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A Geometric Approach to Robust Fault Detection and Isolation of Discrete-time Markovian Jump Systems

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***Abstract*— This paper investigates development of Fault Detection and Isolation (FDI) filters for discrete-time Markovian jump systems (MJS). The notion of a common unobservability subspace is introduced for these systems and an algorithm for finding this subspace is presented. Based on the introduced unobservability subspaces, sufficient conditions for solving the fault detection and isolation problem of MJS systems are explored. A bank of residual generators is designed such that each residual is affected by one fault and is decoupled from the others while the H_∞ norm of the transfer function between the disturbance and the residual signals are less that a prespecified value. Simulation results presented in the paper demonstrate the effectiveness of our proposed FDI algorithm.**

Keywords: Fault detection and isolation (FDI), Markovian jump systems , Unobservability subspaces, Geometric FDI.

I. INTRODUCTION

Modern control systems are becoming increasingly more complex and issues of availability, efficiency, operating safety, and environmental protection concerns are receiving more attention. This requires a fault diagnosis system that is capable of reliably detecting plant, actuator and sensor faults when they occur, and of identifying and isolating the faulty component in the system. In the past three decades, a number of fundamental results on fault detection and isolation (FDI) have been developed [1]–[9].

In another research area, a great deal of attention has been devoted to the Markovian jump systems [10]–[13] which comprise an important class of stochastic systems. This family of systems is generally modeled by a set of linear systems with transition between models determined by a Markov chain taking values in a finite set. Markovian jump systems (MJS) are popular in modeling many practical systems where they may experience abrupt changes in their structures and parameters, which may occur frequently in manufacturing system, economic systems, communication systems, power systems, etc. Recently, Markovian jump systems have also gained interest due to their capabilities in modeling networks that are constructed from sensors, actuators and decision makers [14]–[17].

In recent years, only a few results on FDI of Markovian jump systems have been developed in the literature. In [18], [19], a robust fault detection filter for Markovian jump systems is developed based on an H_∞ filtering problem, in which the residual generator is a Markovian jump system as well. An LMI approach is developed for solving the problem. In [20], a robust fault identification filter for a class of discrete-time Markovian jump systems with mode dependent time-delays and norm bounded uncertainty is developed based on H_∞ optimization technique. In this approach, the generated residual signal is an estimate of the fault signal. However, the problem of fault isolation for a Markovian jump system has not been completely addressed in the above methods.

In this paper, a set of residuals that are based on the dedicated residual scheme [3], [7] is generated by generalizing the geometric FDI results in [3] to Markovian jump systems. The notion of a common unobservability subspace is introduced for MJS systems and an algorithm for constructing the smallest common unobservability subspace containing a given subspace is also proposed. Based on the developed geometric framework, a set of residuals is generated such that each residual is affected by one fault and is decoupled from others. At the same time the effects of disturbances on the residuals are attenuated by using an H_∞ optimization technique and the LMI approach is used for solving this optimization problem. The main contribution of this work is in developing a geometric FDI framework for discrete-time Markovian jump systems.

The remainder of this paper is organized as follows. In section II, a brief background on geometric properties of linear systems and an H_∞ control for Markovian jump systems are reviewed and the notion of a common unobservability subspace is introduced for MJS systems. In section III, an H_∞-based fault detection and isolation strategy for Markovian jump systems is presented. In section IV, the effectiveness and capabilities of our proposed algorithm are shown through simulation results. Conclusions and future work are presented in section V.

The following notation is used throughout this paper. Script letters $\mathcal{X}, \mathcal{U}, \mathcal{Y}, ...$, denote real vector spaces. Matrices and linear maps are denoted by capital italic letters $A, B, C, ...$; the same symbol is used both for a matrix and its map; the zero space, zero vector ,..., are denoted by 0. For any positive integer k, $\mathbf{k}$ denotes the finite set $\{1, 2, ..., k\}$. $\mathcal{B} = \text{Im } B$ denotes the image of B; Ker C denotes the kernel of C. If a map C is epic, then C^{-r} denotes a right inverse of C (i.e., $CC^{-r} = I$)). A subspace $\mathcal{S} \subseteq \mathcal{X}$ is termed A-

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invariant if $A\mathcal{S} \subseteq \mathcal{S}$. For A-invariant subspace $\mathcal{S} \subseteq \mathcal{X}$, $A : \mathcal{S}$ denotes the restriction of A to $\mathcal{S}$, and $A : \mathcal{X}/\mathcal{S}$ denotes the map induced by A on the factor space $\mathcal{X}/\mathcal{S}$. For a linear system (C, A, B), $<$ Ker $C|A >$ denotes the unobservable subspace of (C, A). For a given subspace $\mathcal{L}$, $dim(\mathcal{L})$ denotes the dimension of $\mathcal{L}$. We denote by $||.||$ the standard norm in $\mathbb{R}^n$. For the sequence of second-order random variables $z = (z(0), z(1), z(2), ...)$, $||z||_{2,E}^2 = \sum_{k=0}^{\infty} E(||z(k)||^2)$.

II. BACKGROUND

Consider the linear system

$$\Sigma : \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases}$$

where $x \in \mathcal{X}$ is the state of the system with dimension n, $u \in \mathcal{U}, y \in \mathcal{Y}$ are input and output signals with dimensions m and q, respectively. In the *geometrical approach* to fault detection and isolation certain unobservability subspaces play a central role [3], [21] as defined below.

***Definition** 1:* A subspace $\mathcal{S}$ is a (C, A) *unobservability subspace* (u.o.s.) [1] if $\mathcal{S} =<$ Ker $HC|A + DC >$ for some output injection map $D : \mathcal{Y} \to \mathcal{X}$ and measurement mixing map $H : \mathcal{Y} \to \mathcal{Y}$.

Given an u.o.s. $\mathcal{S}$, a measurement mixing map H can be computed from $\mathcal{S}$ by solving the equation Ker$HC =$ Ker$C + \mathcal{S}$. Let $\underline{D}(\mathcal{S})$ denote the class of all maps $D : \mathcal{Y} \to \mathcal{X}$ such that $(A + DC)\mathcal{S} \subseteq \mathcal{S}$. The notation $\underline{\mathcal{S}}(\mathcal{L})$ refers to the class of (C, A) u.o.s. containing $\mathcal{L} \subseteq \mathcal{X}$. The class of u.o.s. is closed under intersection; therefore, it contains an infimal element $\mathcal{S}^* = \inf \underline{\mathcal{S}}(\mathcal{L})$. In [3] an algorithm for computing $\mathcal{S}^*$ is proposed. The following lemma provides a generic property of the (C, A) u.o.s.

***Lemma** 2.1:* Let A, C and L be arbitrary matrices of dimensions $n \times n$, $q \times n$ and $n \times k$, respectively. Provided that $k < q$, then the following equality holds generically

$$\mathcal{S}^* = \mathcal{L} \tag{1}$$

In the following, certain results on H_∞ disturbance attenuation of Markovian jump systems are reviewed and the notion of common unobservability subspaces is introduced for Markovian jump systems. Consider the following Markovian jump system

$$\begin{aligned} x(k+1) &= A(\lambda_k)x(k) + B(\lambda_k)u(k) + B_d(\lambda_k)d(k) \\ &\quad + \sum_{l=1}^{L} L_l(\lambda_k)m_l(k) \\ y(k) &= C(\lambda_k)x(k) + D_d(\lambda_k)d(k) \\ x(0) &= x_0, \quad r_0 = i_0 \end{aligned} \tag{2}$$

where $x \in \mathcal{X}$ is the state of the system with dimension n, $u \in \mathcal{U}, y \in \mathcal{Y}$ are input and output signals with dimensions m and q, respectively, $d(k) \in \mathbb{R}^p$ is the unknown input, $m_i \in \mathcal{M}_i$ are the fault modes with dimension k_i and L_i's are fault signatures. The fault modes together with the fault signatures may be used to model the effects of actuator faults, sensor faults and system faults on the dynamics of the system. It is assumed that $d(k)$ is l_2-norm bounded, $\{\lambda_k\}$ is a discrete Markov process taking values in a finite set $\Psi = \{1, ..., N\}$ with transition probability matrix $\Pi = (\pi_{ij})_{i,j\in\Psi}$ and π_{ij} is defined as $\pi_{ij} = Pr\{\lambda_{k+1} = j|\lambda_k = i\}$ where $\sum_{j=1}^{N} \pi_{ij} = 1$. The matrices $A(\lambda_k)$, $B(\lambda_k)$, $B_d(\lambda_k)$, $L_l(\lambda_k), l = 1, ..., N$, $C(\lambda_k)$, $D_d(\lambda_k)$ are known constant matrices for all $\lambda_k = i \in \Psi$. We denote the matrices associated with $\lambda_k = i$ by

$$\begin{aligned} &A(\lambda_k) = A_i, \quad B(\lambda_k) = B_i, \quad B_d(\lambda_k) = B_{di}, \\ &C(\lambda_k) = C_i, \quad D_d(\lambda_k) = D_{di}, \quad L_l(\lambda_k) = L_{li}, l \in \mathbf{L} \end{aligned}$$

First, we define a common unobservability subspace for the MJS system (2).

***Definition** 2:* A subspace $\mathcal{S}$ is a common *unobservability subspace* for system (2) if there exist output injection maps $D_i : \mathcal{Y} \to \mathcal{X}$ and measurement mixing maps $H_i : \mathcal{Y} \to \mathcal{Y}$ such that $\mathcal{S} =<$ Ker $H_iC_i|A_i + D_iC_i >$ for $i \in \Psi$.

The notation $\underline{\mathcal{S}}^J(\mathcal{L})$ refers to the class of common u.o.s. for the jump system (2) containing $\mathcal{L} \subseteq \mathcal{X}$. It is clear that $\mathcal{X} \subset \underline{\mathcal{S}}^J(\mathcal{L})$ which is the trivial element of $\underline{\mathcal{S}}^J(\mathcal{L})$. However, $\underline{\mathcal{S}}^J(\mathcal{L})$ has a nontrivial element if $\bigcap_{i=1}^{N} \underline{\mathcal{S}}_i(\mathcal{L}) \neq \{\mathcal{X}\}$ where $\underline{\mathcal{S}}_i(\mathcal{L})$ is the class of (C_i, A_i) u.o.s. containing $\mathcal{L}$.

***Lemma** 2.2:* The class $\underline{\mathcal{S}}^J(\mathcal{L})$ is closed under intersection.

Proof: Let $\mathcal{S}_1$ and $\mathcal{S}_2$ be any two elements of $\underline{\mathcal{S}}^J(\mathcal{L})$ and $\mathcal{S} = \mathcal{S}_1 \cap \mathcal{S}_2$, then since $\underline{\mathcal{S}}_i(\mathcal{L})$ is closed under intersection, then $\mathcal{S} \in \underline{\mathcal{S}}_i(\mathcal{L}),\ i \in \Psi$; hence $\mathcal{S} \in \bigcap_{i=1}^{N} \underline{\mathcal{S}}_i(\mathcal{L})$. ■

Based on Lemma (2.2), the class $\underline{\mathcal{S}}^J(\mathcal{L})$ admits an infimal element $\mathcal{S}^{J^*}$. The following algorithm can be used for constructing the infimal element $\mathcal{S}^{J^*} = \inf \underline{\mathcal{S}}^J(\mathcal{L})$.

***Algorithm** 1:* Consider the sequence of subspaces

$$\mathcal{Z}_{10} = \mathcal{L}; \quad \mathcal{Z}_{1j} = \text{Min } \mathcal{S}(A_j, C_j, \mathcal{Z}_{1,j-1}), \quad j \in \Psi$$

$$\vdots$$

$$\mathcal{Z}_{i0} = \mathcal{Z}_{i-1,N}; \quad \mathcal{Z}_{ij} = \text{Min } \mathcal{S}(A_j, C_j, \mathcal{Z}_{i,j-1}), \quad j \in \Psi$$

where for any $\mathcal{E} \subset \mathcal{X}$, Min $\mathcal{S}(A_j, C_j, \mathcal{E})$ denotes the infimal element $\underline{\mathcal{S}}_i(\mathcal{E})$ which can be found from the algorithm in [3]. When $\mathcal{Z}_{ij} = \mathcal{Z}_{i-1,j}$ for any two indices i, j, the last term of the sequence is $\mathcal{S}^{J^*}$.

To analyze the above algorithm note that every subspace of the sequence contains the previous one (i.e. $\mathcal{Z}_{i,j-1} \subset \mathcal{Z}_{ij},\ j \in \Psi$). The last term of the sequence satisfies

$$\mathcal{Z} = \text{Min } \mathcal{S}(A_j, C_j, \mathcal{Z}), \quad j \in \Psi \tag{3}$$

and hence it is a common unobservability subspace for MJS system (2). It remains to prove that the last term (denoted by $\mathcal{Z}$) is the infimal common unobservability subspace containing the subspace $\mathcal{L}$ (i.e. $\mathcal{S}^{J^*}$). Let $\mathcal{S} \in \underline{\mathcal{S}}^J(\mathcal{L})$, it is clear that $\mathcal{Z}_{10} \subset \mathcal{S}$. By induction, suppose that $\mathcal{Z}_{i,j-1} \subset \mathcal{S}$, since $\mathcal{S}$ is (A_j, C_j) u.o.s. and it contains $\mathcal{Z}_{i,j-1}$, then $\mathcal{S} \in \underline{\mathcal{S}}_j(\mathcal{Z}_{i,j-1})$. Moreover $\mathcal{Z}_{i,j} = \text{Min } \mathcal{S}(A_j, C_j, \mathcal{Z}_{1,j-1}) = \inf \underline{\mathcal{S}}_i(\mathcal{Z}_{i,j-1})$. Hence $\mathcal{Z}_{i,j} \subset \mathcal{S}$ and $\mathcal{Z} \subset \mathcal{S}$ and therefore $\mathcal{Z} = \mathcal{S}^{J^*}$.

The next lemma provides a generic property of the infimal element $\mathcal{S}^{J^*}$.

***Lemma** 2.3:* Let A_i, $C_i, i = 0, ..., N$ and L be arbitrary matrices of dimensions $n \times n$, $q \times n$ and $n \times k$, respectively. Provided that the following equality holds

$$k < q$$

then generically $\mathcal{S}^{J^*}$ satisfies the following equation

$$\mathcal{S}^{J^*} = \mathcal{L} \tag{4}$$

Proof: It follows from the result of Lemma 2.1 that generically

$$\inf \underline{\mathcal{S}}_i(\mathcal{L}) = \mathcal{L} \tag{5}$$

Therefore, in generic $\mathcal{L} \in \bigcap_{i=1}^{N} \underline{\mathcal{S}}_i(\mathcal{L})$ which leads to equation (4). ■

Next, sufficient condition for H_∞ disturbance attenuation of Markovian jump systems is reviewed.

***Definition** 3 ([22]):* System (2) with $u(k) = 0$, $d(k) = 0$, $m_l(k) = 0, l \in \mathbf{L}$ is said to be mean square stable if

$$\lim_{k\to\infty} \mathbb{E}(||x(k)||^2) = 0$$

for any initial condition $x(0) = x_0$ and initial distribution $\lambda_k = i_0 \in \Psi$.

***Lemma** 2.4 ([22]):* Consider the system (2) with $u(k) = 0$ and $m_l(k) = 0, l \in \mathbf{L}$. Let $\gamma > 0$ be a given scalar, then the system is mean square stable when $d = 0$ and under zero initial conditions satisfies

$$||y||_{2,E} < \gamma ||d||_2 \tag{6}$$

if there exist matrices $R_i > 0, i \in \Psi$ such that the following LMIs:

$$\begin{bmatrix} A_i^T \bar{R}_i A_i - R_i & A_i^T \bar{R}_i B_{di} & C_i^T \\ * & -\gamma^2 I + B_{di}^T \bar{R}_i B_{di} & D_{di}^T \\ * & * & -I \end{bmatrix} < 0 \tag{7}$$

hold for $i \in \Psi$ where

$$\bar{R}_i = \sum_{j=1}^{N} \pi_{ij} R_j$$

Moreover, LMI (7) is equivalent to

$$\begin{bmatrix} -R_i & A_i^T \bar{R}_i & 0 & C_i^T \\ * & -\bar{R}_i & \bar{R}_i B_{di} & 0 \\ * & * & -\gamma^2 I & D_{di}^T \\ * & * & * & -I \end{bmatrix} < 0 \tag{8}$$

for $i \in \Psi$.

We are now in a position to formally introduce the robust fault detection and isolation problem considered in this paper.

III. H_∞-BASED FAULT DETECTION AND ISOLATION STRATEGY OF MARKOVIAN JUMP SYSTEMS

The H_∞-based Extended Fundamental Problem in Residual Generation (HEFPRG) for the Markovian system (2) is to design a set of filters that generate L residuals $r_j(k)$ such that a fault in the l-th component $L_l(\lambda_k)$ can only affect the residual $r_l(k)$ and no other residual $r_j(k)(j \neq l)$ and

$$||r_l||_{2,E} < \gamma ||d||_2, \quad l \in \mathbf{L} \tag{9}$$

Specifically, the residual signals $r_l(k),\ l \in \mathbf{L}$ are generated according to the following Markovian jump filters:

$$\begin{aligned} w_l(k+1) =& F_l(\lambda_k) w_l(k) - E_l(\lambda_k) y(k) + K_l(\lambda_k) u(k) \\ r_l(k) =& M_l(\lambda_k) w_l(k) - H_l(\lambda_k) y(k) \end{aligned} \tag{10}$$

The following theorem summarizes our proposed strategy.

***Theorem** 3.1:* The HEFPRG problem defined by expressions (9) and (10) has a solution for the Markovian jump system (2) if there exist common unobservability subspaces

$$\mathcal{S}_l^{J^*} = \inf \underline{\mathcal{S}}^J (\sum_{i=1}^{N} \sum_{j=1, j\neq l}^{L} \mathcal{L}_{ji}), \quad l \in \mathbf{L} \tag{11}$$

such that

$$\mathcal{S}_l^{J^*} \cap \mathcal{L}_{li} = 0, \quad i \in \Psi, l \in \mathbf{L} \tag{12}$$

as well as the matrices T_{li}, and positive-definite matrices R_{li}, $i \in \Psi$, $l \in \mathbf{L}$ such that

$$\begin{bmatrix} -R_{li} & A_{li}^T \bar{R}_{li} + M_{li}^T \bar{T}_{li}^T & 0 & M_{li}^T \\ * & -\bar{R}_{li} & \bar{R}_{li} B_{dli} - \bar{T}_{li} H_{li} D_{di} & 0 \\ * & * & -\gamma^2 I & D_{dli}^T \\ * & * & * & -I \end{bmatrix} < 0 \tag{13}$$

where $\bar{R}_{li} = \sum_{j=1}^{N} \pi_{ij} R_{lj}$, $\bar{T}_{li} = \sum_{j=1}^{N} \pi_{ij} T_{lj}$ and P_l is the canonical projection of $\mathcal{X}$ on $\mathcal{X}/\mathcal{S}_l^{J^*}$, $B_{dli} = -P_l B_{di} - P_l D_{li} D_{di}$, $D_{dli} = -H_{li} D_{di}$, the pairs (M_{li}, A_{li}), $i \in \Psi$, $l \in \mathbf{L}$ are the factor system of the pairs $(C_i, A_i), i \in \Psi$ on $\mathcal{X}/\mathcal{S}_l^{J^*}$ and H_{li} is the solution to Ker $H_{li} C_i = \mathcal{S}_l^{J^*} +$Ker C_i.

Proof: Given the common unobservability subspaces $\mathcal{S}_l^{J^*}$, there exist output map injections D_{li} and measurement mixing map H_{li} $i \in \Psi$, $l \in \mathbf{L}$ such that

$$\mathcal{S}_l^{J^*} =< \text{Ker } H_{li} C_i | A_i + D_{li} C_i >$$

where H_{li} is the solution to Ker $H_{li} C_i = \mathcal{S}_l^{J^*} +$Ker C_i. Let M_{li} be a unique solution to $M_{li} P_l = H_{li} C_i$ and

$$A_{li} = (A_i + D_{li} C_i : \mathcal{X}/\mathcal{S}_l^{J^*})$$

where

$$P_l (A_i + D_{li} C_i) = A_{li} P_l \tag{14}$$

Let T_{li} and R_{li} be the solution to the inequality (13) and define $G_{li} = \bar{R}_{li}^{-1} \bar{T}_{li}$ and $F_{li} = A_{li} + G_{li} M_{li}$, $E_{li} = P_l (D_{li} + P_l^{-r} G_{li} H_{li})$. Let $K_{li} = P_l B_i$. Define $e_l(k) = w_l(k) - P_l x(k)$, then using (10) we have

$$\begin{aligned} e_l(k+1) =& F_{li} w_l(k) - E_{li} y(k) + K_{li} u(k) \\ & - P_l (A_i x(k) + B_i u(k) + B_{di} d(k) + \sum_{l=1}^{L} L_{li} m_l(k)) \\ =& F_{li} w_l(k) - P_l L_{li} m_l(k) - P_l B_{di} d(k) \\ & - P_l (A_i + D_{li} C_i) x(k) - G_{li} H_{li} C_i x(k) - E_{li} D_{di} d(k) \\ =& F_{li} w_l(k) - P_l L_{li} m_l(k) + (B_{dli} - G_{li} H_{li} D_{di}) d(k) \\ & - A_{li} P_l x(k) - G_{li} M_{li} P_l x(k) \\ =& (A_{li} + G_{li} M_{li}) e_l(k) - P_l L_{li} m_l(k) \\ & + (B_{dli} - G_{li} H_{li} D_{di}) d(k) \end{aligned}$$

Note that $P_l L_{ji} = 0, j \neq l, i \in \Psi, j \in \mathbf{L}$, since $\mathcal{L}_{ji} \in \mathcal{S}_l^{J^*}, j \neq l$. Also

$$\begin{aligned} r_l(t) &= M_{li} w_l(k) - H_{li} y(k) \\ &= M_{li} w_l(k) - H_{li} C_i x(k) - H_{li} D_{di} d(k) \\ &= M_{li} e_l(k) + D_{dli} d(k) \end{aligned}$$

Consequently, the error dynamics can be written as

$$\begin{aligned} e_l(k+1) =& (A_{li} + G_{li} M_{li}) e_l(k) - P_l L_{li} m_l(k) \\ &+ (B_{dli} - G_{li} H_{li} D_{di}) d(k) \\ r_l(k) =& M_{li} e_{li}(k) + D_{dli} d(k) \end{aligned} \quad (15)$$

Using Lemma 2.4 and the inequality (8), it follows that the inequality (9) holds. Moreover, from the error dynamics (15), it follows that $r_l(k)$ is only affected by $L_l(\lambda_k)$ and is decoupled from other fault signatures. ■

The major step in generating the residual $r_l(k)$ is to incorporate the image of the fault signatures that requires to be decoupled $(L_{ji}(j \neq l), i \in \Psi)$ in the common unobservability subspace of $r_l(k)$ and then factor out the unobservable subspace in a manner that in the remaining factor space those faults do not appear. The associated necessary condition for this purpose states that the image of L_{li} should not intersect with the unobservable subspace of $r_l(k)$, so that a fault in the l-th component is manifested in the residual $r_l(k)$.

***Corollary** 3.2:* The following condition is necessary for the existence of a solution to HEFPRG problem

$$\mathcal{L}_{li} \bigcap (\sum_{x=1}^{N} \sum_{j=1, j\neq l}^{L} \mathcal{L}_{jx}) = 0, \quad i \in \Psi, \quad l \in \mathbf{L} \quad (16)$$

The generic conditions for existence of the common unobservability subspaces of Theorem 3.1 can now be stated as follows.

***Proposition** 3.3:* Let A_i, C_i, and L_{li}, $i \in \Psi$, $l \in \mathbf{L}$ be arbitrary matrices of dimensions $n \times n$, $q \times n$ and $n \times k_{li}$, respectively, let $v = \sum_{i=1}^{N} \sum_{l=1}^{k} k_{li}$ and $v_l = dim(\sum_{x=1}^{N} \sum_{j=1, j\neq l}^{L} \mathcal{L}_{jx})$, $l \in \mathbf{L}$. The common unobservability subspaces of Theorem 3.1 generically exist if and only if

$$v_l + k_{li} \leq n \quad \forall i \in 1, ..., N \quad (17)$$

and

$$v - \min\{\sum_{i=1}^{N} k_{li}, l \in \mathbf{L}\} < q \quad (18)$$

Proof: (only if) According to the necessary condition in Corollary 3.2, inequality (17) follows immediately. Moreover, if $q < v - \sum_{i=1}^{N} k_{li}$, then generically,

$$\mathcal{S}_l^*(\sum_{x=1}^{N} \sum_{j=1, j\neq l}^{L} \mathcal{L}_{jx}) = \mathcal{X}$$

and

$$\underline{\mathcal{S}}_l^{J^*}(\sum_{x=1}^{N} \sum_{j=1, j\neq l}^{L} \mathcal{L}_{jx}) = \mathcal{X}$$

Therefore inequality (18) is necessary.

(if) Inequality (17) implies that the necessary condition in Corollary 3.2 is satisfied. Also using the result of Lemma 2.3, (18) implies that

$$\inf \underline{\mathcal{S}}^J (\sum_{i=1}^{N} \sum_{j=1, j\neq l}^{L} \mathcal{L}_{ji}) = \sum_{i=1}^{N} \sum_{j=1, j\neq l}^{L} \mathcal{L}_{ji} \quad (19)$$

Hence form Corollary 3.2, it follows that (12) holds. ■

Remark: For Markovian jump systems that the fault signatures are the same for all jump states λ_k, i.e. $L_l(\lambda_k) = L_l$, then equations (11) and (12) may be rewritten as

$$\mathcal{S}_l^{J^*} = \inf \underline{\mathcal{S}}^J (\sum_{j=1, j\neq l}^{L} \mathcal{L}_j) \quad l \in \mathbf{L} \quad (20)$$

and

$$\mathcal{S}_l^{J^*} \cap \mathcal{L}_l = 0, l \in \mathbf{L} \quad (21)$$

and the generic condition for existence of the above u.o.s. is the same as the EFPRG problem in [3].

After constructing the residual signals $r_j(k), j = 1, ..., L$, the last step for a successful fault detection and isolation is the residual evaluation stage which includes determining evaluation functions J_{r_j} and thresholds J_{th_i}. In this paper, evaluation functions and thresholds are selected as

$$J_{r_j}(k) = \sum_{k-k_0}^{k} r_j^T(k) r_j(k), \quad j \in \mathbf{L} \quad (22)$$

$$J_{th_j} = \sup_{d \in l_2, m_j = 0} E(J_{r_j}), \quad j \in \mathbf{L} \quad (23)$$

where k_0 is the length of the evaluation window. Based on the above thresholds and evaluation functions, the occurrence of a fault can be detected and isolated by using the following decision logics

$$J_{r_j} > J_{th_j} \Longrightarrow m_j \neq 0, \quad j = 1, ..., L \quad (24)$$

IV. Numerical Example

To illustrate the effectiveness and capabilities of our proposed FDI algorithm, a numerical example is provided in this section. Consider the Markovian jump system with time-delay (2) that is specified with parameters

$$A_1 = \begin{bmatrix} 0.8 & 0.5 & 1.2 & 0.3 \\ -0.1 & -0.2 & 1.5 & 2 \\ 0.6 & 0.4 & 0.8 & -0.35 \\ 1.1 & 0.5 & 0 & 0.7 \end{bmatrix}, B_1 = \begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 0 & 5 \\ 5 & 2 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, C_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0.4 & 0.5 & 1.2 & 0.3 \\ 0.1 & -0.2 & 0 & 2 \\ 0.6 & 0.4 & 0.8 & -0.35 \\ 1.1 & 0 & 0 & 0 \end{bmatrix},$$

$$B_{d1} = B_{d2} = \begin{bmatrix} 0.4 \\ 0.2 \\ 0.1 \\ 0.2 \end{bmatrix}, D_{d1} = D_{d2} = \begin{bmatrix} 0.1 \\ 0.05 \\ 0.0 \end{bmatrix}$$

and $B_2 = B_1$. The transition probabilities are given by

$$\pi_{11} = 0.4, \quad \pi_{12} = 0.6, \quad \pi_{21} = 0.3, \pi_{22} = 0.7$$

The fault signatures L_1 and L_2 are selected as the first and second columns of the matrix B_1, and hence represent actuator faults for the MJS.

According to the results of Theorem 3.1, first one needs to construct common unobservability subspaces $\mathcal{S}_1^{J^*} = \inf \underline{\mathcal{S}}^J(\mathcal{L}_2)$ and $\mathcal{S}_2^{J^*} = \inf \underline{\mathcal{S}}^J(\mathcal{L}_1)$. These common unobservability subspaces are obtained by using Algorithm 1 as $\mathcal{S}_1^{J^*} = \mathcal{L}_2$ and $\mathcal{S}_2^{J^*} = \mathcal{L}_1$. It can be verified that the two unobservability subspaces satisfy the necessary conditions (12). Once the subspaces $\mathcal{S}_1^{J^*}$ and $\mathcal{S}_2^{J^*}$ are determined, the maps D_{li}, M_{li}, H_{li} and matrices A_{li}, $l = 1, 2$, $i = 1, 2$ can be found according to Theorem 3.1. Using the LMI tools, the gain matrices G_{li}, $l = 1, 2$, $i = 1, 2$ are computed by solving the LMI inequalities (13) for $\gamma = 1$. An H_∞ robust state feedback [22] controller $u(k) = K(\lambda_k)x(t)$ is also designed for the closed-loop system to ensure its stability.

An input disturbance $d(k)$ is assumed to be the band-limited white-noise with power of 0.1. The evaluation window length and sampling time are selected as $k_0 = 50$ and $T_s = 0.1$ second, respectively. The calculated thresholds are $J_{th_1} = 0.35$ and $J_{th_2} = 0.3$. Figure 1 shows the residuals and their evaluation functions corresponding to the healthy operation of the system. As shown in this figure, no false alarm is generated during normal operation of the system. Figure 2 shows the residuals and the evaluation functions corresponding to a fault in the first actuator (u_1) of the system where the gain of the actuator is decreased by 70% at $t = 10$ seconds. This fault can be modeled as $m_1(t) = -0.7u_1(t)$, where $m_1(t)$ is the fault mode of the first actuator. As shown in this figure, the fault is detected and isolated at $t = 12.3$ seconds and the evaluation function of r_2 (i.e. J_{r_2}) remains below its corresponding threshold. Figure 3 shows the residuals and evaluation functions corresponding to a fault in the second actuator where the gain of the actuator is decreased by 40% at $t = 10$ seconds. This fault can be modeled as $m_2(t) = -0.4u_2(t)$, where $m_2(t)$ is the fault mode of the second actuator. As shown in this figure, this fault is detected and isolated at $t = 18.2$ seconds and the evaluation function of r_1 (i.e. J_{r_1}) remains below its corresponding threshold. Figure 4 shows the residuals and the evaluation functions corresponding to concurrent faults in both actuators where 60% loss of effectiveness (gain) is occurred in the first actuator at $t = 5$ seconds and 20% loss of gain is occurred in the second actuator at $t = 10$ seconds. According to this figure, the fault in the first actuator is detected at $t = 8.3$ seconds and the fault in the second actuator is detected at $t = 15.9$ seconds. Figure 5 shows the residuals and the evaluation functions corresponding to an intermittent step fault in the first actuator between $t = 10$ seconds and $t = 20$ seconds. This fault is modeled as $m_1(t) = 0.1, 10 \leq t \leq 20$. As shown in this figure, this intermittent fault can be perfectly detected and isolated. It should be noted that in all the above scenarios the Markovian jump system remains stable and well-behaved, which makes the FDI problem more challenging.

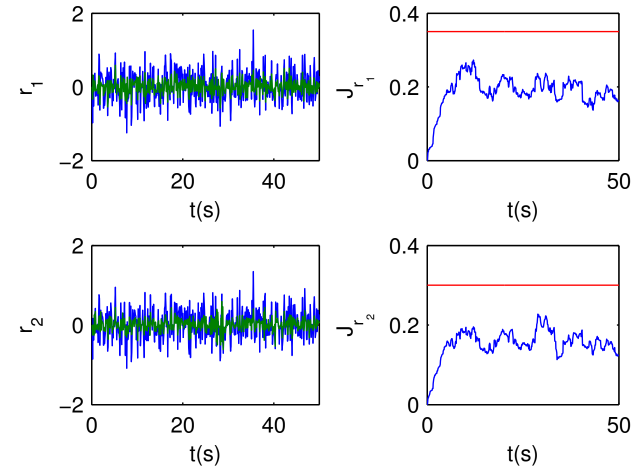

Fig. 1. Residual signals and their evaluation functions corresponding to the normal mode (healthy operation).

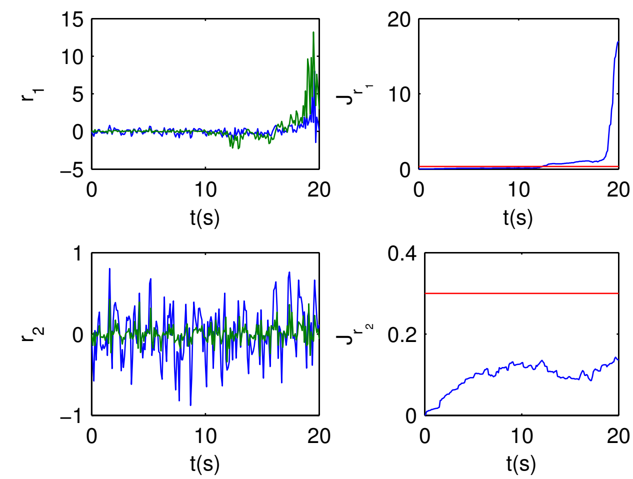

Fig. 2. Residual signals and their evaluation functions corresponding to a fault in the first actuator.

V. Conclusions

A geometric approach to fault detection and isolation for linear Markovian jump systems is developed in this paper. A set of residual signals are generated so that each residual is only affected by one fault and is decoupled from the others while the H_∞ norm of the transfer function between the unknown input (disturbances, uncertainties and modeling errors) and residual signals is less than a given positive value. Simulation results demonstrate and illustrate the effectiveness and capabilities of our proposed method.

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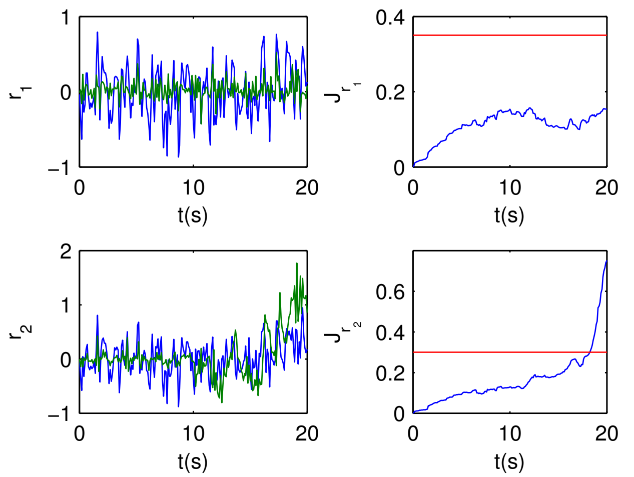

Fig. 3. Residual signals and their evaluation functions corresponding to a fault in the second actuator.

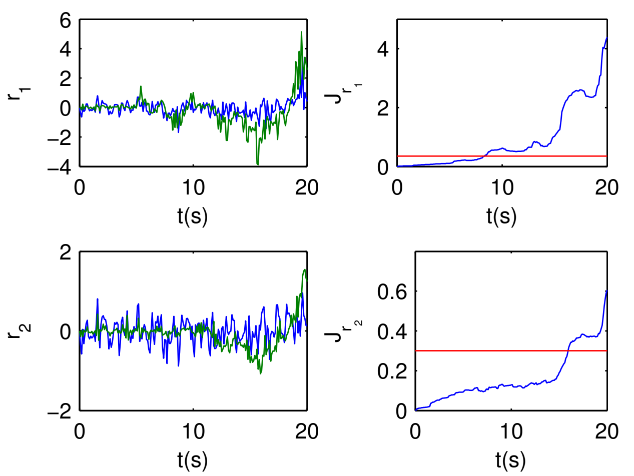

Fig. 4. Residual signals and their evaluation functions corresponding to simultaneous faults in both actuators.

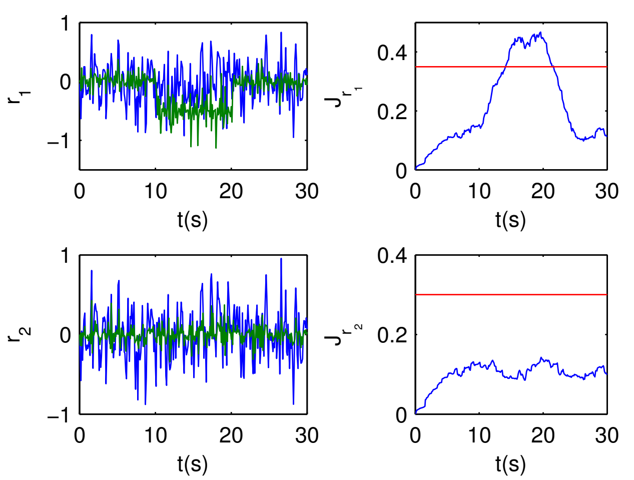

Fig. 5. Residual signals and their evaluation functions corresponding to an intermittent fault in first actuator.

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