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H_∞ Output Tracking Performance Analysis and Controller Design for Networked Control Systems with Packet Dropout

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***Abstract*— This paper studies the problems of H_∞ output tracking performance analysis and controller design for networked control systems (NCSs) with time delay and packet dropout. By defining new Lyapunov function and using the discrete Jensen inequality, linear matrix inequality (LMI)-based H_∞ output tracking performance analysis and controller design are presented. The adoption of the discrete Jensen inequality may reduce the computational complexity of the obtained results. The designed controllers can guarantee asymptotic tracking of prescribed reference outputs while rejecting disturbances. The simulation results illustrate the effectiveness of the proposed H_∞ output tracking controller design.**

I. INTRODUCTION

In many practical systems, the original plant, controller, sensor and actuator are difficult to be located at the same place, they are often connected over network media, giving rise to the so called NCSs. The flexibility and ease of maintenance of a system using network to transfer information is a very appealing goal. However, computer loads, network, sporadic faults, etc. may cause time delay and packet dropout, which might be potential sources to instability and poor performance of NCSs.

Many researchers have studied stability/stabilization for NCSs in the presence of network-induced delay [1], [2], [3]. In [4], a novel model-predictive-control strategy with a timeout scheme and p-step-ahead state estimation was presented to overcome the adverse influences of stochastic time delay and packet losses. For other methods dealing with time delay and packet dropout, see also [5], [6], [7]. Recently, there have been considerable research efforts on H_∞ control for systems with time delay [8], [9], [10].

Synthesizing feedback controllers to achieve asymptotic tracking of prescribed reference outputs while rejecting disturbances is a fundamental problem in control. The main objective of tracking control is to make the output of the plant, via a controller, track the output of a given reference model as close as possible. For the problem of output tracking control, [11] studied the reliable robust tracking controller design problem against actuator faults and control surface impairment for aircraft. [12] solved the tracking and disturbance rejection problem for infinite-dimensional linear systems, with reference and disturbance signals that were finite superpositions of sinusoids. For other results on output tracking control, see also [13], [14], [15] and the references therein.

The study on NCSs and output tracking control keeps attracting considerable attention due to the demands from practical dynamic processes in industry. To the best of our knowledge, however, few works pay attention to the problem of output tracking control for networked control systems except in [16], in which paper the controlled plant and the controller are in continuous time and discrete time, respectively.

In this paper, the discretized controlled plant and the discrete time controller will be considered, and the discrete Jensen inequality is also adopted to avoid introducing any redundant matrices, which are different from the existing results on output tracking control for NCSs. By using LMI-based method, H_∞ output tracking performance analysis and controller design for NCSs with time delay and packet dropout are presented, the designed H_∞ output tracking controllers can ensure the stability of the closed-loop NCSs and optimal tracking performance. The discrete Jensen inequality is adopted for controller design and no any redundant matrices are introduced, so the computational complexity of the obtained results may be reduced compared with the ones having redundant matrices [6], [7], [16].

This paper is organized as follows. Section 2 presents the closed-loop model of H_∞ output tracking NCSs. Section 3 is dedicated to the H_∞ output tracking performance analysis and controller design for NCSs with time delay and packet dropout. The results of numerical simulation are presented in Section 4. Conclusions are stated in Section 5.

Notation. Throughout this paper, M^T represents the transpose of matrix M. I and 0 represent identity matrices and zero matrices with appropriate dimensions, respectively. $*$ denotes the entries of matrices implied by symmetry. Matrices, if not explicitly stated, are assumed to have appropriate dimensions.

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II. PRELIMINARIES AND PROBLEM STATEMENT

Consider a linear time-invariant plant described by

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B_1u(t) + B_2\omega(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{1}$$

where $x(t)$, $u(t)$, $y(t)$, $\omega(t)$ are the state vector, control input vector, measured output, and disturbance input, respectively, and $\omega(t)$ is piecewise constant. A, B_1, B_2, C, D are known constant matrices of appropriate dimensions.

The main objective of this paper is to design a controller, such that the output $y(t)$ of the plant tracks the output of a given reference model as close as possible. Suppose the reference model is described by

$$\begin{aligned} \dot{\hat{x}}(t) &= G\hat{x}(t) + r(t) \\ \hat{y}(t) &= H\hat{x}(t) \end{aligned} \tag{2}$$

where $\hat{x}(t)$ and $r(t)$ are the reference state and the energy bounded reference input, respectively, and $r(t)$ is piecewise constant. G and H are known constant matrices of appropriate dimensions with G Hurwitz.

The state feedback controller takes the following form

$$u(t) = K_1x(t) + K_2\hat{x}(t) \tag{3}$$

where K_1 and K_2 are the state feedback controller gains which will be designed in this paper.

In this paper, we suppose the sensor is clock-driven, both controller and actuator are event-driven. If time delay or packet dropout occur, the latest available control inputs will be used.

The assumption on time delay is as follows.

Assumption 1. The time delay τ_k (τ_k is the sensor-to-actuator time delay) is assumed to be time-varying and can be denoted as $\tau_k = nh + \varepsilon_k$, where n is a positive integer, h is the sampling period, $\varepsilon_k \in \vartheta_1 = \{0,\ h/l,\ 2h/l,\ \cdots,\ (l-1)h/l,\ h\}$, l is a positive integer and $l > 1$.

Considering the definition of ε_k, we have

$$u(t) = \begin{cases} u_{k-n-L_k}, & t \in [kh,\ kh + mh/l) \\ u_{k-n}, & t \in [kh + mh/l,\ (k+1)h] \end{cases} \tag{4}$$

where $m = 0,\ 1,\ \cdots,\ l$, u_{k-n-L_k} is the available control input at the sampling instant kh. Define L_M and L_m as the upper-bound and lower-bound of L_k, respectively, then the maximum of consecutive packet dropout is $L_M - 1$.

Considering the effect of time delay and packet dropout, the discrete time representation of the system (1) is as follows

$$\begin{aligned} x_{k+1} &= \Phi x_k + \Gamma_0(\varepsilon_{k-n})u_{k-n} + \Gamma_1(\varepsilon_{k-n})u_{k-n-L_k} + \Gamma_2\omega_k \\ y_k &= Cx_k + Du_{k-n-L_k} \end{aligned} \tag{5}$$

where $\Phi = e^{Ah}$, $\Gamma_0(\varepsilon_{k-n}) = \int_0^{h-\varepsilon_{k-n}} e^{As}dsB_1$, $\Gamma_1(\varepsilon_{k-n}) = \int_{h-\varepsilon_{k-n}}^{h} e^{As}dsB_1$, $\Gamma_2 = \int_0^h e^{As}dsB_2$, $u_k = K_1x_k + K_2\hat{x}_k$.

Correspondingly, the discrete time representation of the system (2) is as follows

$$\begin{aligned} \hat{x}_{k+1} &= \widetilde{\Phi}\hat{x}_k + \widetilde{\Gamma}\ r_k \\ \hat{y}_k &= H\hat{x}_k \end{aligned} \tag{6}$$

where $\widetilde{\Phi} = e^{Gh}$, $\widetilde{\Gamma} = \int_0^h e^{Gs}ds$.

Therefore, from (5)-(6), we can get the following augmented closed-loop system

$$\begin{aligned} \xi_{k+1} &= \Psi_1\xi_k + \Psi_2K\xi_{k-n} + \Psi_3K\xi_{k-n-L_k} + \Psi_4\nu_k \\ e_k &= \widetilde{\Psi}_1\xi_k + DK\xi_{k-n-L_k} \end{aligned} \tag{7}$$

where $\xi_k = \begin{bmatrix} x_k \\ \hat{x}_k \end{bmatrix}$, $e_k = y_k - \hat{y}_k$, $\nu_k = \begin{bmatrix} \omega_k \\ r_k \end{bmatrix}$, $\Psi_1 = \begin{bmatrix} \Phi & 0 \\ 0 & \widetilde{\Phi} \end{bmatrix}$, $\Psi_2 = \begin{bmatrix} \Gamma_0(\varepsilon_{k-n}) \\ 0 \end{bmatrix}$, $\Psi_3 = \begin{bmatrix} \Gamma_1(\varepsilon_{k-n}) \\ 0 \end{bmatrix}$, $\Psi_4 = \begin{bmatrix} \Gamma_2 & 0 \\ 0 & \widetilde{\Gamma} \end{bmatrix}$, $\widetilde{\Psi}_1 = [C\ \ -H]$, $K = [K_1\ K_2]$.

Then, the tracking requirements are expressed as follows (see [16])

i) The augmented closed-loop system in (7) with $\nu_k = 0$ is asymptotically stable

ii) The effect of ω_k and r_k on the tracking error e_k is attenuated below a desired level in the H_∞ sense. More specifically, it is required that

$$||e_k||_2 < \gamma||\nu_k||_2$$

for all nonzero $\nu_k \in L_2[0,\ \infty)$ under zero initial condition, where $\gamma > 0$. We say the H_∞ output tracking performance is achieved if the above two requirements are met.

Based on the discrete-time state equation (7), we will study the problems of H_∞ output tracking performance analysis and controller design in this paper.

The following discrete Jensen inequality will be used in the sequel.

Lemma 1 [17]. For any constant positive semi-definite symmetric matrix $M \in R^{m*m}$, two positive integers β_1 and β_2 satisfying $\beta_2 \geq \beta_1 \geq 1$, the following inequality holds

$$\begin{aligned} &-(\beta_2 - \beta_1 + 1)\textstyle\sum_{i=\beta_1}^{\beta_2} \psi^T(i)M\psi(i) \\ &\quad \leq -\left(\textstyle\sum_{i=\beta_1}^{\beta_2} \psi(i)\right)^T M\left(\textstyle\sum_{i=\beta_1}^{\beta_2} \psi(i)\right) \end{aligned} \tag{8}$$

III. H_∞ OUTPUT TRACKING PERFORMANCE ANALYSIS AND CONTROLLER DESIGN

This section is concerned with the problems of H_∞ output tracking performance analysis and controller design.

For the augmented closed-loop system (7), we suppose the controller gains K_1 and K_2 are given. The following theorem presents the conditions under which the system (7) achieves the H_∞ output tracking performance γ.

Theorem 1. For given controller gains K_1 and K_2, if there exist symmetric positive definite matrices P, Q_1, Q_2, Q_3, Q_4, Z_1, Z_2, Z_3, Z_4, scalar $\gamma > 0$, such that the following LMIs hold for every feasible value of ε_{k-n} ($\varepsilon_{k-n} \in \vartheta_1$)

$$\begin{bmatrix} \Lambda_{11} & Z_1 & 0 & Z_4 & Z_2 & 0 \\ * & \Lambda_{22} & 0 & 0 & 0 & 0 \\ * & * & \Lambda_{33} & Z_3 & Z_3 & 0 \\ * & * & * & \Lambda_{44} & 0 & 0 \\ * & * & * & * & \Lambda_{55} & 0 \\ * & * & * & * & * & -\gamma I \\ * & * & * & * & * & * \\ * & * & * & * & * & * \\ * & * & * & * & * & * \end{bmatrix}$$

$$\left.\begin{matrix} \Psi_1^T P & (\Psi_1 - I)^T M & \widetilde{\Psi}_1^T \\ K^T\Psi_2^T P & K^T\Psi_2^T M & 0 \\ K^T\Psi_3^T P & K^T\Psi_3^T M & K^T D^T \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \Psi_4^T P & \Psi_4^T M & 0 \\ -P & 0 & 0 \\ * & -M & 0 \\ * & * & -\gamma I \end{matrix}\right] < 0 \qquad (9)$$

where

$$\begin{aligned} \Lambda_{11} &= -P + Q_1 + (L_M - L_m + 1)Q_2 + Q_3 \\ &\quad + Q_4 - Z_1 - Z_2 - Z_4 \\ \Lambda_{22} &= -Q_1 - Z_1 \\ \Lambda_{33} &= -Q_2 - 2Z_3 \\ \Lambda_{44} &= -Q_4 - Z_3 - Z_4 \\ \Lambda_{55} &= -Q_3 - Z_2 - Z_3 \\ M &= n^2 Z_1 + (n+L_M)^2 Z_2 + (L_M - L_m)^2 Z_3 + (n+L_m)^2 Z_4 \end{aligned}$$

then the augmented closed-loop system described by (7) is asymptotically stable with H_∞ output tracking performance γ.

Proof: Let us consider the following Lyapunov function

$$V_k = V_{1k} + V_{2k} + V_{3k} + V_{4k} + V_{5k} + V_{6k} + V_{7k} + V_{8k} + V_{9k} + V_{10k} \qquad (10)$$

where

$$\begin{aligned} V_{1k} &= \xi_k^T P \xi_k \\ V_{2k} &= \sum_{i=k-n}^{k-1} \xi_i^T Q_1 \xi_i \\ V_{3k} &= \sum_{i=k-n-L_k}^{k-1} \xi_i^T Q_2 \xi_i \\ V_{4k} &= \sum_{i=-n-L_M+1}^{-n-L_m} \sum_{j=k+i}^{k-1} \xi_j^T Q_2 \xi_j \\ V_{5k} &= \sum_{i=k-n-L_M}^{k-1} \xi_i^T Q_3 \xi_i \\ V_{6k} &= \sum_{i=k-n-L_m}^{k-1} \xi_i^T Q_4 \xi_i \\ V_{7k} &= n \sum_{i=-n}^{-1} \sum_{j=k+i}^{k-1} \eta_j^T Z_1 \eta_j \\ V_{8k} &= (n+L_M) \sum_{i=-n-L_M}^{-1} \sum_{j=k+i}^{k-1} \eta_j^T Z_2 \eta_j \\ V_{9k} &= (L_M - L_m) \sum_{i=-n-L_M}^{-n-L_m-1} \sum_{j=k+i}^{k-1} \eta_j^T Z_3 \eta_j \\ V_{10k} &= (n+L_m) \sum_{i=-n-L_m}^{-1} \sum_{j=k+i}^{k-1} \eta_j^T Z_4 \eta_j \end{aligned}$$

P, Q_1, Q_2, Q_3, Q_4, Z_1, Z_2, Z_3, Z_4 are symmetric positive definite matrices and $\eta_j = \xi_{j+1} - \xi_j$.

From Lemma 1, we can see that

$$\begin{aligned} -n \sum_{j=k-n}^{k-1} \eta_j^T Z_1 \eta_j &\le -\Big(\sum_{j=k-n}^{k-1} \eta_j\Big)^T Z_1 \Big(\sum_{j=k-n}^{k-1} \eta_j\Big) \\ &= -(\xi_k - \xi_{k-n})^T Z_1 (\xi_k - \xi_{k-n}) \end{aligned} \qquad (11)$$

$$\begin{aligned} -(n+L_M) &\sum_{j=k-n-L_M}^{k-1} \eta_j^T Z_2 \eta_j \\ &\le -\Big(\sum_{j=k-n-L_M}^{k-1} \eta_j\Big)^T Z_2 \Big(\sum_{j=k-n-L_M}^{k-1} \eta_j\Big) \\ &= -(\xi_k - \xi_{k-n-L_M})^T Z_2 (\xi_k - \xi_{k-n-L_M}) \end{aligned} \qquad (12)$$

$$\begin{aligned} -(n+L_m) &\sum_{j=k-n-L_m}^{k-1} \eta_j^T Z_4 \eta_j \\ &\le -\Big(\sum_{j=k-n-L_m}^{k-1} \eta_j\Big)^T Z_4 \Big(\sum_{j=k-n-L_m}^{k-1} \eta_j\Big) \\ &= -(\xi_k - \xi_{k-n-L_m})^T Z_4 (\xi_k - \xi_{k-n-L_m}) \end{aligned} \qquad (13)$$

$$\begin{aligned} -(L_M - L_m) &\sum_{j=k-n-L_M}^{k-n-L_m-1} \eta_j^T Z_3 \eta_j \\ = &-(L_M - L_m) \sum_{j=k-n-L_k}^{k-n-L_m-1} \eta_j^T Z_3 \eta_j \\ &-(L_M - L_m) \sum_{j=k-n-L_M}^{k-n-L_k-1} \eta_j^T Z_3 \eta_j \\ \le &-\Big(\sum_{j=k-n-L_k}^{k-n-L_m-1} \eta_j\Big)^T Z_3 \Big(\sum_{j=k-n-L_k}^{k-n-L_m-1} \eta_j\Big) \\ &-\Big(\sum_{j=k-n-L_M}^{k-n-L_k-1} \eta_j\Big)^T Z_3 \Big(\sum_{j=k-n-L_M}^{k-n-L_k-1} \eta_j\Big) \\ = &-(\xi_{k-n-L_m} - \xi_{k-n-L_k})^T Z_3 (\xi_{k-n-L_m} - \xi_{k-n-L_k}) \\ &-(\xi_{k-n-L_k} - \xi_{k-n-L_M})^T Z_3 (\xi_{k-n-L_k} - \xi_{k-n-L_M}) \end{aligned} \qquad (14)$$

Define $\triangle V_k = V_{k+1} - V_k$, then

$$\triangle V_{1k} = \xi_{k+1}^T P \xi_{k+1} - \xi_k^T P \xi_k \qquad (15)$$

$$\triangle V_{2k} = \xi_k^T Q_1 \xi_k - \xi_{k-n}^T Q_1 \xi_{k-n} \qquad (16)$$

$$\begin{aligned} \triangle V_{3k} \le\ & \xi_k^T Q_2 \xi_k - \xi_{k-n-L_k}^T Q_2 \xi_{k-n-L_k} \\ &+ \sum_{i=k-n-L_{k+1}+1}^{k-n-L_m} \xi_i^T Q_2 \xi_i \end{aligned} \qquad (17)$$

$$\triangle V_{4k} = (L_M - L_m)\xi_k^T Q_2 \xi_k - \sum_{i=k-n-L_M+1}^{k-n-L_m} \xi_i^T Q_2 \xi_i \qquad (18)$$

$$\triangle V_{5k} = \xi_k^T Q_3 \xi_k - \xi_{k-n-L_M}^T Q_3 \xi_{k-n-L_M} \qquad (19)$$

$$\triangle V_{6k} = \xi_k^T Q_4 \xi_k - \xi_{k-n-L_m}^T Q_4 \xi_{k-n-L_m} \qquad (20)$$

$$\begin{aligned} \triangle V_{7k} &= n \sum_{i=-n}^{-1} [\eta_k^T Z_1 \eta_k - \eta_{k+i}^T Z_1 \eta_{k+i}] \\ &= n^2 (\xi_{k+1} - \xi_k)^T Z_1 (\xi_{k+1} - \xi_k) - n \sum_{j=k-n}^{k-1} \eta_j^T Z_1 \eta_j \end{aligned} \qquad (21)$$

$$\begin{aligned}\triangle V_{8k} &= (n+L_M)\sum_{i=-n-L_M}^{-1}[\eta_k^T Z_2\eta_k - \eta_{k+i}^T Z_2 \eta_{k+i}]\\ &= (n+L_M)^2(\xi_{k+1}-\xi_k)^T Z_2(\xi_{k+1}-\xi_k)\\ &\quad -(n+L_M)\sum_{j=k-n-L_M}^{k-1}\eta_j^T Z_2\eta_j \end{aligned} \tag{22}$$

$$\begin{aligned}\triangle V_{9k} &= (L_M-L_m)\sum_{i=-n-L_M}^{-n-L_m-1}[\eta_k^T Z_3\eta_k - \eta_{k+i}^T Z_3 \eta_{k+i}]\\ &= (L_M-L_m)^2(\xi_{k+1}-\xi_k)^T Z_3(\xi_{k+1}-\xi_k)\\ &\quad -(L_M-L_m)\sum_{j=k-n-L_M}^{k-n-L_m-1}\eta_j^T Z_3\eta_j \end{aligned} \tag{23}$$

$$\begin{aligned}\triangle V_{10k} &= (n+L_m)\sum_{i=-n-L_m}^{-1}[\eta_k^T Z_4\eta_k - \eta_{k+i}^T Z_4 \eta_{k+i}]\\ &= (n+L_m)^2(\xi_{k+1}-\xi_k)^T Z_4(\xi_{k+1}-\xi_k)\\ &\quad -(n+L_m)\sum_{j=k-n-L_m}^{k-1}\eta_j^T Z_4\eta_j \end{aligned} \tag{24}$$

Combining (7), (11)-(24) together, we have

$$\Delta V_k \leq \tilde{\xi}_k^T \Lambda \tilde{\xi}_k \tag{25}$$

where

$$\tilde{\xi}_k = \begin{bmatrix} \xi_k \\ \xi_{k-n} \\ \xi_{k-n-L_k} \\ \xi_{k-n-L_m} \\ \xi_{k-n-L_M} \\ v_k \end{bmatrix}, \Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} & \Lambda_{14} & \Lambda_{15} & \Lambda_{16} \\ * & \Lambda_{22} & \Lambda_{23} & \Lambda_{24} & \Lambda_{25} & \Lambda_{26} \\ * & * & \Lambda_{33} & \Lambda_{34} & \Lambda_{35} & \Lambda_{36} \\ * & * & * & \Lambda_{44} & \Lambda_{45} & \Lambda_{46} \\ * & * & * & * & \Lambda_{55} & \Lambda_{56} \\ * & * & * & * & * & \Lambda_{66} \end{bmatrix}$$

and

$$\begin{aligned}
\Lambda_{11} &= \Psi_1^T P\Psi_1 - P + Q_1 + (L_M - L_m + 1)Q_2 + Q_3 + Q_4\\
&\quad - Z_1 - Z_2 - Z_4 + (\Psi_1 - I)^T M(\Psi_1 - I)\\
\Lambda_{12} &= \Psi_1^T P\Psi_2 K + (\Psi_1 - I)^T M\Psi_2 K + Z_1\\
\Lambda_{13} &= \Psi_1^T P\Psi_3 K + (\Psi_1 - I)^T M\Psi_3 K\\
\Lambda_{14} &= Z_4\\
\Lambda_{15} &= Z_2\\
\Lambda_{16} &= \Psi_1^T P\Psi_4 + (\Psi_1 - I)^T M\Psi_4\\
\Lambda_{22} &= K^T\Psi_2^T P\Psi_2 K + K^T\Psi_2^T M\Psi_2 K - Q_1 - Z_1\\
\Lambda_{23} &= K^T\Psi_2^T P\Psi_3 K + K^T\Psi_2^T M\Psi_3 K\\
\Lambda_{24} &= \Lambda_{25} = 0\\
\Lambda_{26} &= K^T\Psi_2^T P\Psi_4 + K^T\Psi_2^T M\Psi_4\\
\Lambda_{33} &= K^T\Psi_3^T P\Psi_3 K + K^T\Psi_3^T M\Psi_3 K - Q_2 - 2Z_3\\
\Lambda_{34} &= Z_3\\
\Lambda_{35} &= Z_3\\
\Lambda_{36} &= K^T\Psi_3^T P\Psi_4 + K^T\Psi_3^T M\Psi_4\\
\Lambda_{44} &= -Q_4 - Z_3 - Z_4\\
\Lambda_{45} &= \Lambda_{46} = 0\\
\Lambda_{55} &= -Q_3 - Z_2 - Z_3\\
\Lambda_{56} &= 0\\
\Lambda_{66} &= \Psi_4^T P\Psi_4 + \Psi_4^T M\Psi_4\\
M &= n^2 Z_1 + (n+L_M)^2 Z_2 + (L_M - L_m)^2 Z_3 + (n+L_m)^2 Z_4
\end{aligned}$$

To notice that the available control input at the instant kh is u_{k-n-L_k}, for any nonzero ξ_k, we have

$$\gamma^{-1}e_k^T e_k - \gamma v_k^T v_k = \tilde{\xi}_k^T \Xi \tilde{\xi}_k$$

where Ξ is omitted here for space limit. Then

$$\gamma^{-1}e_k^T e_k - \gamma v_k^T v_k + \Delta V_k \leq \tilde{\xi}_k^T \tilde{\Lambda} \tilde{\xi}_k$$

where $\tilde{\Lambda} = \Lambda + \Xi$. Using the Schur complement, one can see that (9) is equivalent to $\tilde{\Lambda} < 0$, then for any nonzero $\tilde{\xi}_k$, if (9) is feasible, we have $\gamma^{-1}e_k^T e_k - \gamma v_k^T v_k + \Delta V_k < 0$.

Since $\gamma^{-1}e_k^T e_k - \gamma v_k^T v_k + \Delta V_k < 0$, then

$$\gamma^{-1}e_k^T e_k - \gamma v_k^T v_k < -\Delta V_k$$

Summing up both sides of the above inequality for $k=0$ to $k=p$, using the zero initial condition and the character that the disturbance input v_k has limited energy, we have

$$\sum_{k=0}^{p}||e_k||^2 < \gamma^2\sum_{k=0}^{p}||v_k||^2 - \gamma V_{p+1}$$

the above inequality holds for all p, let $p \to \infty$, we have

$$||e_k||_2^2 < \gamma^2||v_k||_2^2$$

If $v_k = 0$, (9) can ensure the asymptotic stability of the system described by (7), and if $v_k \neq 0$, we have $||e_k||_2 < \gamma||v_k||_2$, so if the LMIs in (9) are feasible, the system described by (7) is asymptotically stable with H_∞ output tracking performance γ, this completes the proof. ∎

In the controller gains K_1 and K_2 are unknown, we will study the problem of H_∞ output tracking controller design for the system (7).

Theorem 2. If there exist symmetric positive definite matrices N, $\tilde{Q}_1$, $\tilde{Q}_2$, $\tilde{Q}_3$, $\tilde{Q}_4$, $\tilde{Z}_1$, $\tilde{Z}_2$, $\tilde{Z}_3$, $\tilde{Z}_4$, matrix V, scalar $\gamma > 0$, such that the following LMIs hold for every feasible value of ε_{k-n} ($\varepsilon_{k-n} \in \vartheta_1$)

$$\left[\begin{array}{cccccccc}
\tilde{\Lambda}_{11} & \tilde{Z}_1 & 0 & \tilde{Z}_4 & \tilde{Z}_2 & 0 & N\Psi_1^T & N\Psi_1^T - N\\
* & \tilde{\Lambda}_{22} & 0 & 0 & 0 & 0 & V^T\Psi_2^T & V^T\Psi_2^T\\
* & * & \tilde{\Lambda}_{33} & \tilde{Z}_3 & \tilde{Z}_3 & 0 & V^T\Psi_3^T & V^T\Psi_3^T\\
* & * & * & \tilde{\Lambda}_{44} & 0 & 0 & 0 & 0\\
* & * & * & * & \tilde{\Lambda}_{55} & 0 & 0 & 0\\
* & * & * & * & * & -\gamma I & \Psi_4^T & \Psi_4^T\\
* & * & * & * & * & * & -N & 0\\
* & * & * & * & * & * & * & \tilde{\Lambda}_{88}\\
* & * & * & * & * & * & * & *\\
* & * & * & * & * & * & * & *\\
* & * & * & * & * & * & * & *\\
* & * & * & * & * & * & * & *
\end{array}\right.$$

$$\begin{bmatrix} N\Psi_1^T-N & N\Psi_1^T-N & N\Psi_1^T-N & N\widetilde{\Psi}_1^T \\ V^T\Psi_2^T & V^T\Psi_2^T & V^T\Psi_2^T & 0 \\ V^T\Psi_3^T & V^T\Psi_3^T & V^T\Psi_3^T & V^TD^T \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \Psi_4^T & \Psi_4^T & \Psi_4^T & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \widetilde{\Lambda}_{99} & 0 & 0 & 0 \\ * & \widetilde{\Lambda}_{10,10} & 0 & 0 \\ * & * & \widetilde{\Lambda}_{11,11} & 0 \\ * & * & * & -\gamma I \end{bmatrix} < 0 \tag{26}$$

where

$$\begin{aligned} \widetilde{\Lambda}_{11} &= -N+\widetilde{Q}_1+(L_M-L_m+1)\widetilde{Q}_2+\widetilde{Q}_3 \\ &\quad +\widetilde{Q}_4-\widetilde{Z}_1-\widetilde{Z}_2-\widetilde{Z}_4 \\ \widetilde{\Lambda}_{22} &= -\widetilde{Q}_1-\widetilde{Z}_1 \\ \widetilde{\Lambda}_{33} &= -\widetilde{Q}_2-2\widetilde{Z}_3 \\ \widetilde{\Lambda}_{44} &= -\widetilde{Q}_4-\widetilde{Z}_3-\widetilde{Z}_4 \\ \widetilde{\Lambda}_{55} &= -\widetilde{Q}_3-\widetilde{Z}_2-\widetilde{Z}_3 \\ \widetilde{\Lambda}_{88} &= n^{-2}(\widetilde{Z}_1-2N) \\ \widetilde{\Lambda}_{99} &= (n+L_M)^{-2}(\widetilde{Z}_2-2N) \\ \widetilde{\Lambda}_{10,10} &= (L_M-L_m)^{-2}(\widetilde{Z}_3-2N) \\ \widetilde{\Lambda}_{11,11} &= (n+L_m)^{-2}(\widetilde{Z}_4-2N) \end{aligned}$$

then with the control law

$$u_k = K_1x_k+K_2\hat{x}_k, \quad K=VN^{-1}$$

the augmented closed-loop system described by (7) is asymptotically stable with H_∞ output tracking performance γ.

Proof: Using the Schur complement, one can see that (9) is equivalent to

$$\begin{bmatrix} \Lambda_{11} & Z_1 & 0 & Z_4 & Z_2 & 0 & \Psi_1^T & \Psi_1^T-I & \Psi_1^T-I & \Psi_1^T-I & \Psi_1^T-I & \widetilde{\Psi}_1^T \\ * & \Lambda_{22} & 0 & 0 & 0 & 0 & K^T\Psi_2^T & K^T\Psi_2^T & K^T\Psi_2^T & K^T\Psi_2^T & K^T\Psi_2^T & 0 \\ * & * & \Lambda_{33} & Z_3 & Z_3 & 0 & K^T\Psi_3^T & K^T\Psi_3^T & K^T\Psi_3^T & K^T\Psi_3^T & K^T\Psi_3^T & K^TD^T \\ * & * & * & \Lambda_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \Lambda_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\gamma I & \Psi_4^T & \Psi_4^T & \Psi_4^T & \Psi_4^T & \Psi_4^T & 0 \\ * & * & * & * & * & * & -P^{-1} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -n^{-2}Z_1^{-1} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \Lambda_{99} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Lambda_{10,10} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & \Lambda_{11,11} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -\gamma I \end{bmatrix} < 0 \tag{27}$$

where Λ_{11}, Λ_{22}, Λ_{33}, Λ_{44}, Λ_{55} are the same as the ones in (9), and

$$\begin{aligned} \Lambda_{99} &= -(n+L_M)^{-2}Z_2^{-1} \\ \Lambda_{10,10} &= -(L_M-L_m)^{-2}Z_3^{-1} \\ \Lambda_{11,11} &= -(n+L_m)^{-2}Z_4^{-1} \end{aligned}$$

Pre- and post-multiply (27) by $diag(P^{-1},\ P^{-1},\ P^{-1},\ P^{-1},\ P^{-1},\ I,\ I,\ I,\ I,\ I,\ I,\ I)$ and $diag(P^{-1},\ P^{-1},\ P^{-1},\ P^{-1},\ P^{-1},\ I,\ I,\ I,\ I,\ I,\ I,\ I)$, define $P^{-1}=N$, $P^{-1}Q_iP^{-1}=\widetilde{Q}_i$, $P^{-1}Z_iP^{-1}=\widetilde{Z}_i$ $(i=1,\ 2,\ 3,\ 4)$, $P^{-1}K^T=V^T$, then (27) is equivalent to

$$\begin{bmatrix} \widetilde{\Lambda}_{11} & \widetilde{Z}_1 & 0 & \widetilde{Z}_4 & \widetilde{Z}_2 & 0 & N\Psi_1^T & N\Psi_1^T-N & N\Psi_1^T-N & N\Psi_1^T-N & N\Psi_1^T-N & N\widetilde{\Psi}_1^T \\ * & \widetilde{\Lambda}_{22} & 0 & 0 & 0 & 0 & V^T\Psi_2^T & V^T\Psi_2^T & V^T\Psi_2^T & V^T\Psi_2^T & V^T\Psi_2^T & 0 \\ * & * & \widetilde{\Lambda}_{33} & \widetilde{Z}_3 & \widetilde{Z}_3 & 0 & V^T\Psi_3^T & V^T\Psi_3^T & V^T\Psi_3^T & V^T\Psi_3^T & V^T\Psi_3^T & V^TD^T \\ * & * & * & \widetilde{\Lambda}_{44} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & \widetilde{\Lambda}_{55} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\gamma I & \Psi_4^T & \Psi_4^T & \Psi_4^T & \Psi_4^T & \Psi_4^T & 0 \\ * & * & * & * & * & * & -N & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -n^{-2}Z_1^{-1} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & \Lambda_{99} & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & \Lambda_{10,10} & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & \Lambda_{11,11} & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -\gamma I \end{bmatrix} < 0 \tag{28}$$

where $\widetilde{\Lambda}_{11}$, $\widetilde{\Lambda}_{22}$, $\widetilde{\Lambda}_{33}$, $\widetilde{\Lambda}_{44}$, $\widetilde{\Lambda}_{55}$ are the same as the ones in (26).

On the other hand, for symmetric positive definite matrices P and Z_i, we have $(P-Z_i)Z_i^{-1}(P-Z_i)\geq 0$, which is equivalent to $-Z_i^{-1}\leq P^{-1}Z_iP^{-1}-2P^{-1}$. From the definition of $\widetilde{Z}_i$ and N, we can see that if (26) is satisfied, (28) is also feasible. So if (26) is satisfied, the augmented closed-loop system described by (7) with $K=VN^{-1}$ is asymptotically stable with H_∞ output tracking performance γ, this completes the proof. ■

Remark 1. Theorem 2 presents LMI-based sufficient conditions for H_∞ output tracking controller design, which can be effectively solved via Matlab LMI control toolbox.

In the following, we will illustrate the effectiveness of the proposed design method by an example.

IV. SIMULATION RESULTS AND DISCUSSION

Example 1. To illustrate the effectiveness of the proposed H_∞ output tracking controller design, we present an open loop unstable system as follows

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0.1615 & 0.1487 \\ -0.1021 & 0.1696 \end{bmatrix} x(t) + \begin{bmatrix} -0.1660 \\ 0.1066 \end{bmatrix} u(t) \\ &\quad + \begin{bmatrix} 0.2066 \\ -0.2104 \end{bmatrix} \omega(t) \\ y(t) &= \begin{bmatrix} 0.0724 & -0.0074 \end{bmatrix} x(t) - 0.2455u(t) \end{aligned} \tag{29}$$

The reference model is described as follows

$$\begin{aligned} \dot{\hat{x}}(t) &= -\hat{x}(t)+r(t) \\ \hat{y}(t) &= 0.6\hat{x}(t) \end{aligned} \tag{30}$$

In this example, we suppose $n=1$, $L_M=2$, L_m=1. If constant sampling period h is adopted, suppose $h=0.05s$ and $\varepsilon_{k-n} \in \{0.2h,\ 0.5h, 0.8h\}$. The initial state of the augmented system (7) is $\xi_0=[0\quad 0\quad 0]^T$. For simplicity, suppose the packets at the instants 0, $2h$, $4h$, $\cdots$ are transferred to the actuator successfully, that is one packet is dropped among two packets.

If constant sampling period h is adopted, by solving the LMIs presented in Theorem 2, we obtain the minimum H_∞ output tracking performance $\gamma=0.8347$ with the controller gain $K=[K_1\quad K_2]=[-0.7118\ \ -19.9618\ 0.9730]$.

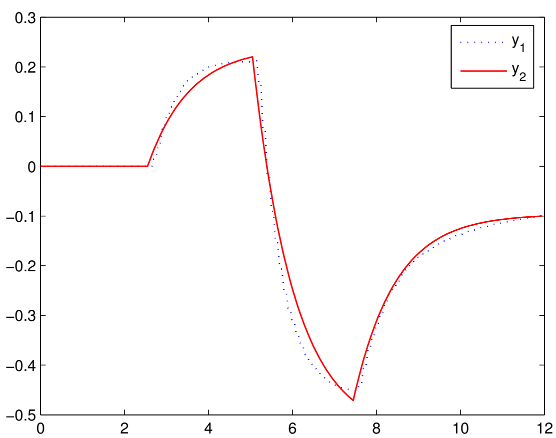

Fig. 1. Outputs y_k and $\hat{y}_k$ for ε_k (32)

Suppose the input signals are given as follows

$$\omega(t)=\begin{cases}0, & 0s\le t<2.55s\\ -0.4, & 2.55s\le t<5.05s\\ 0.8, & 5.05s\le t<7.55s\\ 0.3, & 7.55s\le t<11.95s\end{cases}$$
$$r(t)=\begin{cases}0, & 0s\le t<2.55s\\ 0.4, & 2.55s\le t<5.05s\\ -0.9, & 5.05s\le t<7.55s\\ -0.16, & 7.55s\le t<11.95s\end{cases} \tag{31}$$

For control input u_k which is transferred to the actuator successfully, suppose the time varying part ε_k of τ_k is given as follows

$$\varepsilon_k=\begin{cases}0.2h, & 0s\le t<2.55s\\ 0.5h, & 2.55s\le t<5.05s\\ 0.8h, & 5.05s\le t<7.55s\\ 0.5h, & 7.55s\le t<11.95s\end{cases} \tag{32}$$

The output y_k (denoted as y_1) of the system (29) and $\hat{y}_k$ (denoted as y_2) of the reference model (30) are pictured in Fig. 1.

From Fig. 1, we can see that the output y_k of the system (29) tracks the reference signal $\hat{y}_k$ generated by the reference model (30) well in the H_∞ sense, which illustrates the effectiveness of the proposed H_∞ output tracking controller design.

V. CONCLUSIONS

In this paper, the problems of H_∞ output tracking performance analysis and controller design for NCSs with time delay and packet dropout have been investigated. By using the discrete Jensen inequality, LMI-based H_∞ output tracking performance analysis and controller design for NCSs are presented. The designed output tracking controllers can guarantee asymptotic tracking of prescribed reference outputs while rejecting disturbances. Numerical example has illustrated the effectiveness of the proposed H_∞ output tracking controller design.

REFERENCES

[1] W. Zhang, M. S. Branicky, and S. M. Phillips, Stability of networked control systems , *IEEE Control Systems Magazine*, vol. 21, no. 1, 2001, pp. 84-99.

[2] G. C. Walsh, H. Ye, and L. Bushnell, Stability analysis of networked control systems, *IEEE Transactions on Control Systems Technology,* vol. 10, no. 3, 2002, pp. 438-446.

[3] H. Ishii and B. Francis, Stabilization with control networks, *Automatica,* 38, 2002, pp. 1745-1751.

[4] W. J. Kim, K. Ji, and A. Ambike, Networked real-time control strategies dealing with stochastic time delays and packet losses, *Proc. of the American Control Conference,* 2005, pp. 621-626.

[5] B. Sinopoli, L. Schenato, M. Franceschetti, K. Poolla, and S. S. Sastry, Time varying optimal control with packet losses, *Proc. of the 43rd IEEE Conference on Decision and Control*, 2004, pp. 1938-1943.

[6] H. J. Gao and T. W. Chen, New results on stability of discrete-time systems with time-varying state delay, *IEEE Transactions on Automatic Control,* vol. 52, no. 2, 2007, pp. 328-334.

[7] M. Wu, Y. He, J. H. She, and G. P. Liu, Delay-dependent criteria for robust stability of time-varying delay systems, *Automatica*, 40, 2004, pp. 1435-1439.

[8] S. Y. Xu and T. W. Chen, H_∞ output feedback control for uncertain stochastic systems with time-varying delays, *Automatica,* 40, 2004, pp. 2091-2098.

[9] D. K. Kim, P. G. Park, and J. W. Ko, Output-feedback H_∞ control of systems over communication networks using a deterministic switching system approach, *Automatica,* 40, 2004, pp. 1205-1212.

[10] F. W. Yang, Z. D. Wang, Y. S. Hung, and M. Gani, H_∞ control for networked systems with random communication delays, *IEEE Transactions on Automatic Control,* vol. 51, no. 3, 2006, pp. 511-518.

[11] F. Liao, J. L. Wang, and G. H. Yang, Reliable robust flight tracking control: an LMI approach, *IEEE Transactions on Control Systems Technology*, vol. 10, no. 1, 2002, pp. 76-89.

[12] R. Rebarber and G. Weiss, Internal model based tracking and disturbance rejection for stable well-posed systems, *Automatica,* 39, 2003, pp. 1555-1569.

[13] N. C. Shieh, K. Liang, and C. Mao, Robust output tracking control of an uncertain linear system via a modified optimal linear-quadratic method, *Journal of Optimization Theory and Applications*, vol. 117, no. 3, 2003, pp. 649-659.

[14] A. B. Açlkmeşe and M. Corless, Robust output tracking for uncertain/nonlinear systems subject to almost constant disturbances, *Automatica,* 38, 2002, pp. 1919-1926.

[15] N. C. Shieh, P. C. Tung, and C. L. Lin, Robust output tracking control of a linear brushless DC motor with time-varying disturbances, *IEE Proc. Electric Power Applications*, vol. 149, no. 1, 2002, pp. 39-45.

[16] H. J. Gao and T. W. Chen, H_∞ model reference control for networked feedback systems, *Proc. of the 45th IEEE Conference on Decision and Control,* 2006, pp. 5591-5596.

[17] X. F. Jiang, Q. L. Han, and X. H. Yu, Stability criteria for linear discrete-time systems with interval-like time-varying delay, *Proc. of the American Control Conference*, 2005, pp. 2817-2822.