and stochastic in nature. This fact makes the task of keeping inventory on an appropriate and constant level an *impossible mission*. If inventory levels fall below certain values, there exists the risk of losing sales when demand grows beyond the expected figures. On the other hand, if the inventory levels are kept too high, maintenance cost are usually higher due to the larger volume of resources that are kept in stock, the larger space required, and the higher devaluation and maintenance costs. Therefore, an effective supply chain is managed with an aim at keeping a high level of costumer satisfaction while minimizing costs and maximizing profits [2]. Results of savings achieved by best-in-class companies, as a result of improving their supply chain operations, amount 5-6% of sales [3].

Although research in the inventory management area is not novel, it was recently when the control systems comunity have paid attention to this topic and some dynamic inventory control technics have appear. In an excellent revision of Ortega and Lin [4], some major research efforts for applying control theoretic methods to production inventory systems are presented. Some previous research works have also proposed systems to stabilize the inventory level as is the case of John *et al.* [5] and Disney and Towill [1]. More recently, the works of Grubbström and Wikner [6], Samanta and Al-Araimi [7], and Rivera and Pew [2] have explicitly included dynamical controllers, such as PID, on the supply chain, and have obtained promising results.

In this paper a new approach to the APIOBPCS is presented. The proposed structure is a simple dynamical control system whose main objective is to keep the inventory level at a desired set-point, in spite of demand fluctuations includes a PID controller and an estimator of the demand given by a Kalman filter. It is assumed that demand signal is constituted by a cyclic component and a Poisson-like stochastic perturbation. The one-step ahead prediction of the demand is generated by a Dual Joint EKF [8], which identifies the parameters of a Volterra time-series equation. Lead times of the production processes are considered as a pure delay. A control error analysis is also performed for the proposed control system, in order to obtain error bounds

This work was supported by the National Council for Scientific and Technical Research (CONICET), the National Agency for Scientific and Technological Promotion (ANPCyT) under grants PAV-TIC-076 and PICT/04 21592.

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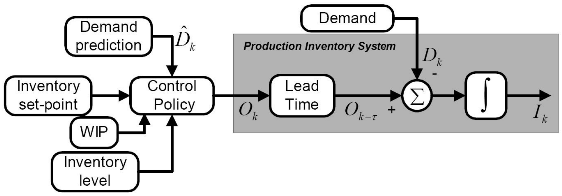

Fig. 1. Basic production-inventory model.

for control error as well as to probe controller stability. This procedure is also useful to determine the desired inventory level for a given prediction error. Finally, simulation results show good performance of the proposed controller and better results than other known techniques.

II. PRODUCTION-INVENTORY SYSTEM MODEL

The dynamics of an inventory system can be represented by a simple difference equation:

$$I(k+1) = I(k) + O(k-\tau) - D(k) \tag{1}$$

where, $I_{(k)}$ is the net inventory level, τ represents the order fulfilment time, $O_{(k-\tau)}$ is the prior orders made τ-days before, and $D_{(k)}$ the demand signal. The order fulfilment time, $O_{(k)}$ is generated by a reorder policy.

Traditionally, reorder policies have been based on Economic Order Quantity (EOQ) approaches, such as the $(\hat{s}, \hat{S})$ policy (when the inventory level becomes equal to or less than $\hat{s}$, order up to the level $\hat{S}$). EOQ approaches are widely used but they are not efficient enough, mainly because they are static laws and do not have into account the demand fluctuations not only as constant signal but also as a temporal one.

On the other hand, APIOBPCS models have shown to perform well, stabilizing the dynamic system and reducing the bullwhip effect. Bullwhip effect refers to the scenario where orders to the suppliers tends to have larger fluctuations than sales to the buyer and this distortion propagates and amplifies itself when going upstream [1], [9]. A basic production-inventory system based on the APIOBPCS scheme has four main components: the *inventory*, that can be modeled as an integrator, the *production process*, that has been modeled in this paper as a finite time delay, the *reorder policy*, and the *demand predictor*. In addition, there are four fundamental information flows [6], namely *demand*, *inventory level*, *work-in-progress* (WIP), and *demand prediction*. Most of the order decision rules are based on one or more of these flows. That is:

$$O(k) = f[I(k), \hat{d}, WIP]. \tag{2}$$

Figure 1 presents a schematic model of the flows and components of a simple production-inventory system.

III. DEMAND ESTIMATION

A key aspect in the inventory management area is the demand estimation. For the sake of simplicity, usually demand is considered constant, or at least known. In real life, demand is the opposite: variable and stochastic. However, due to the probabilistic characteristic of the demand, some useful information, such as variance, mean and trends can be exploded to obtain a forecast of the demand. In this work, the demand is supposed to be cyclic, modeling a seasonal demand, adding a stochastic component given by a Poisson noise. For simplicity, a fixed order fulfillment time is assumed.

A. Volterra Models

The demand over time can be thought as a time-serie, represented by a nonlinear autoregressive model. One way to model it is by mean of a Volterra equation. The finite-dimensional discrete-time Volterra model used in this paper is a single-input, single-output model, relating an input sequence $\{d(k-i)\}$, to an output sequence $\{\hat{d}(k)\}$ [10].

$$\hat{d}(k) = d_0 + \sum_{i=1}^{30} \theta_i d(k-i) \tag{3}$$

where d_0 and θ_i are the model parameters, $\hat{d}_{(k)}$ is the actual estimated demand and $d_{(k-i)}$ are past values of the demand. The values of the unknown parameters will be found by a Kalman Filter.

B. Joint Extended Kalman Filter

The Kalman filter is characterized by a set of equations that synthesizes an optimal estimator of predictor-corrector type in the sense of minimizing the estimate error covariance $\mathbf{P}(k)$. In this particular case, a Joint Extended Kalman Filter [8] was used to solve the dual problem of simultaneously estimating the state and the model parameters θ from the noisy demand signal. To make the Volterra time-serie into a Markovian process its necessary to model the demand given by a the Volterra equation (3) as that given by the general nonlinear auto-regression system (4)

$$\begin{bmatrix} x_{(k)} \\ x_{(k-1)} \\ \vdots \\ x_{(k-M+1)} \end{bmatrix} = \begin{bmatrix} f(x_{(k-1)}\dots, x_{(k-M)}, \theta) \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \ddots & 0 & \vdots \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_{(k-1)} \\ \vdots \\ x_{(k-M)} \end{bmatrix} \end{bmatrix} + \begin{bmatrix} \upsilon_{(k-1)} \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{4}$$
$$y_{(k)} = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \mathbf{x}_{(k)} + \eta_{(k)}$$

where $f(x_{(k-1)}\dots, x_{(k-M)}, \theta_{(k-1)})$ is the mentioned Volterra model, and υ and η are the process and measurement noises respectively. The joint EKF approach to determine the unknown parameters θ consists in augmenting the state vector $\mathbf{x}$ with the parameters vector $\theta_{(k)}$. By doing this, a new state vector $\mathbf{z}_{(k)} = [\mathbf{x}_{(k)}^T, \theta_{(k)}^T]^T$ is obtained. Then, estimation is done recursively by writing the state-space equations for the *joint* state as

$$\begin{aligned} \begin{bmatrix} \mathbf{x}_{(k)} \\ \theta_{(k)} \end{bmatrix} &= \begin{bmatrix} \mathbf{F}(\mathbf{x}_{(k-1)}, \theta_{(k-1)}) \\ \mathbf{I}\theta_{(k-1)} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} \upsilon_{(k-1)} \\ y_{(k)} &= \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}_{(k)} \\ \theta_{(k)} \end{bmatrix} + \eta_{(k)} \end{aligned} \tag{5}$$

and running a EKF on the joint state-space to produce the simultaneous estimates of the states $\mathbf{x}_{(k)}$ and θ.

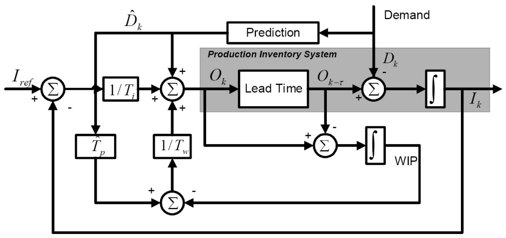

Fig. 2. Ordering system incorporating WIP feedback.

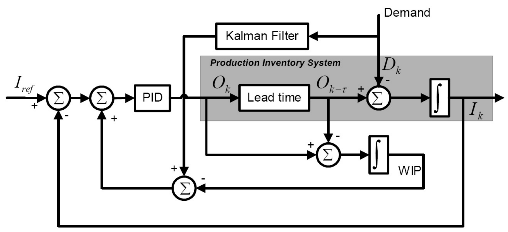

Fig. 3. Proposed PID-APIOBPCS controller.

Once the model parameters d_0 and θ have been estimated, they are used together with the model to get a prediction of on step ahead. This predicted state vector is then used for the PID-APIOBPCS reorder policy.

IV. PID-APIOBPCS-BASED INVENTORY LEVEL CONTROL

In contrast to the APIOBPCS analyzed by Disney and Towill [1], our approach also includes in the control loop a PID-controller and the demand prediction is generated by a joint dual EKF. We call this approach a PID-APIOBPCS model.

APIOBPCS has the main advantage over the other reorder policies of including in the decision rule the value of the WIP. A scheme of the APIOBPCS is shown in Fig. 2, and the reorder policy equations are given by (6),

$$O(k) = \hat{d}(k) + \frac{[I_{ref}(k) - I(k)]}{T_i} + \frac{[dWIP(k) - WIP(k)]}{T_w} \quad (6)$$

$$WIP(k) = WIP(k-1) + O(k) - O(k-\tau)$$
$$dWIP(k) = T_p\hat{d}(k)$$

where, $\hat{d}(k)$ is the estimated demand, and $I_{ref}(k)$ is the inventory level reference. Constant T_i is related to the time to adjust the inventory level, $\hat{T}_p$ is the estimate of the production lead time, and T_w is the time needed to adjust WIP.

As it can be seen, this reorder policy has no dynamics in its structure. This means that the overall system can present over-elongations, steady state errors, instability.

On the other hand, the approach of using only a PID as suggested in Grubbstöm and Wikner [6], and in Rivera and Pew [2] to model an order decision rule does not involve an explicit forecasting unit to estimate demand. So, fusing both controllers, it is possible to obtain a new structure and control law. The proposed control schema can be seen in Fig. 3.

$$O(k) = O(k-1) + K_P\left[e(k) - e(k-1)\right] + K_I e(k-1) + \\ + K_D\left[e(k) - 2e(k-1) + e(k-2)\right] \quad (7)$$

$$e(k) = (I_{ref}(k) - I(k)) + (dWIP(k) - WIP(k))$$
$$WIP(k) = WIP(k-1) + O(k) - O(k-\tau)$$
$$dWIP(k) = \hat{d}(k)$$

Equations (7) represent the reorder policy for the PID-APIOBPCS controller. As it can be seen the reorder policy involves the same variables as the APIOBPCS method, but in this case with the advantages of using a PID controller. The inclusion of a PID is not a capricious choice; according to Kunreuther [11], top level managers are found to act in a three-terms-control mode, similarly to a PID controller, using memory of past results (integral term), anticipating trends (derivative term), and as well as a proportional term for their future decisions.

Therefore, as shown in Fig. 3, the proposed controller has the basic elements of the APIOBPCS, demand forecast, and WIP compensation, and the PID controller is used as a decision rule maker. It is worth to note that, in this case, the PID actions are physically limited, that is, actions should not take values above 200 and below to 0. That is because we assume that the production system saturates when orders are greater than 200, and order with negative represent backorders, that is, items that are sent back from the inventory to the production process.

V. CONTROL ERROR ANALYSIS

As a way to probe the overall behavior of the system, an error analysis is presented. If control errors of the closed-loop control system are *ultimately bounded* [12], then the entire system has stability under a certain perturbation as demand estimate error.

The analysis is performed by using the Input-Output transfer function model of the system. Considering the demand as an input signal, the inventory system is a Multiple-Input Single-Output (MISO). For the stability analysis we first consider the control system presented in Disney and Towill [1], which structure is shown in Fig. 2, and modeling the production process as a first order dynamic system instead of a pure delay.

Therefore, the transfer functions of the closed-loop model are obtained by applying the superposition theorem [13], [14]. That is,

$$G_{I,I_{ref}} = \frac{1/T_i G_1 G_i}{1 + G_1 G_i 1/T_i}; \quad G_{I,D} = \frac{G_i}{1 + G_1 G_i 1/T_i}; \quad (8)$$
$$G_{I,\hat{D}} = \left(1 + \frac{\bar{T}_p}{T_w}\right)\frac{G_1 G_i}{1 + G_1 G_i 1/T_i};$$

where,

$$G_1 = \frac{1}{T_p s + (1 + T_p/T_w)} \qquad \text{and} \qquad G_i = \frac{1}{s}.$$

In (8) , $G_{I,I_{ref}}$, $G_{I,\hat{D}}$ and $G_{I,D}$ represents the transfer functions relating the inventory output (I) to the desired inventory level (I_{ref}), the output to the estimated demand ($\hat{D}$), and output to the demand (D) respectively. G_1 is just an intermediate auxiliary transfer function. Then, the system output can be expressed as,

$$I = \frac{1/T_i G_1 G_i}{1+G_1 G_i 1/T_i} I_{ref} - \frac{G_i}{1+G_1 G_i 1/T_i} D + \frac{(1+\bar{T}_p/T_w) G_1 G_i}{1+G_1 G_i 1/T_i} \hat{D}. \tag{9}$$

Therefore, by using (9), and after some mathematical manipulation, equations (10), it is possible to obtain an expression of the inventory control errors as a function of the demand estimate error as is shown in (11).

$$\begin{aligned} (1 + G_1 G_i 1/T_i) I = (1/T_i G_1 G_i) I_{ref} - G_i D + \\ \left(1 + \frac{\bar{T}_p}{T_w}\right) G_1 G_i \hat{D} \\ (1 + G_1 G_i 1/T_i)(I - I_{ref}) + I_{ref} = G_i(\hat{D} - D) - G_i \hat{D} + \\ \left(1 + \frac{\bar{T}_p}{T_w}\right) G_1 G_i \hat{D} \end{aligned} \tag{10}$$

$$\begin{aligned} (1 + G_1 G_i 1/T_i) E_{Inv} + I_{ref} = G_i E_{Dem} - G_i \hat{D} + \\ \left(1 + \frac{\bar{T}_p}{T_w}\right) G_1 G_i \hat{D}, \end{aligned}$$

$$\begin{aligned} E_{Inv} = \frac{G_i}{1 + G_1 G_i 1/T_i} E_{Dem} - \frac{G_i}{1 + G_1 G_i 1/T_i} \hat{D} + \\ \left(1 + \frac{\bar{T}_p}{T_w}\right) \frac{G_1 G_i}{1 + G_1 G_i 1/T_i} \hat{D} - \frac{1}{1 + G_1 G_i 1/T_i} I_{ref}. \end{aligned} \tag{11}$$

The maximum error E_{Inv}, independently of the values of E_{Dem}, $\hat{D}$ and I_{ref}, will be achieved when the transfer function operators have their maximum values. These maximum values can be obtained by using ∞-Norm ($\|\cdot\|_\infty$), defined as $\|H(s)\|_\infty = \max_\omega |H(j\omega)|$, [15] in (11). Then, applying norm properties, and taking into account the values used in the model ($T_p = \hat{T}_p = T_w = T_i = 1$), (11) can be reduced to (12)

$$\begin{aligned} \|E_{Inv}\|_\infty \leq \left\| \tfrac{G_i}{1+G_1 G_i 1/T_i} \right\|_\infty \|E_{Dem}\|_\infty - \left\| \tfrac{G_i}{1+G_1 G_i 1/T_i} \right\|_\infty \|\hat{D}\ \|_\infty + \\ \left\| \left(1+\tfrac{\bar{T}_p}{T_w}\right) \tfrac{G_1 G_i}{1+G_1 G_i 1/T_i} \right\|_\infty \|\hat{D}\|_\infty + \left\| \tfrac{1}{1+G_1 G_i 1/T_i} \right\|_\infty \|I_{ref}\|_\infty. \end{aligned} \tag{12}$$

Finally, we can obtain a boundary for the inventory control errors for the APIOBPCS control system as

$$\|E_{Inv}\|_\infty \leq 2 \|E_{Dem}\|_\infty + 1.1533 \|I_{ref}\|_\infty. \tag{13}$$

A similar procedure can be performed for the proposed PID-APIOBPCS presented in this work. Resulting,

$$G_{I,I_{ref}} = \frac{G_{1PID} G_i}{1 + G_{1PID} G_i}; \quad G_{I,\hat{D}} = \frac{G_{1PID} G_i}{1 + G_{1PID} G_i} \tag{14}$$
$$G_{I,D} = \frac{G_i}{1 + G_{1PID} G_i};$$

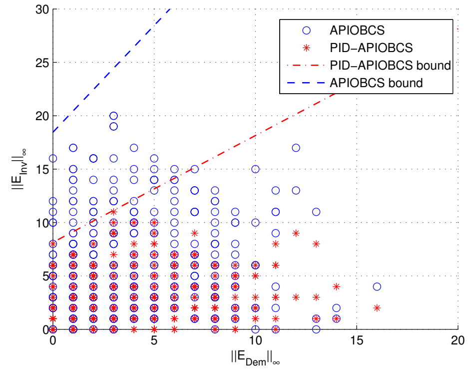

Fig. 4. Errors are bounded

where,

$$G_{1PID} = \frac{G_{PID} G_p}{1 + G_{PID} G_p T_p}; \quad G_{PID} = \frac{K_d s^2 + K_p s + K_i}{s};$$
$$G_p = \frac{1}{T_p s + 1} \qquad \text{and} \qquad G_i = \frac{1}{s}. \tag{15}$$

Then, the expression of the system output as a function of signals I_{ref}, D and $\hat{D}$ is,

$$I = \frac{G_{1PID} G_i}{1 + G_{1PID} G_i} I_{ref} - \frac{G_i}{1 + G_{1PID} G_i} D + \frac{G_{1PID} G_i}{1 + G_{1PID} G_i} \hat{D}. \tag{16}$$

Once again, by taking ∞-Norm and the norm properties, using typical values for $K_p = (30); K_d = (1); K_i = (10)$; $T_p = 1$, and performing the same steps as in the APIOBPCS case, the equation that relates the demand estimate errors to the inventory level errors can be obtained by

$$\|E_{Inv}\|_\infty \leq 1 \|E_{Dem}\|_\infty + 1.0185 \|I_{ref}\|_\infty. \tag{17}$$

Then, evaluating the expression given by (13) and (17) in a graphical interpretation is possible to analyze the stability problem for the inventory control system.

As it can be seen in Fig. 4, inventory level errors are bounded for both cases by the straight line, given by equations (13) and (17). In inventory models, the desired inventory level is usually arbitrarily chosen, based on demand requirements and storage capabilities. The equations above presented can be used to set the value of the desired inventory level (the value of the abscise) at an arbitrarily low value, provided that the out-of-stocks are avoided. In this figure it is clear that for the APIOBPCS model, the minimum inventory level must be chosen around 16 units, while for the PID-APIOBPCS that value can be as low as 8 units. This is an important aspect in the inventory problem. In addition, for both cases the inventory level error is bounded by the demand prediction error, but in the case of the PID-APIOBPCS, errors in the demand prediction have less effects on the inventory level. In order to prevent out-of-stock situations, the desired inventory level should be used as a design parameter and should be chosen looking at the prediction error, that is, *the higher this error is, the higher the desired inventory level must be chosen.*

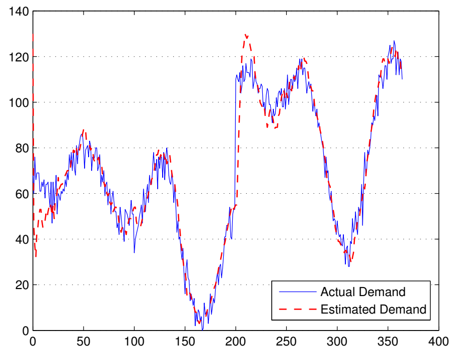

Fig. 5. Demand signal.

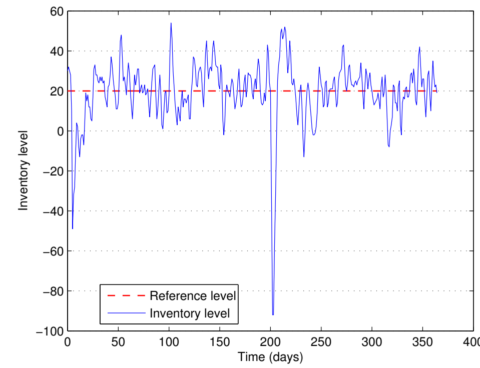

Fig. 6. Inventory level for the APIOBPCS method

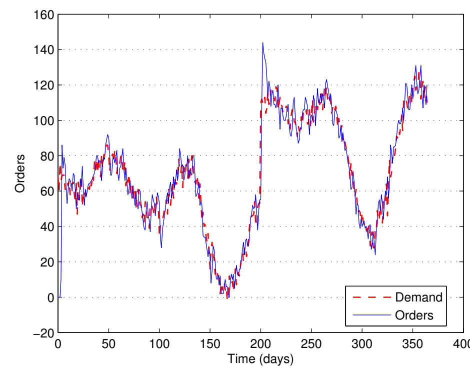

Fig. 7. Demand and Orders for the APIOBPCS method

VI. SIMULATION STUDIES

In order to show the performance of the proposed inventory controller, as well as the stability properties obtained in the preceding theoretical development, a simulation study has been carried out using a Matlab-Simulink model. The joint Dual Extended Kalman filter was implemented in a Matlab S-Function, using the model explained in Section III. Noise covariance for the Kalman filter, $\mathbf{R}^{\upsilon}$ and $\mathbf{R}^{\eta}$ were used as design parameters, and set to 10 and 40 respectively. The time-series model used to approximate the demand is that given by Eq. (3), and its forecasting is performed by the EKF. The demand signal was generated by a sum of sin and cos terms, with different amplitudes, phases and frequencies. A Poisson noise, with $\lambda = 10$, was also added to the seasonal signals. For all simulation runs, the inventory level set-point was set to 20 units, and the PID action limited to a maximum of 200 units, assuming that this is the capacity of the production system. In addition, demand signal has been added with an extra term, representing sudden stochastic changes on the value of demand.

Figure 5 shows the output of the demand estimator. Note that except when actual demand has sudden changes the demand estimator perform well.

A. APIOBPCS reorder policy

In this point is presented the APIOBPCS control system performance. The gain values were all set to one, due to the fact that those values are related to production and lead times. This means that system takes 1 day to adjust de WIP ($T_w = 1$) and the inventory ($T_i = 1$) and it has an estimated lead-time of 1 day ($\hat{T}_p = 1$). In this simulation, desired inventory level was set to 20. Results presented in Fig. 6 shows a good performance for this control system. The inventory level stays stable around 20 units, and seldom falls below 0. There are some peaks, caused by abrupt changes in demand, but they are canceled in around 7 days. In Fig. 7, demand and generated orders are compared. It can be seen that orders follow the demand, meaning that the inventory is close to its reference.

B. PID-APIOBPCS reorder policy

Finally, the proposed control system is tested under simulation. PID parameters were set to $K_P = 30; K_I = 1; K_D = 10$. These values give a good response in terms of dampness and speed. The desired inventory level was, again, set to 20 units. The results are presented in Fig. 8 and Note in this case that the inventory level is more stable. There are some peaks in the inventory level, but they are are canceled in around 5 days. These peaks are due to the sudden changes in the demand. There are, also, some inventory level values below zero, but in this case are less than in the previous one. This values can be interpreted as lost sales. So, as conclusion, in this case it is kept an acceptable level of lost sales, for a relatively low desired inventory level. Figure 9 presents the comparison between demand and generated orders. In this case orders tray to follow demand closer than the previous case. This is the reason for having a noisy-like reorder signal.

C. Results evaluation

To show the advantage of the proposed controller, the results for the different controllers are compared following the next procedure. Suppose that the inventory cost function is defined as in (18), where LS_i means *Lost Sales* and represent the number of units below zero in the inventory level, IL_i is the inventory level at any sample time and

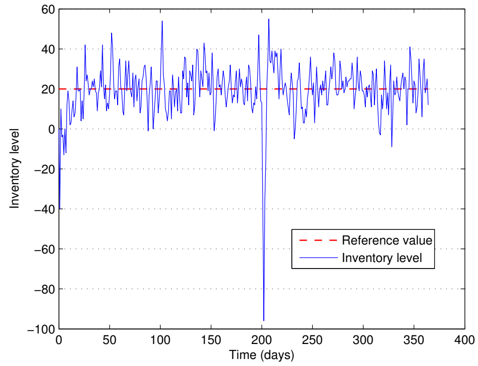

Fig. 8. Inventory for PID-APIOBPCS controller.

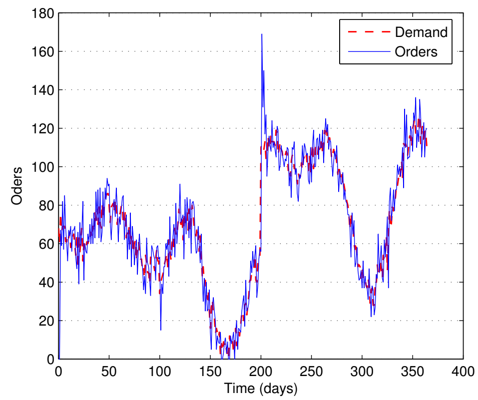

Fig. 9. Orders from PID-APIOBPCS controller.

E is the absolute value of the error between the desired inventory level and its actual level. The first term of the functional is the cost of lost sales, and represent not only an economic quantity but also a measure on service quality and customer satisfaction level. The second one, represent the cost of inventory daily maintenance. The last term, just penalizes the system for not being able to keep the inventory at the desired value. Table I shows the results after applying this cost funcional for each control system approach, with $T_f = 730 = 2\,\text{years}$. Note that proposed controller, PID-APIOBPCS, shows the lowest cost. These results agree with the conclusions obtained from previous experiments, where the PID-APIOBPCS shows a better performance than the other approaches.

$$C = \sum_{i=0}^{T_f} \{3LS_i + 0.3IL_i + 0.1E\} \tag{18}$$

TABLE I

INVENTORY COST SYSTEM

Policy / Costs	LS costs	IL costs	Error	**Total Cost**
APIOBPCS	2109	4294	1101	7585
PID-APIOBPCS	1305	4131	1084	6520

VII. CONCLUSIONS

We have presented an approach and a systematic design methodology to obtain a reorder policy for inventory system based on the APIOBPCS control scheme. The new reorder policy includes a PID controller and an estimate of the demand prediction using a joint dual EKF. This new approach control is called PID-APIOBPCS. An explicit evaluation of control error in terms of the demand prediction error and design parameters was performed. To show the practical feasibility and performance of the proposed control algorithm as well as stability properties obtained in the present work, a simulation study was carried out for a production-inventory system. The results show the practical feasibility and good performance of the proposed approach to production-inventory systems. Future research will include more complex models for the production-inventory systems, such as multiple-echelon and multiple-products production-inventory systems. Besides, the inclusion in the design methodology technics of optimal control to obtain an optimum operative condition for the controller, as well as, for the planning of the desired inventory level. Controllers should be also able to deal with saturation problems and include backordering. Improvement on the demand prediction is also a pending issue.

REFERENCES

[1] S. Disney and D. Towill, "On the bullwhip and inventory variance produced by an ordering policy," *The International Journal of Management Science*, vol. 31, 2003.

[2] E. Rivera and M. D. Pew, "Evaluating pid control for supply chain management: A freshman design project," *CDC-ECC '05. 44th IEEE Conference on Decision and Control.*, 2005.

[3] SimulationDynamics, "Importance of supply chain management," http://www.simulationdynamics.com /Sc /SupplyChainImportance.htm, 2003.

[4] M. Ortega and L. Lin, "Control theory applications to the production-inventory problem: a review," *International Journal of Production Research*, vol. 42, no. 11, 2004.

[5] S. John, M. M. Naim, and D. R. Towill, "Dynamic analysis of a wip compensated decision support system," *International Journal of Manufacturing System Design*, vol. 1, no. 4, 1994.

[6] R. W. Grubbström and J. Wikner, "Inventory trigger control policies developed in terms of control theory," *International Journal of Production Economics*, vol. 45, August 1996.

[7] B. Samanta and S. A. Al-Araimi, "An inventory control model using fuzzy logic," *International Journal of Production Economics*, vol. 73, 2001.

[8] E. A. Wan and R. van der Merwe, "The unscented kalman filter for nonlinear estimation," in *Proc. of IEEE Symposium 2000 (AS-SPCC), Lake Louise, Alberta,Canada, Oct. 2000*, Oct 2000.

[9] R. D. H. Warburton, "An analytical investigation of the bullwhip effect," *Production and Operations Management*, vol. 13, no. 2, 2004.

[10] F. J. Doyle, R. K. Pearson, and B. A. Ogunnaike, *Identification and Control Using Volterra Models*, E. Sontag and M. Thoma, Eds. Springer Verlag, 2002.

[11] H. Kunreuther, "Extensions of bowman's theory on managerial decision making," *Management Science*, vol. 15, 1969.

[12] J. L. Salle and S. Lefschetz, *Stability by Liapunov's Direct Method*, A. Press, Ed. Academic Press, 1961.

[13] K. Ogata, *Modern Control Engineering*, T. Edition, Ed. Prentice-Hall, 1997.

[14] B. C. Kuo, *Digital Control Systems*, S. E. Van Valkenburg, Ed. Holt, Rinehart and Winston, Inc., 1980.

[15] M. Vidyasagar, *Nonlinear Systems Analysis*, S. Edition, Ed. Prentice-Hall, 1993.