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Observer Based Tracking Control for Switched Linear Systems with Time-Delay

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***Abstract*— Observer based tracking control for switched linear systems with time-delay is investigated in this paper. Sufficient condition for the solvability of the tracking control problem is given for the case that the state of system is not available. The design of a switching control law based on measured output instead of the state information is considered. Lyapunov-Krasovskii functional method is utilized to the stability analysis and controller design for the switched linear time-delay systems. The Variation-of-constants formula and linear operator theory are used to conquer the difficulties caused by the estimation error and exotic disturbance. By using linear matrix inequalities techniques, the controller design problem can be solved efficiently. The numerical example shows the effectiveness of the switching control laws.**

I. Introduction

As an important class of systems, time-delay systems are ubiquitous in chemical process, aerodynamics, and communication network (see, e.g.,[3], [5], [8]). It is well known that time-delay is great source of instability and poor performance. Therefore, how to deal with time-delay has been a hot topic in the control area [4], [6], [10].

On the other hand, switching control provides a new technique to the stability analysis and control synthesis for complex control systems (e.g., nonlinear systems, uncertain or parameter varying systems). Due to their significance both in theory development and practical applications, switched systems have been attracting considerable attention during the last decade [7], [9], [12], [16]. Two key problems in the study of switched systems are the stability analysis and control synthesis, it has been shown that Lyapunov functions as useful tools, can deal with the stability problems for switched systems, although certain switching laws incorporated with compatible information sometimes should be designed [1], [7]. Since switched systems with time-delay have strong engineering background, special attention has been attracted, and several useful results have been reported in the literature such as the issues on stability analysis [12], [15], optimal control [14], and so on. The importance of the study of tracking control for switched systems with time-delay arises from the extensive applications in robot tracking control [17], guided missile tracking control, etc. However, to the authors' best knowledge, up to now, the issue of tracking control, which has been well addressed for non-switched systems without delay [11], has been rarely investigated for switched systems with time-delay.

In this paper, we investigate the problem of observer based tracking control for switched linear systems with time-delay. Sufficient condition for the solvability of the tracking control problem is given for the case that the state of a system is unmeasurable. The design of a switching control law based on measured output instead of the state information is considered. We use multiple Lyapunov function technique to design tracking controllers and a switching law such that the observer based H_∞ model reference tracking performance is satisfied. The methods in [13] are extended to the design of the observer based switched tracking control laws, and some results of functional differential equations (see in [5]) such as the Variation-of-constants formula and linear operator theories are utilized to conquer the difficulties caused by the estimation error and exotic disturbance. The feasibility of the problem can be realized by convex optimization techniques and linear matrix inequalities. Finally, the simulation example show the validity of the proposed design method.

II. Problem formulation and preliminaries

In this paper, we use $P > 0$ $(\geq, <, \leq 0)$ to denote a positive definite (semi-definite, negative definite, semi-negative definite) matrix P. The superscript "T" stands for matrix transpose; and the symmetric terms in a matrix are denoted by $*$, $\mathbb{R}^n$ denotes the n dimensional Euclidean space; $L_2[0,\infty)$ is the space of square integrable functions on $[0,\infty)$, $\mathscr{L}_1^{\text{loc}}([\varrho,\infty),\mathbb{R}^n)$ is the space of locally Lebesgue integrable vector valued functions on $[\varrho,\infty)$. For given $\tau > 0$, let $\mathbb{R}_+ = [0,+\infty]$ and $C_n = C([-\tau,0],\mathbb{R}^n)$ be the Banach Space of continuous mapping from $([-\tau,0],\mathbb{R}^n)$ to $\mathbb{R}^n$ with topology of uniform convergence. Let $x_t \in C_n$ be defined by $x_t(\theta) = x(t+\theta), \theta \in [-\tau,0]$. $\|\cdot\|$ denotes the usual 2-norm and $\|x_t\|_{cl} = \sup_{-\tau\leq t\leq 0}\|x(t+\theta)\|$.

Consider the switched linear system with time-delay

$$\begin{cases} \dot{x}(t) = A_\sigma x(t) + D_\sigma x(t-\tau) + B_\sigma u(t) + \omega(t), \\ \phi_\sigma(\theta) = x(t+\theta), \quad \theta \in [t_j-\tau, t_j],\ x(0)=0, \\ y(t) = C_\sigma x(t), \qquad t \in [0,\infty),\ j = 0,1,\cdots. \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^p$ is the control input, $\omega(t) \in \mathbb{R}^n$ is bounded exogenous disturbance which belong to $L_2[0,\infty)$ and $\mathscr{L}_1^{\text{loc}}([\varrho,\infty),\mathbb{R}^n)$, respectively; $y(t) \in \mathbb{R}^q$ is the output, $\phi_\sigma(t)$ is the continuous vector valued function specifying the initial state of each

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subsystem, $\tau > 0$ is the constant, the right continuous function $\sigma(t) : [0, \infty) \to \underline{N} \triangleq \{1, 2, \cdots, N\}$ is the switching signal which can be characterized by the switching sequence $\Sigma = \{x_0; (i_0, t_0), (i_1, t_1), \cdots, (i_j, t_j), \cdots | i_j \in \underline{N}, j = 0, 1, \cdots\}$. Moreover, $\sigma(t) = i$ implies that the i-th subsystem (A_i, D_i, B_i, C_i) is active, where A_i, D_i, B_i and C_i are constant matrices of appropriate dimensions, $i \in \underline{N}$. For simplicity, we denote $\sigma := \sigma(t)$.

Definition 1. The system (1) is said to be exponentially stabilizable under control law $u = u(t)$ and switching signal $\sigma = \sigma(t)$, if the solution $x(t)$ of switched system (1) through $(t_0, \phi) \in \mathbb{R}_+ \times C_n$ satisfies

$$\|x(t)\| \leq \kappa \|x_{t_0}\|_{cl} e^{-\lambda(t-t_0)}, \ \forall t \geq t_0$$

for some constants $\kappa \geq 0$ and $\lambda > 0$.

Suppose that the state observer is with the form

$$\begin{aligned}\dot{\hat{x}}(t) =& A_\sigma \hat{x}(t) + D_\sigma \hat{x}(t-\tau) + B_\sigma u(t) \\ &+ L_\sigma(y(t) - \hat{y}(t)) \qquad (2a)\\ \hat{y}(t) =& C_\sigma \hat{x}(t) \qquad (2b)\end{aligned}$$

in which $y(t)$ and $\sigma(t)$ are the measurable output and switching signal of system (1), respectively. The matrices $L_1, L_2, \cdots, L_N \in \mathbb{R}^{n \times q}$ are to be determined later.

Given a reference model and performance index as

$$\dot{x}_r(t) = A_r x_r(t) + r(t), \quad x_r(0) = 0, \qquad (3)$$

$$\int_0^\infty e_r^T(t) e_r(t) dt < \gamma^2 \int_0^\infty \varpi^T(t) \varpi(t) dt, \qquad (4)$$

where $x_r(t) \in \mathbb{R}^n$ is reference state, A_r is a Hurwitz matrix, $r(t)$ is reference input which belong to $L_2[0, \infty)$ and $\mathscr{L}_1^{\rm loc}([\varrho, \infty), \mathbb{R}^n)$, respectively; $e_r(t) = x(t) - x_r(t)$ denotes the error between the state of system (1) and the reference state; $\varpi(t) = (\omega^T(t), r^T(t))^T$, $\gamma > 0$ is disturbance attenuation level.

Define the difference between the real state and the observer state, the observer state and the reference state as

$$\begin{aligned}e(t) &= x(t) - \hat{x}(t),\\ \hat{e}_r(t) &= \hat{x}(t) - x_r(t).\end{aligned}$$

Design the error feedback control law

$$u(t) = K_{\sigma(t)} \hat{e}_r(t). \qquad (5)$$

Subtracting (2a) from (1) gives the error switched system

$$\dot{e}(t) = (A_\sigma - L_\sigma C_\sigma) e(t) + D_\sigma e(t-\tau) + \omega(t). \qquad (6)$$

Now, combining (2a), (3) with (5) and (6), we have the augmented switching linear time-delay system as follows:

$$\dot{e}(t) = (A_\sigma - L_\sigma C_\sigma) e(t) + D_\sigma e(t-\tau) + \omega(t), \qquad (7a)$$

$$\begin{cases}\dot{\hat{x}}(t) = A_\sigma \hat{x}(t) + D_\sigma \hat{x}(t-\tau) + B_\sigma K_\sigma \hat{e}_r(t) \\ \qquad\quad + L_\sigma C_\sigma e(t), \\ \dot{x}_r(t) = A_r x_r(t) + r(t). \end{cases} \qquad (7b)$$

Let

$$\bar{x}(t) = \begin{bmatrix} \hat{x}(t) \\ x_r(t) \end{bmatrix}, \ \bar{D} = \begin{bmatrix} D_\sigma & 0 \\ 0 & 0 \end{bmatrix}, \ f_\sigma(t) = \begin{bmatrix} L_\sigma C_\sigma e(t) \\ r(t) \end{bmatrix},$$

$$\bar{A}_\sigma = \begin{bmatrix} A_\sigma + B_\sigma K_\sigma & -B_\sigma K_\sigma \\ 0 & A_r \end{bmatrix}, \bar{Q} = \begin{bmatrix} I & -I \\ -I & I \end{bmatrix},$$

Then, (7b) can be rewritten as

$$\dot{\bar{x}}(t) = \bar{A}_\sigma \bar{x}(t) + \bar{D}_\sigma \bar{x}(t-\tau) + f_\sigma(t), \qquad (7b')$$

and the corresponding nominal system of (7b') is

$$\dot{\bar{x}}(t) = \bar{A}_\sigma \bar{x}(t) + \bar{D}_\sigma \bar{x}(t-\tau). \qquad (8)$$

Remark 1. When $f_\sigma(t) \in \mathscr{L}_1^{\rm loc}([\varrho, \infty), \mathbb{R}^n)$, by the stepping method in finite interval $[t_j, t_{j+1})$ for each subsystem of (7) and well-defined switching law, the existence and uniqueness of the solution with initial condition for switched linear time-delay system (7) can be obtained.

Definition 2. For system (1), if there exist control input $u = u(t)$ and switching signal $\sigma = \sigma(t)$ such that (7) is asymptotically stable when $\varpi \equiv 0$ and (4) is satisfied when $\varpi \neq 0$ under the initial conditions stated in (1) and (3), then the switched system (1) is said to have observer based H_∞ model reference tracking performance.

To conclude this section, we recall the following lemma.

Lemma 1 [2]. Let M, N be real matrices of appropriate dimensions. Then, for any matrix $Q > 0$ of appropriate dimension and any scalar $\gamma > 0$, it holds that

$$MN + N^T M^T \leq \gamma^{-1} M Q^{-1} M^T + \gamma N^T Q N.$$

III. Performance analysis and controller design

In this section, we will give our main result.

Assumption 1. There exist positive definite matrices X, G and matrices Y_i, such that

$$\begin{aligned}\Theta_i :=& A_i^T X + X A_i - C_i^T Y_i^T - Y_i C_i + G \\ &+ X D_i G^{-1} D_i^T X < 0. \qquad (9)\end{aligned}$$

Remark 2. The above assumption asserts the existence of a common Lyapunov-Krasovskii functional candidate $V(e(t))$ for the switched linear time-delay system

$$\dot{e}(t) = (A_\sigma - L_\sigma C_\sigma) e(t) + D_\sigma e(t-\tau). \qquad (10)$$

In fact, let $L_i = X^{-1} Y_i$, and choose

$$V(e(t)) = e^T(t) X e(t) + \int_{t-\tau}^t e^T(s) G e(s) ds$$

as a Lyapunov-Krasovskii functional candidate. It is easy to show that there exist scalars $\alpha_1 > 0, \alpha_2 > 0$, such that $\alpha_1 \|e(t)\|^2 \leq V(e(t)) \leq \alpha_2 \|e_t\|_{cl}^2$. Moreover,

$$\|e(t)\| \leq \sqrt{\frac{\alpha_2}{\alpha_1}} e^{-\frac{\lambda}{2\alpha_1}(t-t_0)} \|e_{t_0}\|_{cl}$$

holds with λ being the smallest eigenvalue of the matrices Θ_i. This implies that the system (10) is exponentially stable under arbitrary switching.

Before developing conditions for the solvability of observer based tracking control for switched time-delay systems, a preliminary result is presented. The following state constitutes a generalization of Hale's results (see in [5]).

Consider the linear time-delay system (7b') without switching and its homogeneous system

$$\begin{aligned}\dot{x}(t) =& Ax(t) + Dx(t-\tau) + f(t); \qquad (11)\\ \dot{x}(t) =& Ax(t) + Dx(t-\tau). \qquad (12)\end{aligned}$$

We rewrite them with operator form

$$\dot{x}_t = L(t, x_t) + f(t), \quad t \geq \varrho \tag{13}$$

$$\dot{x}_t = L(t, x_t),\ x_\varrho = \phi.\ t \geq \varrho \tag{14}$$

where the operator $L(t, \phi)$ is linear in ϕ, and has the form $L(t, \phi) = A\phi(0) + D\phi(-\tau)$, in which $\phi(\theta) = x(t + \theta), \theta \in (-\tau, 0)$. Suppose there is an $m \in \mathscr{L}_1^{\text{loc}}([\varrho, \infty), \mathbb{R})$ such that

$$|L(t, \phi)| \leq m(t)|\phi| \tag{15}$$

for all $t \in (-\infty, \infty), \phi \in C_n$.

Lemma 2 (*Variation-of-constants* [5]). If L satisfies the hypotheses of condition (15), $x(\varrho, \phi, f)$ denotes the solution of system (11) or (13), and $x(\varrho, \phi, 0)$ is the solution of the corresponding homogeneous system (12) or (14), then

$$x(\varrho, \phi, f)(t) = x(\varrho, \phi, 0)(t) + \int_\varrho^t U(t, s)f(s)ds, \\ x_\varrho = \phi, \quad t \geq \varrho, \tag{16}$$

where $U(t, s)$ is the solution of the equation

$$U(t, s) = \begin{cases} \int_s^t L(u, U_u(\cdot, s))du + I\ a.e.\ \text{in}\ s, t \geq s \\ 0. \qquad s - r \leq t < s \end{cases}$$

in which $U_t(\cdot, s)(\theta) = U(t + \theta, s), -\tau \leq \theta \leq 0$.

For the convenience of using the variation-of-constants formula, some notations are introduced to rewrite the formula.

Denote $x(\varrho, \phi, 0)(t+\theta)$ as $x_t(\varrho, \phi, 0)$, and if $x(\varrho, \phi, 0) \triangleq T(t, \varrho)\phi$, then the $T(t, \varrho)$ is a continuous linear operator. Therefore

$$U_t(\cdot, s) = T(t, s)X_0, \quad X_0(\theta) = \begin{cases} 0, -\tau \leq \theta < 0, \\ I, \theta = 0. \end{cases}$$

With the above notations, the variation-of-constants formula becomes

$$x_t(t, \varrho, \phi, f) = T(t, \varrho)\phi + \int_\varrho^t T(t, s)X_0 f(s)ds,\ t \geq \varrho.$$

Theorem 1. For system (7), suppose that Assumption 1 holds. If there exist positive definite matrices P_i, S, H_i, matrices K_i, and scalars $\alpha_{ij} > 0$, $\eta > 0$, $(i, j \in \underline{N})$, such that the following matrices inequalities

$$\begin{bmatrix} \Xi_i & P_i\bar{D}_i & P_i \\ * & -e^{-\eta\tau}S & 0 \\ * & * & -\rho^2 I \end{bmatrix} < 0 \tag{17}$$

hold, where

$$\Xi_i := \bar{A}_i^T P_i + P_i\bar{A}_i + \eta P_i + S + \bar{Q} + \sum_{j \neq i, j \in \underline{N}} \alpha_{ij}(P_j - P_i),$$

then the feedback controller $u(t) = K_\sigma \hat{e}_r(t)$ for the system (7), such that the observer based H_∞ model reference tracking performance in (1) is guaranteed, the corresponding switching law is given as

$$\sigma(t) = arg \min_{i \in \underline{N}} \{\bar{x}^T(t)P_i\bar{x}(t)\}. \tag{18}$$

Proof. By Schur complement lemma, the condition (17) is equivalent to the following inequalities

$$\begin{bmatrix} \Xi_i + \rho^{-2}P_iP_i & P_i\bar{D}_i \\ * & -e^{-\eta\tau}S \end{bmatrix} < 0. \tag{19}$$

Design Lyapunov-Krasovskii functional candidate as

$$V(\bar{x}(t)) = \bar{x}^T(t)P_{\sigma(t)}\bar{x}(t) + \int_{t-\tau}^t \bar{x}^T(s)e^{-\eta(t-s)}S\bar{x}(s)ds. \tag{20}$$

Obviously, the Lyapunov-Krasovskii functional candidate is positive definite.

First, we prove asymptotic stability of system (7b') with $\varpi(t) \equiv 0$ (noticing that $f_\sigma(t) \neq 0$ in (7b')).

Consider the switched linear time-delay system (7b'). For any $t > 0$, the jth switching instant is denoted by t_j ($j \geq 0$). During any time interval $[t_j, t_{j+1})$, suppose that the ith subsystem is active. Let $\xi(t) = \left[\bar{x}^T(t)\ \bar{x}^T(t - \tau)\right]^T$. The time derivative of $V(\bar{x}(t))$ along the trajectory of (8) is

$$\dot{V}(\bar{x}(t)) + \eta V(\bar{x}(t)) \\ = \xi^T(t) \begin{bmatrix} \bar{A}_i^T P_i + P_i\bar{A}_i + \eta P_i + S & P_i\bar{D}_i \\ * & -e^{-\eta\tau}S \end{bmatrix} \xi(t). \tag{21}$$

According to the condition (19), we have

$$\begin{bmatrix} \bar{A}_i^T P_i + P_i\bar{A}_i + \eta P_i + S & P_i\bar{D}_i \\ * & -e^{-\eta\tau}S \end{bmatrix} < \begin{bmatrix} \Pi_i & 0 \\ 0 & 0 \end{bmatrix}, \tag{22}$$

where

$$\Pi_i := -\rho^{-2}P_iP_i - \bar{Q} - \sum_{j \neq i, j \in \underline{N}} \alpha_{i,j}(P_j - P_i).$$

By virtue of the designed switching law (18), it follows

$$\bar{x}^T(t)(\sum_{j \neq i, j \in \underline{N}} \alpha_{ij}(P_j - P_i))\bar{x}(t) \geq 0, \forall t \in R^{2n}.$$

Also we note that $\bar{Q} \geq 0$. During $[t_j, t_{j+1})$, when $\xi(t) \neq 0$, we easily get

$$\dot{V}(\bar{x}(t)) + \eta V(\bar{x}(t)) < \xi^T(t) \begin{bmatrix} \Pi_i & 0 \\ 0 & 0 \end{bmatrix} \xi(t) \leq 0.$$

Thus, there holds

$$\dot{V}(\bar{x}(t)) \leq -\eta V(\bar{x}(t)), \tag{23}$$

During any $[t_j, t_{j+1})$, (23) gives rise that

$$V(\bar{x}(t)) \leq e^{-\eta(t-t_j)}V(\bar{x}(t_j)). \tag{24}$$

In addition, by the switching law (18), at the switching instant t_j, we have

$$\bar{x}^T(t_j)P_{\sigma(t_j)}\bar{x}(t_j) \leq \lim_{t \to t_j^-} \bar{x}^T(t)P_{\sigma(t)}\bar{x}(t),$$

which implies $V(\bar{x}^T(t_j)) \leq \lim_{t \to t_j^-} V(\bar{x}^T(t))$, by induction on $t_0, t_1, \cdots, t_j$, from (24) we get

$$V(\bar{x}(t)) \leq e^{-\eta(t-t_0)}V(\bar{x}(t_0)).$$

Then we have

$$\|\bar{x}(t)\|^2 \leq \frac{\lambda_M(P_i) + \tau\lambda_M(S)}{\lambda_m(P_i)} e^{-\eta(t-t_0)}\|\bar{x}(t_0)\|_{cl}^2,$$

where $\lambda_m(\cdot)(\lambda_M(\cdot))$ denotes the minimum (maximum) eigenvalue of a symmetric matrix, which implies exponential stability of the nominal systems of (7b'), i.e., the system (8).

By the variation-of-constants formula, since $f_\sigma(t) \in \mathscr{L}_1^{\mathrm{loc}}([\varrho,\infty), R^{2n})$, the solution through (t_0,φ_0) of (7b') can be expressed as follows

$$\bar{x}_t(t,t_0,\varphi_0,f_\sigma) = T(t,t_0)\varphi_0 + \int_{t_0}^{t} T(t,s)X_0(\theta)f_\sigma(s)ds,\ t \geq t_0.$$

Recalling that we have obtained that the nominal system of (7b') is exponentially stable under the conditions of theorem 1, that is, there are constants $\alpha > 0,\ 0 < \kappa \leq 1$ such that, for all $\varrho \in \mathbb{R}$,

$$\|T(t,\varrho)\| \leq \kappa e^{-\alpha(t-\varrho)},\ \|T(t,\varrho)X_0\| \leq \kappa e^{-\alpha(t-\varrho)},\ t > \varrho.$$

Now, we consider the following stepping iterative process from $t_0 = 0$ to t_j :

- when $t \in [t_0, t_1)$, suppose that the i_0th subsystem is active, noticing $\varphi_{i_0} = 0$ there has

$$\begin{aligned}
&\|\bar{x}_t(t,t_0,\varphi_{i_0},f_{i_0})\| \\
&\leq \|T(t,t_0)\|\cdot\|\varphi_{i_0}\| + \int_{t_0}^{t}\|T(t,s)X_0\|\cdot\|f_{i_0}(s)\|ds \\
&\leq \int_{t_0}^{t_1}\kappa e^{-\alpha(t-s)}\|f_{i_0}(s)\|ds;
\end{aligned}$$

- when $t \in [t_1, t_2)$, suppose that the i_1th subsystem is active,

$$\begin{aligned}
&\|\bar{x}_t(t,t_1,\varphi_{i_1},f_{i_1})\| \\
&\leq \|T(t,t_1)\|\cdot\|\varphi_{i_1}\| + \int_{t_1}^{t}\|T(t,s)X_0\|\cdot\|f_{i_1}(s)\|ds \\
&\leq \kappa e^{-\alpha(t-t_1)}\cdot\int_{t_0}^{t_1}\kappa e^{-\alpha(t-s)}\|f_{i_0}(s)\|ds \\
&\quad + \int_{t_1}^{t_2}\kappa e^{-\alpha(t-s)}\|f_{i_1}(s)\|ds;
\end{aligned}$$

- $\cdots\quad\cdots\quad\cdots\quad\cdots\quad\cdots$
- when $t \in [t_j, t_{j+1})$, suppose that the ith subsystem is active,

$$\begin{aligned}
&\|\bar{x}_t(t,t_j,\varphi_i,f_i)\| \\
&\leq \|T(t,t_j)\|\cdot\|\varphi_i\| + \int_{t_j}^{t}\kappa e^{-\alpha(t-s)}f_i(s)ds \\
&\leq \kappa^j e^{-\alpha(t-t_j)-\cdots-\alpha(t-t_1)}\int_{t_0}^{t_1}\kappa e^{-\alpha(t-s)}\|f_{i_0}(s)\|ds \\
&+\kappa^{j-1}e^{-\alpha(t-t_j)-\cdots-\alpha(t-t_2)}\int_{t_1}^{t_2}\kappa e^{-\alpha(t-s)}\|f_{i_1}(s)\|ds \\
&+\cdots+\kappa e^{-\alpha(t-t_j)}\int_{t_{j-1}}^{t_j}\kappa e^{-\alpha(t-s)}\|f_{i_{j-1}}(s)\|ds \\
&+\int_{t_j}^{t}\kappa e^{-\alpha(t-s)}\|f_i(s)\|ds. \qquad (25)
\end{aligned}$$

Note that $\kappa^{j-n}\exp\left\{\sum_{m=0}^{j-1-n}-\alpha(t-t_{j-m})\right\} \leq 1,\ n = 0,\cdots,j-1$, (25) gives

$$\|\bar{x}_t(t,t_j,\varphi_i,f_i)\| \leq \int_{t_0}^{t}\kappa e^{-\alpha(t-s)}\|f_{\sigma(t)}(s)\|ds. \qquad (26)$$

When $\varpi(t) = 0$, note that $f_i(t) = \begin{bmatrix} L_iC_ie(t) \\ 0 \end{bmatrix}$, with $\omega(t) = 0$ Assumption 1 guarantees $e(t) \to 0\ (t \to \infty)$, which in turn gives $f_i(t) \to 0\ (t \to \infty)$, that is, $\forall\epsilon > 0,\ \exists T_1 > 0$, when $t > T_1$, there holds $\|f_i(t)\| < \epsilon$. The boundedness of $f_i(t)$ follows consequently, i. e., $\|f_i(t)\| \leq B_0\,(i \in \underline{N})$ follows. When $t \in [t_j,t_{j+1}) \subset [T_1,\infty)$, (26) gives that

$$\begin{aligned}
&\int_{t_0}^{t}\kappa e^{-\alpha(t-s)}\|f_{\sigma(t)}(s)\|ds \\
&\leq B_0\int_{t_0}^{T_1}\kappa e^{-\alpha(t-s)}ds + \int_{T_1}^{t}\kappa e^{-\alpha(t-s)}ds\cdot\epsilon \\
&\leq \frac{\kappa B_0}{\alpha}e^{-\alpha(t-T_1)} + \frac{\kappa}{\alpha}\epsilon. \qquad (27)
\end{aligned}$$

For $\forall\varepsilon > 0$, choose $T = T_1 + \frac{\ln\varepsilon^{-1}}{\alpha}$ and $\epsilon = \varepsilon$. When $t \in [t_j,t_{j+1}) \subset [T,\infty)$, it follows from (26) and (27) that

$$\|\bar{x}_t(t,t_j,\varphi_i,f_i)\| \leq (\frac{\kappa B_0}{\alpha} + \frac{\kappa}{\alpha})\varepsilon.$$

The above inequality and the continuity of the state trajectory $\bar{x}(t)$ imply that $\bar{x}(t) \to 0$ as $t \to \infty$, this in turn gives rise to that the switched linear time-delay system (7) is asymptotically stable with $\varpi(t) = 0$.

Secondly, we prove under zero initial condition with $\varpi(t) \neq 0$ that $\int_0^\infty e_r^T(t)Qe_r(t)dt < \gamma^2\int_0^\infty \varpi^T(t)\varpi(t)dt$.

Again, assume $\sigma(t) = i,\ t \in [t_j,t_{j+1})$. Differentiating the Lyapunov-Krasovskii functional candidate $V(\bar{x}(t))$ along the trajectory of the system (7b') with $\varpi(t) \neq 0$, we have

$$\dot{V}(\bar{x}(t)) = \xi^T(t)\begin{bmatrix} \bar{A}_i^TP_i + P_i\bar{A}_i + S & P_i\bar{D}_i \\ * & -e^{-\eta\tau}S \end{bmatrix}\xi(t) + 2\bar{x}^T(t)P_if_i(t) - \eta\int_{t-\tau}^{t}\bar{x}^T(s)e^{-\eta(t-s)}S\bar{x}(s)ds. \qquad (28)$$

Using Lemma 1 again, we have

$$2\bar{x}^T(t)P_if_i(t) \leq \rho^{-2}\bar{x}^T(t)P_iP_i\bar{x}(t) + \rho^2f_i^T(t)f_i(t). \qquad (29)$$

Substituting (29) into (28) and taking into account the condition (19), the switching law (18), and the structure of $\bar{Q}$ yields

$$\begin{aligned}
\dot{V}(\bar{x}(t)) &< \xi^T(t)\begin{bmatrix} -\bar{Q} & 0 \\ 0 & 0 \end{bmatrix}\xi(t) + \rho^2f_i^T(t)f_i(t) \\
&\leq -\hat{e}_r^T(t)\hat{e}_r(t) + \rho^2f_i^T(t)f_i(t). \qquad (30)
\end{aligned}$$

Note that $\hat{e}_r(t) = e_r(t) - e(t)$, using Lemma 1 with $Q = diag\{\frac{1}{2},\cdots,\frac{1}{2}\} \in \mathbb{R}^{n\times n}$ gives

$$\begin{aligned}
-\hat{e}_r^T(t)\hat{e}_r(t) &= -e_r^T(t)e_r(t) - e^T(t)e(t) + 2e_r^T(t)e(t) \\
&\leq -e_r^T(t)e_r(t) - e^T(t)e(t) \\
&\quad + e_r^T(t)Qe_r(t) + e^T(t)Q^{-1}e(t) \\
&= -\frac{1}{2}e_r^T(t)e_r(t) + \|e(t)\|^2. \qquad (31)
\end{aligned}$$

Similar to the disposal in above proving, let $e_t(t,t_j,\psi_i,\omega)$ denote the solution of error switched system (6) or (7a) with initial condition (t_j,ψ_i). By the variation-of-constants formula, there has

$$e_t(t,t_j,\psi_i,\omega)=T_1(t,t_j)\psi_i+\int_{t_j}^{t}T_1(t,s)X_0\omega(s)ds,$$

where T_1 is a proper continuous linear operator. Noticing that Assumption 1 guarantees that the error switched system (6) or (7a) is exponentially stable when $\omega(t)=0$, that is, there are constants $\beta>0,\ 0<k\le 1$ such that,

$$\|T_1(t,\varrho)\|\le ke^{-\beta(t-\varrho)},\ \|T_1(t,\varrho)X_0\|\le ke^{-\beta(t-\varrho)},\ t>\varrho.$$

Repeating the stepping iterative process such as in (26) easily gives

$$\|e_t(t,t_j,\psi_i,\omega)\|\le\int_{t_0}^{t}ke^{-\beta(t-s)}\|\omega(s)\|ds. \quad (32)$$

According to Cauchy-Schwartz Inequality, (32) gives

$$\begin{aligned}\|e(t)\|^2&\le\int_{t_0}^{t}k^2e^{-\beta(t-s)}ds\cdot\int_{t_0}^{t}e^{-\beta(t-s)}\|\omega(s)\|^2ds\\&\le\frac{k^2}{\beta}\int_{t_0}^{t}e^{-\beta(t-s)}\|\omega(s)\|^2ds, \quad (33)\end{aligned}$$

Therefore, when $\varpi(t)\neq 0$, there has

$$\begin{aligned}f_i^T(t)f_i(t)&=e^T(t)C_i^TL_i^TL_iC_ie(t)+r^T(t)r(t)\\&<\lambda^2\|e(t)\|^2+r^T(t)r(t) \quad (34)\\&<\frac{\lambda^2k^2}{\beta}\int_{t_0}^{t}e^{-\beta(t-s)}\|\omega(s)\|^2ds+r^T(t)r(t).\end{aligned}$$

where $\lambda=\max_{i\in\underline{N}}\{\lambda_M(L_iC_i)\}$,

Substituting (31), (33), (34) into (30), we can obtain

$$\begin{aligned}\dot V(\bar x(t))<&-\frac{1}{2}e_r^T(t)e_r(t)+\rho^2r^T(t)r(t)\\&+\frac{(\rho^2\lambda^2+1)k^2}{\beta}\int_{t_0}^{t}e^{-\beta(t-s)}\|\omega(s)\|^2ds. \quad (35)\end{aligned}$$

Integrating (35) from zero to ∞ , we get

$$\begin{aligned}&\int_0^\infty\sum_{i_j\in\underline{N}}\dot V(\bar x(t))dt=\sum_{j=0}^{\infty}\sum_{i_j\in\underline{N}}\int_{t_{i_j}}^{t_{i_{j+1}}}\dot V(\bar x(t))dt\\&<-\frac{1}{2}\int_0^\infty e_r^T(t)e_r(t)dt+\rho^2\int_0^\infty r^T(t)r(t)dt\\&\quad+\frac{(\rho^2\lambda^2+1)k^2}{\beta}\int_0^\infty\int_0^t e^{-\beta(t-s)}\|\omega(s)\|^2dsdt\\&=-\frac{1}{2}\int_0^\infty e_r^T(t)e_r(t)dt+\rho^2\int_0^\infty r^T(t)r(t)dt\\&\quad+\frac{(\rho^2\lambda^2+1)k^2}{\beta^2}\int_0^\infty\|\omega(t)\|^2dt. \quad (36)\end{aligned}$$

Let $\frac{k^2}{\beta^2-\lambda^2k^2}<\rho^2$. There has

$$\begin{aligned}&\int_0^\infty\sum_{i_j\in\underline{N}}\dot V(\bar x(t))dt\\&<-\frac{1}{2}\int_0^\infty e_r^T(t)e_r(t)dt+\rho^2\int_0^\infty\varpi^T(t)\varpi(t)dt. \quad (37)\end{aligned}$$

Again, taking the switching law (18) into account, on the switching instant t_j, it holds

$$V(\bar x(t_j))\le V(\bar x(t_j^-)). \quad (38)$$

Substituting (38) into the expansion form of the left side of (37) with $\gamma^2=2\rho^2$ yields

$$\begin{aligned}&\lim_{t_f\to\infty}V(\bar x(t_f))-V(\bar x(t_0))\\&\le\lim_{t_f\to\infty}[V(\bar x(t_f))-V(\bar x(t_{f-1}))+V(\bar x(t_{f-1}^-))\\&\quad-V(\bar x(t_{f-2}))+\cdots+V(\bar x(t_1^-))-V(\bar x(t_0))]\\&=\int_0^\infty\sum_{i_j\in\underline{N}}\dot V(\bar x(t))dt=\sum_{j=0}^{\infty}\sum_{i_j\in\underline{N}}\int_{t_{i_j}}^{t_{i_{j+1}}}\dot V(\bar x(t))dt\\&<-\frac{1}{2}\int_0^\infty e_r^T(t)e_r(t)dt+\frac{1}{2}\gamma^2\int_0^\infty\varpi^T(t)\varpi(t)dt.\end{aligned}$$

By the zero initial condition and the positive definiteness of $V(\bar x(t))$, that $\int_0^\infty e_r^T(t)e_r(t)dt<\gamma^2\int_0^\infty\varpi^T(t)\varpi(t)dt$ with $\varpi(t)\neq 0$ holds. This end the proof. □

IV. NUMERICAL EXAMPLE

Consider the systems (1) and the reference system (3) with

$$A_1=\begin{bmatrix}-1.5&-1.2\\-1.2&1\end{bmatrix},\ D_1=\begin{bmatrix}-0.5&0.8\\-0.1&-0.4\end{bmatrix},\ B_1=\begin{bmatrix}-0.1\\-0.3\end{bmatrix};$$

$$A_2=\begin{bmatrix}1.5&-1\\-1&-2.3\end{bmatrix},\quad D_2=\begin{bmatrix}-0.3&-0.2\\0.1&-0.3\end{bmatrix},\ B_2=\begin{bmatrix}-1.3\\-0.1\end{bmatrix};$$

$$A_r=\begin{bmatrix}-1.5&-1.2\\2&-0.2\end{bmatrix},C_1=\begin{bmatrix}-0.1&0.5\end{bmatrix},C_2=\begin{bmatrix}1.3&-0.7\end{bmatrix}.$$

First, by Assumption 1, we have the candidate observer gains via arbitrary switching as

$$L_1=\begin{bmatrix}-39.3909\\41.7974\end{bmatrix},\quad L_2=\begin{bmatrix}12.9363\\-11.9392\end{bmatrix}.$$

Consider the closed-loop systems (7). We adopt the parameters below: $\gamma=1,\ \tau=3$. Solving the inequality (17) by using LMIs, we get

$P_1=\begin{bmatrix}\tilde P_1^{-1}&0\\0&\tilde P_1^{-1}\end{bmatrix},P_2=\begin{bmatrix}\tilde P_2^{-1}&0\\0&\tilde P_2^{-1}\end{bmatrix}$, in which

$\tilde P_1=\begin{bmatrix}0.5426&-0.2007\\ *&0.4784\end{bmatrix},\tilde P_2=\begin{bmatrix}0.5026&-0.0527\\ *&0.7023\end{bmatrix};$

$S=\begin{bmatrix}S_1&0\\0&S_1\end{bmatrix}$, where $S_1=\begin{bmatrix}6.5442&-0.0466\\ *&6.4185\end{bmatrix}$

$K_1=\begin{bmatrix}4.0344&14.3356\end{bmatrix},K_2=\begin{bmatrix}-4.4402&-1.6902\end{bmatrix}.$

According to theorem 1, the switching control law are given by

$$\sigma(t)=arg\min_{i\in\underline{N}}\{\tilde x^T(t)P_i\tilde x(t)\},\ u(t)=K_{\sigma(t)}\hat e_r(t).$$

When $\omega(t)=0$, the error switched system (6) is exponentially stability under arbitrary switching (see in Fig.1). With $t_f=40,\ r(t)=\omega(t)=\frac{\sin t}{t}$, the simulation results are given in Fig.2 and Fig.3.

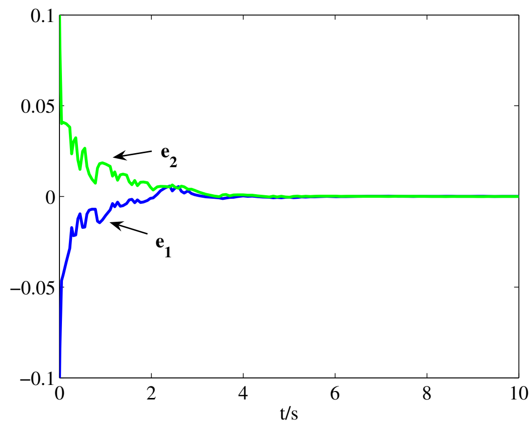

Fig. 1. Error state $e(t)$ via arbitrary switching.

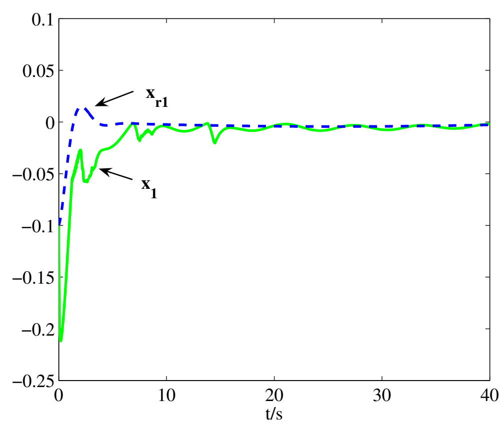

Fig. 2. State x_1 tracking the reference state x_{r1}.

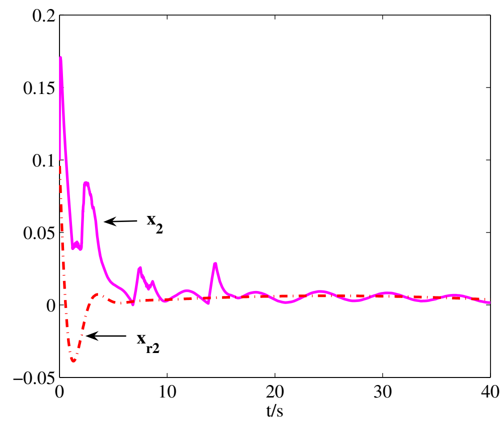

Fig. 3. State x_2 tracking the reference state x_{r2}.

V. CONCLUSIONS

In this paper, observer based tracking control for switched linear systems with time-delay is investigated. The possibility of designing switching control law based on measured output instead of the state information is considered when the state is not available. The Variation-of-constants formula and linear operator theory are used to conquer the difficulties caused by the estimation error and exotic disturbance, and multiple Lyapunov function technique is utilized to design switched tracking controllers such that the observer based H_∞ model reference tracking performance is satisfied. Meanwhile, the controller design problem can be solved efficiently by using linear matrix inequalities and convex optimization techniques. The numerical example show the feasibility and validity of the proposed design methods. However, due to the difficulties of switched system with time-delay, the cases of time varying delay and delay depended criteria for switched linear and nonlinear systems are not investigated yet, this constitutes our next work in the future.

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