Control of Helicopters' Formation Using Non-Iterative Nonlinear Model Predictive Approach

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***Abstract*— A non-iterative Nonlinear Model Predictive Controller (NMPC) for formation control of helicopters is proposed and validated through simulations. The method is based on minimizing the error of geometrical formation parameters specifically designed for helicopters. These parameters are used to form desired three-dimensional (3D) configurations among members of a helicopter group. This approach is tested for both initializing and maintaining the desired formation. Also, simulation has been conducted considering the presence of environmental disturbances and model uncertainties. Compared to the similar approaches, the method has a substantially smaller computational cost. In addition, it is shown that unlike the conventional NMPC optimization methods, the presented framework does not require any iteration. This method inherently possesses the same computational cost for all the time steps throughout the whole time period of the flight scenario. These features make this framework a suitable choice for implementation for formation control of helicopter groups.**

I. INTRODUCTION

In recent years, formation control of autonomous vehicles has become a challenging interdisciplinary research topic. Different aspects of this problem such as navigation of robot swarms [1], [2], collaborative maneuvers [3], [4], [5], [6], [7], payload transportation [8], initialization and reconfiguration [9], [10], inter-telecommunication and information flow [11], [12] have been studied by researches from diverse scientific fields. In this research, we consider the formation control problem for a group of helicopter agents. Helicopters have excellent maneuvering capabilities. However, their complex and non-minimum phase dynamical behavior makes their formation control a challenging problem.

Some aspects of the formation of aerial vehicles in general and helicopters in particular have been addressed in previous researches. The formation strategy considered here is based on the well-known leader-follower technique, which is successfully applied in the former researches. This technique is used mostly for control of systems with two dimensional (2D) maneuvering capabilities. Specially the mobile robots have been a matter of interests for many researchers [5], [7], [13], [14]. Generally, a leader-follower framework must fulfill two main requirements. Firstly, the members must be able to track each other sequentially. Secondly, the framework must be able to integrate the collision avoidance feature. In order to uniquely define the desired pattern in the working environment, two base schemes are required. These schemes, whose simple 2D versions were introduced for mobile robots [13], are known as the $l-\alpha$ and the $l-l$ schemes. The $l-\alpha$ scheme is required to maintain the distance of one helicopter with respect to another neighboring leader. The $l-l$ scheme is used to constrain one agent with respect to two neighboring leaders. Different versions of these schemes are developed by researchers based on robot capabilities and the required performance of the formation scenario [5], [14], [15].

Nonlinear Model Predictive Control (NMPC) has become one of the most attractive methods for researchers to tackle the formation control problem. Initially developed for the fairly slow systems such as industrial chemical processes, and reactors and plants, the MPC method provides a simple, robust and flexible optimum control framework for an integrated system through a time horizon. Up to recent years, the large computational cost of this method prevented researchers and engineers to implement it for the fast control systems such as manipulators, mobile robots or aerial vehicles.

In addition, the fact that the optimization process of the method results in unequal calculation times was considered as a drawback for utilization of NMPC controllers, specifically for nonlinear systems with large amount of nonlinearity through their entire working range.

Several attempts has been reported to provide fast algorithms for NMPC controllers. In [16], the researchers developed a control framework by expanding both the outputs and control commands of the nonlinear system using limited terms of the Taylor series. To demonstrate the application, the model is adopted for the flight dynamics and used to control different flying systems. Another work is reported in [10], where researchers design a Multi-Input-Multi-Output (MIMO) predictive control strategy. The stability of this approach is achieved by penalizing the end state in the framework. This controller is tailored to be used in the reconfigurable control systems that are exposed to actuator saturations. The application of this controller is demonstrated by short-term control of a civil aircraft. Finding a nonlinear model for dynamical systems with a desired accuracy is not always achievable. The work reported in [17] explains the application of pseudo-partial derivatives method to dynamically linearize the nonlinear model through the control horizon. The input-output data are used for calculation of the derivatives and in steady state conditions, the method is shown to have a zero tracking error performance.

In this research, we used the GMRES/Continuation method for the problem of formation control of helicopters. This method is a combination of the Continuation method for finding dynamic response of smooth systems [18] and

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the Generalized Minimum Residual Method (GMRES) [19], which is a fast solver for systems of linear equations. In the research reported in [20] and developed further in [21], the combination of these two techniques for solving the nonlinear receding horizon control problems is presented. The method is shown to be a suitable solution for real-time application of receding horizon control for fast response systems.

The current research is the first work in which the problem of formation control is tackled through the continuation/GMRES method. In the next sections, first we introduce the nonlinear dynamic model of the helicopter used in our work. Next, we briefly introduce our formation schemes, which will be used in the design procedure of the controller. After a brief introduction of both the continuation and the GMRES methods, a control framework is developed based on a combination of these two techniques. Finally, the performance of the method is studied through some simulated flight scenarios. The method is also analyzed for associated computational cost of the real-time implementation and a comparison is made with the former analysis of classical gradient descent method.

II. Dynamics of A Helicopter

The simplified dynamic equations of a helicopter, which are detailed enough for control development for quasi-steady maneuvers, are introduced. The aerodynamic tractions are assumed to be the control inputs. The spatial configuration of the helicopter is shown in the Fig. 1. Using the Newton-Euler equations of motion, one can link the absolute linear and angular accelerations of the helicopter to the aerodynamic tractions exerted by the main and tail rotors. The inertial position of the helicopter is defined by vector $\mathbf{p}_I$, where the index I indicates the vector is expressed in the inertial frame $\{I\}$. The equations for the linear acceleration of the helicopter can be formed as the following:

$$\ddot{\mathbf{p}}_I = \frac{1}{m}\left[\mathbf{R}_{IB}(\mathbf{f}_{dB} + \mathbf{f}_{aB}) + \mathbf{f}_{iI}\right] \quad (1)$$

where $\mathbf{R}_{IB}$ is the transformation between the inertial and the body frame B defined by a successive roll (ϕ), pitch (θ), and yaw (ψ) rotations. The vectors $\mathbf{f}_{dB}$, $\mathbf{f}_{aB}$, $\mathbf{f}_{iI}$ are representing the drag forces, the aerodynamic tractions produced by the main and tail rotors, and the weight of the helicopter, respectively. As the subscript B indicates, these vectors are expressed in the body frame $\{B\}$. Considering the rotational kinematics of the helicopter, the following relation hold between the helicopter's angular velocity vector and the rates of the Euler angles:

$$\boldsymbol{\omega}_B = \mathbf{E}\boldsymbol{\gamma}_B \quad (2)$$

where

$$\mathbf{E} = \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \cos\theta\sin\phi \\ 0 & -\sin\phi & \cos\theta\cos\phi \end{bmatrix} \quad (3)$$

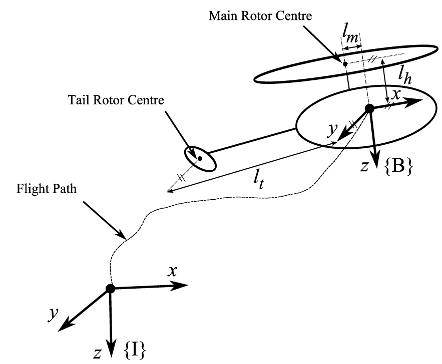

Fig. 1. The Spatial Configuration of the Helicopter

and $\boldsymbol{\gamma}_B = [\dot{\phi}, \dot{\theta}, \dot{\psi}]^T$. The equation for rotational dynamics of the helicopter can be formulated as follows:

$$\dot{\boldsymbol{\omega}}_B = \mathbf{I^{-1}}(\mathbf{t} - \boldsymbol{\omega}_B^T \times \mathbf{I}\boldsymbol{\omega}_B) \quad (4)$$

The vector $\mathbf{t}$ consists of the moments caused by the aerodynamic tractions applied to the helicopter fuselage. $\mathbf{I}$ is the helicopter's moment of inertia tensor. The Eqs. (1)-(4) are used to form the nonlinear relationship for the helicopter dynamics:

$$\begin{bmatrix} \dot{\mathbf{p}}_I \\ \ddot{\mathbf{p}}_I \\ \boldsymbol{\gamma}_B \\ \dot{\boldsymbol{\omega}}_B \end{bmatrix} = \begin{bmatrix} \mathbf{0}_{3\times4} \\ \frac{1}{m}\mathbf{R}_{IB}\mathbf{A}_1 \\ \mathbf{0}_{3\times4} \\ \mathbf{I}^{-1}\mathbf{A}_2 \end{bmatrix} \mathbf{u} + \begin{bmatrix} \dot{\mathbf{p}}_I \\ \frac{1}{m}(\mathbf{R}_{IB}\mathbf{f}_{dB} + \mathbf{f}_{iI}) \\ \mathbf{E}^{-1}\boldsymbol{\omega}_B \\ -\mathbf{I^{-1}}(\boldsymbol{\omega}_B^T \times \mathbf{I}\boldsymbol{\omega}_B) \end{bmatrix} \quad (5)$$

where the input vector, $\mathbf{u} = [M_\phi, M_\theta, T_M, T_T]^T$, consists of the main rotor's roll moment, pitch moment, and thrust; and the tail rotor thrust, respectively. The matrices $\mathbf{A}_1$ and $\mathbf{A}_2$ are utilized to rearrange the equations of motion:

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix} \quad \mathbf{A}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -l_r & 0 \\ 0 & 0 & -k_m & l_t \end{bmatrix} \quad (6)$$

III. Definition of Formation Parameters

Two schemes are developed that allow the use of triangular building blocks to uniquely define any formation mesh with triangular cells. The number of formation parameters (4) for the schemes is chosen to match the number of control inputs for a helicopter (4). Here, we briefly formulate the parameters of each of these schemes in terms of the helicopters' state.

A. The $l-\alpha$ Scheme

In order to constraint two helicopters with respect to each other, we try to formulate the relative spatial position of two points named as control point. For each member of the group, we define a control point, which is located on the helicopter's main axis with an offset of d with respect to the main rotor center. The following vector relation is held for the follower's control point position (see Fig. 2):

$$\mathbf{p}_{c_1} + \mathbf{d}_1 + \mathbf{l}_{12} + \mathbf{z}_{12} = \mathbf{p}_{c_2} + \mathbf{d}_2 \quad (7)$$

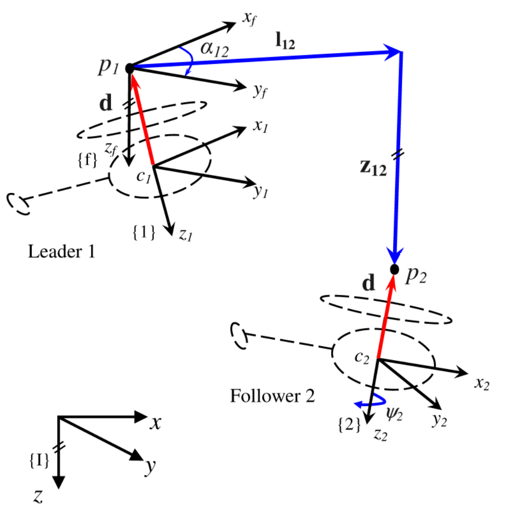

Fig. 2. Formation Parameters of Two Neighboring Helicopters

In the above relation, the vectors $\mathbf{p}_{c_1}$ and $\mathbf{p}_{c_2}$ indicate the position vectors of the leader and the follower helicopters, respectively. The vector $\mathbf{l}_{12} + \mathbf{z}_{12}$ is the relative distance between the leader and the follower control points. The vector $\mathbf{d}_i$ is the relative position of the control point with respect to center of gravity of helicopter i. Representing the sum of the last two terms on the left hand side of the Eq. (7) with $\mathbf{p}_{12}$ and expressing all the terms in the inertial frame, one can write:

$$\mathbf{p}_{12} = \mathbf{R}_{If}^{-1} \left(\mathbf{p}_{c_2} - \mathbf{p}_{c_1} + \mathbf{R}_{I2}\mathbf{d} - \mathbf{R}_{I1}\mathbf{d} \right) \qquad (8)$$

In Eq. (8), the matrices $\mathbf{R}_{I1}$ and $\mathbf{R}_{I2}$ represent the transformation between the inertial coordinate system and the leader ($\{1\}$) and the follower ($\{2\}$) body coordinate frames, respectively. The frame f is defined attached to the leader's control point, with its xy-plane always parallel to the horizontal plane of the inertial frame, and its x_f is in the $x_1 y_1$-plane of the helicopter's body frame. The matrix $\mathbf{R}_{If}$ is defined as a transformation between the frame f and the inertial system. Using the elements of the vector $\mathbf{p}_{12}$, three geometrical parameters of the $l - \alpha$ formation scheme are defined as follow:

$$l_{12} = \sqrt{p_{12_x}^2 + p_{12_y}^2} \quad \alpha_{12} = \arctan(p_{12_y}/p_{12_x}) \qquad (9)$$

$$z_{12} = p_{12_z} \quad \psi_2 = \psi_2 \qquad (10)$$

In Eq. (10), we have also defined the yaw angle of the follower helicopter as our fourth control parameter. This results in a balanced input-output relation, where the dimension of the control input vector is equal to the number of control output parameters. As a result, the $l - \alpha$ formation scheme output is assembled in the following form:

$$\mathbf{y} = \mathbf{g}(\mathbf{x}) = \left[\begin{array}{cccc} l_{12} & \alpha_{12} & z_{12} & \psi_2 \end{array} \right]^T \qquad (11)$$

B. The $l - l$ Scheme

Fig. 3 shows the configuration of the $l - l$ scheme. As for the $l - \alpha$ case, here, we constrain the control point of one helicopter with respect to the control points of two other neighbors. In order to keep the ability of each agents to link with two different schemes simultaneously, the definition of the control point has been kept consistent for both cases. The following relation is held for point p_3, which is the control point of the follower:

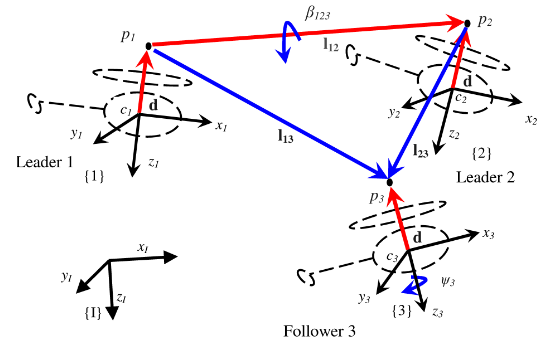

Fig. 3. Formation Parameters of Three Neighboring Helicopters

$$\mathbf{p}_{c_1} + \mathbf{l}_{13} + \mathbf{d}_1 = \mathbf{p}_{c_3} + \mathbf{d}_3 \qquad (12)$$

Here, the vector $\mathbf{l}_{13}$ is the relative distance between the control points of the follower and the leader 3. The definition of the other vectors are the same as the definition for the $l-\alpha$ scheme. The first two formation parameters are defined as the length of the vectors $\mathbf{l}_{13}$ and $\mathbf{l}_{23}$ with the following formulations:

$$l_{13} = |\mathbf{l}_{13}| = |\mathbf{p}_{c_3} + \mathbf{R}_{I3}\mathbf{d} - \mathbf{p}_{c_1} - \mathbf{R}_{I1}\mathbf{d}| \qquad (13)$$

$$l_{23} = |\mathbf{l}_{23}| = |\mathbf{p}_{c_3} + \mathbf{R}_{I3}\mathbf{d} - \mathbf{p}_{c_2} - \mathbf{R}_{I2}\mathbf{d}| \qquad (14)$$

The third parameter, named β_{123} is defined as the angle between the plane of $p_1 p_2 p_3$ and the plane that is formed by the vectors $\mathbf{l}_{12}$ and the axis $\mathbf{z}_I$. Defining the normal vectors of theses two planes by $\mathbf{n}_1$ and $\mathbf{n}_2$, respectively, we can write:

$$\mathbf{n}_1 = \mathbf{l}_{13} \times \mathbf{l}_{12} \quad \mathbf{n}_2 = \mathbf{z}_I \times \mathbf{l}_{12} \qquad (15)$$

$$\beta_{123} = \arccos \frac{\mathbf{n_1}.\mathbf{n_2}}{|\mathbf{n_1}||\mathbf{n_2}|} \qquad (16)$$

Defining ψ_3 as the follower's yaw angle, the following vector represents the parameters of the $l - l$ control scheme:

$$\mathbf{y} = \mathbf{g}(\mathbf{x}) = \left[\begin{array}{cccc} l_{13} & l_{23} & \beta_{123} & \psi_3 \end{array} \right]^T \qquad (17)$$

Note that the defined formation parameters are, in general, a function of time. This feature must be exploited to plan a change in formation such that no collision happens.

IV. Design of the Formation Controller

In this research, the continuation/GMRES method ([20] and [21]) is used as the design platform for formation control of the helicopter groups. The method combines both the continuation and GMRES approaches to come up with an NMPC framework with an acceptable computational load. The former is used to express a desired trace for the control input trajectory and designed to guarantee the convergence of the control force to a value that optimizes a well-defined cost function. The latter is used to form a fast method for integrating the trace path of the input. The main idea is to approximate the time derivative of the input vector through substituting its equivalent from the complete difference expansion of the input. The complete difference is approximated using the desired trend defined by the computational method. Through this approach, the input trace equation is converted into a system of linear algebraic relations, which are solved for the time trend of the input vector usung the GMRES algorithm. The GMRES method itself is a fast numeric solver for linear systems of algebraic equations based on minimizing the norm of the residual.

As a first advantage, by significantly reducing the computational cost, this method makes the NMPC framework a feasible solution to be used for fast response systems. Second, by eliminating the iteration in the optimization process, the method equalizes the required computational time for all the time steps through the horizon.

In the following sections, first we briefly introduce the NMPC formation framework for the helicopter groups. Then, we develop different subroutins of the GMRES/Continuation method for implementation in the helicopter groups.

A. Nonlinear Model Predictive Control Framework

First, we discretize the nonlinear helicopter dynamics (Eq. 5) and its output model (Eqs. 11 or 17) in the following form:

$$\mathbf{x}_{i+1} = \mathbf{f}(\mathbf{x}_i, \mathbf{u}_i) \quad \mathbf{y}_i = \mathbf{g}(\mathbf{x}_i) \tag{18}$$

where $\mathbf{f}(\mathbf{x}_i, \mathbf{u}_i) = \mathbf{x}_i + \mathbf{f}_c(\mathbf{x}_i, \mathbf{u}_i)\Delta t$. Here, $\mathbf{f}_c(\mathbf{x}_i, \mathbf{u}_i)$ is the right hand side of Eq. (5), and Δt is the horizon time step.

In the NMPC approach, we are interested to find a set of control inputs for a time horizon that optimizes the value of a cost function. For formation control of the helicopters, the cost function penalizes the deviation, $\mathbf{e}$, of the formation parameters from their desired values, in addition to the excessive values for the control forces $\mathbf{u}$. The future stepwise control input $\mathbf{u}_i$ ($i = 0, ..., N_t - 1$) must be found such that the cost function is minimized. The relation between the states and the control inputs (the discretized dynamics of the helicopter) must affect the solution to the optimization problem. This relation is incorporated by defining costates $\boldsymbol{\lambda}_i$'s, and defining the cost function as:

$$\begin{aligned} J =& \frac{1}{2}\mathbf{e}_{N_t}^T \mathbf{P} \mathbf{e}_{N_t} + \sum_{i=0}^{N_t-1} [\frac{1}{2}(\mathbf{e}_i^T \mathbf{Q} \mathbf{e}_i + \mathbf{u}_i^T \mathbf{R} \mathbf{u}_i] \\ &+ \boldsymbol{\lambda}_{i+1}^T(-\mathbf{x}_{i+1} + \mathbf{f}(\mathbf{x}_i, \mathbf{u}_i)) \end{aligned} \tag{19}$$

where $\mathbf{e} = \mathbf{y} - \mathbf{y}_d$. In Eq. (19), N_t is the horizon length and $\mathbf{P}$, $\mathbf{Q}$ and $\mathbf{R}$ are the weight matrices.

It should be noted that, in this article, we assumed no constraint in the cost function. However, it is easy to incorporate the saturation of the control inputs as constraints. Also, one could incorporate obstacle avoidance by adding a potential function, penalizing the helicopter's distance to obstacles, to the cost function (19).

To simplify the notation, we define the term H as follows:

$$\begin{aligned} H(\mathbf{x}_i, \mathbf{e}_i, \mathbf{u}_i) =& \frac{1}{2}\left(\mathbf{e}_i^T \mathbf{Q} \mathbf{e}_i + \mathbf{u}_i^T \mathbf{R} \mathbf{u}_i\right) \\ &+ \boldsymbol{\lambda}_{i+1}^T \mathbf{f}(\mathbf{x}_i, \mathbf{u}_i) \end{aligned} \tag{20}$$

This results in the following simplified form for J:

$$J = \frac{1}{2}\mathbf{e}_{N_t}^T \mathbf{P} \mathbf{e}_{N_t} - \boldsymbol{\lambda}_{N_t}^T \mathbf{x}_{N_t} + \sum_{i=0}^{N_t-1} [H(\mathbf{x}_i, \mathbf{e}_i, \mathbf{u}_i) - \boldsymbol{\lambda}_i^T \mathbf{x}_i] \tag{21}$$

Since $\boldsymbol{\lambda}_i$'s are being multiplied by zero terms in Eq. (19) [see Eq. (18)], they can be arbitrarily selected to simplify the calculations. They are defined as follows:

$$\boldsymbol{\lambda}_{N_t}^T = \mathbf{e}_{N_t}^T \mathbf{P} \frac{\partial \mathbf{y}_{N_t}}{\partial \mathbf{x}_{N_t}} \tag{22}$$

$$\boldsymbol{\lambda}_i^T = \mathbf{e}_i^T \mathbf{Q} \frac{\partial \mathbf{y}_i}{\partial \mathbf{x}_i} + \frac{\partial \mathbf{f}(\mathbf{x}_i, \mathbf{u}_i)}{\partial \mathbf{x}_i} \quad i = N_t - 1, ..., 0 \tag{23}$$

Taking a complete difference of the cost function and plugging in the above terms for the costates result in the following simplified form for the term dJ:

$$dJ = \frac{\partial H_0}{\partial \mathbf{y}_0} d\mathbf{y}_0 + \sum_{i=0}^{N_t-1} \frac{\partial H_i}{\partial \mathbf{u}_i} d\mathbf{u}_i \tag{24}$$

where

$$\frac{\partial H_i}{\partial \mathbf{u}_i} = \mathbf{u}_i^T \mathbf{R} + \boldsymbol{\lambda}_{i+1}^T \frac{\partial \mathbf{f}(\mathbf{x}_i, \mathbf{u}_i)}{\partial \mathbf{u}_i} \tag{25}$$

B. The Continuation/GMRES Method

In this section, we introduce the continuation method that is utilized to solve the optimization problem in a non-iterative manner. Continuation method is the mean that enables one to exactly trace the optimum solution throughout the control time. The key point is to constrain the input dynamics and express the dependency of the optimum solution with time. As the first step, we define the following vectors by assembling the control and state vectors through the whole horizon in the $\mathbf{U}$ and $\mathbf{X}$ vectors, respectively:

$$\mathbf{U} = [\mathbf{u}_0^T, \ldots, \mathbf{u}_{N_t-1}^T]^T, \mathbf{X} = [\mathbf{x}_0^T, \ldots, \mathbf{x}_{N_t-1}^T]^T \tag{26}$$

Note that we discretized the time from now to N_t time steps ahead with the time step of Δt. Hence, each of the indices 0 to $N_t - 1$ represent the corresponding snap shot in the whole interval of $[t \ \ldots \ t + (N_t - 1)\Delta t]$. From the previous section, the optimum answer of the cost function presented in Eq. (21) is a set of control inputs that results in vanishing the following vector function (see Eq. 24):

$$\mathbf{F}(\mathbf{U}(t), \mathbf{X}(t)) = \begin{bmatrix} \frac{\partial H}{\partial \mathbf{u}}(\mathbf{x}_0, \mathbf{u}_0, \boldsymbol{\lambda}_1) \\ \frac{\partial H}{\partial \mathbf{u}}(\mathbf{x}_1, \mathbf{u}_1, \boldsymbol{\lambda}_2) \\ \vdots \\ \frac{\partial H}{\partial \mathbf{u}}(\mathbf{x}_{N_t-1}, \mathbf{u}_{N_t-1}, \boldsymbol{\lambda}_{N_t}) \end{bmatrix} = \mathbf{0} \tag{27}$$

In Eq. (27), the general form of the function $\mathbf{F}$ for a receding horizon control problem is defined. Here, $\partial H/\partial \mathbf{u}$ can be

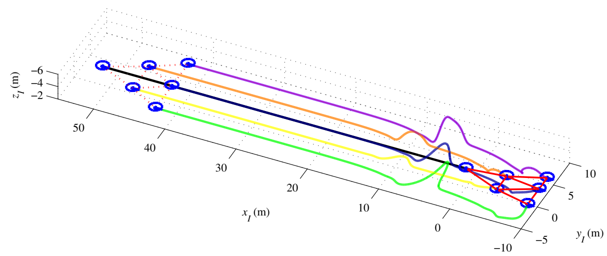

Fig. 4. Triangular Formation Maneuver of 6 Helicopters

determined using Eq. (25). The following dynamic behavior is suggested for the above equation:

$$\dot{\mathbf{F}}(\mathbf{U}(t), \mathbf{X}(t)) = -\zeta \mathbf{F}(\mathbf{U}(t), \mathbf{X}(t)) \tag{28}$$

where $\mathbf{U}(0)$ is chosen to satisfy $\mathbf{F}(\mathbf{U}(0), \mathbf{X}(0)) = \mathbf{0}_{n\times 1}$ and $\zeta > 0$. The above equation possesses an stable response and, with the assumed initial condition, it guarantees that the vector $\mathbf{F}$ remains zero through time. Expanding the term $\dot{\mathbf{F}}$ in Eq. (28) results in:

$$\left(\frac{\partial \mathbf{F}}{\partial \mathbf{U}}\right) \dot{\mathbf{U}} = \left(-\zeta \mathbf{F} - \frac{\partial \mathbf{F}}{\partial \mathbf{X}} \dot{\mathbf{X}} - \frac{\partial \mathbf{F}}{\partial t}\right) \tag{29}$$

The GMRES method is used to find the value of $\dot{\mathbf{U}}$ that minimizes the error between the two sides of Eq. (29). To speed up the calculation of $\partial \mathbf{F}/\partial \mathbf{U}$, $\partial \mathbf{F}/\partial \mathbf{X}$, and $\partial \mathbf{F}/\partial t$ a forward difference approximation is used [21]. Once $\dot{\mathbf{U}}$ is determined, the new control command is calculated using $\mathbf{U}(t + \Delta t) = \mathbf{U}(t) + \dot{\mathbf{U}}(t)\Delta t$. The first element of $\mathbf{U}$ is applied to the system. Then, $\mathbf{U}$ is shifted ($\mathbf{u}_k = \mathbf{u}_{k+1}, k = 0, ..., N_t - 2$, $\mathbf{u}_{N_t-1} = \mathbf{u}_{N_t-1}$) and used as an initial guess for the next step in time.

V. Simulation Results

In this section, the simulation result of the application of the method for formation control of helicopters is presented. As an illustrative example, these schemes are used to bring a group of 6 helicopters into a triangular formation in the presence of the parameter uncertainties and disturbances. The helicopters start from rest with an initial error in formation. We used the specifications of the Ikarus ECO electric model helicopter provided in [22].

The result is demonstrated in Fig. 4. Except the leader that is flying at the front, all the group members are exposed to both 20% parameter uncertainty and a wind gust. The wind gust exists between the 20^{th} and 30^{th} seconds of the 100 second flight scenario. It should be noted that the change of the flying path for the whole group of followers results in a least deviation from the desired triangular pattern of the 6 helicopters.

VI. Real-time Feasibility of the Control Schemes

For real-time applications, the computational cost of the method must be within the available processing power used for similar flight control applications. In this section, we are going to study our purposed algorithm in terms of its required FLoating point OPerations (FLOPs) and compare it with the processing power of commercial processing units available for autonomous flight controls.

Overall, we can write the following set of equations for the required FLOPs for the $l - \alpha$ and $l - l$ controllers, respectively:

$$OP = 6063N_t + 4006k_{max}N_t + 21k_{max} + 32 + \sum_{k=1}^{k_{max}} [6k^2 + 60N_t k + 7k] \quad \text{for } l - \alpha \tag{30}$$

$$OP = 6879N_t + 4544k_{max}N_t + 21k_{max} + 32 + \sum_{k=1}^{k_{max}} [6k^2 + 60N_t k + 7k] \quad \text{for } l - l \tag{31}$$

In these equations, OP is the number of required FLOPs for the routine that calculates the control commands at each sampling time.

The first two terms in Eq. (30) and (31) are resulted from assembling and disassembling the input control horizon from series of vectors to one vector and vice versa. The summation term is resulted from the calculation inside the GMRES routine. The other terms are resulted from initialization section of the algorithm and overhead terms of the routines. Note that the difference of the $l - \alpha$ and $l - l$ computation cost is caused by the difference in the computation cost of $\mathbf{y}_i$ and $\frac{\partial \mathbf{y}_i}{\partial \mathbf{x}_i}$ terms for each schemes.

As can be seen, the FLOPs formulas are nonlinear relations in terms of k_{max} and N_t. If we examine these relations, we can see that the computation cost is more sensitive to the horizon length than to the iteration number. This is an expected result, because all the computations must be repeated for all of the horizon steps. Also, a large k_{max} has a substantial effect on the calculation effort via the summation term in Eq. (30) and (31). However, the GMRES method converges to an acceptable result using a small number for k_{max}.

This term appears in the second term of the Eq. (30) and (31) along with N_t, which dominates the computation cost of both the controllers. Examining the two equations (30) and (31), we can see that both the first and second terms of the $l - l$ possess 13% higher computation cost compared to that of the $l-\alpha$ controller. Using the mentioned parameters, the resulting OP of both the $l - \alpha$ and $l - l$ controller for different values of horizon length and iteration number are calculated and recorded in Table I. For a feasible implementation of the controllers in real-time systems, the following relation must be true, $OP \leq CP.\Delta t$, where CP indicates the computing performance of the processor.

To compare the computation cost of both the $l - \alpha$ and $l-l$ controllers, with the capabilities of the available computational power, we consider a 1.8 GHz Intel Pentium M 755 processor that can be found on PC104 embedded computers that are available in our laboratory. The assumed processor has 1 Giga Floating Operations Per Second (GFLOPS) calculation capacity.

TABLE I

CALCULATED FLOPS NUMBER OF THE $l-\alpha$ AND $l-l$ CONTROLLERS FOR DIFFERENT HORIZON LENGTH AND ITERATION NUMBER

k_{max}	N_t	$l-\alpha$	$l-l$
2	5	20629	23319
	20	82339	93099
5	5	21676	24366
	20	86086	96846

The iteration number, the horizon number of steps, and the sampling time for the simulation presented in this paper are $k_{\max} = 2$, $N_t = 20$, and $\Delta t = 0.01$ second. To calculate the control input for the l – l scheme for one real-time second 9309900 FLOPs are required (Table I). This is 107.4 times less than the processing power of the assumed processor that does 1 GFLOPS. This ensures real-time feasibility of the purposed approach.

The same simulation has been previously done in [23], [24] using the gradient descent method approach, which solves Eq. (27) for $\mathbf{U}$ using the iterative Newton's method. The iteration number required for convergence varied throughout the simulation with the maximum number being 29 for the l – l scheme where the wind disturbance was at its maximum. This means that the gradient decent method has been 14.5 times slower than the C/GMRES method for the presented simulation.

VII. CONCLUSION

Geometrical formation control schemes, tailored specifically for autonomous helicopters, were introduced in this paper. The geometrical schemes provide building blocks with which any three-dimensional formation mesh can be defined for arbitrary number of helicopters. Nonlinear Model Predictive Method (NMPC) has been used to design formation controllers to maintain the formation. Although not shown in the paper, obstacle avoidance can be achieved by adding potential functions that penalize the distance of the helicopter to obstacles to the NMPC optimization cost function, and the input saturation can be addressed by adding Lagrange multipliers and the constraints to the NMPC Hamiltonian function. In this paper, the efficient Continuation/GMRES (Generalized Minimum REsidual) method has been used to lower the computation load to an applicable level. The applicability of this method for formation control has been shown by number of FLOPs (FLoating point OPerations) analysis required for the real-time calculations.

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