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Packet Dropout Compensation for Networked Control Systems: A Multiple Communication Channels Method

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Abstract— **This paper is concerned with the problem of H_∞ controller design for networked control systems (NCSs) with time delay and packet dropout. The idle communication channels are made full use to compensate the negative influences of time delay and packet dropout. By defining new Lyapunov function, linear matrix inequality (LMI)-based H_∞ controller design is presented, and the merit of the proposed design methods lies in their less conservativeness, which is achieved by avoiding the utilization of bounding inequalities for cross products of vectors. The simulation results illustrate the effectiveness of the proposed multiple communication channels-based time delay and packet dropout compensation.**

I. INTRODUCTION

Networked control systems (NCSs) have received increasing attentions in recent years. However, the insertion of the communication network will lead to time delay and data packet dropout inevitably, which might be potential sources to instability and poor performance of NCSs.

Many researchers have studied stability/stabilization of networked control systems in the presence of network-induced delay [1]-[3]. Based on remote control and local control strategy, a class of hybrid multi-rate control models with uncertainties and multiple time-varying delays was formulated in [4], and their robust stability properties were also investigated. For other results dealing with time delay specifically, see also [5]-[10].

Besides the network-induced time delay, data packet dropout is also an important issue for NCSs. [11]-[12] studied the problem of stabilization of NCSs with packet dropout, and time varying optimal control with packet dropout was studied in [13]. There have also been considerable research efforts on H_∞ control for systems with time-delay [14]-[18].

As we can see, the problem of time delay and packet dropout compensation is seldom discussed in the papers listed above. Recently, there are some preliminary results on compensating for time delay and packet dropout. In [19], an estimator was used to reconstruct an approximation to the undelayed plant state. By using a buffer in the actuator node and a state estimator in the controller node, [20] presented LMI-based sufficient condition for the stability of NCSs, but the problem of controller design was not discussed in [19] and [20]. By using prediction-based method, [21] and [22] studied the problem of time delay compensation for NCSs. [23] was concerned with the design of NCSs with random network delay in the feedback channel and gave stability criteria of closed-loop networked predictive control systems.

Just as we can see, the compensation methods presented above are usually based on estimation or prediction. If the prediction-based method is used, an augmented state vector is usually defined (see [22], [23]), and the introduction of the augmented state vector will introduce some conservativeness since the common positive definite matrix P with special structure is needed (see [22]).

On the other hand, one of the most important characters of NCSs is the sharing of the communication channels, if the sampled data from the sensor are transmitted through different communication channels which are idle, the actuator may still receive control inputs even the network communication in some channels is lost for long time, and the sharing of the idle communication channels will not increase the cost of hardware. In this paper, we will propose a new compensation method which is realized by making sufficient use of the idle communication channels. The discretized controlled plant is considered in this paper, and every communication channel may experience time delay and packet dropout.

This paper is organized as follows. The model of NCSs with multiple communication channels is presented in Section 2. By using LMI-based method, Section 3 presents the H_∞ controller design for NCSs with multiple communication channels, time delay and packet dropout. The results of numerical simulation are presented in Section 4. Conclusions are stated in Section 5.

II. PRELIMINARIES AND PROBLEM STATEMENT

The typical structure of NCSs with multiple communication channels is shown in Fig. 1, where K_i $(i = 1, \cdots, p)$ are controllers corresponding to plant i. The motivation of this paper is to compensate the negative influences of time delay and packet dropout by making full use of the idle communication channels and controllers.

Remark 1. Just as shown in Fig. 1, if the network communication in the channel 1 is lost for long time, the communica-

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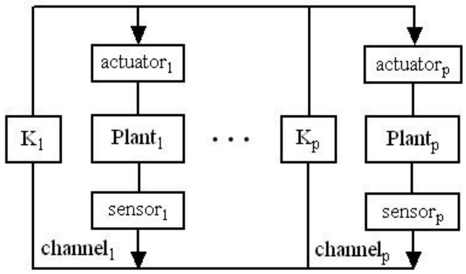

Fig. 1. NCSs with multiple communication channels

tion channels 2, 3, $\cdots$, p (if they are idle) may also be used to transmit control inputs for the plant 1, which will reduce the negative influences of time delay and packet dropout on the plant 1. On the other hand, if the communication channels 2, 3, $\cdots$, p are all busy, then only the channel 1 is used to transmit control inputs for the plant 1, so the NCSs with single communication channel can be viewed as a special case of NCSs with multiple communication channels.

In this paper, we will consider the problem of H_∞ controller design for NCSs which may receive control inputs from two communication channels, and the proposed H_∞ controller design is also applicable for NCSs which may receive control inputs from more than two communication channels.

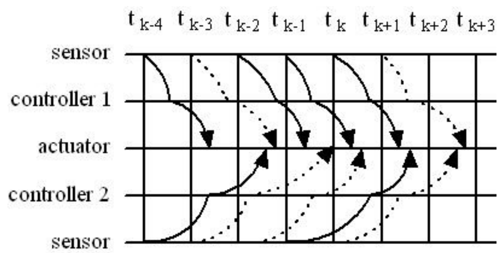

Fig. 2. Timing diagram of signals transmitting

Suppose the control inputs from the controllers 1 and 2 are transmitted through the communication channels 1 and 2, respectively, and the controller 1 is introduced to compensate the negative influences of time delay and packet dropout. Fig. 2 illustrates the timing diagram of signals transmitting of NCSs which may receive control inputs from two communication channels.

The assumption on time delay is as follows.

Assumption 1. The time delay τ_{1k} and τ_{2k} (τ_{1k} and τ_{2k} are the sensor-to-actuator time delay of the channels 1 and 2, respectively) are assumed to be time-varying and can be denoted as $\tau_{1k} = nh + \varepsilon_{1k}$ and $\tau_{2k} = mh + \varepsilon_{2k}$, where n, m are positive integers, h is the sampling period, $\varepsilon_{1k} \in [0,\ h]$, $\varepsilon_{2k} \in [0,\ h]$, $n \leq m$, and $\varepsilon_{1k} \leq \varepsilon_{2k}$.

Consider a linear time-invariant system described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1u(t) + B_2\omega(t) \\ z(t) &= Cx(t) + Du(t) \end{aligned} \tag{1}$$

where $x(t)$, $u(t)$, $z(t)$, $\omega(t)$ are the state vector, control input vector, controlled output, and disturbance input, respectively, and $\omega(t)$ is piecewise constant. A, B_1, B_2, C, D are known constant matrices of appropriate dimensions. Throughout this paper, matrices, if not explicitly stated, are assumed to have appropriate dimensions.

At the sampling instant t_k, suppose the latest available control inputs based on the controller 1 and the controller 2 are u_{k-l_k} and u_{k-r_k}, respectively, and $l_m \leq l_k \leq l_M$, $r_m \leq r_k \leq r_M$, $l_m \leq r_m$, $l_M \leq r_M$.

In this paper, we consider the case that the packet dropout is stochastic and the numbers of consecutive packet dropout in the communication channel 1 and communication channel 2 are upper-bounded by $l_M - n - 1$ and $r_M - m - 1$, respectively. For NCSs with long time delay and packet dropout, if the actuator receives two control inputs u_{k-n} and u_{k-m} during the sampling period $[t_k,\ t_{k+1}]$, the discrete time representation of the system (1) is as follows

$$\begin{aligned} x_{k+1} &= \Phi x_k + \Gamma_{0n}u_{k-n} + \Gamma_{1n}u_{k-l_k} + \Gamma_{0m}u_{k-m} \\ &\quad + \Gamma_{1m}u_{k-r_k} + \Gamma_2\omega_k \\ z_k &= Cx_k + D[pr * u_{k-l_k} + (1-pr) * u_{k-r_k}] \end{aligned} \tag{2}$$

where $\Phi = e^{Ah}$, $\Gamma_{0n} = \int_0^{h-\varepsilon_{1k}} e^{As}dsB_1$, $\Gamma_{1n} = \int_{h-\varepsilon_{1k}}^{h} e^{As}dsB_1$, $\Gamma_{0m} = \int_0^{h-\varepsilon_{2k}} e^{As}dsB_1$, $\Gamma_{1m} = \int_{h-\varepsilon_{2k}}^{h} e^{As}dsB_1$, $\Gamma_2 = \int_0^{h} e^{As}dsB_2$, $u_{k-n} = -K_1x_{k-n}$, $u_{k-l_k} = -K_1x_{k-l_k}$, $u_{k-m} = -K_2x_{k-m}$, $u_{k-r_k} = -K_2x_{k-r_k}$. If the latest available control input at the instant t_k is u_{k-l_k}, then $pr = 1$, and if the latest available one is u_{k-r_k}, $pr = 0$.

Define $-\Gamma_{0n}K_1$, $-\Gamma_{0m}K_2$, $-\Gamma_{1n}K_1$, $-\Gamma_{1m}K_2$ and Γ_2 as Φ_1, Φ_2, Φ_3, Φ_4 and Φ_5, respectively, (2) can be written as follows

$$\begin{aligned} x_{k+1} &= \Phi x_k + \Phi_1x_{k-n} + \Phi_2x_{k-m} + \Phi_3x_{k-l_k} \\ &\quad + \Phi_4x_{k-r_k} + \Phi_5\omega_k \\ z_k &= Cx_k - D[pr * K_1 * x_{k-l_k} + (1-pr) * K_2 * x_{k-r_k}] \end{aligned} \tag{3}$$

then the problem of H_∞ controller design for (1) can be reduced to the corresponding problem for the system (3).

III. H_∞ CONTROLLER DESIGN FOR NCSS

A. NCSs with Multiple Communication Channels

Based on the model presented in (3), we are now in a position to design the feedback gains K_1 and K_2, which can make the system (3) asymptotically stable with the H_∞ norm bound γ.

Theorem 1. For given positive scalars n, m, l_M, l_m, r_M, r_m, if there exist symmetric positive definite matrices $\widetilde{P}$, $\widetilde{Q}_i$ ($i = 1,\ \cdots,\ 6$), $\widetilde{R}_j$ and matrices $\widetilde{M}_j$, $\widetilde{N}_j$, $\widetilde{X}_j$, $\widetilde{Y}_j$, $\widetilde{Z}_j$ ($j = 1,\ \cdots,\ 4$), N, V_1, V_2, scalar $\gamma > 0$, such that the following LMIs hold for every feasible value of pr ($pr = 0$ or $pr = 1$)

$$\begin{bmatrix} \widetilde{\Lambda}_{11} & \widetilde{\Lambda}_{12} & \widetilde{\Lambda}_{13} & \widetilde{\Lambda}_{14} & \widetilde{\Lambda}_{15} & \widetilde{\Lambda}_{16} \\ * & \widetilde{\Lambda}_{22} & \widetilde{\Lambda}_{23} & \widetilde{\Lambda}_{24} & \widetilde{\Lambda}_{25} & \widetilde{\Lambda}_{26} \\ * & * & \widetilde{\Lambda}_{33} & \widetilde{\Lambda}_{34} & \widetilde{\Lambda}_{35} & \widetilde{\Lambda}_{36} \\ * & * & * & \widetilde{\Lambda}_{44} & \widetilde{\Lambda}_{45} & \widetilde{\Lambda}_{46} \\ * & * & * & * & \widetilde{\Lambda}_{55} & \widetilde{\Lambda}_{56} \\ * & * & * & * & * & \widetilde{\Lambda}_{66} \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix}$$

$$\left.\begin{matrix} \widetilde{\Lambda}_{17} & \widetilde{\Lambda}_{18} & -\theta_1\Gamma_2 & NC^T \\ \widetilde{\Lambda}_{27} & \widetilde{\Lambda}_{28} & -\theta_2\Gamma_2 & 0 \\ \widetilde{\Lambda}_{37} & \widetilde{\Lambda}_{38} & -\theta_3\Gamma_2 & \Upsilon_1 \\ \widetilde{\Lambda}_{47} & \widetilde{\Lambda}_{48} & -\theta_4\Gamma_2 & 0 \\ \widetilde{\Lambda}_{57} & \widetilde{\Lambda}_{58} & -\theta_5\Gamma_2 & 0 \\ \widetilde{\Lambda}_{67} & \widetilde{\Lambda}_{68} & -\theta_6\Gamma_2 & \Upsilon_2 \\ \widetilde{\Lambda}_{77} & \widetilde{\Lambda}_{78} & -\theta_7\Gamma_2 & 0 \\ * & \widetilde{\Lambda}_{88} & -\theta_8\Gamma_2 & 0 \\ * & * & -\gamma I & 0 \\ * & * & * & -\gamma I \end{matrix}\right] < 0 \qquad (4)$$

$$\begin{bmatrix} -\widetilde{X}_j & -\widetilde{Y}_j & -\widetilde{M}_j \\ * & -\widetilde{Z}_j & -\widetilde{N}_j \\ * & * & -\widetilde{R}_j \end{bmatrix} < 0, \quad j = 1,\ 2,\ 3,\ 4 \qquad (5)$$

where

$$\begin{aligned}
\widetilde{\Lambda}_{11} &= -\widetilde{P} + \widetilde{Q}_1 + n\widetilde{R}_1 + (l_M - l_m + 1)\widetilde{Q}_2 + l_M\widetilde{R}_2 + \widetilde{Q}_3 \\
&\quad + \widetilde{Q}_4 + m\widetilde{R}_3 + (r_M - r_m + 1)\widetilde{Q}_5 + r_M\widetilde{R}_4 + \widetilde{Q}_6 \\
&\quad + \widetilde{M}_1 + \widetilde{M}_1^T + n\widetilde{X}_1 + \widetilde{M}_2 + \widetilde{M}_2^T + l_M\widetilde{X}_2 + \widetilde{M}_3 + \widetilde{M}_3^T \\
&\quad + m\widetilde{X}_3 + \widetilde{M}_4 + \widetilde{M}_4^T + r_M\widetilde{X}_4 - \theta_1\Phi N^T - \theta_1 N\Phi^T \\
\widetilde{\Lambda}_{12} &= -\widetilde{M}_1 + \widetilde{N}_1^T + n\widetilde{Y}_1 + \theta_1\Gamma_{0n}V_1 - \theta_2 N\Phi^T \\
\widetilde{\Lambda}_{13} &= \theta_1\Gamma_{1n}V_1 - \theta_3 N\Phi^T \\
\widetilde{\Lambda}_{14} &= -\widetilde{M}_2 + \widetilde{N}_2^T + l_M\widetilde{Y}_2 - \theta_4 N\Phi^T \\
\widetilde{\Lambda}_{15} &= -\widetilde{M}_3 + \widetilde{N}_3^T + m\widetilde{Y}_3 + \theta_1\Gamma_{0m}V_2 - \theta_5 N\Phi^T \\
\widetilde{\Lambda}_{16} &= \theta_1\Gamma_{1m}V_2 - \theta_6 N\Phi^T \\
\widetilde{\Lambda}_{17} &= -\widetilde{M}_4 + \widetilde{N}_4^T + r_M\widetilde{Y}_4 - \theta_7 N\Phi^T \\
\widetilde{\Lambda}_{18} &= -n\widetilde{R}_1 - l_M\widetilde{R}_2 - m\widetilde{R}_3 - r_M\widetilde{R}_4 + \theta_1 N^T - \theta_8 N\Phi^T \\
\widetilde{\Lambda}_{22} &= -\widetilde{Q}_1 - \widetilde{N}_1 - \widetilde{N}_1^T + n\widetilde{Z}_1 + \theta_2\Gamma_{0n}V_1 + \theta_2 V_1^T\Gamma_{0n}^T \\
\widetilde{\Lambda}_{23} &= \theta_2\Gamma_{1n}V_1 + \theta_3 V_1^T\Gamma_{0n}^T \\
\widetilde{\Lambda}_{24} &= \theta_4 V_1^T\Gamma_{0n}^T \\
\widetilde{\Lambda}_{25} &= \theta_2\Gamma_{0m}V_2 + \theta_5 V_1^T\Gamma_{0n}^T \\
\widetilde{\Lambda}_{26} &= \theta_2\Gamma_{1m}V_2 + \theta_6 V_1^T\Gamma_{0n}^T \\
\widetilde{\Lambda}_{27} &= \theta_7 V_1^T\Gamma_{0n}^T \\
\widetilde{\Lambda}_{28} &= \theta_2 N^T + \theta_8 V_1^T\Gamma_{0n}^T \\
\widetilde{\Lambda}_{33} &= -\widetilde{Q}_2 + \theta_3\Gamma_{1n}V_1 + \theta_3 V_1^T\Gamma_{1n}^T \\
\widetilde{\Lambda}_{34} &= \theta_4 V_1^T\Gamma_{1n}^T \\
\widetilde{\Lambda}_{35} &= \theta_3\Gamma_{0m}V_2 + \theta_5 V_1^T\Gamma_{1n}^T \\
\widetilde{\Lambda}_{36} &= \theta_3\Gamma_{1m}V_2 + \theta_6 V_1^T\Gamma_{1n}^T \\
\widetilde{\Lambda}_{37} &= \theta_7 V_1^T\Gamma_{1n}^T \\
\widetilde{\Lambda}_{38} &= \theta_3 N^T + \theta_8 V_1^T\Gamma_{1n}^T \\
\widetilde{\Lambda}_{44} &= -\widetilde{Q}_3 - \widetilde{N}_2 - \widetilde{N}_2^T + l_M\widetilde{Z}_2 \\
\widetilde{\Lambda}_{45} &= \theta_4\Gamma_{0m}V_2 \\
\widetilde{\Lambda}_{46} &= \theta_4\Gamma_{1m}V_2 \\
\widetilde{\Lambda}_{47} &= 0 \\
\widetilde{\Lambda}_{48} &= \theta_4 N^T \\
\widetilde{\Lambda}_{55} &= -\widetilde{Q}_4 - \widetilde{N}_3 - \widetilde{N}_3^T + m\widetilde{Z}_3 + \theta_5\Gamma_{0m}V_2 + \theta_5 V_2^T\Gamma_{0m}^T \\
\widetilde{\Lambda}_{56} &= \theta_5\Gamma_{1m}V_2 + \theta_6 V_2^T\Gamma_{0m}^T \\
\widetilde{\Lambda}_{57} &= \theta_7 V_2^T\Gamma_{0m}^T \\
\widetilde{\Lambda}_{58} &= \theta_5 N^T + \theta_8 V_2^T\Gamma_{0m}^T \\
\widetilde{\Lambda}_{66} &= -\widetilde{Q}_5 + \theta_6\Gamma_{1m}V_2 + \theta_6 V_2^T\Gamma_{1m}^T \\
\widetilde{\Lambda}_{67} &= \theta_7 V_2^T\Gamma_{1m}^T \\
\widetilde{\Lambda}_{68} &= \theta_6 N^T + \theta_8 V_2^T\Gamma_{1m}^T \\
\widetilde{\Lambda}_{77} &= -\widetilde{Q}_6 - \widetilde{N}_4 - \widetilde{N}_4^T + r_M\widetilde{Z}_4 \\
\widetilde{\Lambda}_{78} &= \theta_7 N^T \\
\widetilde{\Lambda}_{88} &= \widetilde{P} + n\widetilde{R}_1 + l_M\widetilde{R}_2 + m\widetilde{R}_3 + r_M\widetilde{R}_4 + \theta_8 N + \theta_8 N^T \\
\Upsilon_1 &= -pr * V_1^T D^T \\
\Upsilon_2 &= -(1 - pr)V_2^T D^T
\end{aligned}$$

then with the controller gains

$$K_1 = V_1 N^{-T}, \quad K_2 = V_2 N^{-T}$$

the system described by (3) is asymptotically stable with H_∞ norm bound γ.

Proof: Let us consider the following Lyapunov function

$$\begin{aligned} V_k = {} & V_{1k} + V_{2k} + V_{3k} + V_{4k} + V_{5k} + V_{6k} + V_{7k} \\ & + V_{8k} + V_{9k} + V_{10k} + V_{11k} + V_{12k} + V_{13k} \end{aligned} \qquad (6)$$

where

$$\begin{aligned}
V_{1k} &= x_k^T P x_k \\
V_{2k} &= \sum_{i=k-n}^{k-1} x_i^T Q_1 x_i \\
V_{3k} &= \sum_{i=-n}^{-1} \sum_{j=k+i}^{k-1} \eta_j^T R_1 \eta_j \\
V_{4k} &= \sum_{i=k-l_k}^{k-1} x_i^T Q_2 x_i \\
V_{5k} &= \sum_{i=-l_M+1}^{-l_m} \sum_{j=k+i}^{k-1} x_j^T Q_2 x_j \\
V_{6k} &= \sum_{i=-l_M}^{-1} \sum_{j=k+i}^{k-1} \eta_j^T R_2 \eta_j \\
V_{7k} &= \sum_{i=k-l_M}^{k-1} x_i^T Q_3 x_i \\
V_{8k} &= \sum_{i=k-m}^{k-1} x_i^T Q_4 x_i \\
V_{9k} &= \sum_{i=-m}^{-1} \sum_{j=k+i}^{k-1} \eta_j^T R_3 \eta_j \\
V_{10k} &= \sum_{i=k-r_k}^{k-1} x_i^T Q_5 x_i \\
V_{11k} &= \sum_{i=-r_M+1}^{-r_m} \sum_{j=k+i}^{k-1} x_j^T Q_5 x_j \\
V_{12k} &= \sum_{i=-r_M}^{-1} \sum_{j=k+i}^{k-1} \eta_j^T R_4 \eta_j \\
V_{13k} &= \sum_{i=k-r_M}^{k-1} x_i^T Q_6 x_i
\end{aligned}$$

and P, Q_1, $\cdots$, Q_6, R_1, $\cdots$, R_4 are symmetric positive definite matrices, $\eta_j = x_{j+1} - x_j$.

Define $\triangle V_k = V_{k+1} - V_k$, then

$$\triangle V_{1k} = x_{k+1}^T P x_{k+1} - x_k^T P x_k \qquad (7)$$

$$\triangle V_{2k} = x_k^T Q_1 x_k - x_{k-n}^T Q_1 x_{k-n} \qquad (8)$$

$$\triangle V_{3k} = \sum_{i=-n}^{-1} (\eta_k^T R_1 \eta_k - \eta_{k+i}^T R_1 \eta_{k+i})$$
$$= n(x_{k+1} - x_k)^T R_1 (x_{k+1} - x_k) - \sum_{i=k-n}^{k-1} \eta_i^T R_1 \eta_i \quad (9)$$

$$\triangle V_{4k} \le x_k^T Q_2 x_k + \sum_{i=k-l_M+1}^{k-l_m} x_i^T Q_2 x_i - x_{k-l_k}^T Q_2 x_{k-l_k} \quad (10)$$

$$\triangle V_{5k} = \sum_{i=-l_M+1}^{-l_m} (x_k^T Q_2 x_k - x_{k+i}^T Q_2 x_{k+i})$$
$$= (l_M - l_m) x_k^T Q_2 x_k - \sum_{i=k-l_M+1}^{k-l_m} x_i^T Q_2 x_i \quad (11)$$

$$\triangle V_{6k} = \sum_{i=-l_M}^{-1} (\eta_k^T R_2 \eta_k - \eta_{k+i}^T R_2 \eta_{k+i})$$
$$= l_M (x_{k+1} - x_k)^T R_2 (x_{k+1} - x_k) - \sum_{i=k-l_M}^{k-1} \eta_i^T R_2 \eta_i \quad (12)$$

$$\triangle V_{7k} = x_k^T Q_3 x_k - x_{k-l_M}^T Q_3 x_{k-l_M} \quad (13)$$

Similarly, we have

$$\triangle V_{8k} = x_k^T Q_4 x_k - x_{k-m}^T Q_4 x_{k-m} \quad (14)$$

$$\triangle V_{9k} = m(x_{k+1} - x_k)^T R_3 (x_{k+1} - x_k) - \sum_{i=k-m}^{k-1} \eta_i^T R_3 \eta_i \quad (15)$$

$$\triangle V_{10k} \le x_k^T Q_5 x_k + \sum_{i=k-r_M+1}^{k-r_m} x_i^T Q_5 x_i - x_{k-r_k}^T Q_5 x_{k-r_k} \quad (16)$$

$$\triangle V_{11k} = (r_M - r_m) x_k^T Q_5 x_k - \sum_{i=k-r_M+1}^{k-r_m} x_i^T Q_5 x_i \quad (17)$$

$$\triangle V_{12k} = r_M (x_{k+1} - x_k)^T R_4 (x_{k+1} - x_k) - \sum_{i=k-r_M}^{k-1} \eta_i^T R_4 \eta_i \quad (18)$$

$$\triangle V_{13k} = x_k^T Q_6 x_k - x_{k-r_M}^T Q_6 x_{k-r_M} \quad (19)$$

To notice that

$$\Theta_1 = 2\begin{bmatrix} x_k^T & x_{k-n}^T \end{bmatrix} \begin{bmatrix} M_1 \\ N_1 \end{bmatrix} (x_k - x_{k-n} - \sum_{i=k-n}^{k-1} \eta_i) = 0 \quad (20)$$

$$\Theta_2 = n\begin{bmatrix} x_k^T & x_{k-n}^T \end{bmatrix} \begin{bmatrix} X_1 & Y_1 \\ * & Z_1 \end{bmatrix} \begin{bmatrix} x_k \\ x_{k-n} \end{bmatrix} - \sum_{i=k-n}^{k-1} \begin{bmatrix} x_k^T & x_{k-n}^T \end{bmatrix} \begin{bmatrix} X_1 & Y_1 \\ * & Z_1 \end{bmatrix} \begin{bmatrix} x_k \\ x_{k-n} \end{bmatrix} = 0 \quad (21)$$

$$\Theta_3 = 2\begin{bmatrix} x_k^T & x_{k-l_M}^T \end{bmatrix} \begin{bmatrix} M_2 \\ N_2 \end{bmatrix} (x_k - x_{k-l_M} - \sum_{i=k-l_M}^{k-1} \eta_i) = 0 \quad (22)$$

$$\Theta_4 = l_M\begin{bmatrix} x_k^T & x_{k-l_M}^T \end{bmatrix} \begin{bmatrix} X_2 & Y_2 \\ * & Z_2 \end{bmatrix} \begin{bmatrix} x_k \\ x_{k-l_M} \end{bmatrix} - \sum_{i=k-l_M}^{k-1} \begin{bmatrix} x_k^T & x_{k-l_M}^T \end{bmatrix} \begin{bmatrix} X_2 & Y_2 \\ * & Z_2 \end{bmatrix} \begin{bmatrix} x_k \\ x_{k-l_M} \end{bmatrix} = 0 \quad (23)$$

$$\Theta_5 = 2\begin{bmatrix} x_k^T & x_{k-m}^T \end{bmatrix} \begin{bmatrix} M_3 \\ N_3 \end{bmatrix} (x_k - x_{k-m} - \sum_{i=k-m}^{k-1} \eta_i) = 0 \quad (24)$$

$$\Theta_6 = m\begin{bmatrix} x_k^T & x_{k-m}^T \end{bmatrix} \begin{bmatrix} X_3 & Y_3 \\ * & Z_3 \end{bmatrix} \begin{bmatrix} x_k \\ x_{k-m} \end{bmatrix} - \sum_{i=k-m}^{k-1} \begin{bmatrix} x_k^T & x_{k-m}^T \end{bmatrix} \begin{bmatrix} X_3 & Y_3 \\ * & Z_3 \end{bmatrix} \begin{bmatrix} x_k \\ x_{k-m} \end{bmatrix} = 0 \quad (25)$$

$$\Theta_7 = 2\begin{bmatrix} x_k^T & x_{k-r_M}^T \end{bmatrix} \begin{bmatrix} M_4 \\ N_4 \end{bmatrix} (x_k - x_{k-r_M} - \sum_{i=k-r_M}^{k-1} \eta_i) = 0 \quad (26)$$

$$\Theta_8 = r_M\begin{bmatrix} x_k^T & x_{k-r_M}^T \end{bmatrix} \begin{bmatrix} X_4 & Y_4 \\ * & Z_4 \end{bmatrix} \begin{bmatrix} x_k \\ x_{k-r_M} \end{bmatrix} - \sum_{i=k-r_M}^{k-1} \begin{bmatrix} x_k^T & x_{k-r_M}^T \end{bmatrix} \begin{bmatrix} X_4 & Y_4 \\ * & Z_4 \end{bmatrix} \begin{bmatrix} x_k \\ x_{k-r_M} \end{bmatrix} = 0 \quad (27)$$

On the other hand, from the system (3), we have

$$\Theta_9 = 2[\theta_1 x_k^T S + \theta_2 x_{k-n}^T S + \theta_3 x_{k-l_k}^T S + \theta_4 x_{k-l_M}^T S + \theta_5 x_{k-m}^T S + \theta_6 x_{k-r_k}^T S + \theta_7 x_{k-r_M}^T S + \theta_8 x_{k+1}^T S].[x_{k+1} - \Phi x_k - \Phi_1 x_{k-n} - \Phi_2 x_{k-m} - \Phi_3 x_{k-l_k} - \Phi_4 x_{k-r_k} - \Phi_5 \omega_k] = 0 \quad (28)$$

Combining (7)-(28) together, we have

$$\Delta V_k + \Theta_1 + \cdots + \Theta_9 \le \xi_k{}^T \Lambda \xi_k + \sum_{i=k-n}^{k-1} \xi_{k1}^T \Lambda_1 \xi_{k1} + \sum_{i=k-l_M}^{k-1} \xi_{k2}^T \Lambda_2 \xi_{k2} + \sum_{i=k-m}^{k-1} \xi_{k3}^T \Lambda_3 \xi_{k3} + \sum_{i=k-r_M}^{k-1} \xi_{k4}^T \Lambda_4 \xi_{k4} \quad (29)$$

where

$$\xi_k^T = \begin{bmatrix} x_k^T & x_{k-n}^T & x_{k-l_k}^T & x_{k-l_M}^T & x_{k-m}^T & x_{k-r_k}^T & x_{k-r_M}^T & x_{k+1}^T & \omega_k^T \end{bmatrix}$$

$$\xi_{k1} = \begin{bmatrix} x_k \\ x_{k-n} \\ \eta_i \end{bmatrix}, \quad \xi_{k2} = \begin{bmatrix} x_k \\ x_{k-l_M} \\ \eta_i \end{bmatrix}$$

$$\xi_{k3} = \begin{bmatrix} x_k \\ x_{k-m} \\ \eta_i \end{bmatrix}, \quad \xi_{k4} = \begin{bmatrix} x_k \\ x_{k-r_M} \\ \eta_i \end{bmatrix}$$

$$\Lambda_j = \begin{bmatrix} -X_j & -Y_j & -M_j \\ * & -Z_j & -N_j \\ * & * & -R_j \end{bmatrix}, \quad j = 1,\ 2,\ 3,\ 4 \quad (30)$$

and Λ is omitted here for briefness. By some matrix manipulations, we can prove easily that $||z||_2^2 < \gamma^2 ||\omega||_2^2$ (the details are omitted here).

If the disturbance input $\omega_k = 0$, (4) and (5) can ensure the asymptotic stability of the system described by (3), and if $\omega_k \neq 0$, we have $||z||_2^2 < \gamma^2 ||\omega||_2^2$, so if (4) and (5) are satisfied, the system described by (3) with $K_1 = V_1 N^{-T}$, $K_2 = V_2 N^{-T}$ is asymptotically stable with H_∞ norm bound γ, this completes the proof. ■

Remark 2. Just as shown in the proof of Theorem 1, (20)-(28) are introduced to avoid the utilization of bounding inequalities for cross products of vectors, which may introduce less conservativeness. On the other hand, Theorem 1 presents LMI-based sufficient conditions for H_∞ controller design, which can be effectively solved via Matlab LMI control toolbox.

The above given H_∞ controller design is also applicable for NCSs with single communication channel, which will be studied in the following.

B. NCSs with Single Communication Channel

Compared with the H_∞ controller design for NCSs with multiple communication channels, suppose only the communication channel 2 and the controller 2 are used, and the control input available at the sampling instant t_k is u_{k-r_k}, then the discrete time representation of the system (1) is as follows

$$\begin{aligned} x_{k+1} &= \Phi x_k + \Gamma_{0m} u_{k-m} + \Gamma_{1m} u_{k-r_k} + \Gamma_2 \omega_k \\ z_k &= C x_k + D u_{k-l_k} \end{aligned} \tag{31}$$

where $\Phi = e^{Ah}$, $\Gamma_{0m} = \int_0^{h-\varepsilon_{2k}} e^{As} ds B_1$, $\Gamma_{1m} = \int_{h-\varepsilon_{2k}}^{h} e^{As} ds B_1$, $\Gamma_2 = \int_0^h e^{As} ds B_2$, $u_{k-m} = -K_2 x_{k-m}$, $u_{k-r_k} = -K_2 x_{k-r_k}$.

Similar to Theorem 1, the following corollary presents the H_∞ controller design for NCSs with single communication channel.

Corollary 1. For given positive scalars m, r_M, r_m, if there exist symmetric positive definite matrices $\widetilde{P}$, $\widetilde{Q}_i$ $(i = 1,\ 2,\ 3)$, $\widetilde{R}_j$ and matrices $\widetilde{M}_j$, $\widetilde{N}_j$, $\widetilde{X}_j$, $\widetilde{Y}_j$, $\widetilde{Z}_j$ $(j = 1,\ 2)$, N, V, scalar $\gamma > 0$, such that the following LMIs hold

$$\begin{bmatrix} \widetilde{\Lambda}_{11} & \widetilde{\Lambda}_{12} & \widetilde{\Lambda}_{13} & \widetilde{\Lambda}_{14} & \widetilde{\Lambda}_{15} & -\theta_1\Gamma_2 & NC^T \\ * & \widetilde{\Lambda}_{22} & \widetilde{\Lambda}_{23} & \widetilde{\Lambda}_{24} & \widetilde{\Lambda}_{25} & -\theta_2\Gamma_2 & 0 \\ * & * & \widetilde{\Lambda}_{33} & \widetilde{\Lambda}_{34} & \widetilde{\Lambda}_{35} & -\theta_3\Gamma_2 & -V^T D^T \\ * & * & * & \widetilde{\Lambda}_{44} & \widetilde{\Lambda}_{45} & -\theta_4\Gamma_2 & 0 \\ * & * & * & * & \widetilde{\Lambda}_{55} & -\theta_5\Gamma_2 & 0 \\ * & * & * & * & * & -\gamma I & 0 \\ * & * & * & * & * & * & -\gamma I \end{bmatrix} < 0 \tag{32}$$

$$\begin{bmatrix} -\widetilde{X}_j & -\widetilde{Y}_j & -\widetilde{M}_j \\ * & -\widetilde{Z}_j & -\widetilde{N}_j \\ * & * & -\widetilde{R}_j \end{bmatrix} < 0, \quad j = 1,\ 2,\ 3,\ 4 \tag{33}$$

where

$$\begin{aligned}
\widetilde{\Lambda}_{11} &= -\widetilde{P} + \widetilde{Q}_1 + m\widetilde{R}_1 + (r_M - r_m + 1)\widetilde{Q}_2 \\
&\quad + r_M\widetilde{R}_2 + \widetilde{Q}_3 + \widetilde{M}_1 + \widetilde{M}_1^T + m\widetilde{X}_1 + \widetilde{M}_2 \\
&\quad + \widetilde{M}_2^T + r_M\widetilde{X}_2 - \theta_1\Phi N^T - \theta_1 N\Phi^T \\
\widetilde{\Lambda}_{12} &= -\widetilde{M}_1 + \widetilde{N}_1^T + m\widetilde{Y}_1 + \theta_1\Gamma_{0m}V - \theta_2 N\Phi^T \\
\widetilde{\Lambda}_{13} &= \theta_1\Gamma_{1m}V - \theta_3 N\Phi^T \\
\widetilde{\Lambda}_{14} &= -\widetilde{M}_2 + \widetilde{N}_2^T + r_M\widetilde{Y}_2 - \theta_4 N\Phi^T \\
\widetilde{\Lambda}_{15} &= -m\widetilde{R}_1 - r_M\widetilde{R}_2 + \theta_1 N^T - \theta_5 N\Phi^T \\
\widetilde{\Lambda}_{22} &= -\widetilde{Q}_1 - \widetilde{N}_1 - \widetilde{N}_1^T + m\widetilde{Z}_1 + \theta_2\Gamma_{0m}V + \theta_2 V^T\Gamma_{0m}^T \\
\widetilde{\Lambda}_{23} &= \theta_2\Gamma_{1m}V + \theta_3 V^T\Gamma_{0m}^T \\
\widetilde{\Lambda}_{24} &= \theta_4 V^T\Gamma_{0m}^T \\
\widetilde{\Lambda}_{25} &= \theta_2 N^T + \theta_5 V^T\Gamma_{0m}^T \\
\widetilde{\Lambda}_{33} &= -\widetilde{Q}_2 + \theta_3\Gamma_{1m}V + \theta_3 V^T\Gamma_{1m}^T \\
\widetilde{\Lambda}_{34} &= \theta_4 V^T\Gamma_{1m}^T \\
\widetilde{\Lambda}_{35} &= \theta_3 N^T + \theta_5 V^T\Gamma_{1m}^T \\
\widetilde{\Lambda}_{44} &= -\widetilde{Q}_3 - \widetilde{N}_2 - \widetilde{N}_2^T + r_M\widetilde{Z}_2 \\
\widetilde{\Lambda}_{45} &= \theta_4 N^T \\
\widetilde{\Lambda}_{55} &= \widetilde{P} + m\widetilde{R}_1 + r_M\widetilde{R}_2 + \theta_5 N + \theta_5 N^T
\end{aligned}$$

then with the control law

$$u_k = -K_2 x_k, \quad K_2 = VN^{-T}$$

the system described by (31) is asymptotically stable with H_∞ norm bound γ.

Remark 3. Theorem 1 and Corollary 1 present the H_∞ controller design for NCSs with multiple communication channels and single communication channel, respectively, and the multiple communication channels method may provide better H_∞ performance, which will be illustrated by an example in Section 4.

In the following, we will illustrate the effectiveness of the proposed design methods by an example.

TABLE I

THE H_∞ NORM BOUNDS (DIFFERENT ε_{2k})

ε_{2k}	$0.2h$	$0.3h$	$0.4h$	$0.5h$	h
γ_m	0.1019	0.1023	0.1024	0.1024	0.1025
γ_s	0.2289	6.3034	-	-	-

TABLE II

THE H_∞ NORM BOUNDS (DIFFERENT r_M)

r_M	5	8	11	13	14
γ_m	0.1019	0.1043	0.1063	0.1075	0.1081
γ_s	0.2289	0.4669	1.6496	29.8510	-

IV. SIMULATION RESULTS AND DISCUSSION

Example 1. To illustrate the effectiveness of the multiple communication channels-based time delay and packet dropout compensation, we present an open loop unstable system as follows

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} -0.3954 & -0.1070 \\ -0.0993 & -0.0131 \end{bmatrix} x(t) + \begin{bmatrix} 0.2631 \\ 0.2951 \end{bmatrix} u(t) \\ &\quad + \begin{bmatrix} -0.2756 \\ -0.2649 \end{bmatrix} \omega(t) \\ z(t) &= \begin{bmatrix} -0.0670 & -0.3057 \end{bmatrix} x(t) + 0.0091u(t) \end{aligned} \tag{34}$$

If the proposed multiple communication channels method is used, suppose the sampling period $h = 0.1s$, $n = 1$, $l_M = 3$, $l_m = n$, $m = 2$, $r_M = 5$, $r_m = m$, $\theta_1 = \cdots = \theta_7 = 1$, $\theta_8 = 100$, $\varepsilon_{1k} = 0.2h$. If single communication channel is used, suppose $m = 2$, $r_M = 5$, $r_m = m$, $\theta_1 = \cdots = \theta_4 = 1$, $\theta_5 = 100$. Denote γ_m and γ_s as the H_∞ norm bounds corresponding to the multiple communication channels method and the single communication channel method, respectively, by solving the LMIs in Theorem 1 and Corollary 1, we can get the H_∞ norm bounds corresponding to different ε_{2k} (see Table 1, '-' denotes that the LMIs are infeasible).

If ε_{1k} and ε_{2k} are constant and $\varepsilon_{1k} = \varepsilon_{2k} = 0.2h$, for NCSs with multiple communication channels, suppose $n = 1$, $l_M = 3$, $l_m = n$, $m = 2$, $r_m = m$, $\theta_1 = \cdots = \theta_7 = 1$, $\theta_8 = 100$. For

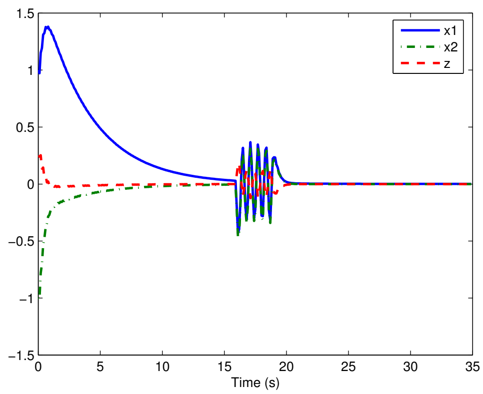

Fig. 3. State response and controlled output

NCSs with single communication channel, suppose $m = 2$, $r_m = m$, $\theta_1 = \cdots = \theta_4 = 1$, $\theta_5 = 100$, by solving the LMIs in Theorem 1 and Corollary 1, we can get the H_∞ norm bounds corresponding to different r_M (see Table 2).

From Table 1 and Table 2, one can see that the proposed multiple communication channels method may provide better H_∞ performance than the single communication channel-based method.

Suppose the initial state of the system is $x_0 = [1 - 1]^T$ and the control inputs based on plant states $x_0, x_2, x_4, \cdots$ are transferred to the actuator successfully, while the control inputs based on plant states $x_1, x_3, x_5, \cdots$ are dropped. If the proposed multiple communication channels method is used, suppose $n = 1$, $l_M = 3$, $l_m = n$, $m = 2$, $r_M = 5$, $r_m = m$, $\theta_1 = \cdots = \theta_7 = 1$, $\theta_8 = 100$, $\varepsilon_{1k} = 0.2h$, $\varepsilon_{2k} = 0.3h$, solving the LMIs in Theorem 1, we can get the controller gains $K_1 = [0.6595\ \ 10.6846]$, $K_2 = [0.0069\ \ 0.1052]$, during the time interval $[15.9s,\ 18.9s)$, the disturbance inputs $10sin(j)$ ($j = 1, 2, \cdots, 30$) are added into the system, the plant state response and controlled output are pictured in Fig. 3.

Fig. 3 illustrates the effectiveness of the proposed H_∞ controller design for NCSs with multiple communication channels.

V. CONCLUSIONS

This paper studies the problem of H_∞ controller design for NCSs with multiple communication channels. The idle communication channels and controllers are made sufficient use to compensate the negative influences of time delay and packet dropout. By defining new Lyapunov function and avoiding the utilization of bounding inequalities for cross products of vectors, LMI-based sufficient conditions for H_∞ controller design are presented, and the merit of the proposed design methods lies in their less conservativeness. The simulation results have illustrated the effectiveness of the proposed multiple communication channels-based H_∞ controller design.

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