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Distributed Leadless Coordination for Networks of Second-Order Agents With Time-Delay on Switching Topology

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***Abstract*— In this paper, we consider distributed leadless coordination in networks of second-order agents with time-delay and switching topology. We first perform a model transformation and turn the original system into an equivalent system. Then based on this equivalent system, we derive sufficient conditions in terms of linear matrix inequalities (LMIs) that guarantee all agents asymptotically reach consensus. Finally, simulation results are provided to show the effectiveness of the obtained theoretical results.**

I. INTRODUCTION

In recent years, distributed coordination of multiple agents has attracted considerable attention in many fields such as biology, physics, robotics and control engineering. Many results have been obtained [1]-[14]. In [1], Vicsek *et al.* proposed a simple model for phase transition of a group of self-driven particles and numerically demonstrated complex dynamics of the model. In [2], the alignment of a network of agents with switching topology that is periodically connected is analyzed. Moreau used a set-valued Lyapunov approach to study consensus problems with unidirectional time-dependent communication links [3]. Moreover, Olfati-Saber *et al.* systematically solved the average-consensus problem with directed interconnection graphs or time-delays [4]. Lin *et al.* extended the results of [4] to the case of time-delay and switching topology [5]. Also, the authors of [6]-[8] introduced a set of control laws that enable the second-order agents to generate stable flocking motion, while Ren *et al.* proposed a second-order protocol and provided sufficient conditions in the case of fixed topology [10]. However, very little work has considered the effects of time-delay for second-order multi-agent systems. Recently, the leader-follower consensus problem with time-delay has been studied in [11], and frequency domain analysis has been used for delayed multi-agent systems [13].

In this paper, we focus on distributed leadless coordination in directed networks of second-order agents with time-delay and switching topology. In doing the analysis, we first perform a model transformation and turn the original system into an equivalent system. Then, based on this equivalent system, we introduce a common Lyapunov function and use a dimension reduction approach to give sufficient conditions in terms of LMIs under which all agents asymptotically reach consensus.

The paper is organized as follows. In Section II, with help of basic results in graph theory, we define the consensus problem and then a linear protocol is given in networks of agents with double integrator dynamics. The stability analysis is shown for delayed multi-agent systems with switching topology in section III. Section IV gives numerical simulations, and finally, some conclusions are drawn in section V.

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II. GRAPH THEORY AND CONSENSUS PROTOCOL

A. Graph theory

At first, we introduce some preliminary knowledge of graph theory for the following analysis (referring to [15]). Let $G(\mathcal{V}, \varepsilon, \mathcal{A})$ be a directed graph of order n, where $\mathcal{V} = \{\mathrm{s}_1, \cdots, \mathrm{s}_n\}$ is the set of nodes, $\varepsilon \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges and $\mathcal{A} = [a_{ij}]$ is a weighted adjacency matrix. The node indexes belong to a finite index set $\mathcal{I} = \{1, 2, \cdots, n\}$. An edge of G is denoted by $e_{ij} = (\mathrm{s}_i, \mathrm{s}_j)$. The adjacency elements associated with the edges are positive, i.e., $e_{ij} \in \varepsilon \Leftrightarrow a_{ij} > 0$. Moreover, it is assumed that $a_{ii} = 0$ for all $i \in \mathcal{I}$. Correspondingly, the Laplacian associated with the directed graph is defined as $L = [l_{ij}]$, where $l_{ii} = \sum_j a_{ij}$ and $l_{ij} = -a_{ij}$, $i \neq j$. The set of neighbors of node s_i is denoted by $N_i = \{\mathrm{s}_j \in \mathcal{V} : (\mathrm{s}_i, \mathrm{s}_j) \in \varepsilon\}$. A directed path is a sequence of ordered edges of the form $(\mathrm{s}_{i_1}, \mathrm{s}_{i_2}), (\mathrm{s}_{i_2}, \mathrm{s}_{i_3}), \cdots$, where $\mathrm{s}_{i_j} \in \mathcal{V}$ and

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$(\mathrm{s}_{i_j}, \mathrm{s}_{i_{j+1}}) \in \varepsilon$. If a directed graph has the property that $a_{ij} = a_{ji}$ for any $i, j \in \mathcal{I}$, the directed graph is called undirected graph. If there is a directed path from every node to every other node, the graph is said to be strongly connected (connected for undirected graph).

Definition 1: (Balanced Graphs) [4] The node of a directed graph $G(\mathcal{V}, \varepsilon, \mathcal{A})$ is balanced if and only if its in-degree and out-degree are equal, i.e., $d_o(\mathrm{s}_i) = d_i(\mathrm{s}_i)$, where $d_o(\mathrm{s}_i) = \sum_j a_{ij}, d_i(\mathrm{s}_i) = \sum_j a_{ji}$. A graph $G(\mathcal{V}, \varepsilon, \mathcal{A})$ is called balanced if and only if all of its nodes are balanced. If the graph is balanced, then $\mathbf{1}_n^T L = 0$.

Definition 2: (Balanced Matrix) A square matrix $M \in \mathbb{R}^{n\times n}$ is said to be a balanced matrix if and only if $\mathbf{1}_n^T M = \mathbf{0}$ and $M\mathbf{1}_n = \mathbf{0}$.

Evidently, the Laplacian of any balanced graph is a balanced matrix.

Lemma 1: [4] If a directed graph G is strongly connected, then its Laplacian L has the following properties:
(1) Zero is one eigenvalue of L, and $\mathbf{1}_n$ is the corresponding eigenvector, i.e., $L\mathbf{1}_n = 0$.
(2) The rest $n-1$ eigenvalues all have positive real-parts.

B. Consensus and Protocol

Suppose that the multi-agent system under consideration consists of n agents. Each agent is regarded as a node in a directed graph, G. Each edge $(\mathrm{s}_j, \mathrm{s}_i) \in \varepsilon(G(t))$ corresponds to an available information channel from agent s_i to agent s_j at time t. Moreover, each agent updates its current state based upon the information received from its neighbors. Suppose the ith agent ($i \in \mathcal{I}$) has the dynamics as follows:

$$\begin{array}{rcl} \dot{x}_i & = & v_i \\ m_i \dot{v}_i & = & u_i, \end{array} \tag{1}$$

where x_i is the position state, v_i is the speed state, m_i is the mass and u_i is the control input. Without loss of generality, we assume $m_1 = m_2 = \cdots = m_n = 1$.

We say protocol u_i asymptotically solves the consensus problem, *i.e.*, the agreement of the position states, if and only if the states of agents satisfy

$$\lim_{t\to+\infty} (x_i - x_j) = 0, \qquad \lim_{t\to+\infty} v_i = 0, \tag{2}$$

for all $i, j \in \mathcal{I}$.

In this paper, we are interested in discussing the consensus problem for networks of agents with switching topology and time-delay, where the information passes through each edge with time-delay τ. To solve this problem, we use the following linear consensus protocol,

$$u_i(t) = -2k_1 v_i + \sum_{\mathrm{s}_j \in N_i} a_{ij}(x_j(t-\tau) - x_i(t-\tau)), \tag{3}$$

where $2k_1 > 0$ denotes velocity damping gain.

Let

$$\xi = [x_1, \bar{v}_1, \cdots, x_n, \bar{v}_n]^T, \quad \bar{v}_i = \frac{v_i}{k_1} + x_i,$$

$$A = \begin{bmatrix} -k_1 & k_1 \\ k_1 & -k_1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ \frac{1}{k_1} & 0 \end{bmatrix}.$$

Using protocol (3) the network dynamics is

$$\dot{\xi} = (I_n \otimes A)\xi - (L_\sigma \otimes B)\xi(t-\tau), \tag{4}$$

where L_σ is the Laplacian of the graph G_σ, $'\otimes'$ denotes the Kronecker product and σ denotes the switching signal that determines the topology.

III. THE MAIN RESULTS

In this section, we will provide the stability analysis for directed multi-agent networks with switching topology and time-delay using LMIs.

Let us give some lemmas first for the following analysis.

Lemma 2: Consider the following matrix

$$\Psi_n = \begin{bmatrix} n-1 & -1 & \cdots & -1 \\ -1 & n-1 & \cdots & -1 \\ \vdots & \vdots & \ddots & \vdots \\ -1 & -1 & \cdots & n-1 \end{bmatrix} \in \mathbb{R}^{n\times n}.$$

The following statements hold.

(1) The eigenvalues of Ψ_n are n with multiplicity $n-1$ and 0 with multiplicity 1. The vectors $\mathbf{1}_n^T$ and $\mathbf{1}_n$ are the left and the right eigenvectors of Ψ_n associated with the zero eigenvalue respectively.

(2) There exists an orthogonal matrix $U_n \in \mathbb{R}^{n\times n}$ such that

$$U_n^T \Psi_n U_n = \begin{bmatrix} nI_{n-1} & \mathbf{0}_{(n-1)\times 1} \\ \mathbf{0}_{1\times(n-1)} & 0 \end{bmatrix}$$

and the last column of U_n is $\frac{\mathbf{1}_n}{\sqrt{n}}$. Let $\Xi \in \mathbb{R}^{n\times n}$ be a balanced matrix, then

$$U_n^T \Xi U_n = \begin{bmatrix} * & \mathbf{0}_{(n-1)\times 1} \\ \mathbf{0}_{1\times(n-1)} & 0 \end{bmatrix}.$$

Actually, Ψ_n can be viewed as the Laplacian of a complete undirected graph. By Lemma 1, we know $\mathbf{1}_n$ is the eigenvector of Ψ_n associated with the zero eigenvalue. So it is easy to see that the entries of the last row and the last column of the matrix $U_n^T \Xi U_n$ are all zeros.

Lemma 3: Let $E \in \mathbb{R}^{n\times n}$ be a symmetric positive semi-definite balanced matrix and $rank(E) = n-1$. For any $y \in \mathbb{R}^n$ with $\sum_i y_i = 0$, $y^T E y > 0$ if and only if $y \neq 0$.

Proof: Since $\sum_i y_i = 0$ and the last row of U_n^T is $\frac{\mathbf{1}_n^T}{\sqrt{n}}$, then $\bar{y} = U_n^T y = [\,* \quad \cdots \quad * \quad 0\,]^T$. Since E is a positive semi-definite balanced matrix and $rank(E) = n-1$, then

$$U_n^T E U_n = \begin{bmatrix} \bar{E} & \mathbf{0}_{(n-1)\times 1} \\ \mathbf{0}_{1\times(n-1)} & 0 \end{bmatrix}, \bar{E} > 0.$$

It follows that

$$\begin{aligned} y^T E y &= y^T U_n U_n^T E U_n U_n^T y \\ &= \bar{y}^T \begin{bmatrix} \bar{E} & \mathbf{0}_{(n-1)\times 1} \\ \mathbf{0}_{1\times(n-1)} & 0 \end{bmatrix} \bar{y}. \end{aligned}$$

This implies that $y^T E y > 0$ if and only if $y \neq 0$.

Lemma 2 and Lemma 3 provide a dimension reduction approach for balanced matrices. In what follows, we will use this property to construct a common Lyapunov function and give sufficient conditions in terms of LMIs.

Lemma 4: (Schur Complement) [17] For a given symmetric matrix S with the form $S = [S_{ij}]$, $S_{11} \in \mathbb{R}^{r\times r}, S_{12} \in \mathbb{R}^{r\times(n-r)}, S_{21} \in \mathbb{R}^{(n-r)\times r}, S_{22} \in \mathbb{R}^{(n-r)\times(n-r)}$, then, $S < 0$ if and only if $S_{11} < 0$, $S_{22} - S_{21}S_{11}^{-1}S_{12} < 0$ or $S_{22} < 0$, $S_{11} - S_{12}S_{22}^{-1}S_{21} < 0$.

Theorem 1: Consider a directed network of agents with time-delay τ and switching topology G_σ that is kept strongly connected and balanced. Given protocol (3), (2) holds if there exist symmetric positive matrices $\bar{P} \in \mathbb{R}^{(2n-1)\times(2n-1)}$, $\bar{Q} \in \mathbb{R}^{(2n-1)\times(2n-1)}$, $R \in \mathbb{R}^{2n\times 2n}$ satisfying

$$H = \begin{bmatrix} H_{11} & H_{12} & H_{13} \\ H_{12}^T & H_{22} & \mathbf{0}_{(n-1)\times n} \\ H_{13}^T & \mathbf{0}_{n\times(n-1)} & -\tau R \end{bmatrix} < 0 \qquad (5)$$

where

$$\begin{aligned} H_{11} &= \bar{P}\bar{\Phi}_\sigma + \bar{\Phi}_\sigma^T \bar{P} + \bar{Q} + \tau \Xi_2 \bar{U}^T R \bar{U} \Xi_2^T, \\ H_{12} &= -\tau \Xi_2 \bar{U}^T R \bar{U} \Xi_1, \\ H_{22} &= -\bar{Q} + \tau \Xi_1^T \bar{U}^T R \bar{U} \Xi_1, \\ H_{13} &= \tau \bar{P} \Xi_1 \bar{U}^T, \\ \Xi_1 &= \bar{U}^T (L_\sigma \otimes B) \bar{U}, \quad \Xi_2 = \bar{U}^T (I_n \otimes A) \bar{U} \\ \bar{\Phi}_\sigma &= \bar{U}^T (I_n \otimes A - L_\sigma \otimes B) \bar{U} \end{aligned}$$

and $\bar{U}$ is the first $2n-1$ columns of U_{2n} with U_{2n} as defined in Lemma 2.

Proof: Since

$$\mathbf{1}_{2n}^T [(I_n \otimes A)\xi(t) - (L_\sigma \otimes B)\xi(t-\tau)] = \mathbf{0}_{2n}^T$$

and

$$[(I_n \otimes A)\xi(t) - (L_\sigma \otimes B)\xi(t-\tau)]^T \mathbf{1}_{2n} = \mathbf{0}_{2n},$$

we have $\sum_i (\dot{x}_i + \dot{\bar{v}}_i) = 0$. Hence, $\sum_i (x_i + \bar{v}_i)$ is an invariant quantity.

Let $\beta = \frac{1}{2n}\sum_i [x_i(0) + \bar{v}_i(0)]$. Then $\xi(t)$ can be decomposed into $\xi(t) = \beta \mathbf{1}_{2n} + \delta(t)$, where $\delta(t) \in \mathbb{R}^{2n}$ satisfies $\sum_i \delta_i(t) = 0$, and the vector $\delta(t)$ is orthogonal to $\mathbf{1}_{2n}$. Thus (3) can be transformed into the following equation:

$$\dot{\delta}(t) = (I_n \otimes A)\delta(t) - (L_\sigma \otimes B)\delta(t-\tau). \qquad (6)$$

Define a Lyapunov function for system (6) as follows

$$\begin{aligned} V &= \delta^T(t) P \delta(t) + \int_{t-\tau}^{t} \delta^T(s) Q \delta(s) ds \\ &+ \int_{-\tau}^{0}\int_{t+\theta}^{t} \dot{\delta}^T(s) R \dot{\delta}(s) ds d\theta, \end{aligned}$$

where $R \in \mathbb{R}^{2n\times 2n}$ is a positive definite matrix, $P, Q \in \mathbb{R}^{2n\times 2n}$ are symmetric balanced positive semi-definite matrices and $rank(P) = rank(Q) = 2n-1$.

By Lemma 3, $\delta^T(t) P \delta(t) > 0$ holds if and only if $\delta(t) \neq 0$, which means $V > 0$ if $\delta(t) \neq 0$.

Differentiating V along the trajectory of (6) leads to

$$\begin{aligned} \dot{V} &= 2\delta^T(t) P [(I_n \otimes A)\delta(t) - (L_\sigma \otimes B)\delta(t-\tau)] \\ &+ \delta^T(t) Q \delta(t) - \delta^T(t-\tau) Q \delta(t-\tau) \\ &+ \tau \dot{\delta}^T(t) R \dot{\delta}(t) - \int_{t-\tau}^{t} \dot{\delta}^T(\theta) R \dot{\delta}(\theta) d\theta. \end{aligned}$$

Since $\delta(t-\tau) = \delta(t) - \int_{t-\tau}^{t} \dot{\delta}(s) ds$, and for any symmetric positive definite matrix $\bar{R} \in \mathbb{R}^{2n\times 2n}$ and any $x, y \in \mathbb{R}^{2n}$,

$$2x^T y \leq x^T \bar{R}^{-1} x + y^T \bar{R} y \qquad (7)$$

we have

$$\begin{aligned} & -2\delta^T(t) P (L_\sigma \otimes B) \delta(t-\tau) \\ =& -2\delta^T(t) P (L_\sigma \otimes B) \delta(t) \\ +& \int_{t-\tau}^{t} 2((L_\sigma \otimes B)^T P^T \delta(t))^T \dot{\delta}(s) ds \\ \leq& -2\delta^T(t) P (L_\sigma \otimes B) \delta(t) + \int_{t-\tau}^{t} \dot{\delta}^T(s) R \dot{\delta}(s) ds \\ +& \tau \delta^T(t) P (L_\sigma \otimes B) R^{-1} (L_\sigma \otimes B)^T P \delta(t) \end{aligned}$$

This yields

$$\begin{aligned}\dot{V} \;\le\;& 2\delta^T(t)P\Phi_\sigma\delta(t) + \delta^T(t)Q\delta(t)\\ &+ \tau\delta^T(t)P(L_\sigma\otimes B)R^{-1}(L_\sigma\otimes B)^TP\delta(t)\\ &- \delta^T(t-\tau)Q\delta(t-\tau)\\ &+ \tau[(I_n\otimes A)\delta(t)-(L_\sigma\otimes B)\delta(t-\tau)]^TR\\ &\quad[(I_n\otimes A)\delta(t)-(L_\sigma\otimes B)\delta(t-\tau)]\end{aligned}$$

where $\Phi_\sigma = I_n\otimes A - L_\sigma\otimes B$.

Let $\bar{\delta}(t) = \bar{U}^T\delta(t)$, $\bar{P} = \bar{U}^TP\bar{U}$, $\bar{Q} = \bar{U}^TQ\bar{U}$. Noting that $\sum_i \delta_i(t) = 0$ and P, Q, Φ_σ, $I_n\otimes A$ and $L_\sigma\otimes B$ are all balanced matrices and by Lemma 2, we have

$$\begin{aligned}\delta^T(t)U_{2n} &= [\,\bar{\delta}^T(t)\quad 0\,],\\ U_{2n}^TPU_{2n} &= diag\{\bar{P},0\},\\ U_{2n}^T\Phi_\sigma U_{2n} &= diag\{\bar{\Phi}_\sigma,0\},\\ U_{2n}^TQU_{2n} &= diag\{\bar{Q},0\},\\ U_{2n}^T(L_\sigma\otimes B)U_{2n} &= diag\{\Xi_1,0\},\\ U_{2n}^T(I_n\otimes A)U_{2n} &= diag\{\Xi_2,0\}.\end{aligned}$$

Consequently,

$$\begin{aligned}&\dot{V}\\ \le\;& \bar{\delta}^T(t)(\bar{P}\bar{\Phi}_\sigma + \bar{\Phi}_\sigma^T\bar{P} + \bar{Q} + \tau\bar{P}\Xi_1\bar{U}^TR^{-1}\bar{U}\\ \times\;& \Xi_1^T\bar{P} + \tau\Xi_2\bar{U}^TR\bar{U}\Xi_2^T)\bar{\delta}(t) - 2\tau\bar{\delta}^T(t)(\Xi_2\\ \times\;& \bar{U}^TR\bar{U}\Xi_1)\bar{\delta}(t-\tau) + \bar{\delta}^T(t-\tau)(-\bar{Q}\\ +\;& \tau\Xi_1^T\bar{U}^TR\bar{U}\Xi_1)\bar{\delta}(t-\tau)\end{aligned} \tag{8}$$

Rewriting the Lyapunov function V, we have

$$\begin{aligned}V \;=\;& \bar{\delta}^T(t)\bar{P}\bar{\delta}(t) + \int_{t-\tau}^{t}\bar{\delta}^T(s)\bar{Q}\bar{\delta}(s)ds\\ &+ \int_{-\tau}^{0}\int_{t+\theta}^{t}\dot{\bar{\delta}}^T(s)\bar{U}^TR\bar{U}\dot{\bar{\delta}}(s)dsd\theta,\end{aligned}$$

Observing that $\bar{P} > 0$, $\bar{Q} > 0$ and $\bar{U}^TR\bar{U} > 0$, it is easy to see that there exist positive scalars β_1 and β_2 such that

$$\beta_1\|\bar{\delta}(t)\|^2 \le V(t) \le \beta_2 \sup_{\theta\in[-2\tau,0]}\|\bar{\delta}(t+\theta)\|^2.$$

Then, a sufficient condition for $\dot{V} < 0$ is that

$$\bar{H} = \begin{bmatrix}\bar{H}_{11} & H_{12}\\ H_{12}^T & H_{22}\end{bmatrix} < 0$$

where

$$\begin{aligned}\bar{H}_{11} \;=\;& \bar{P}\bar{\Phi}_\sigma + \bar{\Phi}_\sigma^T\bar{P} + \bar{Q} + \tau\bar{P}\Xi_1\bar{U}^TR^{-1}\bar{U}\Xi_1^T\bar{P}\\ &+ \tau\Xi_2\bar{U}^TR\bar{U}\Xi_2^T,\\ H_{12} \;=\;& -\tau\Xi_2\bar{U}^TR\bar{U}\Xi_1,\\ H_{22} \;=\;& -\bar{Q} + \tau\Xi_1^T\bar{U}^TR\bar{U}\Xi_1.\end{aligned}$$

Further, by Lemma 4, $\bar{H} < 0$ holds if and only if $H < 0$. This completes the proof.

Remark 1: Let λ_{max} denote the largest eigenvalue of L_σ ($t\in[0,+\infty)$). By Lemma 1, $\lambda_{max} > 0$ under the assumption of Theorem 1. Further, after simple calculations, it is not hard to see that if $k_1 > 0$ and $4k_1^2 > \lambda_{max}$, $\bar{\Phi}_\sigma + \bar{\Phi}_\sigma^T < 0$. Take $\bar{P} = \gamma_1 I_{2n-1}$, $\bar{Q} = \gamma_2 I_{2n-1}$ and $\gamma_1\lambda_{min} > \gamma_2$ where $\gamma_1 > 0, \gamma_2 > 0$ and λ_{min} denotes the smallest eigenvalue of $-\bar{\Phi}_\sigma - \bar{\Phi}_\sigma^T$. Then $\bar{H} < 0$ holds for sufficiently small τ. This means $H < 0$ is always feasible for sufficiently small τ and $4k_1^2 > \lambda_{max}$.

Remark 2: Since L_σ is time-varying, the matrix inequality (5) should be satisfied for all the possible graphs.

Corollary 1: Consider a directed network of agents with switching topology G_σ that is kept strongly connected and balanced. Given protocol (3) with time-delay $\tau = 0$, (2) holds if there exists a symmetric positive definite matrix $\bar{P}\in\mathbb{R}^{(2n-1)\times(2n-1)}$ satisfying

$$\bar{P}\bar{\Phi}_\sigma + \bar{\Phi}_\sigma^T\bar{P} < 0. \tag{9}$$

Theorem 2: Consider a directed network of agents with time-varying delay and switching topology G_σ that is kept strongly connected and balanced. Given protocol (3) with time-varying delay $\tau(t) < d$ and the derivative $\dot{\tau}(t) < d_1 < 1$, (2) holds if there exist symmetric positive definite matrices $\bar{P}\in\mathbb{R}^{(2n-1)\times(2n-1)}$, $\bar{Q}\in\mathbb{R}^{(2n-1)\times(2n-1)}$, $R\in\mathbb{R}^{2n\times 2n}$ satisfying

$$\hat{H} = \begin{bmatrix}\hat{H}_{11} & \hat{H}_{12} & \hat{H}_{13}\\ \hat{H}_{12}^T & \hat{H}_{22} & \mathbf{0}_{(n-1)\times n}\\ \hat{H}_{13}^T & \mathbf{0}_{n\times(n-1)} & -d(1-d_1)R\end{bmatrix} < 0 \tag{10}$$

where

$$\begin{aligned}\hat{H}_{11} &= \bar{P}\bar{\Phi}_\sigma + \bar{\Phi}_\sigma^T\bar{P} + \bar{Q} + d\Xi_2\bar{U}^TR\bar{U}\Xi_2^T,\\ \hat{H}_{12} &= -d\Xi_2\bar{U}^TR\bar{U}\Xi_1,\\ \hat{H}_{22} &= -\bar{Q}(1-d_1) + d\Xi_1\bar{U}^TR\bar{U}\Xi_1,\\ \hat{H}_{13} &= d\bar{P}\Xi_1\bar{U}^T.\end{aligned}$$

The proof of Theorem 2 is very similar to that of Theorem 1 and hence omitted.

Remark 3: In this paper, we only consider the single time-delay case. Actually, all the results can be extended to the multiple time-delay case following the lines of the proof of Theorem 1.

IV. SIMULATION RESULTS

In this section, numerical simulations will be given to illustrate the theoretical results obtained in the previous sections. These simulations are performed with four agents. Fig. 1 denotes four different graphs. Moreover, the weight of each edge is 1 and the protocol parameter k_1 is taken as $k_1 = 1$.

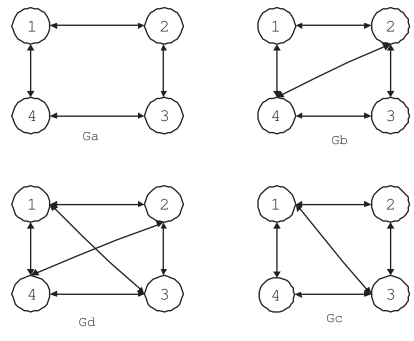

Fig.1 Four different graphs

Fig.2 shows the switching sequences of the network topology, and it starts at G_a, and switches to the next state every 0.01 second. By using Theorem 1, it is solved that for time-delay $\tau = 0.43$, a feasible solution is

$$\bar{P} = \begin{bmatrix} 22.62 & 1.63 & -8.08 & 5.89 & -1.70 & -7.23 & 5.58 \\ 1.63 & 10.76 & -1.27 & 3.20 & -1.47 & 0.88 & -6.86 \\ -8.08 & -1.27 & 25.52 & -3.85 & -12.52 & 0.29 & 2.01 \\ 5.89 & 3.20 & -3.85 & 11.75 & -1.41 & 1.45 & -2.11 \\ -1.70 & -1.47 & -12.52 & -1.41 & 21.05 & 1.46 & -2.64 \\ -7.23 & 0.88 & 0.29 & 1.45 & 1.46 & 34.83 & 4.65 \\ 5.58 & -6.86 & 2.00 & -2.11 & -2.64 & 4.65 & 32.57 \end{bmatrix}$$

$$\bar{Q} = \begin{bmatrix} 13.55 & -0.13 & -6.45 & 6.21 & -4.80 & 1.15 & 4.32 \\ -0.13 & 7.39 & 1.34 & 3.61 & -2.39 & 4.07 & -8.66 \\ -6.45 & 1.34 & 12.45 & 0.95 & -6.65 & -4.76 & -2.80 \\ 6.21 & 3.61 & 0.95 & 6.50 & -6.66 & 0.26 & -2.42 \\ -4.80 & -2.39 & -6.66 & -6.67 & 14.01 & 2.53 & -2.49 \\ 1.15 & 4.07 & -4.76 & 0.26 & 2.53 & 19.26 & -0.97 \\ 4.32 & -8.66 & -2.80 & -2.42 & -2.49 & -0.97 & 16.90 \end{bmatrix}$$

$$R = \begin{bmatrix} 28.07 & 0.83 & -4.27 & 0.21 & -3.24 & 0.50 & -4.27 & 0.21 \\ 0.83 & 5.70 & 0.19 & 5.05 & 0.50 & 5.37 & 0.19 & 5.05 \\ -4.27 & 0.19 & 28.06 & 0.81 & -4.29 & 0.19 & -3.14 & 0.48 \\ 0.21 & 5.05 & 0.81 & 5.71 & 0.21 & 5.05 & 0.48 & 5.37 \\ -3.24 & 0.50 & -4.29 & 0.21 & 28.11 & 0.83 & -4.29 & 0.21 \\ 0.50 & 5.37 & 0.19 & 5.05 & 0.83 & 5.70 & 0.19 & 5.05 \\ -4.28 & 0.19 & -3.14 & 0.48 & -4.29 & 0.19 & 28.06 & 0.81 \\ 0.21 & 5.05 & 0.48 & 5.38 & 0.21 & 5.05 & 0.81 & 5.71 \end{bmatrix}.$$

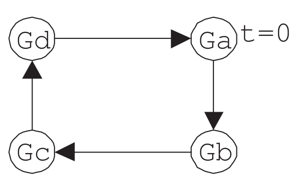

Fig.2 Switching sequences of the network topology

Fig.3 depicts the state trajectories for this switching network with $\tau = 0.43$.

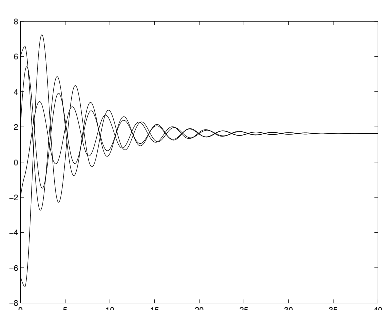

Fig.3(a) Position trajectories of the network

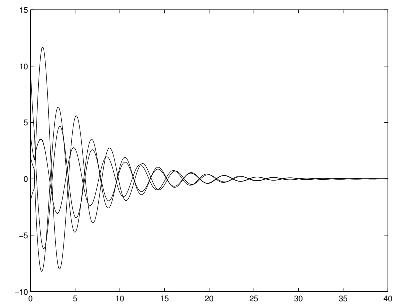

Fig.3(b) Velocity trajectories of the network

Clearly, one can observe that all agents converge to a common value and their speeds reduce to zero; that is, all agents achieve consensus.

V. Conclusions

In this paper, distributed coordination has been considered for networks of second-order agents with time-delay and switching topology. We first perform a model transformation and turn the original system into an equivalent system; then, based on this equivalent system, we introduce a common Lyapunov function and use a dimension reduction approach to give sufficient conditions in terms of LMIs under which all agents asymptotically reach consensus. Simulation results are provided to show the effectiveness of the obtained theoretical results.

It is worth noting that the communication topology considered is assumed to be strongly connected, and future research could be directed towards considering jointly-connected topology case, which is more general. In addition, the dynamics of each agent considered in this paper is for continuous-time, and all the methods introduced in this paper could also be used for discrete-time case, which deserves further investigation.

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