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An Analytical Model for Automated Packaging Lines Design

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Abstract— **The problem of deriving a probabilistic mathematical model of an industrial production line is addressed in this paper. In particular, the analysis of a Tetra Pak liquid foodstuff packaging line has been developed as application of the modeling theory. Tetra Pak's package forming machine works on the basis of a continuous process which cannot be interrupted without production wasting. Production buffer are used to decouple package forming machine with following (downstream) machines (e.g. straw applicator, film wrapper, etc.) to avoid forming machine forced stoppage because downstream machine stoppage. The paper presents a new analytical approach for the packaging forming machine restart control policy based on buffer level.**

I. Introduction

Automated production lines represent a key choice in production systems when high demand volumes and constant quality assurance have to be addressed. Among the different manufacturing fields, food packaging industry is certainly one of the primary user of such production systems.

Briefly, an automated production line consists in a series of machines forced to work in a predefined chain, each of them performing specific operations on the raw material at the highest possible speed. As a primary consequence of such a particular layout, the performance of the whole line is not only determined by the performances of the machines themselves, but also by the way they are linked each other.

This latter aspect is a natural consequence of the sequential disposition of the machines in the line, which implies that when a machine goes down other machines can incur in idle state. In particular, an *operational* machine could be found in idle state as a consequence of failures occurred

- in the upstream, determining an interruption of the ingoing product flow, thus causing a *starvation*;
- in the downstream, determining a stop of the outgoing product flow, thus a *block* of the machine as a consequence of the impossibility to discharge products.

To mitigate the effects of such harmful interactions, buffers are allocated along the line to act as decoupling points to sustain the flow for a determined time span, when failures on machines occur. Hence, the optimal design of an automated production line cannot prescind from the correct understanding of the interactions between the machines and the determination of the right position and capacity of the buffers ([2][3][4]).

Since several years, scientists have address the problem of buffer dimensioning and allocation, and one of the most attractive and effective way adopted is the mathematical modeling. By means of mathematical models, an analytic relation between the buffers structuring and the throughput of the line can be established, thus precise and useful insights can be obtained to address the optimal dimensioning (see [1] for a comprehensive representation).

The most important methodology adopted considers the line as a continuous time Markov process, to allow also the modeling of inhomogeneous lines (i.e. those in which machines can have different productivities). A generic line is decomposed in a series of *two machines one buffer* sub-problems, for which analytical solution have been obtained in literature for the canonical case [5], while also some other particular cases are under study [6]. Provided that analytical models for each two machines one buffer sub-problem are given, the performance parameters of the whole line can be computed by means of iterative procedures adopting a decomposition technique, as pointed out by the literature [7][8][9][10][11][12]. Hence, the importance to have well performing mathematical models to correctly represent the behavior of each two-machines sub-system is emphasized.

In this paper, the canonical two machine one buffer model developed in [5] has been extended to take into account the particular behavior of the filling machine of Tetra Pak automated packaging lines.

In such packaging lines, the filling machine (also named *filler*) constitutes the most important and critical part, being the one that executes the packaging formation and filling process. To guarantee world-class constant sealing and aseptic conditions together with highest production speed, the packaging formation and filling process is executed in a continuous manner. Hence, when a failure in the downstream blocks the possibility to discharge packages, an interruption in the filling process occurs causing a stop of the filler and an *outage cost* related to the succeeding restart phase. This firstly implies that the filler cannot work in an intermittent manner or at a speed lower than the canonical one, and secondly that interruption in the downstream flow have to be reduced to a bare minimum.

A first attempt to produce a mathematical model for the two-machine one-buffer problem (see Figure 1) able to take into account this particular behavior can be found in [13]. A restart level L has been introduced by the authors to impose a forced block state on the filler each time the buffer reaches the maximum level N. The forced block state on the filler is then removed when the buffer level gets back L. This policy aims to prevent situations in which the filler restarts when the buffer level is still high, then reducing the probability to have a further block if the downstream interrupts again for

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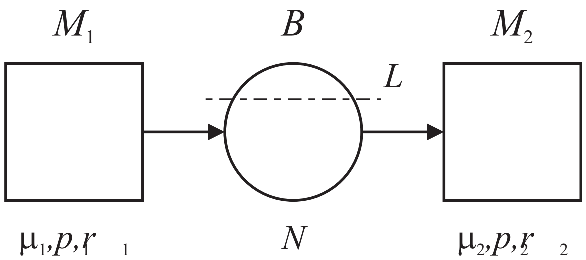

Fig. 1. Two machines system with restart level.

the time being.

The present work improves [13] by introducing some new boundary conditions and providing an analytical solution to the mathematical formulation. Finally, some numerical results show the effects of the restart level variation on the throughput of the system, thus the optimal value can be determined.

The remaining part is organized as follows. In Section II the mathematical model is presented while in Section III the solution is obtained. Finally, Section IV reports the results of numerical applications and Section V points out some concluding remarks.

II. System model

The two machine finite buffer problem, with M_1's restart controlled by buffer level, is modeled as a continuous time, mixed state Markov process. The system is depicted in Figure 1.

The variables μ_i, p_i, and r_i represent the production rate, the failure rate, and the restoration rate, respectively, $\forall\, i = 1, 2$.

Moreover, the condition $\mu_2 > \mu_1$ is imposed to the system. In fact, in real packaging lines the machines downstream of the filler (the so-called *distribution equipment*, here ideally represented by M_2) are less reliable than the filler itself, thus a value of μ_2 higher than μ_1 allows the buffer to work at low load levels.

The system state is defined as

$$\mathcal{S} = \big(x, \beta, \alpha_1, \alpha_2, t\big)\,, \tag{1}$$

being x the buffer level, $\beta = \{0, 1\}$ a binary parameter identifying the forced block state of the first machine, $\alpha_i = \{0, 1\}$ the repair state of the machine $i = \{1, 2\}$, and t the time variable.

In a generic time interval δt, the variation in the buffer level involved by the machines behavior is $\big((1-\beta)\alpha_1\mu_1 - \alpha_2\mu_2\big)\delta t$, if x is far enough to its boundaries 0 and N.

When the buffer reaches the level N, the first machine can not discharge products and consequently goes blocked, that is, it could process units ($\alpha_1 = 1$) but it has to stop production as a consequence of the impossibility to send products in the downstream. Moreover, as said in Section I, to reduce the number of stops of M_1, an immediate restart is prevented by putting M_1 in the forced block state ($\beta = 1$) and maintaining it blocked until the buffer level decreases to a predefined value $L \in [0, N]$. As an additional consequence, while $\beta = 1$, M_1 can not go down since operational dependent failures are assumed. While $\beta = 0$, the probability of failure of M_1 at time $t + \delta t$, provided that $\alpha_1(t) = 1$, is $p_1\delta t$.

On the other side, M_2 can consume products at its nominal rate μ_2 only if the buffer is not empty, otherwise it is forced to slow down its speed to μ_1 (remember the hypothesis $\mu_2 > \mu_1$). In this case the probability of failure of M_2 at time $t + \delta t$, provided that $\alpha_2(t) = 1$, is $p_2^b\delta t$, where

$$p_2^b = \frac{\mu_1}{\mu_2} p_2\,, \tag{2}$$

since a failure rate proportional to machine operating speed is assumed. When the buffer is not empty, such a probability is $p_2\delta t$.

Finally, the probability to have a restoration at time $t + \delta t$ of a machine i failed in t $\big(\alpha_i(t) = 0\big)$ is $r_i\delta t$.

The model comprises a set of equations that represent the behavior of the system. Let $\mathbf{p}(x, \beta, \alpha_1, \alpha_2, t)$ be the probability of being in state $(x, \beta, \alpha_1, \alpha_2, t)$ and $f(x, \beta, \alpha_1, \alpha_2, t)$ be the probability density.

It is advisable to distinguish two groups of equations, the one related to the boundary states and the other related to the intermediate buffer levels.

A. Boundary behavior

The previous literature examines only two kinds of boundary states: the first kind of boundary equations refers to the situation where the buffer is empty $x = 0$, the second kind to the situation where the buffer is full $x = N$.

The present study significantly extends previous literature by considering two different dynamics: the first one ($\beta = 0$) holds when the first machine is not forced to be blocked; the second one ($\beta = 1$) takes over when the buffer reaches the value N and M_1 is then put in the forced blocked state. This latter dynamics lasts until the buffer level decreases to the value L.

We then introduce a new kind of boundary equations related to the situation where the buffer level is equal to L and the system passes from the first dynamics to the second one (i.e. from $\beta = 0$ to $\beta = 1$).

Let us examine the equations to represent the probability of finding the system in a given boundary state.

Lower Boundary – x = 0*:* The equations that describe the behavior of the system at the lower boundary are very similar to those investigated in the previous literature. In this case the parameter β is included in the definition of the system state nevertheless its value is fixed to zero when the buffer is empty.

Hence, the equations related to the lower boundary are only stated and not derived in the following (the reader is referred to [1] for more details).

- *Boundary-to-Boundary Equations*

$$\frac{d}{dt}\mathbf{p}(0,0,0,0) = -(r_1 + r_2)\mathbf{p}(0,0,0,0)\,, \quad (3)$$
$$\mathbf{p}(0,0,1,0) = 0\,. \quad (4)$$

- *Interior-to-Boundary Equations*

$$\frac{d}{dt}\mathbf{p}(0,0,0,1) = r_2\mathbf{p}(0,0,0,0) - r_1\mathbf{p}(0,0,0,1) + + p_1\mathbf{p}(0,0,1,1) + \mu_2 f(0,0,0,1)\,, \quad (5)$$
$$\frac{d}{dt}\mathbf{p}(0,0,1,1) = -(p_1 + p_2^b)\mathbf{p}(0,0,1,1) + + r_1\mathbf{p}(0,0,0,1) + (\mu_2 - \mu_1) f(0,0,1,1)\,. \quad (6)$$

- *Boundary-to-Interior Equations*

$$\mu_1 f(0,0,1,0) = r_1\mathbf{p}(0,0,0,0) + p_2^b\mathbf{p}(0,0,1,1)\,. \quad (7)$$

Upper Boundary – x = N: It is important to note that the variable β changes instantaneously from 0 to 1 when the buffer level reaches the value N: as soon as this situation occurs M_1 is put in the forced blocked state. Thus, the concern is about the passage from the first dynamics, when $\beta = 0$, to the second one, when $\beta = 1$.

- *Boundary-to-Boundary Equations*

$$\mathbf{p}(N,1,0,0) = 0\,. \quad (8)$$
$$\mathbf{p}(N,1,1,1) = 0\,, \quad (9)$$

  The equations (8) and (9) result from the following consideration: the buffer can fill up only if the first machine is up and the second one is down; otherwise, as a consequence of the hypothesis $\mu_2 > \mu_1$, the buffer level can only decreases and the upper boundary $x = N$ can not be reached.
- *Interior-to-Boundary Equations*
  To be in state $(N,1,1,0)$ at time $t+\delta t$ the system could have been only in one of two sets of states at time t. It could have been in state $(N,1,1,0)$ with no repair of the second machine (the first could not have failed since it was blocked) or else in any interior state $(x,0,1,0)$, where $N - \mu_1\delta t \le x < N$, if repair of the second machine or failure of the first did not occur.
  Symbolically, ignoring the second order terms,

$$\mathbf{p}(N,1,1,0,t+\delta t) = (1 - r_2\delta t)\mathbf{p}(N,1,1,0,t) + + \int_{N-\mu_1\delta t}^{N} f(x,0,1,0,t)dx\,,$$

  It is not necessary to consider transitions directly from states like $(x,0,1,1)$, since, if the second machine is working in t, the buffer level cannot reach N in $t+\delta t$. As $\delta t \to 0$, the equation becomes

$$\frac{d}{dt}\mathbf{p}(N,1,1,0) = -r_2\mathbf{p}(N,1,1,0) + \mu_1 f(N,0,1,0)\,. \quad (10)$$

- *Boundary-to-Interior Equations*
  The only possible internal states reachable from the upper boundary are those with $\beta = 1$ and $\alpha_1 = 1$ because the first machine is forced to be blocked and can not fail. In addition, it is possible to leave the upper boundary $x = N$ only by repairing the second machine, then it results $\alpha_2 = 1$ and the buffer level decreases according to the productivity of the second machine (μ_2). To be in the state $(x,1,1,1)$ at time $t+\delta t$ the system can have been at the boundary state $(N,1,1,0)$ some time during the time interval $(t, t+\delta t)$, then

$$\int_{N}^{N-\mu_2\delta t} f(x,1,1,1,t+\delta t)dx = \int_{t}^{t+\delta t} r_2\mathbf{p}(N,1,1,0,s)ds\,.$$

  Letting $\delta t \to 0$, the equation becomes

$$\mu_2 f(N,1,1,1) = r_2\mathbf{p}(N,1,1,0)\,. \quad (11)$$

Middle Boundary – x = L and β = 1: The states representing the second dynamics ($\beta = 1$) and characterized by a buffer level $x = L$, can be conceptually considered equivalent to the states representing the first dynamics ($\beta = 0$) with $x = 0$. Hence, the states with $x = L$ and $\beta = 1$ are the lower boundary states of the second dynamics.

The equations that are useful in the mathematical modeling of such a system are the following.

- *Interior-to-Boundary Equations*

$$\frac{d}{dt}\mathbf{p}(L,1,1,1) = -p_2\mathbf{p}(L,1,1,1) + \mu_2 f(L,1,1,1)\,. \quad (12)$$

  The (12) follows by the same reasonings used in defining the (6). Obviously in this case the parameters related to the first machine, which is in the forced block state, are omitted.

- *Boundary-to-Interior Equations*
  To be in the state $(x,0,0,1)$ at time $t+\delta t$, where $L - \mu_2\,\delta t \le x < L$, the only possible boundary state of the second dynamics in which the system could have been in t is $(L,1,1,1)$ with the first machine failing and the second not failing during δt.
  Accounting only the first order terms in δt:

$$\mu_2 f(L,0,0,1) = p_1\mathbf{p}(L,1,1,1)\,. \quad (13)$$

B. *Intermediate buffer level*

The transition equations represent the behavior of the system at intermediate storage levels, that is, when the buffer is neither empty nor full. The set of equations reported below characterizes the system when the forced block state in M_1 is not reached. This is the case in which the parameter β equals 0, thus those equations are the same as the ones reported in [1].

$$\frac{\partial f}{\partial t}(x,0,1,1) = -(p_1+p_2)f(x,0,1,1)+ + (\mu_2-\mu_1)\frac{\partial f}{\partial x}(x,0,1,1)+ + r_1 f(x,0,0,1) + r_2 f(x,0,1,0), \quad (14)$$

$$\frac{\partial f}{\partial t}(x,0,0,0) = -(r_1+r_2)f(x,0,0,0)+ + p_1 f(x,0,1,0) + p_2 f(x,0,0,1), \quad (15)$$

$$\frac{\partial f}{\partial t}(x,0,0,1) = \mu_2\frac{\partial f}{\partial x}(x,0,0,1)+ -(r_1+p_2)f(x,0,0,1)+ + p_1 f(x,0,1,1) + r_2 f(x,0,0,0), \quad (16)$$

$$\frac{\partial f}{\partial t}(x,0,1,0) = -\mu_1\frac{\partial f}{\partial x}(x,0,1,0)+ -(p_1+r_2)f(x,0,1,0)+ + p_2 f(x,0,1,1) + r_1 f(x,0,0,0). \quad (17)$$

In the considered case other equations are needed to model the entire behavior of the system. In fact, when the buffer level is between L and N, M_1 could be or not in the forced block state, hence there is another set of transient equation in which $\beta = 1$. Such equations are defined for $x \in [L, N]$.

When $\beta = 1$, M_1 is operational but in the forced block state, thus failures can not occur. Hence, the only transient states available in such a situation are $(x,1,1,1)$ and $(x,1,1,0)$.

Let us consider the first state $(x,1,1,1)$ representing the situation in which the machine M_2 is operational. The probability of finding such a state with a storage level between x and $x+\delta x$ at time $t+\delta t$ is given by $f(x,1,1,1,t+\delta t)\delta x$, where:

$$f(x,1,1,1,t+\delta t) = (1-p_2\delta t)f(x+\mu_2\delta t,1,1,1,t)+ + r_2\delta t f(x,1,1,0,t) + \circ(\delta t)\,.$$

This derives from the following considerations:

1) If M_2 is operational at time t and the buffer level is $x+\mu_2\delta t$ (with $\delta x = \mu_2\delta t$), then, at time $t+\delta t$, the storage level will be x if failures do not occur in M_2 during δt, thus involving probability $(1-p_2\delta t)$.
2) If M_2 is down at time t, it can be up at time $t+\delta t$ if it will be repaired in δt, thus implying probability $r_2\delta t$. Moreover, there is not variation in the buffer level since M_1 is in the forced block state.
3) States characterized by $\alpha_1 = 0$ (M_1 is down) are not possible since, being M_1 in the forced block state, it can not fail.

With few steps the derivative form can be obtained.

$$f(x,1,1,1,t+\delta t) - f(x,1,1,1,t) = (1-p_2\delta t)f(x+\mu_2\delta t,1,1,1,t)+ - f(x,1,1,1,t) + r_2\delta t f(x,1,1,0,t)\,,$$

$$\lim_{\delta t\to 0}\frac{f(x,1,1,1,t+\delta t)-f(x,1,1,1,t)}{\delta t} = \lim_{\delta t\to 0}\big(-p_2 f(x+\mu_2\delta t,1,1,1,t)\big)+ + \lim_{\delta t\to 0}\big(r_2 f(x,1,1,0,t)\big) + + \lim_{\delta t\to 0}\left(\frac{f(x+\mu_2\delta t,1,1,1,t)-f(x,1,1,1,t)}{\delta t}\right),$$

$$\frac{\partial f}{\partial t}(x,1,1,1,t) = -p_2 f(x,1,1,1,t) + r_2 f(x,1,1,0,t)+ + \mu_2\lim_{\delta x\to 0}\left(\frac{f(x+\delta x,1,1,1,t)-f(x,1,1,1,t)}{\delta x}\right).$$

The final equation is here reported, where the t argument is suppressed.

$$\frac{\partial f}{\partial t}(x,1,1,1) = -p_2 f(x,1,1,1) + r_2 f(x,1,1,0) + + \mu_2\frac{\partial f}{\partial x}(x,1,1,1)\,. \quad (18)$$

The same reasoning is adopted to obtain the other transient equation, reported in the following.

$$\frac{\partial f}{\partial t}(x,1,1,0) = -r_2 f(x,1,1,0) + p_2 f(x,1,1,1)\,. \quad (19)$$

C. Normalization

The normalization equation must be satisfied to assure that the sum of the probabilities of all possible states (transient and boundary) is 1.

$$\sum_{\alpha_1=0}^{1}\sum_{\alpha_2=0}^{1}\left[\int_0^N f(x,0,\alpha_1,\alpha_2)dx + \mathbf{p}(0,0,\alpha_1,\alpha_2)\right]+ + \sum_{\alpha_2=0}^{1}\left[\int_L^N f(x,1,1,\alpha_2)dx\right] + \mathbf{p}(N,1,1,0) = 1\,. \quad (20)$$

D. Throughput of the line

Let P_i be the throughput of the machine i, i.e. the rate at which material leaves the machine i, $\forall i = 1,2$; and let $\pi_{\beta=1}$ be the probability of being in the second dynamics ($\beta = 1$).

Note that material leaves the second machine at rate μ_2 only if the buffer level is different from zero, otherwise the rate is equal to μ_1. Consequently,

$$P_2 = (1-\pi_{\beta=1})\times\left[\mu_2\int_0^N f(x,0,0,1)dx+ + \mu_2\int_0^N f(x,0,1,1)dx + \mu_1\mathbf{p}(0,0,1,1)\right]+ + \pi_{\beta=1}\times\mu_2\int_L^N f(x,1,1,1)dx\,. \quad (21)$$

For what concerns the expression of P_1, it is necessary to consider also that material can not enter the first machine if the machine is forced to be blocked. Thus,

$$P_1 = (1 - \pi_{\beta=1}) \times \mu_1 \left[\int_0^N f(x,0,1,0) + \right.$$
$$\left. + \int_0^N f(x,0,1,1) \big) dx + \mathbf{p}(0,0,1,1) \right]. \qquad (22)$$

III. Solution technique

Since the steady state density equations (14)–(17) are coupled ordinary linear differential equations, the form of the solution is the following:

$$f(x,\beta,\alpha_1,\alpha_2) = (1-\beta) C e^{\lambda x} Y_1^{\alpha_1} Y_2^{\alpha_2} + \\ + \beta C' e^{\lambda' x} {Y'}_2^{\alpha_2}. \qquad (23)$$

Thus, the solution is made up of two contributions related to the two dynamics: the first one valid if $\beta = 0$, i.e. when the first machine is not in the forced block state, the second one valid if $\beta = 1$, i.e. when the first machine is forced to be blocked and the buffer level x is in the interval $[L, N]$. $C, C', \lambda, \lambda', Y_1, Y_2, {Y'}_2$ are parameters to be determined. Note that in the second part of the (23) the term ${Y'}_1^{\alpha_1}$ has been omitted since α_1 is always equal to 1 if $\beta = 1$.

Consider first the case where $\beta = 1$ and the behavior of the system is led by the second dynamics. Equation (23) satisfies the steady state version of the (18)–(19) if:

$${Y'}_2 = \frac{r_2}{p_2}, \qquad (24)$$

$$\lambda' = 0. \qquad (25)$$

Next, let us consider the case where $\beta = 0$ and the behavior of the system is led by the first dynamics. Equation (23) satisfies the steady state version of the (14)–(17) if:

$$\sum_{i=1}^{2} (p_i Y_i - r_i) = 0, \qquad (26)$$

$$-\mu_1 \lambda = (p_1 Y_1 - r_1) \frac{1 + Y_1}{Y_1}, \qquad (27)$$

$$\mu_2 \lambda = (p_2 Y_2 - r_2) \frac{1 + Y_2}{Y_2}. \qquad (28)$$

The three parametric equations (26)–(28) allows to determine the three unknowns λ, Y_1, Y_2.

Equation (26) implies:

$$Y_2 = \frac{r_1 - p_1 Y_1 + r_2}{p_2}. \qquad (29)$$

By substituting the expression of Y_2 in the (28), the expression of λ in term of Y_1 follows:

$$\lambda = -\frac{r_1 - p_1 Y_1}{\mu_2} \frac{p_2 + r_1 - p_1 Y_1 + r_2}{r_1 - p_1 Y_1 + r_2}. \qquad (30)$$

Finally, substituting (30) in (27) we get a single quadratic equation in Y_1:

$$-(\mu_2 - \mu_1) p_1 Y_1^2 + [(\mu_2 - \mu_1)(r_1 + r_2) + \\ -(\mu_2 p_1 + \mu_1 p_2)] Y_1 + \mu_2 (r_1 + r_2) = 0. \qquad (31)$$

Thus, if $\mu_1 \neq \mu_2$, as in the present case where $\mu_2 > \mu_1$, we have two solutions Y_{1j}, to which correspond the solutions Y_{2j} and λ_j with $j = 1, 2$.

The final form of the (23) is then the following:

$$f(x,\beta,\alpha_1,\alpha_2) = (1-\beta) \sum_{j=1}^{2} C_j e^{\lambda_j x} Y_{1j}^{\alpha_1} Y_{2j}^{\alpha_2} + \\ + \beta C' {Y'}_2^{\alpha_2}. \qquad (32)$$

where the parameters $\lambda_j, \lambda', Y_{1j}, Y_{2j}, {Y'}_2$ have already been determined.

It is necessary to determine the three coefficients C_1, C_2, C' in order to complete the solution. Three linear equations are required:

- equation (10) states the passage from the first dynamics, with $\beta = 0$, to the second one, with $\beta = 1$; it implies:

$$\mu_1 \sum_{j=1}^{2} C_j e^{\lambda_j N} Y_{1j} = \mu_2 {Y'}_2 C', \qquad (33)$$

- equations (12) and (13), which establish the passage from the second dynamics to the first one, yield to the following expression:

$$\mu_2 \sum_{j=1}^{2} C_j e^{\lambda_j L} Y_{2j} = -\frac{p_1 \mu_2 {Y'}_2}{p_2} C', \qquad (34)$$

- normalization equations (20) is the last linear equation requested in the three unknowns.

All the parameters have been determined, thus the solution is complete.

IV. Numerical results

A numerical experimentation has been executed in order to demonstrate the effects of the restart level L on the throughput of the two machine system. Table I shows the values of the input data adopted in the computations.

Parameter	Value
N	2300 [units]
μ_1	20000 [units/h]
μ_2	a) 22000 [units/h] b) 22500 [units/h] c) 23000 [units/h] d) 24000 [units/h]
p_1	1 $[h^{-1}]$
p_2	2 $[h^{-1}]$
r_1	15 $[h^{-1}]$
r_2	20 $[h^{-1}]$

TABLE I
Input data

Four cases have been examined by varying production rate of the second machine. For each different case the throughput of the line has been computed as a function of the restart level L, ranging in $[0, N]$. Results are shown in Figure 2.

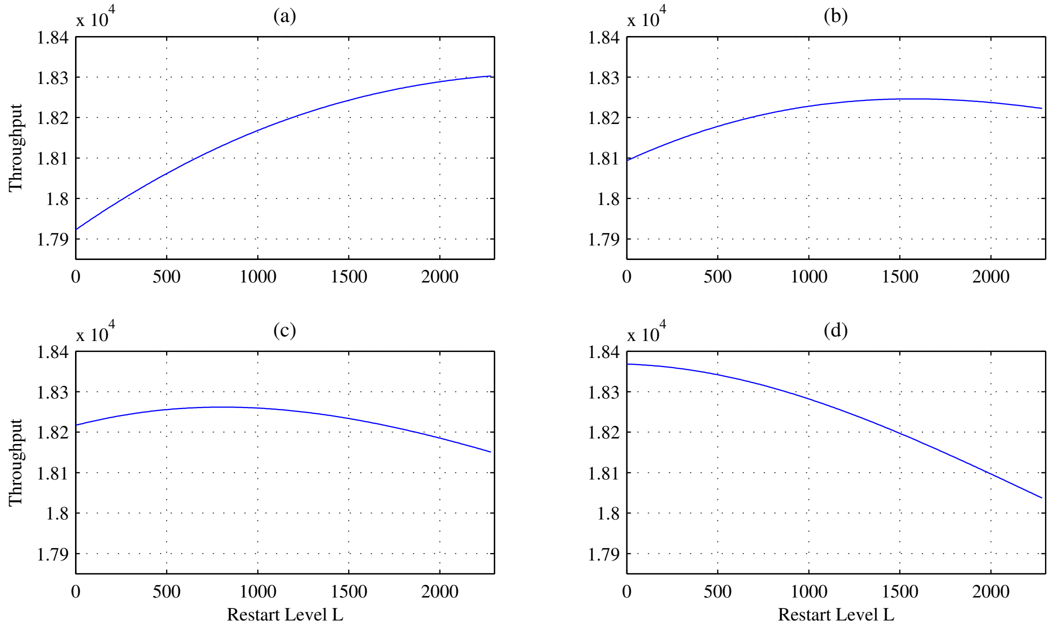

Fig. 2. Numerical examples.

As can be seen, by passing from the case (a) to the case (d), that is, as the difference between μ_1 and μ_2 becomes higher, an inversion in the effects of the restart level L on the throughput of the line occurs.

On one hand, when μ_2 is quite close to μ_1 (case (a)), the time employed to empty the buffer at each M_1 blocking is not recovered by a decrease in blocking ratio, thus best performances are related to high values of L. On the other hand, when μ_2 becomes significantly higher than μ_1, the buffer empties quickly then forced blocks in M_1 last few time. Thus, low levels of L can be adopted since the beneficial effect of a low L on the block ratio reduction is higher than the loss in production time due to the buffer emptying procedure. Finally, cases (b) and (c) reports intermediate conditions.

V. Conclusions

This work addresses particular aspects of automated packaging line design. In such lines, the filling machine forms and fills packages by means of a continuous process, whose interruption causes an outage costs related to the succeeding restart phase. A buffer is adopted to decouple the filling machine by the rest of the line. Nevertheless, if the buffer reaches its maximum level, the filling machine has to stop working.

The paper proposes a mathematical formulation of the system behavior, introducing a restart policy based on buffer level. An analytical solution of the model is also obtained, while some numerical examples are finally discussed.

VI. Acknowledgement

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