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Nonquadratic Lyapunov Function Based Control Law Design for Discrete Fuzzy Systems with State And Input Delays

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Abstract— **This paper deals with the stability analysis and the stabilization control law for a class of discrete Takagi-Sugeno (T-S) fuzzy systems with both state and input delays. Based on the nonquadratic Lyapunov function constructed here, the stability analysis and the design way of stabilization control law are derived in the form of linear matrix inequality (LMI) via a nonparallel distributed compensation (non-PDC) scheme. The new conclusion is also suitable for a PDC law under a special situation. Two numerical examples are supplied to demonstrate the effectiveness of the designed control law. And both theoretical analysis and numerical examples illustrate that these novel sufficient conditions are less conservative than previous results obtained within the quadratic framework.**

I. INTRODUCTION

Fuzzy control based on Takagi-Sugeno (T-S) fuzzy model [9] has attracted considerable attention and is widely applied to a variety of industrial processes. Recently, a great amount efforts have been devoted to stability analysis and systematic design of T-S fuzzy control laws [1-5, 7-10, 12-15]. Most of the works constructed a quadratic Lyapunov function, i.e. $V(\boldsymbol{x}(t)) = \boldsymbol{x}^T(t)\boldsymbol{P}\boldsymbol{x}(t)$ with $\boldsymbol{P} = \boldsymbol{P}^T > \boldsymbol{0}$. Employing the so-called parallel distributed compensation (PDC) scheme [2], this common matrix $\boldsymbol{P}$ for stabilization of all subsystems can be found by solving linear matrix inequality (LMI) constraints with standard numerical softwares.

Moreover, to release the conservatism of the common quadratic Lyapunov function approach for fuzzy control systems, many other works can be found dealing with piecewise quadratic Lyapunov functions (PQLF) [7, 14]. Another significant contribution in this area was given in [1, 4, 5, 8-10, 12, 15]. All of these works tackled stability analysis and stabilization method of both continuous and discrete T-S fuzzy systems in the nonquadratic framework. Among them, parameter-dependent Lyapunov functions (PDLF) [8] (also named as fuzzy Lyapunov functions [4] and multiple Lyapunov functions [5]), piecewise fuzzy Lyapunov functions (PFLF) [9, 15] were studied as well in the interest of obtaining less conservative stability and stabilization conditions.

On the other hand, it is well known that there exist many complex nonlinear systems with time delays in practice such as communication networks. Usually a time delay is frequently a source of instability generating oscillations in many systems. It is thus natural to extend the T-S fuzzy model to the corresponding time-delay model [8, 13]. This paper deals with the stability analysis and the control law design for a class of discrete T-S fuzzy systems with both state and input delays based on a nonquadratic Lyapunov function. By applying a matrix transformation that can be viewed as an extension to the Schur complement, a non-PDC control law is systematically designed as a set of LMIs. The proposed conclusion is also suitable for a PDC law under a special situation. Two examples illustrate the utility and advantage of this nonquadratic approach and non-PDC stabilization method.

The organization of this paper is as follows: Section II provides preliminaries and the formulation of the control problem for discrete T-S fuzzy systems. In Section III, stability analysis of the open-loop delayed discrete T-S systems on the basis of a nonquadratic Lyapunov function is considered. In Section IV a non-PDC control law for stabilization of T-S fuzzy systems is put forward in terms of LMIs. Two numerical examples are given in Section V to illustrate the validity of the design method presented in this paper. And these are followed by some concluding remarks in Section VI.

The notations used in this paper are quite standard: $\mathbb{R}^n$ denotes the $n-$dimensional real Euclidean space; $\mathbb{R}^{n\times m}$ signifies space of $n \times m$ real matrices; $\boldsymbol{I}$ and $\boldsymbol{0}$ represent identity matrix and zero matrix of appropriate dimensions; the superscripts 'T' and '-1' stand for the matrix transpose and inverse respectively; $\boldsymbol{A} > \boldsymbol{0}$ means that $\boldsymbol{A}$ is symmetric and positive definite; $\|\cdot\|$ denotes the spectral norm; $\lambda_{min}(\cdot)$ and $\lambda_{max}(\cdot)$ are the smallest and the largest eigenvalue of a matrix; $diag(\boldsymbol{A}_1, \cdots, \boldsymbol{A}_n)$ refers to a diagonal matrix with $\boldsymbol{A}_i$ as its ith diagonal element.

II. PROBLEM FORMULATION AND PRELIMINARIES

Consider a discrete T-S fuzzy system as follows:
R^i: If $\xi_1(t)$ is M_{i1} and … and $\xi_n(t)$ is M_{in}, then

$$\begin{cases} \boldsymbol{x}(t+1) = \boldsymbol{A}_i\boldsymbol{x}(t) + \bar{\boldsymbol{A}}_i\boldsymbol{x}(t-\tau_1) + \boldsymbol{B}_i\boldsymbol{u}(t) \\ \qquad\qquad + \bar{\boldsymbol{B}}_i\boldsymbol{u}(t-\tau_2), \\ \boldsymbol{x}(s) = \boldsymbol{\phi}(s), \quad s = -\tau, -\tau+1, \dots, 0, \\ \qquad\qquad i = 1, 2, \dots, r. \end{cases} \tag{1}$$

where R^i denotes the ith rule, r is the number of rules, M_{ij} is the fuzzy set, $\boldsymbol{x}(t) \in \mathbb{R}^n$ is the state vector, $\boldsymbol{u}(t) \in \mathbb{R}^m$ is the control input. $\boldsymbol{\xi}(t) = [\xi_1(t), \dots, \xi_n(t)]^T$ is the measurable premise vector, which is a function of states and not supposed to depend on the control input. $\boldsymbol{A}_i, \bar{\boldsymbol{A}}_i \in \mathbb{R}^{n\times n}, \boldsymbol{B}_i, \bar{\boldsymbol{B}}_i \in \mathbb{R}^{n\times m}$. $\tau_1, \tau_2 \in (0, \infty)$ are two real positive constants representing state delay and input delay

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respectively. $\phi(s)$ is a discrete vector-valued initial function on $\{-\tau, -\tau+1, \ldots, 0\}$, where $\tau = max\{\tau_1, \tau_2\}$.

Given a pair of $(\boldsymbol{x}(t), \boldsymbol{u}(t))$, the final output of the fuzzy system as follows may be inferred by using a singleton fuzzifier, product inference and a center-average defuzzifier:

$$\begin{cases} \boldsymbol{x}(t+1) = \sum_{i=1}^{r} h_i(\boldsymbol{\xi}(t))(\boldsymbol{A}_i\boldsymbol{x}(t) + \bar{\boldsymbol{A}}_i\boldsymbol{x}(t-\tau_1) \\ \qquad\qquad + \boldsymbol{B}_i\boldsymbol{u}(t) + \bar{\boldsymbol{B}}_i\boldsymbol{u}(t-\tau_2)), \\ \boldsymbol{x}(s) = \boldsymbol{\phi}(s), \quad s = -\tau, -\tau+1, \ldots, 0. \end{cases} \tag{2}$$

where $h_i(\boldsymbol{\xi}(t))$ denotes the normalized weight for each rule and, $h_i(\boldsymbol{\xi}(t)) = \mu_i(\boldsymbol{\xi}(t))/\sum_{i=1}^{r}\mu_i(\boldsymbol{\xi}(t))$; $\mu_i(\boldsymbol{\xi}(t)) = \prod_{j=1}^{n} M_{ij}(\xi_j(t))$. $M_{ij}(\xi_j(t))$ is the grade of membership function of $\xi_j(t)$ in the fuzzy set M_{ij}. In this paper, we presume that $\mu_i(\boldsymbol{\xi}(t)) \geq 0$ (for i=1, 2, $\ldots$, r) and $\sum_{i=1}^{r}\mu_i(\boldsymbol{\xi}(t)) > 0$ for all t. Thus we get $h_i(\boldsymbol{\xi}(t)) \geq 0$ and $\sum_{i=1}^{r} h_i(\boldsymbol{\xi}(t)) = 1$.

In this paper, the following notations will be employed for simplicity: $\boldsymbol{X}_\xi = \sum_{i=1}^{r} h_i(\boldsymbol{\xi}(t))\boldsymbol{X}_i$, $\boldsymbol{X}_{\xi+1} = \sum_{i=1}^{r} h_i(\boldsymbol{\xi}(t+1))\boldsymbol{X}_i$ and also $\boldsymbol{X}_\xi^{-1} = (\sum_{i=1}^{r} h_i(\boldsymbol{\xi}(t))\boldsymbol{X}_i)^{-1}$. Then the discrete T-S fuzzy system (2) can be described in the following form:

$$\begin{cases} \boldsymbol{x}(t+1) = \boldsymbol{A}_\xi\boldsymbol{x}(t) + \bar{\boldsymbol{A}}_\xi\boldsymbol{x}(t-\tau_1) + \boldsymbol{B}_\xi\boldsymbol{u}(t) \\ \qquad\qquad + \bar{\boldsymbol{B}}_\xi\boldsymbol{u}(t-\tau_2), \\ \boldsymbol{x}(s) = \boldsymbol{\phi}(s), \quad s = -\tau, -\tau+1, \ldots, 0. \end{cases} \tag{3}$$

For the nonquadratic case, the principal results are derived from a property due to [6], which is slightly modified as:

Lemma 1. $\boldsymbol{\Lambda}$ *and* $\boldsymbol{\Theta}$ *are two matrices with proper dimensions. The following conditions are equivalent:*
(i) There exists a symmetric matrix $\boldsymbol{P} > \boldsymbol{0}$ *such that*

$$\boldsymbol{\Lambda}^T\boldsymbol{P}\boldsymbol{\Lambda} - \boldsymbol{\Theta} < \boldsymbol{0}. \tag{4}$$

(ii) There exist a symmetric matrix $\boldsymbol{P} > \boldsymbol{0}$ *and a matrix* $\boldsymbol{\Upsilon}$ *with proper dimension such that*

$$\begin{bmatrix} \boldsymbol{\Theta} & * \\ \boldsymbol{\Upsilon}\boldsymbol{\Lambda} & \boldsymbol{\Upsilon} + \boldsymbol{\Upsilon}^T - \boldsymbol{P} \end{bmatrix} > \boldsymbol{0}. \tag{5}$$

Proof. Multiplying (5) by $\boldsymbol{Y} := [\boldsymbol{I} \quad -\boldsymbol{\Lambda}^T]$ on the left and by $\boldsymbol{Y}^T$ on the right we get (4) which establishes that (ii) implies (i). Then by choosing $\boldsymbol{\Upsilon} = \boldsymbol{\Upsilon}^T = \boldsymbol{P}$ and applying Schur complement (4) can be recovered. Hence (i) implies (ii) and concludes this proof. □

III. BASIC STABILITY ANALYSIS

We first recall the free T-S fuzzy system with time delay, i.e., $\boldsymbol{u} \equiv 0$,

$$\boldsymbol{x}(t+1) = \boldsymbol{A}_\xi\boldsymbol{x}(t) + \bar{\boldsymbol{A}}_\xi\boldsymbol{x}(t-\tau_1). \tag{6}$$

In order to develop the sufficient stability conditions of system (6) via nonquadratic Lyapunov function, first let us define:

$$\boldsymbol{\Omega}_{ij}^{l} = \begin{bmatrix} \boldsymbol{P}_i & * & \boldsymbol{0} & * \\ \boldsymbol{G}_i & \boldsymbol{S} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{S} & * \\ \boldsymbol{A}_i\boldsymbol{G}_j & \boldsymbol{0} & \bar{\boldsymbol{A}}_i\boldsymbol{S} & \boldsymbol{G}_l + \boldsymbol{G}_l^T - \boldsymbol{P}_l \end{bmatrix}, \quad i, j, l \in \{1, \ldots, r\}. \tag{7}$$

In addition, some definitions are given as follows:

Definition 1.[3] For a discrete system $\boldsymbol{X}(k+1) = f(\boldsymbol{X}(k)), \boldsymbol{X}(k) \in \mathbb{R}^n, f(\boldsymbol{X}(k)) \in \mathbb{R}^n$ is a function vector with the property $f(0) = 0$ for all k. If there exists a scalar function $V(\boldsymbol{X}(k))$ continuous in $\boldsymbol{x}(k)$ such that:
a) $V(0) = 0$;
b) $V(\boldsymbol{X}(k)) > 0$ for all $\boldsymbol{X}(k) \neq 0$;
c) $V(\boldsymbol{X}(k)) \to \infty$ as $\|\boldsymbol{X}(k)\| \to \infty$;
d) $L \triangleq V(\boldsymbol{X}(k+1)) - V(\boldsymbol{X}(k)) < 0$ for all $\boldsymbol{X}(k) \neq 0$.
Then the system equilibrium state $\boldsymbol{X}(k) = 0$ is asymptotically stable in the large for all k and, $V(\boldsymbol{X}(k))$ is a Lyapunov function.

Definition 2. The unforced T-S fuzzy system with time delay described by (6) is globally asymptotically stable for any $\tau \in (0, \infty)$ if, there exists a Lyapunov function $V(\boldsymbol{x}(t))$ such that for all $\boldsymbol{x}(t) \neq 0$, the inequality $L \triangleq V(\boldsymbol{x}(t+1)) - V(\boldsymbol{x}(t)) < 0$ is guaranteed.

Theorem 1. *Consider the free T-S fuzzy system with time delay represented by (6). With* $\boldsymbol{\Omega}_{ij}^{l}$ *defined in (7), if there exist matrices* $\boldsymbol{P}_i > \boldsymbol{0}$, $\boldsymbol{S} > \boldsymbol{0}$ *and matrices* $\boldsymbol{G}_i$ *with proper dimensions such that*

$$\boldsymbol{\Omega}_{ii}^{l} > \boldsymbol{0}, \quad i, l \in \{1, \ldots, r\}, \tag{8}$$

$$\boldsymbol{\Omega}_{ij}^{l} + \boldsymbol{\Omega}_{ji}^{l} > \boldsymbol{0}, \quad i < j, \quad i, j, l \in \{1, \ldots, r\}. \tag{9}$$

then the free system (6) is globally asymptotically stable for any $\tau_1 \in (0, \infty)$.

Proof. Let us construct a candidate nonquadratic Lyapunov function with $\boldsymbol{S} > \boldsymbol{0}$, $\boldsymbol{P}_i > \boldsymbol{0}$, and proper dimensional matrices $\boldsymbol{G}_i, i \in \{1, \ldots, r\}$ as follows:

$$\begin{aligned} V(\boldsymbol{x}(t)) &= \boldsymbol{x}^T(t)(\textstyle\sum_{i=1}^{r} h_i(\boldsymbol{\xi}(t))\boldsymbol{G}_i)^{-T}(\sum_{i=1}^{r} h_i(\boldsymbol{\xi}(t))\boldsymbol{P}_i) \\ &\quad (\textstyle\sum_{i=1}^{r} h_i(\boldsymbol{\xi}(t))\boldsymbol{G}_i)^{-1}\boldsymbol{x}(t) + \sum_{i=1}^{\tau_1}\boldsymbol{x}^T(t-i) \\ &\quad \times\boldsymbol{S}^{-1}\boldsymbol{x}(t-i) \\ &= \boldsymbol{x}^T(t)\boldsymbol{G}_\xi^{-T}\boldsymbol{P}_\xi\boldsymbol{G}_\xi^{-1}\boldsymbol{x}(t) + \textstyle\sum_{i=1}^{\tau_1}\boldsymbol{x}^T(t-i)\boldsymbol{S}^{-1} \\ &\quad \times\boldsymbol{x}(t-i). \end{aligned} \tag{10}$$

It is necessary to check first the validity of the candidate Lyapunov function. As $\forall i$, $\boldsymbol{P}_i > \boldsymbol{0}$ and $h_i(\boldsymbol{\xi}(t)) \geq 0$ having a convex sum property, $\boldsymbol{P}_\xi > \boldsymbol{0}$ for every $\boldsymbol{\xi}(t)$. The regularity of $\boldsymbol{G}_i$ is ensured by the last blocks of conditions (8) in the sense that with conditions (8) holding true we have $\boldsymbol{G}_i + \boldsymbol{G}_i^T > \boldsymbol{P}_i > \boldsymbol{0}$. Therefore the existence of $\boldsymbol{G}_\xi^{-1} = (\sum_{i=1}^{r} h_i(\boldsymbol{\xi}(t))\boldsymbol{G}_i)^{-1}$ is guaranteed. Moreover, taking $\boldsymbol{S} > \boldsymbol{0}$ into account, the following inequality can be derived: $\lambda_{min}(\boldsymbol{P}_\xi)\lambda_{max}(\boldsymbol{G}_\xi\boldsymbol{G}_\xi^T)\|\boldsymbol{x}\|^2 \leq V \leq \lambda_{max}(\boldsymbol{P}_\xi)\lambda_{min}(\boldsymbol{G}_\xi\boldsymbol{G}_\xi^T)\|\boldsymbol{x}\|^2 + \tau_1\lambda_{min}(\boldsymbol{S})\|\boldsymbol{x}(t)\|^2$, that ensures V to be a valid Lyapunov function.

The variation of the nonquadratic Lyapunov function (10) is given by

$$\begin{aligned} L &\triangleq V(\boldsymbol{x}(t+1)) - V(\boldsymbol{x}(t)) \\ &= \boldsymbol{x}^T(t+1)\boldsymbol{G}_{\xi+1}^{-T}\boldsymbol{P}_{\xi+1}\boldsymbol{G}_{\xi+1}^{-1}\boldsymbol{x}(t+1) \\ &\quad + \textstyle\sum_{i=1}^{\tau_1}\boldsymbol{x}^T(t+1-i)\boldsymbol{S}^{-1}\boldsymbol{x}(t+1-i) - \boldsymbol{x}^T(t)\boldsymbol{G}_\xi^{-T} \\ &\quad \times\boldsymbol{P}_\xi\boldsymbol{G}_\xi^{-1}\boldsymbol{x}(t) - \textstyle\sum_{i=1}^{\tau_1}\boldsymbol{x}^T(t-i)\boldsymbol{S}^{-1}\boldsymbol{x}(t-i) \\ &= \boldsymbol{x}^T(t)(\boldsymbol{A}_\xi^T\boldsymbol{G}_{\xi+1}^{-T}\boldsymbol{P}_{\xi+1}\boldsymbol{G}_{\xi+1}^{-1}\boldsymbol{A}_\xi + \boldsymbol{S}^{-1} - \boldsymbol{G}_\xi^{-T}\boldsymbol{P}_\xi\boldsymbol{G}_\xi^{-1}) \end{aligned}$$

$$\begin{aligned}&\times\boldsymbol{x}(t)+\boldsymbol{x}^T(t-\tau_1)(\bar{\boldsymbol{A}}_\xi^T\boldsymbol{G}_{\xi+1}^{-T}\boldsymbol{P}_{\xi+1}\boldsymbol{G}_{\xi+1}^{-1}\bar{\boldsymbol{A}}_\xi-\boldsymbol{S}^{-1})\\&\times\boldsymbol{x}(t-\tau_1)+\boldsymbol{x}^T(t)\boldsymbol{A}_\xi^T\boldsymbol{G}_{\xi+1}^{-T}\boldsymbol{P}_{\xi+1}\boldsymbol{G}_{\xi+1}^{-1}\bar{\boldsymbol{A}}_\xi\boldsymbol{x}(t-\tau_1)\\&+\boldsymbol{x}^T(t-\tau_1)\bar{\boldsymbol{A}}_\xi^T\boldsymbol{G}_{\xi+1}^{-T}\boldsymbol{P}_{\xi+1}\boldsymbol{G}_{\xi+1}^{-1}\boldsymbol{A}_\xi\boldsymbol{x}(t)\\=&\begin{bmatrix}\boldsymbol{x}^T(t) & \boldsymbol{x}^T(t-\tau_1)\end{bmatrix}\times\\&\begin{bmatrix}\begin{pmatrix}\boldsymbol{A}_\xi^T\boldsymbol{G}_{\xi+1}^{-T}\boldsymbol{P}_{\xi+1}\boldsymbol{G}_{\xi+1}^{-1}\boldsymbol{A}_\xi\\+\boldsymbol{S}^{-1}-\boldsymbol{G}_\xi^{-T}\boldsymbol{P}_\xi\boldsymbol{G}_\xi^{-1}\end{pmatrix} & *\\ \bar{\boldsymbol{A}}_\xi^T\boldsymbol{G}_{\xi+1}^{-T}\boldsymbol{P}_{\xi+1}\boldsymbol{G}_{\xi+1}^{-1}\boldsymbol{A}_\xi & (2,2)\end{bmatrix}\begin{bmatrix}\boldsymbol{x}(t)\\ \boldsymbol{x}(t-\tau_1)\end{bmatrix},\end{aligned}\tag{11}$$

where $(2,2)=\bar{\boldsymbol{A}}_\xi^T\boldsymbol{G}_{\xi+1}^{-T}\boldsymbol{P}_{\xi+1}\boldsymbol{G}_{\xi+1}^{-1}\bar{\boldsymbol{A}}_\xi-\boldsymbol{S}^{-1}$. Thus $L<0$ indicates

$$\begin{bmatrix}\boldsymbol{A}_\xi^T\boldsymbol{G}_{\xi+1}^{-T}\boldsymbol{P}_{\xi+1}\boldsymbol{G}_{\xi+1}^{-1}\boldsymbol{A}_\xi+\boldsymbol{S}^{-1}-\boldsymbol{G}_\xi^{-T}\boldsymbol{P}_\xi\boldsymbol{G}_\xi^{-1} & *\\ \bar{\boldsymbol{A}}_\xi^T\boldsymbol{G}_{\xi+1}^{-T}\boldsymbol{P}_{\xi+1}\boldsymbol{G}_{\xi+1}^{-1}\boldsymbol{A}_\xi & (2,2)\end{bmatrix}<\boldsymbol{0}.\tag{12}$$

Multiplying left and right by $diag(\boldsymbol{G}_\xi^T,\boldsymbol{S}^T)$ and $diag(\boldsymbol{G}_\xi,\boldsymbol{S})$ to (12) respectively, the following inequality equivalent to (12) can be obtained:

$$\begin{aligned}&\begin{bmatrix}\begin{pmatrix}\boldsymbol{G}_\xi^T\boldsymbol{A}_\xi^T\boldsymbol{G}_{\xi+1}^{-T}\boldsymbol{P}_{\xi+1}\boldsymbol{G}_{\xi+1}^{-1}\boldsymbol{A}_\xi\boldsymbol{G}_\xi\\+\boldsymbol{G}_\xi^T\boldsymbol{S}^{-1}\boldsymbol{G}_\xi-\boldsymbol{P}_\xi\end{pmatrix} & *\\ \boldsymbol{S}^T\bar{\boldsymbol{A}}_\xi^T\boldsymbol{G}_{\xi+1}^{-T}\boldsymbol{P}_{\xi+1}\boldsymbol{G}_{\xi+1}^{-1}\boldsymbol{A}_\xi\boldsymbol{G}_\xi & \boldsymbol{S}^T(2,2)\boldsymbol{S}\end{bmatrix}\\=&\begin{bmatrix}\boldsymbol{G}_\xi^T\boldsymbol{S}^{-1}\boldsymbol{G}_\xi-\boldsymbol{P}_\xi & \boldsymbol{0}\\ \boldsymbol{0} & -\boldsymbol{S}\end{bmatrix}+\begin{bmatrix}\boldsymbol{G}_\xi^T\boldsymbol{A}_\xi^T\\ \boldsymbol{S}^T\bar{\boldsymbol{A}}_\xi^T\end{bmatrix}\boldsymbol{G}_{\xi+1}^{-T}\boldsymbol{P}_{\xi+1}\\&\times\boldsymbol{G}_{\xi+1}^{-1}\begin{bmatrix}\boldsymbol{A}_\xi\boldsymbol{G}_\xi & \bar{\boldsymbol{A}}_\xi\boldsymbol{S}\end{bmatrix}\\<&\,\boldsymbol{0}.\end{aligned}\tag{13}$$

According to Lemma 1, let $\boldsymbol{\Lambda}=\boldsymbol{G}_{\xi+1}^{-1}\begin{bmatrix}\boldsymbol{A}_\xi\boldsymbol{G}_\xi & \bar{\boldsymbol{A}}_\xi\boldsymbol{S}\end{bmatrix}$, (13) can be rewritten in the following form:

$$\begin{bmatrix}\boldsymbol{P}_\xi-\boldsymbol{G}_\xi^T\boldsymbol{S}^{-1}\boldsymbol{G}_\xi & \boldsymbol{0} & *\\ \boldsymbol{0} & \boldsymbol{S} & *\\ \boldsymbol{A}_\xi\boldsymbol{G}_\xi & \bar{\boldsymbol{A}}_\xi\boldsymbol{S} & \boldsymbol{G}_{\xi+1}+\boldsymbol{G}_{\xi+1}^T-\boldsymbol{P}_{\xi+1}\end{bmatrix}>\boldsymbol{0}.\tag{14}$$

Applying Schur complement to (14) we can get

$$\begin{bmatrix}\boldsymbol{P}_\xi & * & \boldsymbol{0} & *\\ \boldsymbol{G}_\xi & \boldsymbol{S} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{S} & *\\ \boldsymbol{A}_\xi\boldsymbol{G}_\xi & \boldsymbol{0} & \bar{\boldsymbol{A}}_\xi\boldsymbol{S} & \boldsymbol{G}_{\xi+1}+\boldsymbol{G}_{\xi+1}^T-\boldsymbol{P}_{\xi+1}\end{bmatrix}>\boldsymbol{0}.\tag{15}$$

When conditions (8) and (9) hold true, $\sum_{l=1}^r h_l(\boldsymbol{\xi}(t+1))(\sum_{i=1}^r h_i^2(\boldsymbol{\xi}(t))\boldsymbol{\Omega}_{ii}^l+\sum_{i=1}^r\sum_{i<j}h_i(\boldsymbol{\xi}(t))h_j(\boldsymbol{\xi}(t))(\boldsymbol{\Omega}_{ij}^l+\boldsymbol{\Omega}_{ji}^l))>\boldsymbol{0}$, which leads to (15) and $L<0$ is ensured. Thus the free system (6) is globally asymptotically stable. □

Remark 1. Theorem 1 is a generalization of the previous quadratic results. Actually, for all $i\in\{1,\cdots,r\}$, the CQLF case can be recovered from using $\boldsymbol{P}^{-1}=\boldsymbol{G}_i=\boldsymbol{P}_i$, while the simple PDLF case can be deduced by choosing $\boldsymbol{G}_i=\boldsymbol{P}_i$, i.e. $V(\boldsymbol{x}(t))=\boldsymbol{x}^T(t)(\sum_{i=1}^r h_i(\boldsymbol{\xi}(t))\boldsymbol{P}_i)^{-1}\boldsymbol{x}(t)+\sum_{i=1}^{\tau_1}\boldsymbol{x}^T(t-i)\boldsymbol{S}^{-1}\boldsymbol{x}(t-i)$.

Remark 2. The conditions (9) and (10) in Theorem 1 are all LMIs, which can be readily checked by using standard numerical software packages, for example, LMI toolbox in MATLAB.

Remark 3. Theorem 1 is less conservative than the quadratic case due to the presence of the extra degree of freedom provided by the introduction of matrices $\boldsymbol{G}_i$ which are not even constrained to be symmetric, that leads to the Lyapunov matrices $\boldsymbol{P}_i$ is not involved in any product with the dynamic matrices $\boldsymbol{A}_i$ and delay state matrices $\bar{\boldsymbol{A}}_i$. However, this relaxed condition is obtained at the cost of the increment of LMIs. Noting that in Theorem 1 the number of LMIs is $r^2(1+r)/2$ instead of r^2 in the PDLF case and r in the CQLF case.

IV. STABILIZATION CONTROL LAW DESIGN VIA NON-PDC SCHEME

Consider a non-PDC control law as

$$\begin{aligned}\boldsymbol{u}(t)&=-(\textstyle\sum_{i=1}^r h_i(\boldsymbol{\xi}(t))\boldsymbol{F}_i)(\sum_{j=1}^r h_j(\boldsymbol{\xi}(t))\boldsymbol{G}_j)^{-1}\boldsymbol{x}(t)\\&=-\boldsymbol{F}_\xi\boldsymbol{G}_\xi^{-1}\boldsymbol{x}(t).\end{aligned}\tag{16}$$

Then the closed-loop T-S fuzzy system composed of (3) and (16) is described by

$$\begin{cases}\boldsymbol{x}(t+1)=(\boldsymbol{A}_\xi-\boldsymbol{B}_\xi\boldsymbol{F}_\xi\boldsymbol{G}_\xi^{-1})\boldsymbol{x}(t)+\bar{\boldsymbol{A}}_\xi\boldsymbol{x}(t-\tau_1)\\ \qquad -\bar{\boldsymbol{B}}_\xi\boldsymbol{F}_\xi\boldsymbol{G}_\xi^{-1}\boldsymbol{x}(t-\tau_2),\\ \boldsymbol{x}(s)=\boldsymbol{\phi}(s),\quad s=-\tau,-\tau+1,\ldots,0.\end{cases}\tag{17}$$

In the balance of this section, based on the nonquadratic Lyapunov function, the control law design for stabilization of the T-S fuzzy system with time delay as (17) via the non-PDC scheme (16) is studied.

Theorem 2. *For a discrete T-S fuzzy system with both state and input delays represented by (17), if there exist symmetric matrices $\boldsymbol{P}_i>\boldsymbol{0},\boldsymbol{S}>\boldsymbol{0},\boldsymbol{Q}>\boldsymbol{0}$, and matrices $\boldsymbol{F}_i,\boldsymbol{G}_i$ with proper dimensions such that Eqs. (8), (9) are satisfied, where*

$$\boldsymbol{\Omega}_{ij}^l=\begin{bmatrix}\boldsymbol{P}_i-\boldsymbol{Q} & * & \boldsymbol{0} & \boldsymbol{0} & *\\ \boldsymbol{G}_i & \boldsymbol{S} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{S} & \boldsymbol{0} & *\\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{Q} & *\\ \boldsymbol{A}_i\boldsymbol{G}_j-\boldsymbol{B}_i\boldsymbol{F}_j & \boldsymbol{0} & \bar{\boldsymbol{A}}_i\boldsymbol{S} & -\bar{\boldsymbol{B}}_i\boldsymbol{F}_j & \boldsymbol{G}_l+\boldsymbol{G}_l^T-\boldsymbol{P}_l\end{bmatrix},\quad i,j,l\in\{1,\ldots,r\},\tag{18}$$

then the non-PDC control law (16) makes the closed-loop T-S fuzzy system (17) globally asymptotically stable.

Proof. First the following nonquadratic Lyapunov function is constructed with proper dimensional matrices $\boldsymbol{G}_i$ and symmetric matrices $\boldsymbol{P}_i>\boldsymbol{0},i\in\{1,\ldots,r\}$, $\boldsymbol{S}>\boldsymbol{0}$, $\boldsymbol{Q}>\boldsymbol{0}$.

$$\begin{aligned}V(\boldsymbol{x}(t))=&\,\boldsymbol{x}^T(t)\boldsymbol{G}_\xi^{-T}\boldsymbol{P}_\xi\boldsymbol{G}_\xi^{-1}\boldsymbol{x}(t)+\textstyle\sum_{i=1}^{\tau_1}\boldsymbol{x}^T(t-i)\\&\times\boldsymbol{S}^{-1}\boldsymbol{x}(t-i)+\textstyle\sum_{i=1}^{\tau_2}\boldsymbol{x}^T(t-i)\boldsymbol{G}_\xi^{-T}\boldsymbol{Q}\\&\times\boldsymbol{G}_\xi^{-1}\boldsymbol{x}(t-i).\end{aligned}\tag{19}$$

The proof of the Lyapunov function's validity is almost the same as in Theorem 1 and is omitted.

Consider the variation of the nonquadratic Lyapunov function (19):

$$
\begin{aligned}
L \triangleq\ & V(\boldsymbol{x}(t+1)) - V(\boldsymbol{x}(t)) \\
=\ & \Big[(\boldsymbol{A}_\xi - \boldsymbol{B}_\xi \boldsymbol{F}_\xi \boldsymbol{G}_\xi^{-1})\boldsymbol{x}(t) + \bar{\boldsymbol{A}}_\xi \boldsymbol{x}(t-\tau_1) - \bar{\boldsymbol{B}}_\xi \boldsymbol{F}_\xi \\
& \times \boldsymbol{G}_\xi^{-1}\boldsymbol{x}(t-\tau_2)\Big]^T \boldsymbol{G}_{\xi+1}^{-T}\boldsymbol{P}_{\xi+1}\boldsymbol{G}_{\xi+1}^{-1}\big[(\boldsymbol{A}_\xi - \boldsymbol{B}_\xi \boldsymbol{F}_\xi \\
& \times \boldsymbol{G}_\xi^{-1})\boldsymbol{x}(t) + \bar{\boldsymbol{A}}_\xi \boldsymbol{x}(t-\tau_1) - \bar{\boldsymbol{B}}_\xi \boldsymbol{F}_\xi \boldsymbol{G}_\xi^{-1}\boldsymbol{x}(t-\tau_2)\Big] \\
& + \textstyle\sum_{i=1}^{\tau_1} \boldsymbol{x}^T(t+1-i)\boldsymbol{S}^{-1}\boldsymbol{x}(t+1-i) \\
& + \textstyle\sum_{i=1}^{\tau_2} \boldsymbol{x}^T(t+1-i)\boldsymbol{G}_{\xi+1}^{-T}\boldsymbol{Q}\boldsymbol{G}_{\xi+1}^{-1}\boldsymbol{x}(t+1-i) \\
& - \boldsymbol{x}^T(t)\boldsymbol{G}_\xi^{-T}\boldsymbol{P}_\xi \boldsymbol{G}_\xi^{-1}\boldsymbol{x}(t) - \textstyle\sum_{i=1}^{\tau_1} \boldsymbol{x}^T(t-i)\boldsymbol{S}^{-1} \\
& \times \boldsymbol{x}(t-i) - \textstyle\sum_{i=1}^{\tau_2} \boldsymbol{x}^T(t-i)\boldsymbol{G}_\xi^{-T}\boldsymbol{Q}\boldsymbol{G}_\xi^{-1}\boldsymbol{x}(t-i) \\
=\ & \begin{bmatrix} \boldsymbol{x}^T(t) & \boldsymbol{x}^T(t-\tau_1) & \boldsymbol{x}^T(t-\tau_2) \end{bmatrix} \\
& \times \boldsymbol{\Psi}_{\xi(\xi+1)} \begin{bmatrix} \boldsymbol{x}^T(t) \\ \boldsymbol{x}^T(t-\tau_1) \\ \boldsymbol{x}^T(t-\tau_2) \end{bmatrix},
\end{aligned}
\tag{20}
$$

where

$$
\boldsymbol{\Psi}_{\xi(\xi+1)} = \begin{bmatrix} (1,1) & * & * \\ \bar{\boldsymbol{A}}_\xi^T \boldsymbol{\Phi}_{\xi+1}\boldsymbol{M}_\xi & \bar{\boldsymbol{A}}_\xi^T \boldsymbol{\Phi}_{\xi+1}\bar{\boldsymbol{A}}_\xi - \boldsymbol{S}^{-1} & * \\ -\boldsymbol{N}_\xi^T \boldsymbol{\Phi}_{\xi+1}\boldsymbol{M}_\xi & -\boldsymbol{N}_\xi^T \boldsymbol{\Phi}_{\xi+1}\bar{\boldsymbol{A}}_\xi & (3,3) \end{bmatrix}
$$

and

$$
\begin{aligned}
&(1,1) = \boldsymbol{M}_\xi^T \boldsymbol{\Phi}_{\xi+1}\boldsymbol{M}_\xi - \boldsymbol{\Phi}_\xi + \boldsymbol{S}^{-1} + \boldsymbol{G}^{-T}\boldsymbol{Q}\boldsymbol{G}^{-1}, \\
&(3,3) = \boldsymbol{N}_\xi^T \boldsymbol{\Phi}_{\xi+1}\boldsymbol{N}_\xi - \boldsymbol{G}^{-T}\boldsymbol{Q}\boldsymbol{G}^{-1}, \\
&\boldsymbol{M}_\xi = \boldsymbol{A}_\xi - \boldsymbol{B}_\xi \boldsymbol{F}_\xi \boldsymbol{G}_\xi^{-1},\ \boldsymbol{N}_\xi = \bar{\boldsymbol{B}}_\xi \boldsymbol{F}_\xi \boldsymbol{G}_\xi^{-1}, \\
&\boldsymbol{\Phi}_\xi = \boldsymbol{G}_\xi^{-T}\boldsymbol{P}_\xi \boldsymbol{G}_\xi^{-1}.
\end{aligned}
$$

Thus from $L < \boldsymbol{0}$, we have $\boldsymbol{\Psi}_{\xi(\xi+1)} < \boldsymbol{0}$. Pre-multiplying by $diag(\boldsymbol{G}_\xi^T, \boldsymbol{S}^T, \boldsymbol{G}_\xi^T)$ and post-multiplying by $diag(\boldsymbol{G}_\xi, \boldsymbol{S}, \boldsymbol{G}_\xi)$ to $\boldsymbol{\Psi}_{\xi(\xi+1)}$, the following inequality equivalent to $\boldsymbol{\Psi}_{\xi(\xi+1)} < \boldsymbol{0}$ can be deduced:

$$
\begin{aligned}
& diag(-\boldsymbol{P}_\xi + \boldsymbol{G}_\xi^T \boldsymbol{S}^{-1}\boldsymbol{G}_\xi + \boldsymbol{Q}, \quad -\boldsymbol{S}, \quad -\boldsymbol{Q}) \\
& + \begin{bmatrix} (\boldsymbol{M}_\xi \boldsymbol{G}_\xi)^T \\ (\bar{\boldsymbol{A}}_\xi \boldsymbol{S})^T \\ -(\boldsymbol{N}_\xi \boldsymbol{G}_\xi)^T \end{bmatrix} \boldsymbol{\Phi}_{\xi+1} \begin{bmatrix} \boldsymbol{M}_\xi \boldsymbol{G}_\xi & \bar{\boldsymbol{A}}_\xi \boldsymbol{S} & -\boldsymbol{N}_\xi \boldsymbol{G}_\xi \end{bmatrix} < \boldsymbol{0}.
\end{aligned}
\tag{21}
$$

Select $\boldsymbol{\Lambda} = \boldsymbol{G}_{\xi+1}^{-1}\begin{bmatrix} \boldsymbol{M}_\xi \boldsymbol{G}_\xi & \bar{\boldsymbol{A}}_\xi \boldsymbol{S} & -\boldsymbol{N}_\xi \boldsymbol{G}_\xi \end{bmatrix}$, (21) can be rewritten in the following form according to Lemma 1:

$$
\begin{bmatrix} \begin{pmatrix} -\boldsymbol{G}_\xi^T \boldsymbol{S}^{-1}\boldsymbol{G}_\xi \\ +\boldsymbol{P}_\xi - \boldsymbol{Q} \end{pmatrix} & \boldsymbol{0} & \boldsymbol{0} & * \\ \boldsymbol{0} & \boldsymbol{S} & \boldsymbol{0} & * \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{Q} & * \\ \boldsymbol{M}_\xi \boldsymbol{G}_\xi & \bar{\boldsymbol{A}}_\xi \boldsymbol{S} & -\boldsymbol{N}_\xi \boldsymbol{G}_\xi & \boldsymbol{\Xi} \end{bmatrix} > \boldsymbol{0}, \tag{22}
$$

where $\boldsymbol{\Xi} = \boldsymbol{G}_{\xi+1} + \boldsymbol{G}_{\xi+1}^T - \boldsymbol{P}_{\xi+1}$. Then utilizing Schur complement to (22) yields

$$
\begin{aligned}
& \begin{bmatrix} \boldsymbol{P}_\xi & * & \boldsymbol{0} & \boldsymbol{0} & * \\ \boldsymbol{G}_\xi & \boldsymbol{S} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{S} & \boldsymbol{0} & * \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{Q} & * \\ \boldsymbol{A}_\xi \boldsymbol{G}_\xi - \boldsymbol{B}_\xi \boldsymbol{F}_\xi & \boldsymbol{0} & \bar{\boldsymbol{A}}_\xi \boldsymbol{S} & -\bar{\boldsymbol{B}}_\xi \boldsymbol{F}_\xi & \boldsymbol{\Xi} \end{bmatrix} \\
& = \textstyle\sum_{l=1}^{r} h_l(\boldsymbol{\xi}(t+1)) \big(\sum_{i=1}^{r} h_i^2(\boldsymbol{\xi}(t))\boldsymbol{\Omega}_{ii}^l \\
& \quad + \textstyle\sum_{i=1}^{r}\sum_{i<j} h_i(\boldsymbol{\xi}(t))h_j(\boldsymbol{\xi}(t))(\boldsymbol{\Omega}_{ij}^l + \boldsymbol{\Omega}_{ji}^l)\big) > \boldsymbol{0}.
\end{aligned}
\tag{23}
$$

Obviously, (23) can be guaranteed when (8) and (9) hold true. Therefore $L < 0$ is ensured to make the system (17) asymptotically stable. The proof is concluded. □

Corollary 1. *The discrete T-S fuzzy system with both state and input delays represented by (17) is globally asymptotically stable if there exist symmetric matrices $\boldsymbol{P}_i > \boldsymbol{0}, \boldsymbol{S} > \boldsymbol{0}$ and matrices $\boldsymbol{F}_i, \boldsymbol{G}$ with proper dimensions satisfying Eqs. (8) and (9), where*

$$
\begin{aligned}
& \boldsymbol{\Omega}_{ij}^l = \\
& \begin{bmatrix} \boldsymbol{P}_i - \boldsymbol{Q} & * & \boldsymbol{0} & \boldsymbol{0} & * \\ \boldsymbol{G} & \boldsymbol{S} & \boldsymbol{0} & \boldsymbol{0} & * \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{S} & \boldsymbol{0} & * \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{Q} & * \\ \boldsymbol{A}_i\boldsymbol{G} - \boldsymbol{B}_i\boldsymbol{F}_j & \boldsymbol{0} & \bar{\boldsymbol{A}}_i\boldsymbol{S} & -\bar{\boldsymbol{B}}_i\boldsymbol{F}_j & \boldsymbol{G} + \boldsymbol{G}^T - \boldsymbol{P}_l \end{bmatrix}, \\
& \qquad i, j, l \in \{1, \ldots, r\}.
\end{aligned}
\tag{24}
$$

Proof. Select $\boldsymbol{G}_i = \boldsymbol{G}_j = \boldsymbol{G}_l = \boldsymbol{G}$, then the conclusion is recovered from Theorem 3. □

Remark 4. Corollary 1 applies PDC law with the feedback gain $\boldsymbol{Y}_i = \boldsymbol{F}_i\boldsymbol{G}^{-1}$ rather than non-PDC law.

V. EXAMPLES

A. Example 1

Consider the unforced system $\boldsymbol{x}(t+1) = \boldsymbol{A}_\xi \boldsymbol{x}(t) + \bar{\boldsymbol{A}}_\xi \boldsymbol{x}(t-\tau_1)$ with

$$
\boldsymbol{A}_1 = \begin{bmatrix} -0.31 & 1 \\ 0 & 0.95 \end{bmatrix}, \quad \boldsymbol{A}_2 = \begin{bmatrix} -0.09 & 0 \\ 0.8 & -0.2 \end{bmatrix};
$$

$$
\bar{\boldsymbol{A}}_1 = \begin{bmatrix} 0.012 & 0.014 \\ 0 & 0.015 \end{bmatrix}, \quad \bar{\boldsymbol{A}}_2 = \begin{bmatrix} 0.01 & 0 \\ 0.01 & 0.015 \end{bmatrix}.
$$

If one uses common Lyapunov function $V(\boldsymbol{x}(t)) = \boldsymbol{x}^T(t)\boldsymbol{P}\boldsymbol{x}(t) + \sum_{i=1}^{\tau_1} \boldsymbol{x}^T(t-i)\boldsymbol{S}\boldsymbol{x}(t-i)$, then the corresponding conditions can be described as Theorem 6 in [13], that is

$$
\begin{bmatrix} \boldsymbol{A}_i^T\boldsymbol{P}\boldsymbol{A}_i - \boldsymbol{P} + \boldsymbol{S} & \boldsymbol{A}_i^T\boldsymbol{P}\bar{\boldsymbol{A}}_i \\ \bar{\boldsymbol{A}}_i^T\boldsymbol{P}\boldsymbol{A}_i & \bar{\boldsymbol{A}}_i^T\boldsymbol{P}\bar{\boldsymbol{A}}_i - \boldsymbol{S} \end{bmatrix} < \boldsymbol{0}, \quad i = 1, 2. \tag{25}
$$

It can be checked that LMIs (25) are infeasible by employing MATLAB LMI Control Toolbox. However, a feasible solution for LMIs (8) and (9) is as follows:

$$
\boldsymbol{S} = \begin{bmatrix} 11.7476 & 4.7434 \\ 4.7434 & 13.8537 \end{bmatrix},
$$

$$
\boldsymbol{P}_1 = \begin{bmatrix} 1.5572 & 0.1440 \\ 0.1440 & 1.0270 \end{bmatrix}, \quad \boldsymbol{G}_1 = \begin{bmatrix} 2.0994 & 0.6365 \\ 0.1664 & 0.9726 \end{bmatrix},
$$

$$
\boldsymbol{P}_2 = \begin{bmatrix} 1.5760 & 0.6287 \\ 0.6287 & 1.9052 \end{bmatrix}, \quad \boldsymbol{G}_2 = \begin{bmatrix} 1.3127 & 0.5776 \\ 0.6854 & 1.5661 \end{bmatrix}.
$$

Thus the stability of this unforced system is guaranteed by Theorem 1 presented in this paper while can not be ensured by Theorem 6 in [13]. As expected, results based on nonquadratic Lyapunov functions are less conservative than those based on common quadratic Lyapunov functions.

B. Example 2

Consider a nonlinear discrete-time system with both state and input delays as follows:

$$\begin{cases} x_1(t+1) = x_1(t) + 0.02x_1(t)x_2(t) + 0.3x_1(t-\tau_1) \\ \qquad\qquad +0.1x_2(t-\tau_1) + 6u(t) + 0.6u(t-\tau_2); \\ x_2(t+1) = 0.05x_1^2(t) - 0.5x_2(t) + 0.2x_2(t-\tau_1) \\ \qquad\qquad +6u(t) + 0.6u(t-\tau_2). \end{cases} \tag{26}$$

In (26) $x_1(t)$ is measurable and $x_1(t) \in [-2, 3]$. Define $F_1^1(x_1(t)) = (x_1(t)+2)/5$ and $F_1^2(x_1(t)) = (3 - x_1(t))/5$. In this way, the nonlinear model can be exactly represented by the following two rules of discrete T-S fuzzy model with $h_1(\boldsymbol{\xi}(t)) = F_1^1(x_1(t)), h_2(\boldsymbol{\xi}(t)) = F_1^2(x_1(t))$:

$$Rule\ 1: If\ x_1(t)\ is\ MAX,\ then$$
$$\boldsymbol{x}(t+1) = \begin{bmatrix} 1 & 0.06 \\ 0.15 & -0.5 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 0.3 & 0.1 \\ 0 & 0.2 \end{bmatrix} \boldsymbol{x}(t-\tau_1) + \begin{bmatrix} 6 \\ 6 \end{bmatrix} \boldsymbol{u}(t) + \begin{bmatrix} 0.6 \\ 0.6 \end{bmatrix} \boldsymbol{u}(t-\tau_2),$$
$$Rule\ 2: If\ x_1(t)\ is\ MIN,\ then$$
$$\boldsymbol{x}(t+1) = \begin{bmatrix} 1 & -0.04 \\ -0.1 & -0.5 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 0.3 & 0.1 \\ 0 & 0.2 \end{bmatrix} \boldsymbol{x}(t-\tau_1) + \begin{bmatrix} 6 \\ 6 \end{bmatrix} \boldsymbol{u}(t) + \begin{bmatrix} 0.6 \\ 0.6 \end{bmatrix} \boldsymbol{u}(t-\tau_2).$$

The corresponding result with Theorem 2 applied is

$$\boldsymbol{S} = \begin{bmatrix} 10.3032 & -1.8612 \\ -1.8612 & 12.2324 \end{bmatrix}, \boldsymbol{Q} = \begin{bmatrix} 1.5893 & -0.6909 \\ -0.6909 & 2.8423 \end{bmatrix},$$

$$\boldsymbol{P}_1 = \begin{bmatrix} 6.6972 & -1.9155 \\ -1.9155 & 9.7065 \end{bmatrix}, \boldsymbol{G}_1 = \begin{bmatrix} 5.0055 & -1.9599 \\ -1.6116 & 7.0788 \end{bmatrix},$$

$$\boldsymbol{F}_1 = [0.5744 \quad -0.4143],$$

$$\boldsymbol{P}_2 = \begin{bmatrix} 6.5781 & -2.0005 \\ -2.0005 & 9.7089 \end{bmatrix}, \boldsymbol{G}_2 = \begin{bmatrix} 4.8290 & -1.4620 \\ -2.1834 & 7.1223 \end{bmatrix},$$

$$\boldsymbol{F}_2 = [0.5052 \quad -0.4015].$$

The initial conditions for the simulation are given by $\boldsymbol{x}(1) = [2 \quad -1]^T$. Figs. 1-3 show the closed-loop results with control law (16).

VI. Conclusion

In this paper, some new stability and stabilization conditions for discrete time-delayed fuzzy systems have been put forward by defining a nonquadratic Lyapunov function and adopting a matrix transformation. These conditions can be readily solved by using available numerical software. The stability analysis of the open-loop fuzzy system demonstrated that former quadratic cases could be implied by the nonquadratic Lyapunov function approach presented here, based on which a stabilization control law for closed-loop T-S systems associated with non-PDC scheme has been derived. Two numerical examples illustrated the advantage of the proposed function approach. However, this nonquadratic framework still needs a lot of calculation. Therefore it is a challenging work calling for further investigation to reduce such computational burdens.

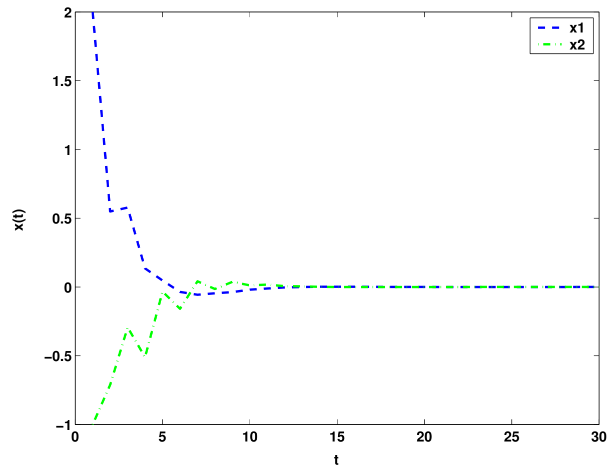

Fig. 1. Evolutions of the state variables x_1, x_2.

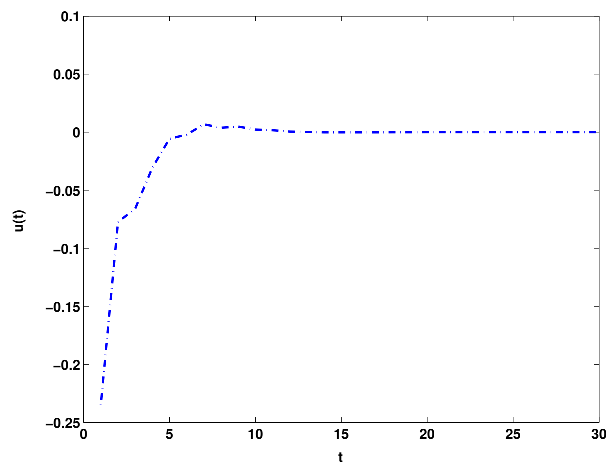

Fig. 2. Evolutions of the control signal u.

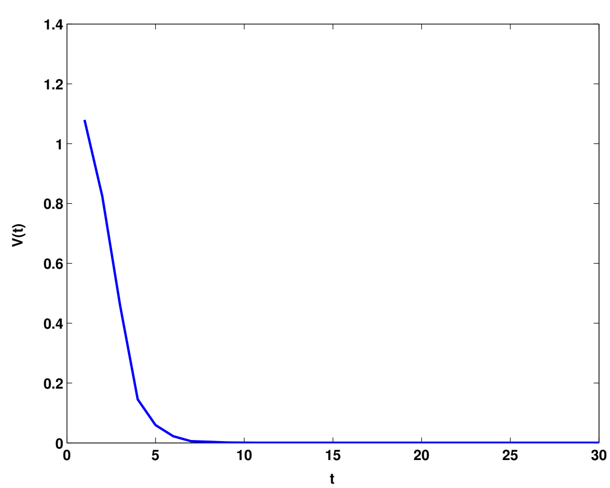

Fig. 3. Lyapunov function.

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