2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

Real-time compensation of hysteresis in a piezoelectric-stack actuator tracking a stochastic reference

F. Weibel, Y. Michellod, Ph. Mullhaupt and D. Gillet
École Polytechnique Fédérale de Lausanne, Switzerland

***Abstract*— This paper presents a convenient way to invert the classical Preisach model to compensate the hysteresis of a piezoelectric stack actuator in real-time. The advantage of the proposed method lies the possibility to track a stochastic signal and compensate the hysteresis in real-time. Experimental results show a reduction of the RMS tracking error by 67% to 90% by using the compensation algorithm designed.**

I. INTRODUCTION

Piezoelectric actuators are nowadays widely used in nanometer-accuracy positioning systems and are appreciated for their high precision, stiffness, and fast response. Nevertheless, the hysteresis of these actuators remains a major limitation to their precision. Such a degradation of the performance cannot be tolerated in applications such as atomic-force microscopes [12], or high-accuracy optical systems.

In the context of the conception of a new optical Differential-Delay Line for the Very Large Telescope Interferometer (VLTI) at the European Southern Observatory (ESO) facility in Chile, a dual-stage system has been designed [14]. The fine stage of this system is composed of a piezoelectric-stack actuator that compensates both for atmospheric disturbances and for the positioning errors of the coarse stage. The coarse stage is controlled by a stepper motor. So as to improve the bandwidth and the precision of the system, a feed-forward loop containing an inverse model of the hysteresis can be added to the controller of the fine stage.

Different methods have been proposed to model the hysteresis appearing in piezoelectric actuators, but the most popular one remains the classical Preisach model. It has been adapted using electromagnetism theory by Ge and Jouaneh [2]. Some improvements of this method have also been proposed, such as the generalized Preisach model, which relaxes the congruency property [4]. However Hu and Ben Mrad showed that the congruency property is already satisfied whenever either no load or a constant load is applied to the actuator [10]. Moreover, it is possible to add a neural network to facilitate the on-line implementation of the model [6].

Ge and Jouaneh developed a compensation method based on the Preisach model [3]. However, this method is not suitable for a real-time compensation of a stochastic signal as it is required for the above setup in Chile. In addition, in the case of the atmospheric disturbances, no information is given at the beginning of the compensation about the future piezo expansion. Since very few studies have investigated such a problem [11], a method in order to invert the classical Preisach model is proposed in this paper and validated on a piezoelectric stack actuator. First, the classical Preisach model is described, implemented, and validated through different simulations. The model is then inverted and validated boths in simulation and on the real system for different input signals. Experimental results show a reduction of the RMS tracking error by 67% to 90% whenever the compensation algorithm is added using open-loop control.

II. CLASSICAL PREISACH MODEL

The basic idea of the Preisach model lies in the description of the hysteresis through an infinite number of operators $\gamma_{\alpha\beta}[u(t)]$ (Fig. 1a). For piezoelectric actuators, $\gamma_{\alpha\beta}[u(t)]$ is set to $+1$ if the input $u(t)$ exceeds the switching value α or to 0 if the input $u(t)$ is below the switching value β. The operators are multiplied by a weighting function $\mu(\alpha, \beta)$ and connected in parallel (Fig. 1b). Such a representation takes into account the fact that the hysteresis is a nonlinearity with nonlocal memory effect, which means that the current displacement of the actuator, namely $x(t)$, depends upon the history of the input voltage $u(t)$. The classical Preisach model can then be mathematically written as:

$$x(t) = \iint_{\alpha>\beta} \mu(\alpha, \beta)\gamma_{\alpha\beta}[u(t)]\, d\alpha\, d\beta \qquad (1)$$

This equation can be interpreted thanks to a limiting triangle T_0, also called $\alpha - \beta$ diagram. It is defined in such a way that $u_{max} \geq \alpha \geq \beta \geq u_{min}$, where u_{max} and u_{min} are the limiting values of the input voltage $u(t)$. The surface S^+, which corresponds to the operators $\gamma_{\alpha\beta}[u(t)]$ set to $+1$, grows from bottom to top when the hysteresis is in an ascending loop and decreases from right to left in

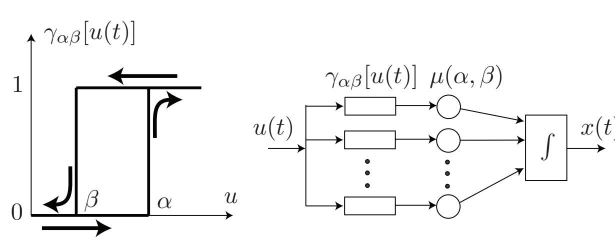

Fig. 1. (a) Hysteresis operator $\gamma_{\alpha\beta}[u(t)]$. (b) Block diagram of the Preisach model.

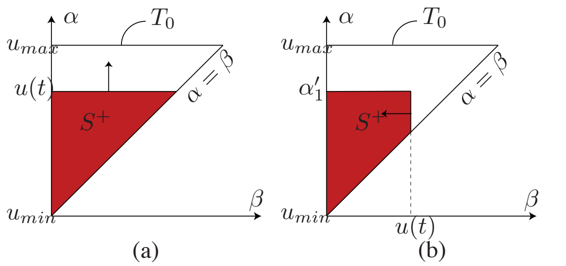

Fig. 2. Limiting triangle T_0 for (a) an ascending loop and (b) a descending loop.

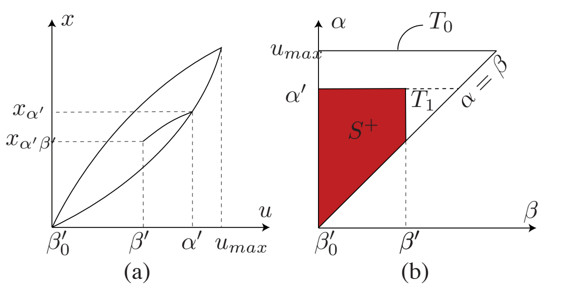

Fig. 3. (a) Expansion of the actuator for a input voltage increasing until α' and then decreasing until β'. (b) Limiting triangle T_0 related to (a).

a descending loop (Fig. 2). Since the operator $\gamma_{\alpha\beta}[u(t)]$ is equal to 0 out of the surface S^+, Equation (1) can be written as:

$$x(t) = \iint_{S^+} \mu(\alpha,\beta)\, d\alpha\, d\beta \tag{2}$$

So as to both simplify the calculation and suppress the double integration, the Preisach function is defined as follows:

$$X(\alpha',\beta') = x_{\alpha'} - x_{\alpha'\beta'} \tag{3}$$

where $x_{\alpha'}$ is the piezoelectric expansion on the major ascending branch for an input voltage α', and $x_{\alpha'\beta'}$ is the piezoelectric expansion on the first order reversal curve for an input voltage β' (Fig. 3). α' and β' represent the maxima, resp. the minima, of the input voltage $u(t)$. Figure 3b shows that Equation (3) can also be written as:

$$X(\alpha',\beta') = \iint_{T_1} \mu(\alpha,\beta)\, d\alpha\, d\beta \tag{4}$$

If the hysteresis loop contains several extrema, the surface S^+ is composed of several trapezoidal regions S_k (Fig. 4). All the extrema α'_k and β'_k that depend on the past values of the input voltage $u(t)$ are stored in the history. For the region S_1, the following equation is deduced:

$$\iint_{S_1} \mu(\alpha,\beta)\, d\alpha\, d\beta = X(\alpha'_1,\beta'_0) - X(\alpha'_1,\beta'_1) \tag{5}$$

The other regions are calculated in the same way. Because the integration on the surface S^+ is the sum of the integrations on all the surfaces S_k, the total piezoelectric expansion

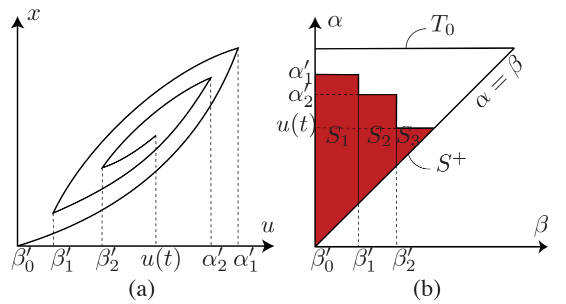

Fig. 4. (a) Hysteresis loop with several extrema α' and β'. (b) Limiting triangle T_0 related to (a).

$x(t)$ for an input voltage $u(t)$ is determined thanks to (2), depending on the current slope of $u(t)$:

$\dot{u}(t) > 0$

$$\begin{aligned} x(t) &= \sum_{k=1}^{N} [X(\alpha'_k,\beta'_{k-1}) - X(\alpha'_k,\beta'_k)] \\ &\quad + X(u(t),\beta'_N) \end{aligned} \tag{6}$$

$\dot{u}(t) < 0$

$$\begin{aligned} x(t) &= \sum_{k=1}^{N-1} [X(\alpha'_k,\beta'_{k-1}) - X(\alpha'_k,\beta'_k)] \\ &\quad + X(\alpha'_N,\beta'_{N-1}) - X(\alpha'_N,u(t)) \end{aligned} \tag{7}$$

where N is the number of maxima α'_k and minima β'_k that are stored.

So as to compute the values $X(\alpha',\beta')$, a mesh of α and β is created within T_0. The reference values $X(\alpha,\beta)$ are measured on the piezoelectric actuator for all α and β of the mesh and stored at each corresponding node (Fig. 5). Once the cell in which a given pair (α',β') lies is determined, the corresponding value $X(\alpha'_i,\beta'_j)$ is computed using a bilinear-spline interpolation:

$$X(\alpha',\beta') = a_{00} + a_{10}\alpha' + a_{01}\beta' + a_{11}\alpha'\beta' \tag{8}$$

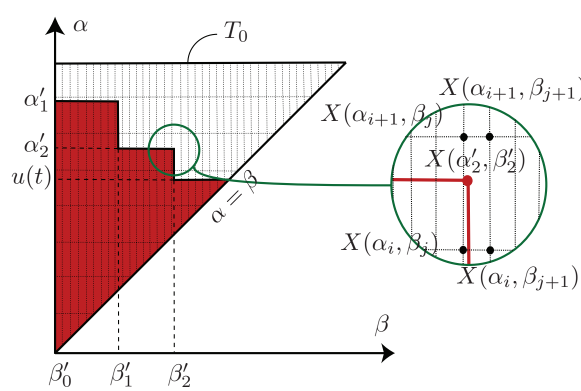

Fig. 5. Division of the limiting triangle T_0 into a finite number of rectangles and triangles.

For every $X(\alpha', \beta')$, the interpolation coefficients a_{00}, a_{10}, a_{01} and a_{11}, are obtained through the same spline interpolation based on the values of the nodes surrounding the cell $X(\alpha_i, \beta_j)$, $X(\alpha_i, \beta_{j+1})$, $X(\alpha_{i+1}, \beta_j)$ and $X(\alpha_{i+1}, \beta_{j+1})$ (Fig. 5). The expansion is determined using either (6) or (7).

To work properly, the Preisach model still needs an additional property, namely the wipe-out property. It allows to erase the pair $(\alpha'_N, \beta'_{N-1})$ from the history once $u(t)$ exceeds α'_N. Similarly, the pair (α'_N, β'_N) can be erased from the history once $u(t)$ becomes smaller than β'_N. This avoids the excessive growing of the stored values.

III. INVERSE PREISACH MODEL

So as to compensate the hysteresis of the actuator, the Preisach model has to be inverted. In other words, the voltage $u(t)$ that produces the desired expansion $x(t)$ must be determined, based on the model. This inversion is complicated by the fact that the hysteresis is a nonlinearity with a nonlocal memory. We propose a novel approach to solve this inversion problem. This is achieved by modifying Equations (6) and (7) so as to express the voltage $u(t)$ as a function of the desired expansion $x(t)$. The history of the hysteresis must however be carefully taken into account. The cases of either ascending or descending branches are treated separately.

- $\dot{u}(t) > 0$

If t_0 is defined as the time at which the input voltage reaches a local minimum, the expansion is:

$$x(t_0) = \sum_{k=1}^{N} [X(\alpha'_k, \beta'_{k-1}) - X(\alpha'_k, \beta'_k)] \qquad (9)$$

where all values $X(\alpha'_k, \beta'_{k-1})$ and $X(\alpha'_k, \beta'_k)$ are already stored in the history. As the voltage grows, the expansion is obtained with (6). Combining this result together with (9), the following relation holds:

$$X(u(t), \beta'_N) = x(t) - x(t_0) \qquad (10)$$

Equations (8) and (10) lead to the voltage

$$u(t) = \frac{x(t) - x(t_0) - a_{01}\beta'_N - a_{00}}{a_{10} + a_{11}\beta'_N} \qquad (11)$$

The only remaining problem is that the interpolation coefficients, a_{00}, a_{10}, a_{01} and a_{11}, depend on the cell which contains the value $X(u(t), \beta'_N)$. Nevertheless, it can be solved by reasoning in the $\alpha - \beta$ diagram, as illustrated in Fig. 6. When the voltage is at his local minimum, the value $X(u(t), \beta'_N)$ is located on the straight line $\alpha = \beta$ and is equal to zero. As the voltage grows, the point that contains $X(u(t), \beta'_N)$ moves up on the vertical line $\beta = \beta'_N$. By calculating the values $X(\alpha, \beta'_N)$ that lies at the intersection of the line $\beta = \beta'_N$ with the horizontal lines of the mesh, the cell which should contain the value $X(u(t), \beta'_N)$ can be determined. As the interpolation coefficients are known for each cell, the voltage $u(t)$ is the only remaining unknown value and is obtained thanks to (11).

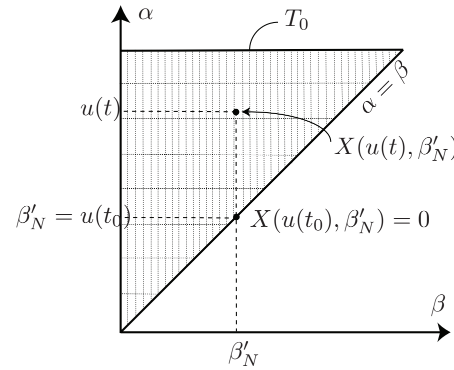

Fig. 6. Inverse Preisach model principle when $\dot{u}(t) > 0$.

- $\dot{u}(t) < 0$

The case of a descending branch is quite similar to the one treated above. t_0 is now the time when the input voltage is at a local maximum and the expansion is calculated as follows:

$$\begin{aligned} x(t_0) \quad = \quad & \sum_{k=1}^{N} [X(\alpha'_k, \beta'_{k-1}) - X(\alpha'_k, \beta'_k)] \\ & + X(\alpha'_N, \beta'_{N-1}) \end{aligned} \qquad (12)$$

As the voltage decreases, the expansion is obtained with (7) and the following relation can then be deduced:

$$X(\alpha'_N, u(t)) = x(t_0) - x(t) \qquad (13)$$

Equations (8) and (13) are then combined to find the needed voltage:

$$u(t) = \frac{x(t_0) - x(t) - a_{10}\alpha'_N - a_{00}}{a_{01} + a_{11}\alpha'_N} \qquad (14)$$

As for the ascending case, the value $X(\alpha'_N, u(t))$ is initially located on the straight line $\alpha = \beta$ and is equal to zero (Fig. 7). However the point that contains $X(\alpha'_N, u(t))$ moves to the left on the line $\alpha = \alpha'_N$. The corresponding

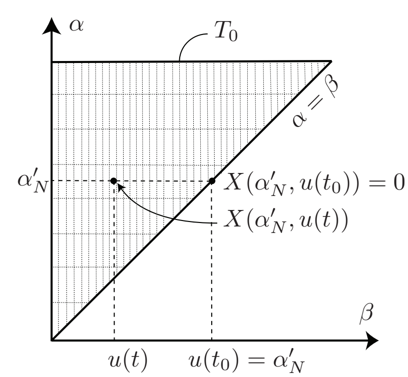

Fig. 7. Inverse Preisach model principle when $\dot{u}(t) < 0$.

cell is found thanks to the values $X(\alpha'_N, \beta)$ that lie at the intersections of the line $\alpha = \alpha'_N$ with the vertical lines of the mesh. Equation (14) can finally be used to find the voltage $u(t)$.

At each time step, the previous voltage $u(t)$ and expansion $x(t)$ are stored. The extrema α' and β' are defined when the difference between the current and the previous value of $x(t)$ changes its sign. As for the classical Preisach model, the wipe-out property can be used to simplify the storage of the extrema.

IV. RESULTS

A. Experimental setup and procedure

The piezoelectric stack actuator S-325 from PI is used to realize all the tests for a stroke between 0 and 30 μm and an input voltage of 0 to 100V. A VME industrial rack controls the input voltage of the actuator and the power is supplied by an amplifier E-505 from PI. A laser interferometer (Agilent 10897B) with 1.25 nm resolution is used to measure the displacement of the actuator which is provided to the VME rack. The measurement and control signal have both a sampling rate of 8 kHz.

To identify the Preisach model, the expansion of the actuator, x_α, is measured for a 1 Hz sinusoidal-input voltage of 30 different amplitude values, corresponding to 30 maxima α (Fig. 8). The expansion for 150 values β are then taken on the descending branch for the larger amplitude, 145 for the second larger and so on until 5 are left for the smaller one. Thanks to these measurements, all the Preisach functions $X(\alpha, \beta)$ can be calculated with (3) and stored in a 31x151 triangular matrix. The interpolation coefficients a_{ij} can also be calculated offline with (8) and saved in four 30x150 matrix. This allows minimizing the number of operations that have to be calculated in real-time. The values α', β' and $X(\alpha', \beta')$ are saved in the history as static vectors. A variable k points on those vectors and allows to simply add new values to the history without any dynamical vectors. The wipe-out property, which consists in erasing a pair (α', β') and its corresponding value $X(\alpha', \beta')$, is then performed by decrementing the variable k. At each time step, the position of each value α' and β' in the limiting triangle T_0 is determined by comparing them with the reference values α and β. As the interpolation coefficients a_{ij} are stored in the history, Equations (6) to (8), for the hysteresis modeling, and (9) to (14), for its compensation, do not require a large amount of computing power.

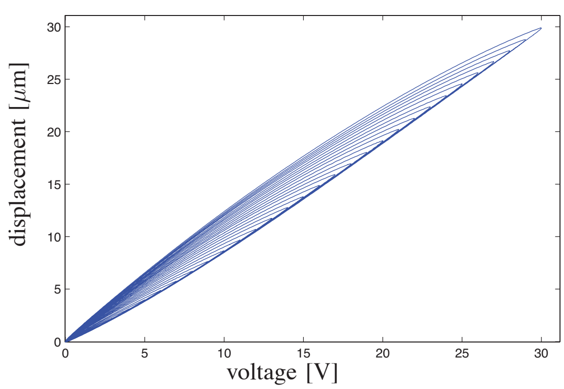

Fig. 8. Hysteresis loops used to identify the Preisach model of the piezoelectric actuator. The voltage is the one before the amplifier.

B. Simulation results

First of all, the model created is compared to the measures collected on the system to be validated. Fig. 9 and 10 illustrate how well the model fits the system for a sinusoidal input signal of 1 Hz. The modeling error is approximately equal to 75 nm rms. The maximum error is about 150 nm, which is only 1.1% of the total stroke.

The model validated is then inverted with the method developed in Section III and validated in simulation. The compensation algorithm is placed before the model and a stochastic signal is used as an input to this system. The stochastic signal, whose frequency content lies between 0

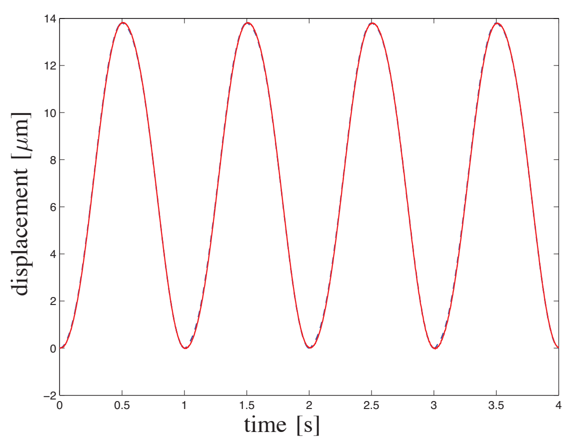

Fig. 9. Measured (continuous line) and simulated (dashed line) actuator displacement for a 1 Hz sinusoidal input voltage with an amplitude of 15 V. The signals perfectly match.

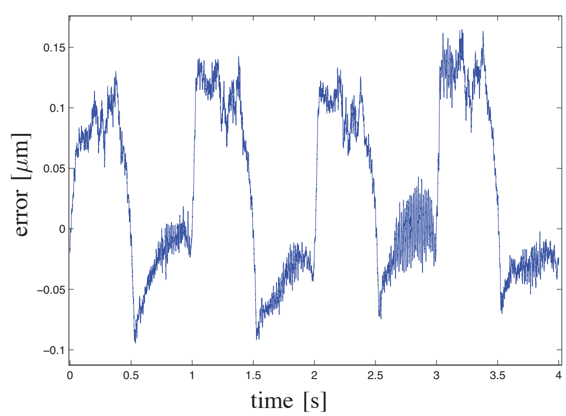

Fig. 10. Modeling error for a 1 Hz sinusoidal input voltage.

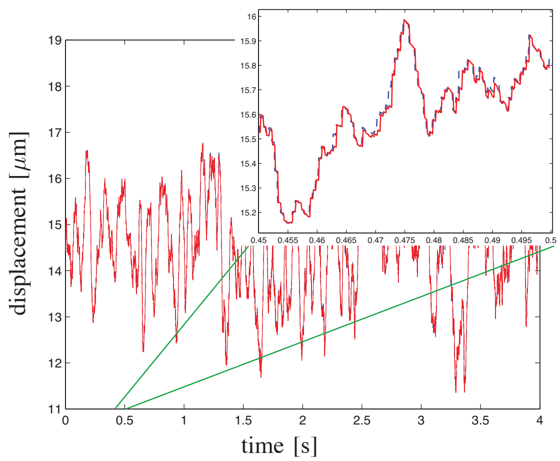

Fig. 11. Simulated actuator displacement (continuous line) for a stochastic reference (dashed line).

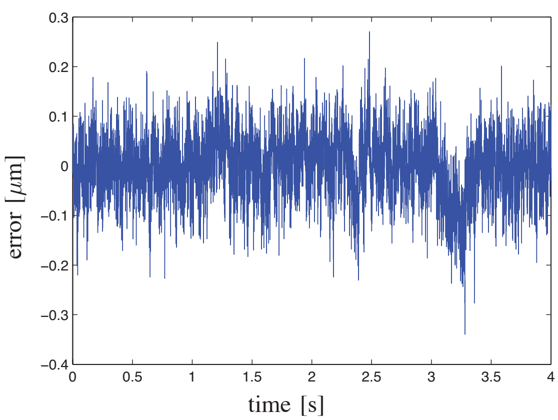

Fig. 12. Simulated tracking error for a stochastic reference.

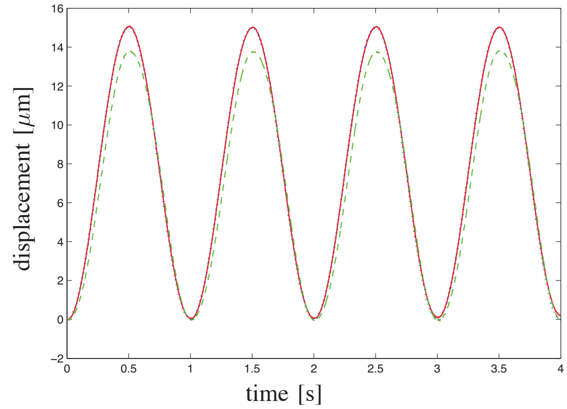

Fig. 13. Open-looped tracking of a 1 Hz sinusoidal reference (dotted line) with and without the compensation algorithm (continuous and dashed line).

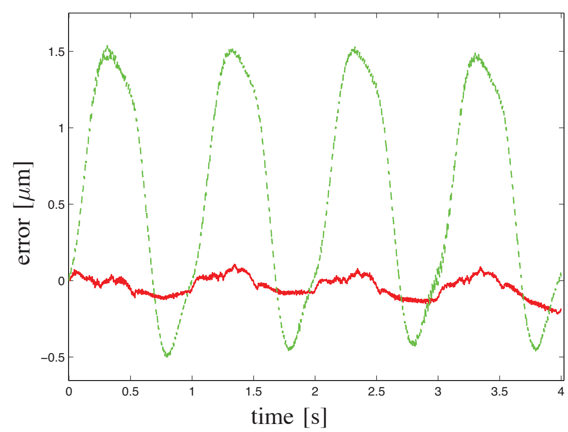

Fig. 14. Tracking error for a 1 Hz sinusoidal reference with and without the compensation algorithm (continuous and dashed line).

and 200 Hz, is chosen so as to significantly represent the perturbations that the system will have to compensate. Figures 11 and 12 show the manner in which the output perfectly follows the input, with a delay of only one sampling time. Because the model is static and does not take into account the dynamics of the piezoelectric actuator, the real system will undoubtedly have a small additional delay. Moreover, since the compensation algorithm is directly deduced from the model, the same errors will appear and counterbalance each other. Nevertheless, this simulation shows that the compensation algorithm is not only applicable for a sinusoidal input, but also for dealing with input signals that have a slope with constantly changing sign.

C. Experimentation results

The compensation algorithm is finally validated on the real system with different input signals. All the experimentations are realized in open loop. Comparisons are done between the system controlled with and without the compensation of the hysteresis. For a low frequency input signal, Figures 13 and 14 show a large improvement by using the compensation algorithm. The error decreases from 902 nm rms to approximatively 79 nm rms. The maximum error is about 150 nm, which is only 1.1% of the total stroke. By comparing this result with the one obtained during the modeling procedure (Fig. 10), it can be seen that the errors are quite similar in both cases. This shows that the major part of the error comes from the modeling and not from the inversion. With input signals of higher frequency, the sampling period necessary for the inversion involves a higher error, which is also increased by the dynamics of the piezoelectric actuator. Nevertheless, the compensation algorithm still provides far better results than a simple open-loop control. In fact, the delay due to the dynamics of the actuator stays unchanged, but the amplitude of the piezoelectric expansion is corrected. Tests have been carried out over the full 100 Hz bandwidth.

The response to a stochastic signal plotted in Figure 15 also shows a great improvement obtained by the addition of the compensation algorithm. An error of 44 nm rms can be

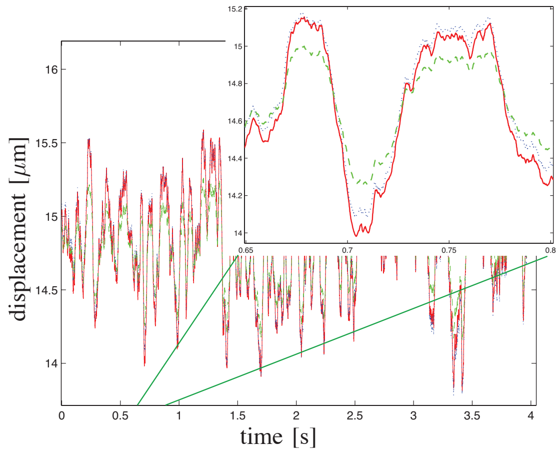

Fig. 15. Open-looped tracking of a stochastic signal (dotted line) with and without the compensation algorithm (continuous and dashed line).

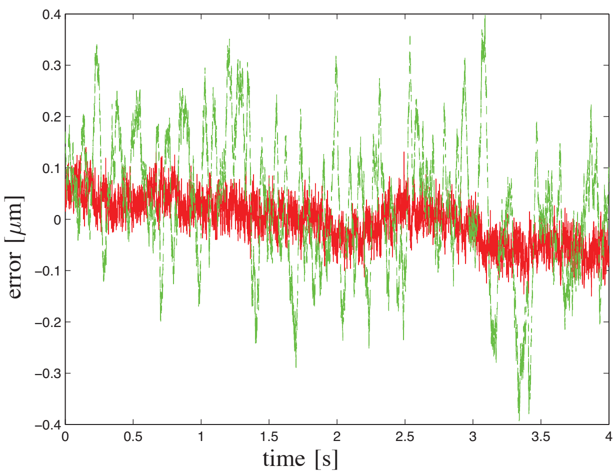

Fig. 16. Tracking error for a stochastic reference with and without the compensation algorithm (continuous and dashed line).

seen in Figure 16 instead of 134 nm rms for a simple open-loop control, that means a diminution of approximatively 67%. The maximum error is about 120 nm, which is 6% of the maximal stroke of 2 μm. In fact the results are promising even if the stochastic signal contains high frequencies that cannot be represented optimally with the model built at low frequency. However, as the amplitude of the input signal diminishes, the dynamics of the piezoelectric are less visible than with a high-amplitude sinusoidal signal. As the effect of the hysteresis slowly decreases with an increasing frequency [5], a dynamic model could be implemented to improve the performances [6]. Since both the signal is stochastic and the future values are unknown, the determination of the frequency or of the current slope can however be particularly tricky.

V. CONCLUSION

This paper presents the Preisach model of hysteresis and a novel method to invert it to realize a tracking control of a piezoelectric stack actuator. The first tests in simulation show that the proposed method allows tracking input signals that have a slope whose sign is changing constantly (see Fig. 11 and 12). The experimentations realized on the real system show good agreement between the desired and measured signals. For a 1 Hz sinusoidal reference, the error decreases of approximatively 90% compared to an open-loop control and represents only 1.1% of the total stroke of the actuator (see Fig. 13 and 14). Good performances are also obtained for a stochastic signal of frequency content comprised between 0 and 200 [Hz]. In this case, Fig. 15 and 16 show a diminution of the tracking error of about 67%, with a maximal error that represents 6% of the stroke of the actuator.

The proposed algorithm can be integrated in a feed-forward loop to improve the system performance. Future work could take into account the dynamics of the actuator so as to both improve the Preisach model and increase the tracking precision.

REFERENCES

[1] I.D. Mayergoyz, Mathematical Models of Hysteresis, *Springer-Verlag*, New York, 1991.
[2] P. Ge and M. Jouaneh, "Modeling hysteresis in piezoceramic actuators", *Precision Engineering*, vol. 17, 1995, pp. 211-221.
[3] P. Ge and M. Jouaneh, "Tracking control of a piezoceramic actuator", *IEEE Transactions on Control Systems Technology*, vol. 4, 1996, pp. 209-216.
[4] P. Ge and M. Jouaneh, "Generalized Preisach model for hysteresis nonlinearity of piezoceramic actuators", *Precision Engineering*, vol. 20, 1997, pp. 99-111.
[5] D. Damjanovic, "Stress and frequency dependence of the direct piezoelectric effect in ferroelectric ceramics", *Journal of Applied Physics*, vol. 82, 1997, pp. 1788-1797.
[6] D. Song and J. Li, "Modeling of piezo actuator's nonlinear and frequency dependent dynamics", *Mechatronics*, vol. 9, 1999, pp. 391-410.
[7] Y. Yu, Z. Xiao, N. Naganathan and R. Dukkipati, "Dynamic Preisach modeling of hysteresis for the piezoceramic actuator system", *Mechanism and Machine Theory*, vol. 37, 2002, pp. 75-89.
[8] Y. Yu, N. Naganathan and R. Dukkipati, "Preisach modeling of hysteresis for piezoceramic actuator system", *Mechanism and Machine Theory*, vol. 37, 2002, pp. 49-59.
[9] R. Ben Mrad and H. Hu, "A model for voltage-to-displacement dynamics in piezoceramic actuators subject to dynamic-voltage excitations", *IEEE/ASME Transactions on Mechatronics*, vol. 7, 2002, pp. 479–489.
[10] H. Hu and R. Ben Mrad, "On the classical Preisach model for hysteresis in piezoceramic actuators", *Mechatronics*, vol. 13, 2003, pp. 85-94.
[11] G. Song, J. Zhao, X. Zhou and J.A. De Abreu-Garcia, "Tracking Control of Piezoceramic Actuator With Hysteresis Compensation Using Inverse Preisach Model", *IEEE/ASME Transactions on Mechatronics*, vol. 10, 2005, pp. 198-209.
[12] K.K. Leang and S. Devasia, "Design of hysteresis-compensating iterative learning control for piezo-positioners: Application to atomic force microscopes", *Mechatronics*, vol. 16, 2006, pp. 141-158.
[13] H.M.S. Georgiou and R. Ben Mrad, "Electromechanical Modeling of Piezoceramic Actuators for Dynamic Loading Applications", *Journal of Dynamic Systems, Measurement and Control : Trans. ASME*, vol. 128, 2006, pp. 558-567.
[14] Y. Michellod, P. Mullhaupt and D Gillet, "Strategy for the Control of a Dual-stage Nano-positioning System with a Single Metrology", *Proceedings on the IEEE Conference on Robotics, Automation and Mechatronics*, 2006.