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Satellite attitude control using three reaction wheels

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Abstract— **This work addresses the attitude control of a satellite by applying MIMO quantitative feedback approach. The objective is to design a set of proper controllers in presence of unknown disturbances and parametric uncertainties for a nonlinear MIMO system. The physical model of satellite utilizes three reaction wheels as actuators. The controller goal is to change the rotational speed of reaction wheels to adjust the satellite in desired course. First, the mathematical model of satellite and its actuators using angular kinematics and kinetic equations is developed. Quantitative feedback theory is then applied to synthesize a set of linear controllers that deals with both nonlinearities in the equations and unknown parameters or disturbance sources. By using basically non-interacting desired outputs and extracting sets of linear time invariant equivalent (LTIE) plants, the controllers set is designed for nine SISO systems. Simulation of closed loop system shows that all desired specifications of closed loop (tracking, stability, disturbance rejection) are robustly satisfied.**

I. INTRODUCTION

Satellite attitude control has been an active research topic for quite sometime. The nonlinear nature of satellite model itself, coupled with the uncertainties both in parameters and disturbances, makes the attitude control problem attractive and challenging. Numerous control design methods have been investigated to achieve control system performance and/or robustness. Recent works on satellite control include linear and nonlinear H_∞ control [1-3], fuzzy-neuro control [4-5], LQR/LRT [6-8], and adaptive control [9] among others. A good review of many control approaches for attitude control is provided in [10].

The use of a reaction wheel as an actuator in satellite control has gained popularity lately [11-13]; some advantages of this type of actuator configuration over others are shown in [12]. A reaction wheel is a device that applies torque to satellite at the control command resulting in changed angular momentum (or angular velocity) of the satellite. However, the presence of disturbances and uncertainties in reaction wheel itself can significantly deteriorate control performance and must be compensated for in the control design [14]. In addition, the nature of torques or forces acting on satellite in general are not thoroughly known and the derived equations may include uncertainties (either modeled or un-modeled), thus making the satellite attitude control arduous. In these circumstances, robust control theory offers an attractive solution for control design.

To robustly control nonlinear and uncertain systems, quantitative feedback control theory (QFT) has emerged as an effective synthesis tool with advantages over other control schemes [15-18] and thus provides a basis for solving the current problem. This work addresses the attitude control of a satellite utilizing three reaction wheels while incorporating uncertainties in the parameters and disturbances. The study synthesizes a control system that stabilizes the satellite in its orbit and orients the satellite in any desired spatial direction relative to the reference frame. The use of three reaction wheels is novel in satellite attitude control that facilitates flexibility in orientation of the satellite and can be duly compared against a single reaction wheel for improved performance.

The structure of paper is as follows. Section II describes the problem formulation and system modeling. Section III describes the controller design based on QFT control law for desired closed loop system specifications. The simulation results are presented in section IV.

II. GOVERNING EQUATIONS

The governing equations of satellite attitude are expressed by angular kinetic and angular kinematics equations. Angular kinetic equations (or Euler equations) express the rate of change in angular velocities due to external torques or disturbances. The angular kinematics equations specify the relationship between absolute angular velocity of the satellite and its orientation in the space [19].

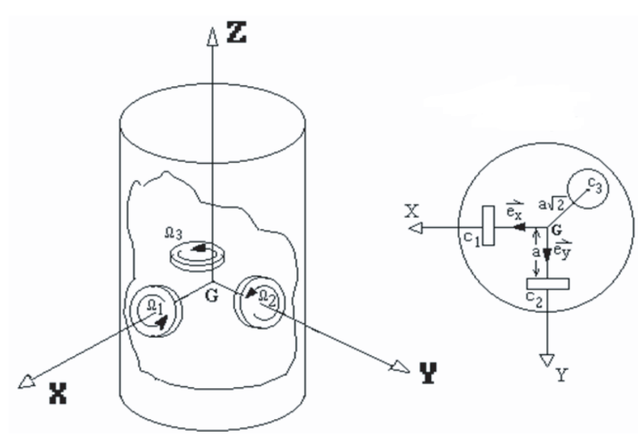

Fig. 1. Satellite schematic with reaction wheels

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Let the components of angular velocity in body coordinate XYZ be given by $(\omega_1, \omega_2, \omega_3)$ as shown in Fig. 1. Using Euler angles to specify angular kinematics, the satellite attitude model is determined as:

$$\begin{aligned}\dot{\psi} &= (\omega_2 \sin\phi + \omega_3 \cos\phi)\sec\theta \\ \dot{\theta} &= \omega_2 \cos\phi - \omega_3 \sin\phi \\ \dot{\phi} &= \omega_1 + \dot{\psi}\sin\theta = \omega_1 + (\omega_2 \sin\phi + \omega_3 \cos\phi)\tan\theta\end{aligned} \tag{1}$$

Here (ψ, θ, ϕ) are the Euler angles that determine the satellite position relative to the reference frame here chosen as Earth.

The angular kinetic equations consist of governing equations of the total system that include equations of satellite with the reaction wheels and equations of satellite actuators alone (the reaction wheels) , which are expressed as

$$\overrightarrow{M} = \dot{\overrightarrow{H}}_G \tag{2}$$

$$\overrightarrow{T} = \overrightarrow{H}_W \tag{3}$$

Here $\overrightarrow{H}_G$ describes the angular momentum of the total system (satellite and reaction wheels) about the satellite center of mass and $\overrightarrow{H}_W$ represents the angular momentum of the reaction wheels relative to the satellite body. The term $\overrightarrow{H}_W$ is a representation of the known or unknown torques on satellite and $\overrightarrow{T}$ is the torque of motor acting on the reaction wheels.

The angular kinetic equations of the system are determined by using total angular momentum with (2) to obtain

$$\left(I_a + 2I_t + 2ma^2 + I_{xx}\right)\dot{\omega}_1 - ma^2\dot{\omega}_2 + I_a\dot{\Omega}_1 + I_a\Omega_3\omega_2 + \left(I_{zz} - I_{yy} + 2ma^2\right)\omega_2\omega_3 - I_a\Omega_2\omega_3 + ma^2\omega_1\omega_3 = M_1 \tag{4a}$$

$$\left(I_a + 2I_t + 2ma^2 + I_{yy}\right)\dot{\omega}_2 - ma^2\dot{\omega}_1 + I_a\dot{\Omega}_2 - I_a\Omega_3\omega_1 + \left(I_{xx} - I_{zz} - 2ma^2\right)\omega_1\omega_3 - ma^2\omega_2\omega_3 + I_a\Omega_1\omega_3 = M_2 \tag{4b}$$

$$\left(I_a + 2I_t + 4ma^2 + I_{zz}\right)\dot{\omega}_3 + I_a\dot{\Omega}_3 + I_a\Omega_2\omega_1 - I_a\Omega_1\omega_2 + \left(I_{yy} - I_{xx}\right)\omega_1\omega_2 + ma^2\left(\omega_2^2 - \omega_1^2\right) = M_3 \tag{4c}$$

In (4), (M_1, M_2, M_3) are unknown disturbance torques acting on the body of the satellite. The principal moments of inertia of the satellite in XYZ direction are given by (I_{xx}, I_{yy}, I_{zz}) respectively whereas (I_a, I_t) are axial and transversal moments of inertia of the reaction wheels. The angular velocity of the i^{th} reaction wheel relative to satellite is given by Ω_i, $(i = 1, 2, 3)$ and the term ma^2 expresses the concentrated inertial moment of the reaction wheel due to the satellite center of mass. Similarly, by applying angular momentum on (3), the governing equations of the actuators are obtained as

$$\begin{aligned}I_a\left(\dot{\omega}_1 + \dot{\Omega}_1\right) &= u_1 \\ I_a\left(\dot{\omega}_2 + \dot{\Omega}_2\right) &= u_2 \\ I_a\left(\dot{\omega}_3 + \dot{\Omega}_3\right) &= u_3\end{aligned} \tag{5}$$

In (5), u_i $(i = 1, 2, 3)$ is the torque applied from the i^{th} motor of the reaction wheel (where the friction force effects in motor are neglected).

In essence, the governing equations of system are presented by (1), (4), and (5), which indicate that the coupled system is highly nonlinear and uncertain, and has three inputs and three outputs. The inputs to the system are torques exerted by reaction wheels (or electric current) and the system outputs are the desired Euler angles.

Spatial torques generated from known or unknown sources such as solar wind pressure, meteoroid impact, and earth oblateness cannot be exactly determined. These values are introduced as disturbances in the closed loop system, and the controller must be able to reject them in addition to the satellite tracking. The physical parameters of the satellite and reaction wheels are given in Table I.

TABLE I
PHYSICAL PROPERTIES OF TOTAL SYSTEM (ALL IN Kgm^2)

I_{xx}	I_{yy}	I_{zz}	I_a	I_t	ma^2
1000	700	400	2	1	1

III. CONTROLLER DESIGN

A. Closed Loop Requirements

Satellite control system requires designing of robust controllers that should meet the following closed loop system specifications:

- The closed loop *must* be stable for the desired input range and unknown limited disturbances' set.
- The closed loop system *must* satisfy the tracking bounds that were initially specified for designing the controller.
- The closed loop system *should* be able to reject all the known and unknown disturbances acting on the system.

The following assumptions are made for the controller design.

- The three reaction wheels used for attitudecontrol are identical and their rotation directions are coincidentwith the principal axes of the satellite.
- The unbalancing effects in reaction wheels are ignored.
- The un-modeled dynamic effects at high frequency are neglected and/or the satellite is considered rigid.

The closed loop specifications suggest that the satellite control system should change angular velocity of the reaction wheels relative to satellite body to aptly respond to the disturbance effects thus adjusting satellite orientation to desired inputs. This implies that the angular velocities of the reaction wheels must be changed continuously to meet the desired closed loop performance specifications.

B. LTIE Plants using QFT

For the nonlinear set of governing equations, the QFT requires deriving a set of linear time invariant (LTIE) plants for the desired input range that captures the dynamic behavior of the original nonlinear system [20]. Here it is assumed that the system inputs are step functions for each angle and desired outputs for closed loop systems are specified as

$$\begin{aligned} y_{ii}(t) &= m(1-\lambda e^{-\sigma t}-(B-\lambda)e^{-\tau t}+(B-1)e^{-\gamma t}) \\ y_{ji}(t) &= \frac{1}{60}mt^3e^{-\alpha t} \\ y_{ki}(t) &= -y_{ji}(t) \end{aligned} \tag{6}$$

where

$$\begin{aligned} B &\triangleq \frac{\gamma(\sigma+\tau)-\tau^2}{(\tau-\gamma)(\gamma-\sigma)} \\ \lambda &\triangleq \frac{-\gamma\tau}{(\sigma-\tau)(\gamma-\sigma)} \end{aligned} \tag{7}$$

and

$$\begin{gathered} 1 \leq (\sigma,\tau,\gamma,\alpha) \leq 3 \\ -0.6\, rad \leq m \leq 0.6\, rad \end{gathered} \tag{8}$$

In (6), y_{ii} is the i^{th}, $(i = 1-3)$ desired output for the closed loop corresponding to the i^{th} step input of amplitude m, y_{ji} and y_{ki} are the j^{th} and k^{th} outputs $(i \neq j \neq k)$ corresponding to the i^{th} input. The index $(1-3)$ corresponds to $(\psi, \theta,$ and $\phi)$ respectively and σ, τ, γ and α represent the range of system eigenvalues. This approach is termed as basically non-interacting [21]. Hence, the set of desired outputs corresponding to ψ are (y_{11}, y_{21}, y_{31}), for θ are (y_{12}, y_{22}, y_{32}) and for ϕ are (y_{13}, y_{23}, y_{33}) respectively.

In order to find sets of LTIE transfer function matrices that are equivalent to the original nonlinear plant, it is required to know the outputs of the system and controller inputs. The outputs are chosen as desired outputs (6-8) which are then substituted in system equations (1), (4) and (5) to solve for the controller inputs to the system by inverse solving differential equations. Consequently, for the set of outputs of (y_{11}, y_{21}, y_{31}), the controller inputs are obtained as (u_{11}, u_{21}, u_{31}). Similar procedure for (y_{12}, y_{22}, y_{32}), and (y_{13}, y_{23}, y_{33}) results in the controller inputs (u_{12}, u_{22}, u_{32}) and (u_{13}, u_{23}, u_{33}) respectively. Now, the columns of two matrices $[\mathbf{U(s)}]$ and $[\mathbf{Y(s)}]$ are formed that are the Laplace transforms of the control signals and the desired outputs respectively. The Laplace transform of the control signal is obtained numerically as in [22-23].

The LTIE plants are expressed as

$$\mathbf{P}_{LTIE} = [\mathbf{Y(s)}]_{3\times 3}[\mathbf{U(s)}]_{3\times 3}^{-1} \tag{9}$$

For the selected range of parameters (8), using above approach, 800 sets of LTIE plants were extracted that fully capture the nonlinear system dynamic behavior for the specified step input range.

C. MIMO Controller and Prefilter

After obtaining the LTIE sets, we want to design a diagonal controller matrix $[\mathbf{G(s)}]_{3\times 3}$ and a prefilter matrix $[\mathbf{F(s)}]_{3\times 3}$ that satisfy the closed loop specifications for all the acquired LTIE plants. Using MIMO QFT design approach [23], we convert our system into a set of nine SISO plants as in Fig. 2. In Fig. 2, G_1, G_2 and G_3 are the diagonal elements of the controller matrix $\mathbf{G(s)}$ and F_{ij} are the elements of the prefilter matrix $\mathbf{F(s)}$ where elements of inverse $\mathbf{P}_{LTIE}$ are

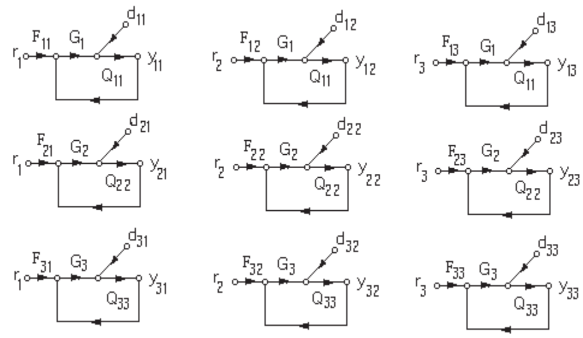

Fig. 2. The equivalent 9 SISO plants for the 3×3 MIMO Plant

expressed as $1/Q_{ij}$. The disturbance d_{ij} is defined in (10) along with the closed loop transfer function for each SISO system T_{ij}

$$\begin{aligned} T_{ij} &= \frac{F_{ij}G_iQ_{ij}+d_{ij}Q_{ij}}{1+G_iQ_{ii}}, \quad i,j = 1,2,3 \\ d_{ij} &= -\sum_{u=1}^{n}\frac{T_{uj}}{Q_{iu}}, \qquad u \neq i \end{aligned} \tag{10}$$

The time domain design specifications for each controller and prefilter are first converted into frequency domain and are then translated into performance bounds in Nichols charts, as shown here only for the case of G_1 and F_1 (only one shown due to space constraints).

1) The output of closed loop transfer functions T_{11}, T_{21} and T_{31} for step input range $\in [-0.6, 0.6]$ *rad* must lie within the originally specified desired outputs in (6-8). This performance restriction results in tracking bounds (11) and disturbance rejection bounds (12) in Nichols charts as

$$a_{11} \leq \left|\frac{F_1G_1Q_{11}}{1+G_1Q_{11}}\right| \leq b_{11} \tag{11}$$

$$\begin{aligned} 0 &\leq \left|\frac{d_{21}}{1+G_1Q_{11}}\right| \leq b_{21} \\ 0 &\leq \left|\frac{d_{31}}{1+G_1Q_{11}}\right| \leq b_{31} \end{aligned} \tag{12}$$

2) T_{11}, T_{21}, and T_{31} must be stable, thus resulting in stability bounds in Nichols charts as,

$$\left|\frac{G_1Q_{11}}{1+G_1Q_{11}}\right| \leq 1.2 \tag{13}$$

All bounds for the first row plotted at various frequencies are shown in Fig. 3.

Now the loop shaping using *MATLAB QFT* Toolbox is conducted manually to acquire the controller structure that satisfies the performance bounds at each frequency. For instance, the nominal loop shape of one of the $1/Q_{11}$ sets is shown in Fig. 4. Following are the obtained diagonal

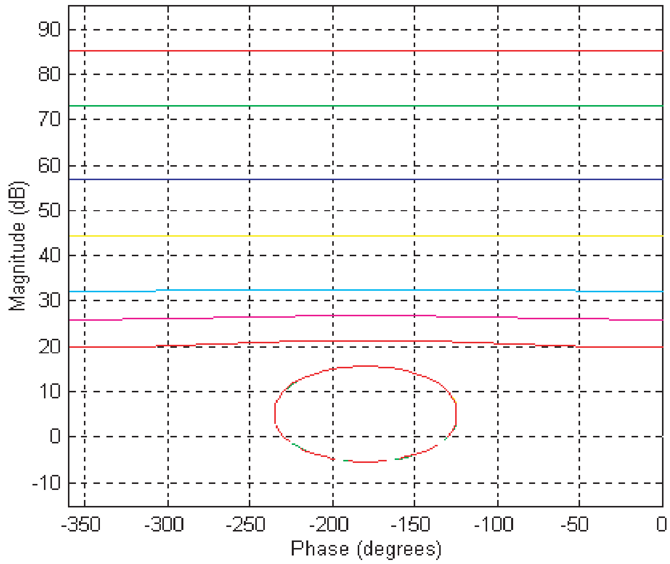

Fig. 3. Stability, tracking and disturbance rejection bounds for the first closed loop row.

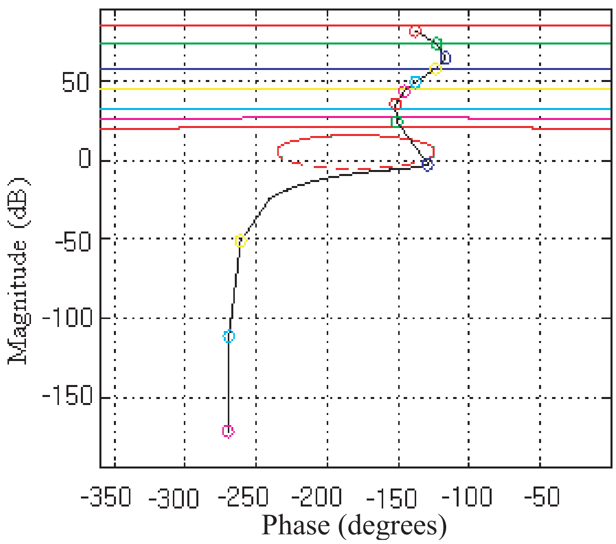

Fig. 4. The Loop shaping for the nominal plant of the first row.

controller **G** and prefilter transfer functions **F** respectively.

$$
\begin{aligned}
G_1 &= \frac{-3477\left(\frac{s}{0.1}+1\right)\left(\frac{s}{24.68}+1\right)}{\left(\frac{s}{1.726}+1\right)\left(\frac{s^2}{213^2}+\frac{0.82s}{213}+1\right)} \\
G_2 &= \frac{-4377\left(\frac{s}{0.15}+1\right)\left(\frac{s}{16.74}+1\right)}{\left(\frac{s}{1.0}+1\right)\left(\frac{s^2}{161^2}+\frac{2\times 0.28s}{161}+1\right)} \\
G_3 &= \frac{-3662\left(\frac{s}{0.7139}+1\right)\left(\frac{s}{11.78}+1\right)}{\left(\frac{s}{2.94}+1\right)\left(\frac{s^2}{48.4^2}+\frac{s}{48.4}+1\right)}
\end{aligned}
\tag{14}
$$

and

$$
\begin{aligned}
F_{11}(s) &= \frac{1}{\left(\frac{s}{0.983}+1\right)\left(\frac{s}{2.24}+1\right)\left(\frac{s}{4.34}+1\right)} \\
F_{22}(s) &= \frac{1}{\left(\frac{s}{0.4652}+1\right)\left(\frac{s^2}{1.788^2}+\frac{2\times 0.707s}{1.788}+1\right)} \\
F_{33}(s) &= \frac{1}{\left(\frac{s}{0.806}+1\right)\left(\frac{s^2}{4}+0.707s+1\right)}
\end{aligned}
\tag{15}
$$

IV. SIMULATION OF CLOSED LOOP RESPONSE

The closed loop system is simulated in both the frequency and time domains. The designed LTIE plants, controller set and pre-filters are implemented in the SIMULINK workspace. The extremes values for closed loop transfer functions T_{ii} are compared with the tolerance bounds and with the maximum value of non-interacting loop in Fig. 4, that show satisfactory results with some very small and insignificant violations in the non-interacting loop. The step responses for all Euler angles in time domain are shown in Fig. (5-7). It is seen that the MIMO controllers are able

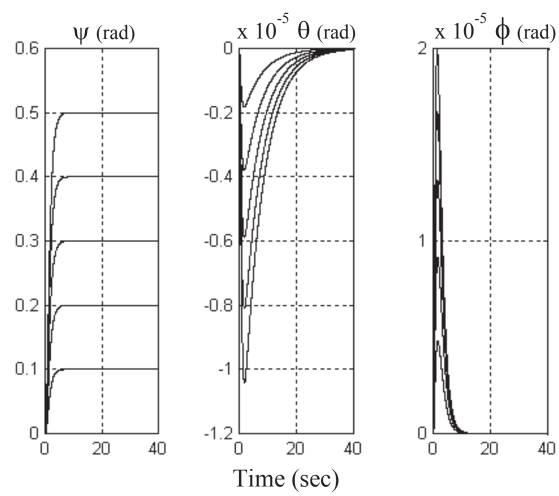

Fig. 5. Closed loop system step response of system for $(\psi, 0, 0)$ input.

to achieve the steady state response relatively quickly. The time response shows that the system is stable in range of desired inputs and also satisfies the tracking and upper bound specifications. While designing the controller, the range of desired inputs was chosen between -0.6 to $+0.6$ *rad* (8), but the system actually showed stability for a broader range. It was observed to be stable in a range of $[-0.9+0.9]$ *rad* and went unstable beyond this range (not shown). The over designed controller was not unexpected since it is inherent in the QFT theory. The effects of low frequency disturbances were also simulated and it was observed that in the closed

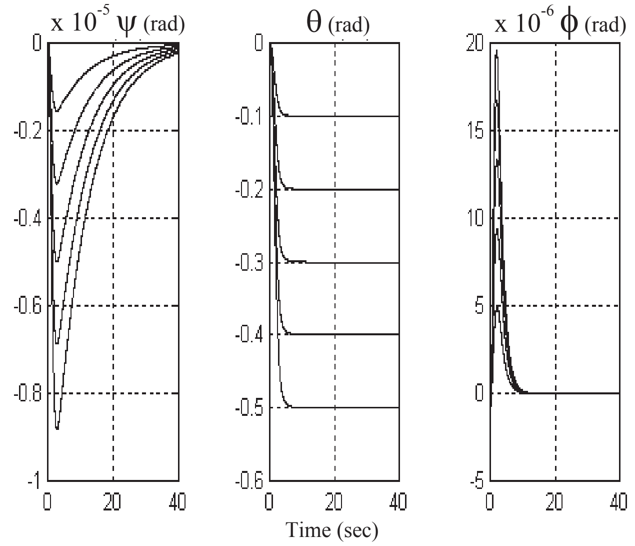

Fig. 6. Closed loop system step response of system for $(0,\theta,0)$ input.

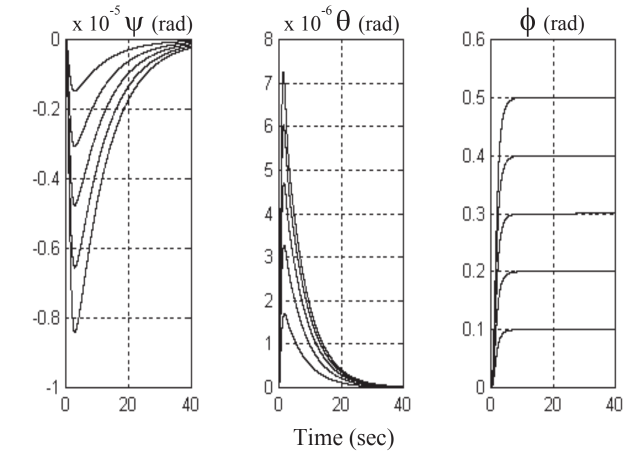

Fig. 7. Closed loop system step response of system for $(0,0,\phi)$ input.

loop response all disturbances from 0 up and including upper frequency of 5 rad/sec were attenuated successfully (not shown).

V. CONCLUSION

The attitude control of a satellite by three reaction wheels using QFT is presented. The nonlinear dynamic model of plant with uncertain inputs was first converted into a set of 800 LTIE plants that fully capture the dynamics of original nonlinear system. Then, the performance specifications of controllers were derived and translated into Nichols charts. Loop shaping was then applied to obtain a diagonal set of controllers along with pre-filters for the nine SISO systems. The results of closed loop response show that the controller set satisfies the closed loop specifications for MIMO system such as tracking, stability and disturbance rejection at each input frequency. The response is observed to lie within the desired bounds. Also the closed loop system is able to reject noise at frequencies higher than 300 rad/sec. Also, if required, the limitation of low frequency noise problem can be addressed by changing the desired specifications of the closed loop and hence the procedure of designing controller can be repeated.

REFERENCES

[1] L. C. G. De Souza, "Design of satellite control system using optimal nonlinear theory," *Mechanics Based Design of Structures and Machines*, vol. 34, n. 4, pp. 351-364, Dec 2006.

[2] D. Prieto and B. Bona, "Orbit and attitude control for the European satellite GOCE," *IEEE Netw. Sens. Control Proc.*, pp. 728-733, 2005.

[3] M. Aliabbasi, H. A. Talebi and M. Karrari, "A satellite attitude controller using nonlinear H_∞ approach," *in Proceedings of the 41st IEEE Conference on Decision and Control*, Las Vegas, Nevada USA, pp. 4078-4083, Dec 2002.

[4] P. Guan, X.J. Liu, F. Lara-Rosano, and B.J. Chen, "Adaptive fuzzy attitude control of satellite based on linearization," *in Proceedings of the 2004 American Control Conference*, vol. 2, pp. 1091-1096, 2004.

[5] M. Belanger and J. De Lafontaine, "Quaternion-based satellite attitude control using fuzzy logic," *in Proceedings of the AAS/AIAA Astrodynamics Conference*, pp. 2701-2711, 2006.

[6] H. Bolandi, F. Bayat, and M. Nasirian, "Attitude control of spinning satellite subject to actuators restriction using eigenstructure assignment," *1st International Symposium on Systems and Control in Aerospace and Astronautics*, pp. 1413-1419, 2006.

[7] C. H. Won, "Satellite structure attitude control with parameter robust risk-sensitive control synthesis," *in Proceedings of the 2004 American Control Conference*, pp. 3538-3543, 2004.

[8] J. Rodden, L. McGovern, J. Higham and X. Price,"Momentum bias for spacecraft attitude," *Advances in the Astronautical Sciences*, vol. 111, pp. 35-44, 2002.

[9] S. N. Singh and W. Yim, "Nonlinear adaptive spacecraft attitude control using solar radiation pressure," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 41, n. 3, pp. 770-779, July, 2005.

[10] E. Silani, and M. Lovera, "Magnetic spacecraft attitude control: a survey and some new results," *Control Engineering Practice*, vol. 13, n. 3, pp. 357-371, Mar 2005.

[11] W. H. Steyn, "A view finder control system for an earth observation satellite," *Aerospace Science and Technology*, vol. 10, n. 3, pp. 248-255, April, 2006.

[12] R. Kristiansen, O. Egeland and P. Johan, "A comparative study of actuator configurations for satellite attitude control," *Modeling, Identification and Control*, vol. 26, n. 4, pp. 201-219, Dec 2005.

[13] S. Y. Yong, "Development of reaction wheels housing for microsatellites," *Aircraft Engineering and Aerospace Technology*, vol. 77, n. 2, pp. 114-121, 2005.

[14] G. Shengmin and C. Hao, "A comparative design of satellite attitude control system with reaction wheel," *in Proceedings of First NASA/ESA Conference on Adaptive Hardware and Systems*, pp. 359-362, 2006.

[15] Y. Chait and C. V. Holt, "Comparison between H-infinity methods and QFT for a SISO plant with both plant uncertainty and performance specifications," *ASME Dyn Syst Control Div Pub. DSC, Recent Developments in Quantitative Feedback Theory*, vol. 24, pp. 33-39, 1990.

[16] J.C. Moreno, A. Banos and M. Berenguel, "Improvements on the computation of boundaries in QFT," *International Journal of Robust and Nonlinear Control*, vol. 16, n. 12, pp. 575-597, Aug 2006.

[17] S. G. Breslin, and M. J. Grimble, "Longitudinal control of an advanced combat aircraft using quantitative feedback theory," *in Proceedings of the American Control Conference*, vol. 1, pp. 113-117, 1997.

[18] S. S. Nudehi and U. Farooq, "Hybrid QFT/$H\infty$ for control of nonlinear systems: An example of position control of a pendulum," *in Proceedings of American Control Conference*, New York City, USA, July 9-13, pp. 2793-2798, 2007.

[19] D. T. Greenwood, *Principles of Dynamics*, New Jersey: Prentice Hall, 1988, Ch. 8.

[20] I. M. Horowitz, "Synthesis of feedback systems with nonlinear time-varying uncertain plants to satisfy quantitative performance specifications," *Proceedings of IEEE*, vol. 64, pp. 123-130, 1976.

[21] O. Yaniv and I. Horowitz, "A quantitative design method for MIMO linear feedback systems having uncertain plants," *International Journal of Control*, vol. 43, n. 2, pp. 401-421, 1986.

[22] I. M. Horowitz, Quantitative Feedback Design Theory, vol. 1. QFT publications: Boulder, CO, 1993.
[23] I. M. Horowitz, “A synthesis theory for linear time-varying feedback systems with plant uncertainty,” *IEEE Transactions On Automatic Control*, vol. AC-20, n. 4, Aug 1975