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Linear Estimation-Based Time Delay and Packet Dropout Compensation for Networked Control Systems

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Abstract— **This paper studies the problem of H_∞ controller design for networked control systems (NCSs) with time delay and packet dropout. A linear estimation-based method is proposed to compensate time delay and packet dropout, a delay switching-based method is also proposed to model the variation of time delay, and H_∞ controller design is presented by using the delay switching-based method and the existing parameter uncertainty-based method. The newly proposed delay switching-based method is proved to be less conservative than the existing parameter uncertainty-based method. The simulation results illustrate the effectiveness of the linear estimation-based time delay and packet dropout compensation.**

I. INTRODUCTION

Networked control systems (NCSs) have received increasing attentions in recent years. Advantages of NCSs include low cost, high reliability, less wiring and easy maintenance, etc. However, the insertion of the communication network will lead to time delay, data packet dropout and disordering inevitably, which make the analysis and design of NCSs complex.

Many researchers have studied stability/stabilization, controller design and performance of NCSs in the presence of network-induced delay [1]-[5]. For other methods dealing with delay specifically, see also [6]-[10]. In [11] and [12], the parameter uncertainty-based method was proposed to deal with time-varying delay. There have also been considerable research efforts on H_∞ control for NCSs [13]-[16].

Since time delay and packet dropout may lead to instability and poor performance of NCSs, it is important to overcome the negative influences of time delay and packet dropout, especially for wireless NCSs. However, how to compensate the negative influences of time delay and packet dropout has not been considered in the papers listed above.

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Recently, there are some preliminary results on the compensation of time delay and packet dropout. [17] presented several recent results on estimation, analysis, and controller synthesis for NCSs. In [18], an estimator was used to reconstruct an approximation to the undelayed plant state. By using a buffer in the actuator node and a state estimator in the controller node, [19] presented LMI-based sufficient condition for the stability of NCSs, but the problem of controller design was not discussed in [18] and [19]. By using prediction-based method, [20] and [21] studied the problem of time delay compensation for NCSs. [22] was concerned with the design of NCSs with random network delay in the feedback channel and gave stability criteria of closed-loop networked predictive control systems.

Just as we can see, the compensation methods presented above are usually based on estimation or prediction. If the prediction-based method is used, an augmented state vector is usually defined (see [20], [22]), and the introduction of the augmented state vector will introduce some conservativeness since the common positive definite matrix P with special structure is needed (see [20]).

This paper proposes a linear estimation-based method to compensate time delay and packet dropout. When compared with design methods without compensation, the proposed compensation method may provide better H_∞ performance. A delay switching-based method is presented to design H_∞ controllers for the system with time delay and packet dropout compensation, and H_∞ controller design using the newly proposed delay switching-based method is proved to be less conservative than the one using the existing parameter uncertainty-based method. Compared with the prediction-based method, since the common positive definite matrix P with special structure is not needed in this paper, the conservativeness of the obtained results may be reduced.

This paper is organized as follows. The linear estimation-based time delay and packet dropout compensation and the delay switching-based method are presented in Section 2. Section 3 is dedicated to H_∞ controller design by using the delay switching-based method. Section 4 presents the H_∞ controller design by using the parameter uncertainty-based method. The results of numerical simulation are presented in Section 5. Conclusions are stated in Section 6.

II. PRELIMINARIES AND PROBLEM STATEMENT

Consider a linear time-invariant plant described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1 u(t) + B_2 \omega(t) \\ z(t) &= C_1 x(t) + D_1 u(t) \end{aligned} \tag{1}$$

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where $x(t)$, $u(t)$, $z(t)$, $\omega(t)$ are the state vector, control input vector, controlled output, and disturbance input, respectively, and $\omega(t)$ is piecewise constant. A, B_1, B_2, C_1, D_1 are known constant matrices of appropriate dimensions. Throughout this paper, matrices, if not explicitly stated, are assumed to have appropriate dimensions.

To compensate time delay and packet dropout, a linear estimator may be added into the system to estimate the delayed and dropped control input packets.

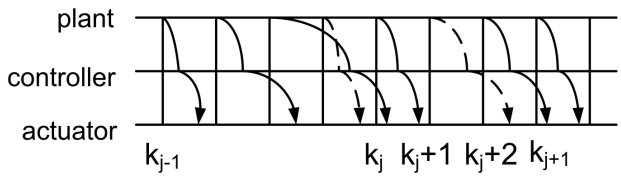

Fig. 1. NCSs with stochastic delay and packet dropout

The main idea of the linear estimation-based time delay and packet dropout compensation is as follows. Suppose h is the length of sampling period, k_jh (where $j = 1,\ 2,\ \cdots$) and $(k_j + m)h$ (where $m = 1,\ 2,\ \cdots$) are the sampling instants, $\tau_{k_{j+i}}$ (where $i = 0,\ 1,\ \cdots$) is the time delay of the control input $u_{k_{j+i}}$, and $\tau_{k_j} \le h$, $\tau_{k_j+m} > h$ or u_{k_j+m} is dropped (denoted by the dashed lines in Fig. 1). Since u_{k_j+m} cannot reach the actuator at the instant $(k_j+m+1)h$, the estimator will estimate u_{k_j+m} at the instant $(k_j+m+1)h$, and the estimated value $\hat{u}_{k_j+m}$ of u_{k_j+m} will be used as the control input, u_{k_j+m} will not be used even if it reaches the actuator eventually.

Remark 1. As shown in Fig. 1, u_{k_j+1} does not reach the actuator at the instant $(k_j+2)h$ for being dropped, the estimator will estimate u_{k_j+1} at the instant $(k_j+2)h$ based on u_{k_j}, while u_{k_j+2} does not reach the actuator at the instant $k_{j+1}h$ for long time delay, the estimator will also estimate u_{k_j+2} at the instant $k_{j+1}h$ based on u_{k_j} and $\hat{u}_{k_j+1}$ ($\hat{u}_{k_j+m}$ denotes the estimated value of u_{k_j+m}).

Suppose the state x_0, x_{k_1}, x_{k_2}, $\cdots$, x_{k_j}, $\cdots$ and the corresponding control inputs based on these states are transferred to the actuator successfully, $L-1$ is the maximum of consecutive packet dropout. Considering that the control inputs will tend to zero when the system reaches a steady state, then the estimated values of the delayed and dropped control inputs are as follows

$$\begin{aligned}
\hat{u}_{k_j+1} &= u_{k_j} - \tfrac{1}{L}u_{k_j} = (1-\tfrac{1}{L})u_{k_j} \\
\hat{u}_{k_j+2} &= \hat{u}_{k_j+1} + (\hat{u}_{k_j+1} - u_{k_j}) = (1-\tfrac{2}{L})u_{k_j} \\
\hat{u}_{k_j+3} &= \hat{u}_{k_j+2} + (\hat{u}_{k_j+2} - \hat{u}_{k_j+1}) = (1-\tfrac{3}{L})u_{k_j} \\
&\vdots \\
\hat{u}_{k_{j+1}-1} &= \hat{u}_{k_{j+1}-2} + (\hat{u}_{k_{j+1}-2} - \hat{u}_{k_{j+1}-3}) \\
&= (1-\tfrac{k_{j+1}-k_j-1}{L})u_{k_j}
\end{aligned} \tag{2}$$

where L is a predefined positive scalar and $k_{j+1} - k_j = 1,\ \cdots,\ L$.

Based on the compensation method presented in (2), we will give the discrete time model of the system (1) in the following.

In this paper, we suppose $\tau_{k_j} \in \{0,\ h/l,\ 2h/l,\ \cdots,\ (l-1)h/l,\ h\}$, where l is a positive scalar, then

$$u(t) = \begin{cases} \hat{u}_{k_j-1}, & t \in [k_jh,\ k_jh + \alpha h/l) \\ u_{k_j}, & t \in [k_jh + \alpha h/l,\ (k_j+1)h] \end{cases} \tag{3}$$

where $\alpha = 0,\ 1,\ \cdots,\ l$.

Just as shown in Fig. 1, u_{k_j} will be used during the interval $[(k_j+1)h,\ (k_j+2)h)$, while $\hat{u}_{k_j+1}$ will be used during the interval $[(k_j+2)h,\ (k_j+3)h)$, etc. Suppose the disturbance inputs $\omega_{k_j} = \omega_{k_j+1} = \cdots = \omega_{k_{j+1}-1}$ for every k_j, then the evolution of plant states can be described as follows

$$\begin{aligned}
x_{k_j+1} &= \Phi x_{k_j} + \Gamma_0^{k_j} u_{k_j} + \Gamma_1^{k_j}\hat{u}_{k_j-1} + \widetilde{\Gamma}\omega_{k_j} \\
&= (\Phi - \Gamma_0^{k_j}K)x_{k_j} - (1 - \tfrac{k_j - k_{j-1}-1}{L})\Gamma_1^{k_j}Kx_{k_{j-1}} + \widetilde{\Gamma}\omega_{k_j} \\
x_{k_j+2} &= \Phi x_{k_j+1} + \Gamma u_{k_j} + \widetilde{\Gamma}\omega_{k_j} \\
&= (\Phi^2 - \Phi\Gamma_0^{k_j}K - \Gamma K)x_{k_j} + (\Phi\widetilde{\Gamma} + \widetilde{\Gamma})\omega_{k_j} \\
&\quad -(1 - \tfrac{k_j-k_{j-1}-1}{L})\Phi\Gamma_1^{k_j}Kx_{k_{j-1}} \\
x_{k_j+3} &= \Phi x_{k_j+2} + \Gamma\hat{u}_{k_j+1} + \widetilde{\Gamma}\omega_{k_j} \\
&= [\Phi^3 - \Phi^2\Gamma_0^{k_j}K - \Phi\Gamma K - (1-\tfrac{1}{L})\Gamma K]x_{k_j} \\
&\quad -(1 - \tfrac{k_j-k_{j-1}-1}{L})\Phi^2\Gamma_1^{k_j}Kx_{k_{j-1}} + (\Phi^2\widetilde{\Gamma} + \Phi\widetilde{\Gamma} + \widetilde{\Gamma})\omega_{k_j} \\
&\vdots \\
x_{k_{j+1}} &= \widetilde{A}_j x_{k_j} + \widetilde{B}_j x_{k_{j-1}} + \widetilde{D}_j\omega_{k_j}
\end{aligned} \tag{4}$$

where $\Phi = e^{Ah}$, $\Gamma_0^{k_j} = \int_0^{h-\tau_{k_j}} e^{As}dsB_1$, $\Gamma_1^{k_j} = \int_{h-\tau_{k_j}}^{h} e^{As}dsB_1$, $\widetilde{\Gamma} = \int_0^h e^{As}dsB_2$, $\Gamma = \int_0^h e^{As}dsB_1$, τ_{k_j} is the time delay of the control input u_{k_j}, and

$$\begin{aligned}
\widetilde{A}_j &= \Phi^{k_{j+1}-k_j} - \Phi^{k_{j+1}-k_j-1}\Gamma_0^{k_j}K - \Phi^{k_{j+1}-k_j-2}\Gamma K \\
&\quad -(1-\tfrac{1}{L})\Phi^{k_{j+1}-k_j-3}\Gamma K - (1-\tfrac{2}{L})\Phi^{k_{j+1}-k_j-4}\Gamma K \\
&\quad -\cdots - \sigma_1\Gamma K \\
\widetilde{B}_j &= -\sigma_2\Phi^{k_{j+1}-k_j-1}\Gamma_1^{k_j}K \\
\widetilde{D}_j &= \Phi^{k_{j+1}-k_j-1}\widetilde{\Gamma} + \Phi^{k_{j+1}-k_j-2}\widetilde{\Gamma} + \cdots + \widetilde{\Gamma} \\
\sigma_1 &= (1 - \tfrac{k_{j+1}-k_j-2}{L}) \\
\sigma_2 &= (1 - \tfrac{k_j-k_{j-1}-1}{L})
\end{aligned} \tag{5}$$

Define $x_{k_{j+1}}$, x_{k_j}, $x_{k_{j-1}}$, ω_{k_j}, u_{k_j}, and z_{k_j} as ξ_{j+1}, ξ_j, ξ_{j-1}, ω_j, u_j, and z_j, respectively. Then

$$\begin{aligned}
\xi_{j+1} &= \widetilde{A}_j\xi_j + \widetilde{B}_j\xi_{j-1} + \widetilde{D}_j\omega_j \\
z_j &= C_1\xi_j + D_1\hat{u}_{k_j-1}
\end{aligned} \tag{6}$$

Just as shown above, the time delay τ_{k_j} switches in the finite set $\vartheta = \{0,\ h/l,\ 2h/l,\ \cdots,\ (l-1)h/l,\ h\}$, so $\Gamma_0^{k_j}$ and $\Gamma_1^{k_j}$ of (5) also switch in finite sets, then the problem of H_∞ controller design for (1) can be reduced to the corresponding problem for the system (6), which will be studied in Section 3.

Remark 2. If the disturbance inputs $\omega_{k_j} \ne \omega_{k_j+1} \ne \cdots \ne \omega_{k_{j+1}-1}$, we can also get the discrete time model of the system (1) by using the method presented in (4), here it is omitted.

The following lemmas will be used in the sequel.

Lemma 1 [7]. Suppose $a \in R^n$, $b \in R^m$, $G \in R^{n*m}$, then for any $X \in R^{n*n}$, $Y \in R^{n*m}$, $Z \in R^{m*m}$ satisfying

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \geq 0$$

the following inequality holds

$$-2a^T Gb \leq \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} X & Y-G \\ Y^T - G^T & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

Lemma 2 [8]. Suppose D, E, and F are real matrices of appropriate dimensions with $||F|| \leq 1$, for any scalar $\varepsilon > 0$, the following inequality holds

$$DFE + E^T F^T D^T \leq \varepsilon DD^T + \varepsilon^{-1} E^T E$$

III. H_∞ CONTROLLER DESIGN BASED ON DELAY SWITCHING

We are now in a position to design the state feedback controller gain K, which can make the system (6) asymptotically stable with the H_∞ norm bound γ.

Theorem 1. For given positive scalars θ_1, θ_2, and θ_3, if there exist symmetric positive definite matrices $\widetilde{P}$, $\widetilde{Z}$, $\widetilde{R}$, and matrices $\widetilde{X}_{11}$, $\widetilde{X}_{22}$, $\widetilde{X}_{33}$, V, N, $\widetilde{X}_{12}$, $\widetilde{X}_{13}$, $\widetilde{X}_{23}$, $\widetilde{Y}_1$, $\widetilde{Y}_2$, $\widetilde{Y}_3$, scalar $\gamma > 0$, such that the following inequalities hold for every feasible value of $k_{j+1} - k_j$ ($k_{j+1} - k_j = 1, \cdots, L$), $\Gamma_0^{k_j}$, and $\Gamma_1^{k_j}$

$$\begin{bmatrix} \widetilde{\Lambda}_{11} & \widetilde{\Lambda}_{12} & \widetilde{\Lambda}_{13} & -\theta_1 \widetilde{D}_j & 0 \\ * & \widetilde{\Lambda}_{22} & \widetilde{\Lambda}_{23} & -\theta_2 \widetilde{D}_j & NC_1^T \\ * & * & \widetilde{\Lambda}_{33} & -\theta_3 \widetilde{D}_j & -\sigma_2 V^T D_1^T \\ * & * & * & -\gamma I & 0 \\ * & * & * & * & -\gamma I \end{bmatrix} < 0 \quad (7)$$

$$\begin{bmatrix} \widetilde{X}_{11} & \widetilde{X}_{12} & \widetilde{X}_{13} & \widetilde{Y}_1 \\ * & \widetilde{X}_{22} & \widetilde{X}_{23} & \widetilde{Y}_2 \\ * & * & \widetilde{X}_{33} & \widetilde{Y}_3 \\ * & * & * & \widetilde{Z} \end{bmatrix} \geq 0 \quad (8)$$

where

$$\begin{aligned} \widetilde{\Lambda}_{11} &= \widetilde{P} + \widetilde{Z} + \theta_1 N + \theta_1 N^T + \widetilde{X}_{11} \\ \widetilde{\Lambda}_{12} &= -\widetilde{Z} - \theta_1 \Psi_1 + \theta_2 N + \widetilde{X}_{12} + \widetilde{Y}_1 \\ \widetilde{\Lambda}_{13} &= -\theta_1 \Psi_2 + \theta_3 N + \widetilde{X}_{13} - \widetilde{Y}_1 \\ \widetilde{\Lambda}_{22} &= -\widetilde{P} + \widetilde{Z} + \widetilde{R} - \theta_2 \Psi_1 - \theta_2 \Psi_1^T + \widetilde{X}_{22} + \widetilde{Y}_2 + \widetilde{Y}_2^T \\ \widetilde{\Lambda}_{23} &= -\theta_2 \Psi_2 - \theta_3 \Psi_1^T + \widetilde{X}_{23} - \widetilde{Y}_2 + \widetilde{Y}_3^T \\ \widetilde{\Lambda}_{33} &= -\widetilde{R} - \theta_3 \Psi_2 - \theta_3 \Psi_2^T + \widetilde{X}_{33} - \widetilde{Y}_3 - \widetilde{Y}_3^T \\ \Psi_1 &= \Phi^{k_{j+1}-k_j} N^T - \Phi^{k_{j+1}-k_j-1} \Gamma_0^{k_j} V - \Phi^{k_{j+1}-k_j-2} \Gamma V \\ &\quad -(1-\tfrac{1}{L}) \Phi^{k_{j+1}-k_j-3} \Gamma V - (1-\tfrac{2}{L}) \Phi^{k_{j+1}-k_j-4} \Gamma V \\ &\quad - \cdots - \sigma_1 \Gamma V \\ \Psi_2 &= -\sigma_2 \Phi^{k_{j+1}-k_j-1} \Gamma_1^{k_j} V \end{aligned} \quad (9)$$

then with the control law

$$u_j = -K\xi_j, \quad K = VN^{-T}$$

the system described by (6) is asymptotically stable with H_∞ norm bound γ.

Proof: Let us consider the following Lyapunov function

$$\begin{aligned} V_j &= V_{1j} + V_{2j} + V_{3j} \\ V_{1j} &= \xi_j^T P \xi_j \\ V_{2j} &= (\xi_j - \xi_{j-1})^T Z (\xi_j - \xi_{j-1}) \\ V_{3j} &= \xi_{j-1}^T R \xi_{j-1} \end{aligned}$$

where P, Z, R are symmetric positive definite matrices. The difference of function V_j along the trajectory of (6) is given by

$$\begin{aligned} \Delta V_{1j} &= \xi_{j+1}^T P \xi_{j+1} - \xi_j^T P \xi_j \\ \Delta V_{2j} &= (\xi_{j+1} - \xi_j)^T Z (\xi_{j+1} - \xi_j) - (\xi_j - \xi_{j-1})^T Z (\xi_j - \xi_{j-1}) \\ \Delta V_{3j} &= \xi_j^T R \xi_j - \xi_{j-1}^T R \xi_{j-1} \\ \Delta V_j &= \Delta V_{1j} + \Delta V_{2j} + \Delta V_{3j} \end{aligned} \quad (10)$$

Define $\tilde{\eta} = [\xi_{j+1}^T, \ \xi_j^T, \ \xi_{j-1}^T, \ \omega_j^T]^T$, for any matrices W and M of appropriate dimensions and positive scalars θ_1, θ_2, and θ_3, we have

$$\begin{aligned} \Pi_1 &= 2\eta^T W[\xi_j - \xi_{j-1} - (\xi_j - \xi_{j-1})] = 0 \\ \Pi_2 &= 2[\theta_1 \xi_{j+1}^T M + \theta_2 \xi_j^T M + \theta_3 \xi_{j-1}^T M] \\ &\quad .[\xi_{j+1} - \widetilde{A}_j \xi_j - \widetilde{B}_j \xi_{j-1} - \widetilde{D}_j \omega_j] = 0 \end{aligned} \quad (11)$$

Define $a = \eta$, $G = W$, $b = \xi_j - \xi_{j-1}$, using the Lemma 1, for any matrices X, Y, Z satisfying $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \geq 0$, we have

$$\begin{aligned} -2\eta^T W(\xi_j - \xi_{j-1}) &\leq \eta^T X \eta + 2\eta^T [Y - W](\xi_j - \xi_{j-1}) \\ &\quad + (\xi_j - \xi_{j-1})^T Z (\xi_j - \xi_{j-1}) \end{aligned} \quad (12)$$

that is

$$\Pi_1 \leq \eta^T X \eta + 2\eta^T Y(\xi_j - \xi_{j-1}) + (\xi_j - \xi_{j-1})^T Z(\xi_j - \xi_{j-1}) \quad (13)$$

By defining

$$X = \begin{bmatrix} X_{11} & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & X_{33} \end{bmatrix}, \ Y = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} \quad (14)$$

we have

$$\Delta V_j + \Pi_1 + \Pi_2 \leq \tilde{\eta}^T \Lambda \tilde{\eta} \quad (15)$$

where

$$\Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & -\theta_1 M\widetilde{D}_j \\ * & \Lambda_{22} & \Lambda_{23} & -\theta_2 M\widetilde{D}_j \\ * & * & \Lambda_{33} & -\theta_3 M\widetilde{D}_j \\ * & * & * & 0 \end{bmatrix} \quad (16)$$

$$\begin{aligned} \Lambda_{11} &= P + Z + \theta_1 M + \theta_1 M^T + X_{11} \\ \Lambda_{12} &= -Z - \theta_1 M\widetilde{A}_j + \theta_2 M^T + X_{12} + Y_1 \\ \Lambda_{13} &= -\theta_1 M\widetilde{B}_j + \theta_3 M^T + X_{13} - Y_1 \\ \Lambda_{22} &= -P + Z + R - \theta_2 M\widetilde{A}_j - \theta_2 \widetilde{A}_j^T M^T + X_{22} + Y_2 + Y_2^T \\ \Lambda_{23} &= -\theta_2 M\widetilde{B}_j - \theta_3 \widetilde{A}_j^T M^T + X_{23} - Y_2 + Y_3^T \\ \Lambda_{33} &= -R - \theta_3 M\widetilde{B}_j - \theta_3 \widetilde{B}_j^T M^T + X_{33} - Y_3 - Y_3^T \end{aligned} \quad (17)$$

If the linear estimation-based packet dropout compensation (2) is used, the available control input at the instant $k_j h$ is $\hat{u}_{k_j - 1} = -\sigma_2 K \xi_{j-1}$, then the controlled output $z_j = C_1 \xi_j - \sigma_2 D_1 K \xi_{j-1}$, for any nonzero ξ_j, we have

$$\gamma^{-1} z_j^T z_j - \gamma \omega_j^T \omega_j = \tilde{\eta}^T \Xi \tilde{\eta}$$

where

$$\Xi = \begin{bmatrix} 0 & 0 & 0 & 0 \\ * & \gamma^{-1}{C_1}^T C_1 & -\sigma_2\gamma^{-1}{C_1}^T D_1 K & 0 \\ * & * & \sigma_2^2\gamma^{-1}K^T D_1^T D_1 K & 0 \\ * & * & * & -\gamma I \end{bmatrix}$$

so

$$\gamma^{-1}z_j^T z_j - \gamma\omega_j^T\omega_j + \Delta V_j + \Pi_1 + \Pi_2 \le \tilde{\eta}^T\widetilde{\Lambda}\tilde{\eta}$$

where $\widetilde{\Lambda} = \Lambda + \Xi$.

In the following, we will prove that $\gamma^{-1}z_j^T z_j - \gamma\omega_j^T\omega_j + \Delta V_j < 0$, that is $\widetilde{\Lambda} < 0$. Using the Schur complement, $\widetilde{\Lambda} < 0$ is equivalent to

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & -\theta_1 M\widetilde{D}_j & 0 \\ * & \Lambda_{22} & \Lambda_{23} & -\theta_2 M\widetilde{D}_j & C_1^T \\ * & * & \Lambda_{33} & -\theta_3 M\widetilde{D}_j & -\sigma_2(D_1K)^T \\ * & * & * & -\gamma I & 0 \\ * & * & * & * & -\gamma I \end{bmatrix} < 0 \quad (18)$$

Pre- and post-multiply (18) by $diag(M^{-1},\ M^{-1},\ M^{-1},\ I,\ I)$ and $diag(M^{-T},\ M^{-T},\ M^{-T},\ I,\ I)$, define $M^{-1} = N$, $KN^T = V$, $M^{-1}PM^{-T} = \widetilde{P}$, $M^{-1}ZM^{-T} = \widetilde{Z}$, $M^{-1}RM^{-T} = \widetilde{R}$, $M^{-1}X_{ij}M^{-T} = \widetilde{X}_{ij}$, $M^{-1}Y_iM^{-T} = \widetilde{Y}_i$, where $i = 1,\ 2,\ 3$ and $i \le j \le 3$. Take the representations of $\widetilde{A}_j$ and $\widetilde{B}_j$ into consideration, we can see that (18) is equivalent to (7). On the other hand, pre- and post-multiply $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \ge 0$ by $diag(M^{-1},\ M^{-1},\ M^{-1},\ M^{-1})$ and $diag(M^{-T},\ M^{-T},\ M^{-T},\ M^{-T})$, one can see that $\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \ge 0$ is equivalent to (8), that is if (7) and (8) are satisfied, we have $\gamma^{-1}z_j^T z_j - \gamma\omega_j^T\omega_j + \Delta V_j < 0$.

Since $\gamma^{-1}z_j^T z_j - \gamma\omega_j^T\omega_j + \Delta V_j < 0$, then

$$\gamma^{-1}z_j^T z_j - \gamma\omega_j^T\omega_j < -\Delta V_j$$

Summing up the both sides of the above inequality for $j = 0$ to $j = n$, using the zero initial condition and the character that the disturbance input ω_j has limited energy, we have

$$\sum_{j=0}^{n} ||z_j||^2 < \gamma^2 \sum_{j=0}^{n} ||\omega_j||^2 - \gamma V_{n+1}$$

the above inequality holds for all n, let $n \to \infty$, we have

$$||z||_2^2 < \gamma^2 ||\omega||_2^2$$

If the disturbance input $\omega_j = 0$, (7) and (8) can ensure the asymptotic stability of system described by (6), and if $\omega_j \ne 0$, we have $||z||_2^2 < \gamma^2||\omega||_2^2$. So if the LMIs (7) and (8) are feasible, the system described by (6) with $K = VN^{-T}$ is asymptotically stable with H_∞ norm bound γ, this completes the proof. ■

When the delay switching-based method is used to design H_∞ controllers, the LMIs presented in Theorem 1 should be satisfied for every feasible value of τ_{k_j} ($\tau_{k_j} \in \vartheta$), which will lead to the increase of computational complexity.

One can also use the existing parameter uncertainty-based method (see [11], [12]) to design H_∞ controllers for the system described by (6).

IV. H_∞ CONTROLLER DESIGN BASED ON PARAMETER-UNCERTAINTY

Suppose τ_{min} and τ_{max} are the minimum and maximum of time delay τ_{k_j}, and define $\bar{\tau} = (\tau_{min} + \tau_{max})/2$, considering the definition of $\Gamma_0^{k_j}$ and $\Gamma_1^{k_j}$, we have

$$\Gamma_0^{k_j} = \int_0^{h-\tau_{k_j}} e^{As}dsB_1 = \int_0^{h-\bar{\tau}} e^{As}dsB_1 + \int_{h-\bar{\tau}}^{h-\tau_{k_j}} e^{As}dsB_1 \quad (19)$$

define $\widehat{\Gamma}_1 = \int_0^{h-\bar{\tau}} e^{As}dsB_1$, $\widehat{D}_1 = I$, $F = \int_{h-\bar{\tau}}^{h-\tau_{k_j}} e^{As}ds$, $E_1 = B_1$, then

$$\Gamma_0^{k_j} = \widehat{\Gamma}_1 + \widehat{D}_1 F E_1 \quad (20)$$

Using the same method presented above, we have

$$\Gamma_1^{k_j} = \int_{h-\tau_{k_j}}^{h} e^{As}dsB_1 = \widehat{\Gamma}_2 + \widehat{D}_2 F E_2 \quad (21)$$

where $\widehat{\Gamma}_2 = \int_{h-\bar{\tau}}^{h} e^{As}dsB_1$, $\widehat{D}_2 = I$, $F = \int_{h-\bar{\tau}}^{h-\tau_{k_j}} e^{As}ds$, $E_2 = -B_1$.

For a specific system, $h - \tau_{k_j} \le h - \tau_{min}$, so $F^TF \le \lambda^2 I$, where λ is a positive scalar. Denote $\sigma_{max}(A)$ as the maximum singular value of matrix A, then (to see [12] for the proof)

$$\lambda = \frac{e^{\sigma_{max}(A)*(h-\tau_{min})} - e^{\sigma_{max}(A)*(h-\bar{\tau})}}{\sigma_{max}(A)} \quad (22)$$

So the system described by (6) can be written as follows

$$\begin{aligned} \xi_{j+1} &= (\widetilde{A}_{j1} + \widetilde{A}_{j2})\xi_j + (\widetilde{B}_{j1} + \widetilde{B}_{j2})\xi_{j-1} + \widetilde{D}_j\omega_j \\ z_j &= C_1\xi_j + D_1\hat{u}_{k_j-1} \end{aligned} \quad (23)$$

where

$$\begin{aligned} \widetilde{A}_{j1} &= \Phi^{k_{j+1}-k_j} - \Phi^{k_{j+1}-k_j-1}\widehat{\Gamma}_1 K - \Phi^{k_{j+1}-k_j-2}\Gamma K \\ &\quad -(1-\tfrac{1}{L})\Phi^{k_{j+1}-k_j-3}\Gamma K - (1-\tfrac{2}{L})\Phi^{k_{j+1}-k_j-4}\Gamma K \\ &\quad - \cdots - \sigma_1\Gamma K \\ \widetilde{A}_{j2} &= -\Phi^{k_{j+1}-k_j-1}\widehat{D}_1 F E_1 K \\ \widetilde{B}_{j1} &= -\sigma_2\Phi^{k_{j+1}-k_j-1}\widehat{\Gamma}_2 K \\ \widetilde{B}_{j2} &= -\sigma_2\Phi^{k_{j+1}-k_j-1}\widehat{D}_2 F E_2 K \end{aligned}$$

We are now in a position to design the feedback gain K, which can make the system (23) asymptotically stable with H_∞ norm bound γ.

Theorem 2. For given positive scalars θ_1, θ_2, θ_3, and λ, if there exist symmetric positive definite matrices $\widetilde{P}$, $\widetilde{Z}$, $\widetilde{R}$, and matrices V, N, $\widetilde{X}_{11}$, $\widetilde{X}_{22}$, $\widetilde{X}_{33}$, $\widetilde{X}_{12}$, $\widetilde{X}_{13}$, $\widetilde{X}_{23}$, $\widetilde{Y}_1$, $\widetilde{Y}_2$, $\widetilde{Y}_3$, scalars $\gamma > 0$, $\varepsilon > 0$, such that the following inequalities hold for every feasible value of $k_{j+1} - k_j$ ($k_{j+1} - k_j = 1,\ \cdots,\ L$)

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} \\ * & \Omega_{22} & 0 \\ * & * & \Omega_{33} \end{bmatrix} < 0 \quad (24)$$

$$\begin{bmatrix} \widetilde{X}_{11} & \widetilde{X}_{12} & \widetilde{X}_{13} & \widetilde{Y}_1 \\ * & \widetilde{X}_{22} & \widetilde{X}_{23} & \widetilde{Y}_2 \\ * & * & \widetilde{X}_{33} & \widetilde{Y}_3 \\ * & * & * & \widetilde{Z} \end{bmatrix} \ge 0 \quad (25)$$

where

$$\Omega_{11}=\begin{bmatrix}\widetilde{\Lambda}_{11} & \widetilde{\Lambda}_{12} & \widetilde{\Lambda}_{13} & -\theta_1\widetilde{D}_j & 0\\ * & \widetilde{\Lambda}_{22} & \widetilde{\Lambda}_{23} & -\theta_2\widetilde{D}_j & NC_1^T\\ * & * & \widetilde{\Lambda}_{33} & -\theta_3\widetilde{D}_j & -\sigma_2V^TD_1^T\\ * & * & * & -\gamma I & 0\\ * & * & * & * & -\gamma I\end{bmatrix}$$

$$\begin{aligned}
\widetilde{\Lambda}_{11}&=\widetilde{P}+\widetilde{Z}+\theta_1N+\theta_1N^T+\widetilde{X}_{11}\\
\widetilde{\Lambda}_{12}&=-\widetilde{Z}-\theta_1\Psi_1+\theta_2N+\widetilde{X}_{12}+\widetilde{Y}_1\\
\widetilde{\Lambda}_{13}&=-\theta_1\Psi_2+\theta_3N+\widetilde{X}_{13}-\widetilde{Y}_1\\
\widetilde{\Lambda}_{22}&=-\widetilde{P}+\widetilde{Z}+\widetilde{R}-\theta_2\Psi_1-\theta_2\Psi_1^T+\widetilde{X}_{22}+\widetilde{Y}_2+\widetilde{Y}_2^T\\
\widetilde{\Lambda}_{23}&=-\theta_2\Psi_2-\theta_3\Psi_1^T+\widetilde{X}_{23}-\widetilde{Y}_2+\widetilde{Y}_3^T\\
\widetilde{\Lambda}_{33}&=-\widetilde{R}-\theta_3\Psi_2-\theta_3\Psi_2^T+\widetilde{X}_{33}-\widetilde{Y}_3-\widetilde{Y}_3^T\\
\Psi_1&=\Phi^{k_{j+1}-k_j}N^T-\Phi^{k_{j+1}-k_j-1}\widehat{\Gamma}_1V-\Phi^{k_{j+1}-k_j-2}\Gamma V\\
&\quad-(1-\tfrac{1}{L})\Phi^{k_{j+1}-k_j-3}\Gamma V-(1-\tfrac{2}{L})\Phi^{k_{j+1}-k_j-4}\Gamma V\\
&\quad-\cdots-\sigma_1\Gamma V\\
\Psi_2&=-\sigma_2\Phi^{k_{j+1}-k_j-1}\widehat{\Gamma}_2V\\
\Omega_{12}&=diag(0,\ V^TE_1^T,\ V^TE_2^T,\ 0,\ 0)
\end{aligned}$$

$$\Omega_{13}=\begin{bmatrix}0 & \theta_1\varepsilon\Phi^{k_{j+1}-k_j-1}\widehat{D}_1 & \theta_1\sigma_2\varepsilon\Phi^{k_{j+1}-k_j-1}\widehat{D}_2 & 0 & 0\\ 0 & \theta_2\varepsilon\Phi^{k_{j+1}-k_j-1}\widehat{D}_1 & \theta_2\sigma_2\varepsilon\Phi^{k_{j+1}-k_j-1}\widehat{D}_2 & 0 & 0\\ 0 & \theta_3\varepsilon\Phi^{k_{j+1}-k_j-1}\widehat{D}_1 & \theta_3\sigma_2\varepsilon\Phi^{k_{j+1}-k_j-1}\widehat{D}_2 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$\Omega_{22}=diag(\underbrace{-\lambda^{-1}\varepsilon I,\ \cdots,\ -\lambda^{-1}\varepsilon I}_{5})$$

$$\Omega_{33}=diag(\underbrace{-\lambda^{-1}\varepsilon I,\ \cdots,\ -\lambda^{-1}\varepsilon I}_{5}) \tag{26}$$

then with the control law

$$u_j=-K\xi_j,\quad K=VN^{-T}$$

the system described by (23) is asymptotically stable with H_∞ norm bound γ.

Proof: Using the same method proposed in Theorem 1, one can see that if (7) and (8) are satisfied, we have $\gamma^{-1}z_j^Tz_j-\gamma\omega_j^T\omega_j+\Delta V_j<0$, and (7) can be written as

$$\Omega+\widetilde{D}\widetilde{F}\widetilde{E}+\widetilde{E}^T\widetilde{F}^T\widetilde{D}^T<0 \tag{27}$$

where Ω is the same as Ω_{11} given in (26), and

$$\widetilde{D}=\begin{bmatrix}0 & \theta_1\Phi^{k_{j+1}-k_j-1}\widehat{D}_1 & \theta_1\sigma_2\Phi^{k_{j+1}-k_j-1}\widehat{D}_2 & 0 & 0\\ 0 & \theta_2\Phi^{k_{j+1}-k_j-1}\widehat{D}_1 & \theta_2\sigma_2\Phi^{k_{j+1}-k_j-1}\widehat{D}_2 & 0 & 0\\ 0 & \theta_3\Phi^{k_{j+1}-k_j-1}\widehat{D}_1 & \theta_3\sigma_2\Phi^{k_{j+1}-k_j-1}\widehat{D}_2 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$\begin{aligned}\widetilde{E}&=diag(0,\ E_1V,\ E_2V,\ 0,\ 0)\\ \widetilde{F}&=diag(F,\ F,\ F,\ F,\ F)\end{aligned} \tag{28}$$

For any scalar $\varepsilon>0$, using the Lemma 2, we have

$$\widetilde{D}\widetilde{F}\widetilde{E}+\widetilde{E}^T\widetilde{F}^T\widetilde{D}^T<\lambda\varepsilon\widetilde{D}\widetilde{D}^T+\lambda\varepsilon^{-1}\widetilde{E}^T\widetilde{E} \tag{29}$$

Using the Schur complement, if the following inequality is feasible, (27) is also feasible.

$$\begin{bmatrix}\Omega & \widetilde{E}^T & \widetilde{D}\\ * & -\lambda^{-1}\varepsilon I & 0\\ * & * & -\lambda^{-1}\varepsilon^{-1}I\end{bmatrix}<0 \tag{30}$$

Pre- and post-multiply (30) by $diag(I,\ I,\ \varepsilon I)$ and $diag(I,\ I,\ \varepsilon I)$, one can see that (30) is equivalent to (24), that is if (24) and (25) are satisfied, we have $\gamma^{-1}z_j^Tz_j-\gamma\omega_j^T\omega_j+\Delta V_j<0$. The rest of this proof is similar to the proof of Theorem 1, here it is omitted. ■

Theorem 3. If the LMIs of Theorem 2 are feasible, then the LMIs of Theorem 1 are also feasible.

Proof: The proof is similar to Theorem 3 of [20], here it is omitted. ■

Remark 3. As shown in Theorem 3, Theorem 1 is less conservative than Theorem 2, and Theorem 2 may provide less computational complexity than Theorem 1, one may choose appropriate method according to the actual requirement.

Remark 4. For the system (6), if no compensation method is adopted and the latest available control inputs are used, then $u_{k_j+1}=u_{k_j},\ u_{k_j+2}=u_{k_j},\ \cdots,\ u_{k_{j+1}-1}=u_{k_j}$, which is corresponding to $L\to\infty$ in (2).

V. SIMULATION RESULTS AND DISCUSSION

To illustrate the merits of the proposed methods, we present an open loop unstable system as follows:

$$\begin{aligned}\dot{x}(t)&=\begin{bmatrix}0.8822 & -0.4090\\ -0.4863 & -0.3902\end{bmatrix}x(t)+\begin{bmatrix}0.7523\\ 0.7399\end{bmatrix}u(t)\\ &\quad+\begin{bmatrix}-0.3379\\ -0.3533\end{bmatrix}\omega(t)\\ z(t)&=\begin{bmatrix}-0.1181 & 0.4057\end{bmatrix}x(t)-0.5074u(t)\end{aligned} \tag{31}$$

Suppose the sampling period of sensor is 0.25s, and the time delay $\tau_{k_j}\in\{0.06s,\ 0.09s,\ 0.12s\}$, the initial state of the system is $x_0=[1-1]^T$, $\theta_1=10$, $\theta_2=1$, $\theta_3=1$, and from (22), we can get $\lambda=0.0359$. For simplicity, suppose the packets at the instants 0, $4h$, $8h$, $\cdots$ are transferred to the actuator successfully, that is 3 packets are dropped among every 4 packets, which means that $L=4$.

For systems without time delay and packet dropout compensation, one can also design the H_∞ controllers by using the same methods proposed in Theorem 1 and Theorem 2. The H_∞ norm bounds corresponding to different cases are shown in Table 1 (γ_1 and γ_2 represent the H_∞ norm bounds got by Theorem 1 with and without time delay and packet dropout compensation, γ_3 and γ_4 represent the H_∞ norm bounds got by Theorem 2 with and without time delay and packet dropout compensation, respectively).

From Table 1, one can see that $\gamma_1<\gamma_3$ and $\gamma_2<\gamma_4$, which show that the H_∞ controller design using the proposed delay switching-based method is less conservative than the one using the existing parameter uncertainty-based method. On the other hand, $\gamma_1<\gamma_2$ and $\gamma_3<\gamma_4$ demonstrate the merits of the proposed time delay and packet dropout compensation.

The maximum admissible sampling periods (MASPs) based on different methods are shown in Table 2 (h_1 and h_2 represents the MASPs got by Theorem 1 with and without time delay and packet dropout compensation, h_3 and h_4 represents the MASPs got by Theorem 2 with and without time delay and packet dropout compensation, respectively).

From Table 2, one can see that $h_1>h_3$ and $h_2>h_4$, which demonstrate the merits of the proposed delay switching-based H_∞ controller design. On the other hand, $h_1>h_2$ and

TABLE I

THE H_∞ NORM BOUNDS

γ_1	γ_2	γ_3	γ_4
0.6028	1.6991	1.2126	20.8469

TABLE II

THE MAXIMUM ADMISSIBLE SAMPLING PERIODS

h_1	h_2	h_3	h_4
0.66	0.43	0.44	0.26

$h_3 > h_4$ demonstrate the merits of the proposed time delay and packet dropout compensation.

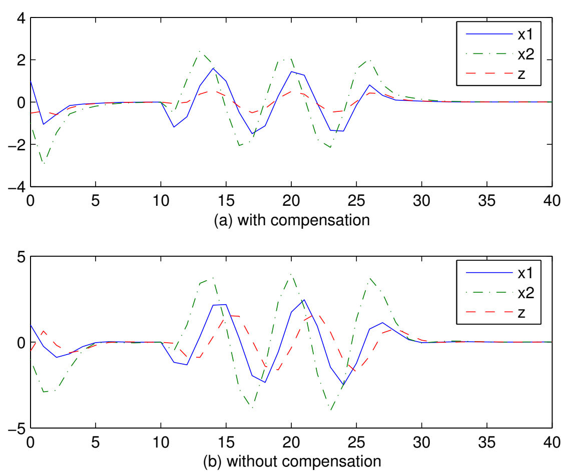

Fig. 2. Curves of state response and controlled output

Suppose the controller gains got by Theorem 1 with and without packet dropout compensation are used (if the compensation method is adopted, $K = [4.0151 \;\; -1.1952]$, otherwise, $K = [2.7169 \; -0.8087]$), and at the instant 10s, the disturbance inputs $3sin(j)$ ($j = 1, 2, \cdots, 15$) are added into the system, the plant state responses and controlled outputs (τ_{k_j}=0.06s) are pictured in Fig. 2. From Fig. 2(a) and Fig. 2(b), one can see that the proposed compensation method may provide good H_∞ performance.

VI. CONCLUSIONS

This paper is concerned with the problem of designing H_∞ controllers for NCSs by using the linear estimation-based time delay and packet dropout compensation. The H_∞ controller design using the newly proposed delay switching-based method is proved to be less conservative than the one using the existing parameter uncertainty-based method. The simulation results have illustrated the effectiveness of the proposed delay switching-based method and the linear estimation-based time delay and packet dropout compensation.

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