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Nonlinear Adaptive Observer Control for a Riser Slugging System in Unstable Wells

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Abstract— **This paper presents a nonlinear observer-based control scheme to stabilize the downhole pressure and eliminate riser slugging. A simple empirical model is developed that describes the qualitative behavior of the downhole pressure in case of severe riser slugging. A new nonlinear adaptive observer is developed for state estimations with the new parameter adaptation law. The controller is designed by an integrator backstepping approach to stabilize the downhole pressure and achieve asymptotic tracking. The stabilization in the unstable region is demonstrated by the proposed control. For the design and implementation of the controller, no knowledge is assumed on the system parameters. It is shown that the proposed controller not only can guarantee asymptotic stability, but also transient performance.**

I. INTRODUCTION

In many hydrocarbon production systems, unstable multiphase flow poses a serious challenge for safe and efficient operation of the field. The stabilization is related to the purpose of attenuating an oscillation phenomenon, called severe slugging, that exists in pipelines carrying multiphase flow. In many cases, the wells and production lines enter a slug flow regime where liquid slugs are followed by gas pockets yielding large oscillations in the flow rate and phase distribution as seen from the outlet of the pipe/well. This alternating flow regime, referred to as severe slugging, poses a serious operational challenge for the downstream process, such as separators, and may cause lower oil production. A schematic diagram of riser slug rig is shown in Figure 1.

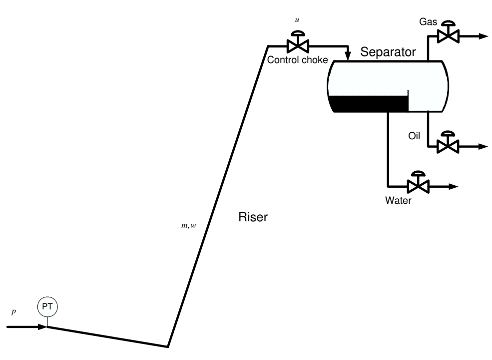

Fig. 1. Schematic diagram of riser slug rig.

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Research on handling severe slugging in pipeline-riser systems has received great attentions in the literature and in the industry. Several remediation strategies has been proposed. These can be categorized as design modifications and operational modifications. In operational modifications, an effective method is to develop control strategies that guarantee attenuation of riser slugging. The motivation for using active feedback control is that one can operate the pipeline/well in an unstable operating region, where the system is open-loop unstable. Several publications describe the use of active feedback control in order to stabilize the flow, see for examples, [1], [2], [3], [4], [5], [6], [7], [8]. Some works used a detailed model and only proved stability linearly, whereas [9] proved nonlinear stability with a simplified model. In [9], a state feedback control design was presented based on linearization method, where the system parameters must be known and all system states are assumed to be known.

In this paper, we will address nonlinear observed-based control of a riser slugging system in the presence of unknown parameters and unmeasured signal. A simple empirical model is developed that describes the qualitative behavior of the downhole pressure in case of severe riser slugging. A new nonlinear observer is developed by using Lyapunov technique to estimate the unmeasured state with the new parameter adaptation law. The controller is designed by an integrator backstepping approach in [10] to stabilize the downhole pressure. The stabilization in the unstable region is demonstrated by the proposed control. For the design and implementation of the controller, there is no apriori information required from the system parameters and thus they can be allowed totally uncertain. It is shown that the proposed controller not only can guarantee asymptotic stability, but also transient performance. The simulation results are presented to illustrate the effectiveness of proposed control scheme.

II. MODEL

For unstable flow, several mechanisms can cause the instability depending on the geometry, fluids and process equipment. In order to understand the underlaying instabilities and to predict the controllability of slugging, a relatively simple model is needed which captures the fundamental dynamics of the system. Furthermore, a simple model may be used to develop a model-based stabilizing control law which more intelligently counteracts the destabilizing mechanisms of slugging. The developed models are different depending on the application and assumptions made and can be found in [7], [8], [9], [11], [12]. [7] presents a relatively simple first principle-based model which captures the main dynamics of a severe slugging flow regime in pipeline-riser systems. The model is able to reproduce observed unstable flow for a particular test case. A schematic of the severe slugging cyclic behavior is shown in 2, where the main phases of the formation of a slug is illustrated. In the first sub-figure, liquid

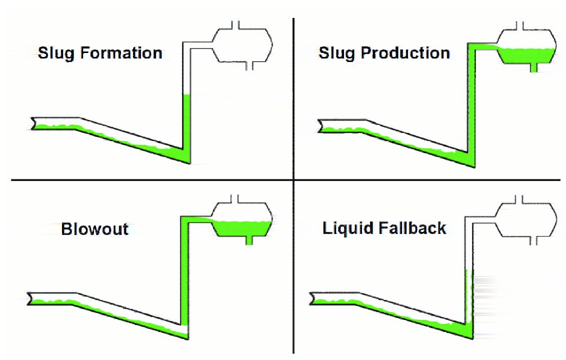

Fig. 2. Schematics of the severe slug cycle in riser systems

blocks the low point of the pipeline-riser system, preventing the gas from passing. Liquid flows from the riser and into the slug by gravity and causes the slug to grow and fill the riser. The pressure in the pipeline increases due to the inlet flow of gas and the increased liquid head. In the "slug production phase" the liquid slug has reached the top of the riser and flows into the separator. The pressure in the pipeline has steadily increased and is now large enough to push the liquid slug out of the riser. When the tail of the liquid slug enters the riser, the pressure drops due to the reduced static head of the liquid column which causes the gas to expand and accelerate the "blow out phase". When the gas has left the riser, the velocities in the riser are too low to carry any liquid up the riser and the process starts over ("liquid fall-back phase").
In this paper, a simplified model structure is presented that describes the qualitative behavior of the downhole pressure in case of severe riser slugging.

$$\dot{p} = w, \quad (1)$$
$$\dot{w} = a_1(\beta - p) + a_2(\zeta - w^2)w, \quad (2)$$

where the states p and w are the down hole pressure in the riser and its time derivative, respectively. The coefficients in (1)–(2) can be explained as follows.

- β: steady state pressure.
- a_1: frequency or stiffness of the system.
- a_2, ζ: local "degree of the stability/instability" and amplitude of the oscillation.

A. The equilibrium downhole pressure β

If the characteristic of the reservoir influx is known, the equilibrium downhole pressure can be derived from the steady-state flow rate characteristics of the choke valve and riser. From the slugging model (1)–(2), the fixed point $(\dot{p}, \dot{w}) = (0, 0)$ gives

$$0 = a_1(\beta - p). \quad (3)$$

Thus

$$\beta = p, \quad (4)$$

which implies that β is the steady state curve as in Figure 3, where β is plotted as a function of the choke opening. In the simplest case, we may assume constant flow rates of liquid and gas from the reservoir. Then

$$\beta(q) = b_0 + b_1 q, \quad (5)$$

where b_0 and b_1 are positive constants and q can be interpreted as proportional to the differential pressure over the production choke.

B. Local Degree of Stability/Instability a_2,ζ

The parameters a_2 and ζ are related to the amplitude of oscillation and stability properties of the fixed point. This can be seen by linearizing system (1)–(2) to get

$$\dot{\Delta p} = \Delta\omega, \quad (6)$$
$$\dot{\Delta\omega} = -a_1\Delta p + a_2\zeta\Delta\omega. \quad (7)$$

The eigenvalues of the system are $\lambda = \frac{a_2\zeta \pm \sqrt{a_2^2\zeta^2 - 4a_1}}{2}$, which means that (assuming $a_1 > 0$ and $a_2 > 0$)

- $\zeta = 0$, bifurcation point.
- $\zeta < 0$, system is stable.
- $\zeta > 0$, system is unstable.

In the simplest case, we may assume constant flow rates of liquid and gas from the reservoir. Then

$$\zeta(q) = c_0 - c_1 q, \quad (8)$$

where c_0/c_1 denotes the bifurcation point and c_0, c_1 are positive constants.

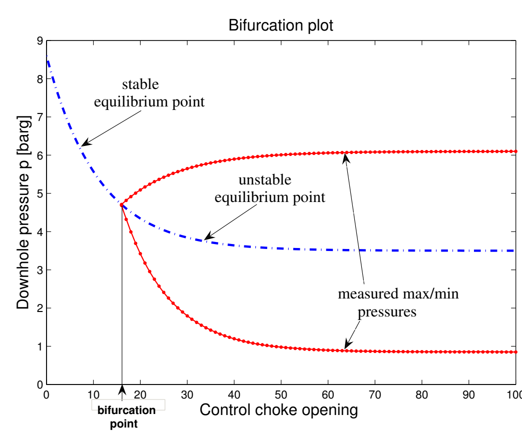

Fig. 3. Bifurcation plot

C. Transportation Delay

The variable q is related to the effect the differential pressure over the production choke. Due to transport delay in the well, a time-lag is expected between application of the control signal to the choke and seeing the effect in (1)–(2). This time-lag is modelled as follows

$$\dot{q} = -\frac{1}{\tau}q + \frac{1}{\tau}\delta, \tag{9}$$

where δ represents the control input.

D. Simplified Model of Riser Slugging

Based on (5) and (8), the system dynamics (1)–(2) and (9) can be assembled into

$$\begin{aligned} \dot{p} &= w, && (10)\\ \dot{w} &= -a_1 p + h(w) + g(w)\,q + f_1 && (11)\\ \dot{q} &= -\frac{1}{\tau}q + \frac{1}{\tau}\delta, && (12) \end{aligned}$$

where $f_1 = a_1 b_0$, the functions h and g are defined as

$$\begin{aligned} h(w) &= a_2 c_0 w - a_2 w^3 = h_0 w - h_1 w^3 && (13)\\ g(w) &= a_1 b_1 - a_2 c_1 w = g_0 - g_1 w. && (14) \end{aligned}$$

The positive constants a_i, b_i and c_i $(i = 1, 2)$ are empirical parameters that are adjusted to produce the right behavior of the downhole pressure p.

The system (10)–(12) can capture some of the qualitative properties in the downhole pressure during riser slugging.

- Decreasing control gain: A characteristic property of riser slugging is that the static gain decreases with choke opening.
- Bifurcation: The model exhibits the characteristic bifurcation that occurs at a certain choke opening c_0/c_1, *i.e.*, the steady-state response of the downhole pressure exhibits changes from a stable point when choke opening is smaller than c_0/c_1 to a stable limit cycle when choke opening is larger that c_0/c_1 (see Figure 3).
- Time lag: The transportation delay between a change in choke opening to the resulting change in downhole pressure p is modeled by simple 1st-order lag.

Our objective is to design a control law for the control input δ which stabilizes p at the desired set-point p_{ref}.

III. Nonlinear Adaptive Observer

Consider that p and q are measured and w is unmeasured, where the parameters h_0, h_1, g_0, g_1, f_1 and a_1 are unknown. The following change of coordinate is defined

$$\xi \triangleq w - l_1 p, \tag{15}$$

where l_1 is a tunable feedback gain. This gives the dynamics

$$\begin{aligned} \dot{\xi} &= \dot{w} - l_1 w\\ &= -a_1 p + h_0 w - h_1 w^3 + g_0 q - g_1 q w + f_1 - l_1 w. \end{aligned} \tag{16}$$

An observer for w must then be implemented with estimated parameter values $\hat{a}_1$, $\hat{h}_0$, $\hat{h}_1$, $\hat{g}_0$, $\hat{g}_1$ and $\hat{f}_1$, according to

$$\dot{\hat{\xi}} = -\hat{a}_1 p + \hat{h}_0 \hat{w} - \hat{h}_1 \hat{w}^3 + \hat{g}_0 q - \hat{g}_1 q \hat{w} + \hat{f}_1 - l_1 \hat{w} \tag{17}$$

$$\hat{w} = \hat{\xi} + l_1 p, \tag{18}$$

1) Resulting error dynamics: First we need to express the resulting error dynamics in appropriate form, incorporating the effect of parameter errors. Defining

$$\begin{aligned} \theta &= [a_1, h_0, h_1, g_0, g_1, f_1]^T, && (19)\\ \tilde{\theta} &= \theta - \hat{\theta}. && (20) \end{aligned}$$

Preparing for subsequent steps, we obtain the following error terms as

$$\begin{aligned} g_1 q w - \hat{g}_1 q \hat{w} &= g_1 q(\tilde{w} + \hat{w}) - \hat{g}_1 q \hat{w}\\ &= \tilde{g}_1 q \hat{w} + g_1 q \tilde{w} && (21)\\ h_1 w^3 - \hat{h}_1 \hat{w}^3 &= h_1 w^3 + \left(h_1 \hat{w}^3 - h_1 \hat{w}^3\right) - \hat{h}_1 \hat{w}^3\\ &= \tilde{h}_1 \hat{w}^3 + h_1\left(w^3 - \hat{w}^3\right). && (22) \end{aligned}$$

With (21), (22), and $\tilde{w} = \tilde{\xi}$, the error dynamics $\tilde{\xi}$ becomes

$$\begin{aligned} \dot{\tilde{\xi}} = {} & -\left(l_1 - (h_0 - g_1 q)\right)\tilde{\xi} - \tilde{a}_1 p + \tilde{h}_0 \hat{w} - \tilde{h}_1 \hat{w}^3 + \tilde{g}_0 q\\ & - \tilde{g}_1 q \hat{w} + \tilde{f}_1 - h_1\left(w^3 - \hat{w}^3\right). \end{aligned} \tag{23}$$

Defining the regressor vector

$$\phi\left(p, \hat{\xi}, q\right) \triangleq \begin{bmatrix} -p\\ \hat{w}\\ -\hat{w}^3\\ q\\ -q\hat{w}\\ 1 \end{bmatrix}, \tag{24}$$

and substituting $w = \xi + l_1 p$ and $\hat{w} = \hat{\xi} + l_1 p$, (23) gives

$$\begin{aligned} \dot{\tilde{\xi}} = {} & -\left(l_1 - (h_0 - g_1 q)\right)\tilde{\xi} + \tilde{\theta}^T \phi\left(p, \hat{\xi}, q\right)\\ & - h_1\left((\xi + l_1 p)^3 - (\hat{\xi} + l_1 p)^3\right). \end{aligned} \tag{25}$$

2) Lyapunov Analysis: Consider the Lyapunov function

$$U\left(\tilde{\xi}, \tilde{\theta}\right) = \frac{1}{2}\tilde{\xi}^2 + \frac{1}{2}\tilde{\theta}^T \mathbf{\Gamma}^{-1} \tilde{\theta}, \tag{26}$$

where $\mathbf{\Gamma}$ is the adaption gain. The time derivative of U gives

$$\begin{aligned} \dot{U} = {} & -\left(l_1 - (h_0 - g_1 q)\right)\tilde{\xi}^2 + \tilde{\theta}^T \phi\left(p, \hat{\xi}, q\right)\tilde{\xi}\\ & - h_1 \tilde{\xi}\left((\xi + l_1 p)^3 - (\hat{\xi} + l_1 p)^3\right) + \tilde{\theta}^T \mathbf{\Gamma}^{-1} \dot{\tilde{\theta}} \end{aligned} \tag{27}$$

Noticing that

$$\begin{aligned} & \tilde{\xi}\left((\xi + l_1 p)^3 - \left(\hat{\xi} + l_1 p\right)^3\right)\\ = {} & \left((\xi + l_1 p) - (\hat{\xi} + l_1 p)\right)\left((\xi + l_1 p)^3 - \left(\hat{\xi} + l_1 p\right)^3\right)\\ \geq {} & 0. \end{aligned} \tag{28}$$

We obtain

$$\begin{aligned} \dot{U} \leq {} & -\left(l_1 - (h_0 - g_1 q)\right)\tilde{\xi}^2\\ & + \tilde{\theta}^T\left(\mathbf{\Gamma}^{-1}\dot{\tilde{\theta}} + \phi\left(p, \hat{\xi}, u\right)\tilde{\xi}\right). \end{aligned} \tag{29}$$

This suggests that we should choose an adaptation law which satisfies

$$\dot{\tilde{\theta}} = -\mathbf{\Gamma}\phi\left(p,\hat{\xi},u\right)\tilde{\xi}. \tag{30}$$

It gives the time-derivative of U

$$\dot{U} \leq -\left(l_1 - \left(h_0 - g_1 q\right)\right)\tilde{\xi}^2. \tag{31}$$

If $q > 0$, the convergence is given by

$$U\left(\tilde{\xi}\left(t\right)\right) \leq U\left(\tilde{\xi}\left(0\right)\right)e^{-2\alpha t}, \tag{32}$$

with convergence rate $\alpha = l_1 - h_0$ that can be made arbitrary fast by increasing the feedback gain $l_1 > h_0$.

3) Adaptation Law: Note that (30) cannot be used for parameter estimation because $\tilde{\xi}$ is unavailable. We introduce a new variable

$$\sigma \triangleq \theta + \eta\left(p,\hat{\xi},q\right), \tag{33}$$

where $\eta\left(\cdot\right)$ is a vector function to be designed to assign σ a desired dynamics. Differentiating σ with respect to time, gives

$$\dot{\sigma} = \frac{\partial\eta}{\partial p}w + \frac{\partial\eta}{\partial\hat{\xi}}\dot{\hat{\xi}} + \frac{\partial\eta}{\partial q}\dot{q}. \tag{34}$$

Let an estimate $\hat{\theta}$ of the parameter vector be given by

$$\begin{aligned} \dot{\hat{\sigma}} &= \frac{\partial\eta}{\partial p}\hat{w} + \frac{\partial\eta}{\partial\hat{\xi}}\dot{\hat{\xi}} + \frac{\partial\eta}{\partial q}\dot{q} && (35)\\ \hat{\theta} &= \hat{\sigma} - \eta\left(p,\hat{\xi},q\right). && (36)\end{aligned}$$

The resulting estimation error

$$\begin{aligned} \tilde{\theta} &= \sigma - \eta\left(p,\hat{\xi},q\right) - \left(\hat{\sigma} - \eta\left(p,\hat{\xi},q\right)\right)\\ &= \sigma - \hat{\sigma} \triangleq \tilde{\sigma} \end{aligned} \tag{37}$$

is then governed by

$$\dot{\tilde{\theta}} = \dot{\tilde{\sigma}} = \frac{\partial\eta}{\partial p}\tilde{w} = \frac{\partial\eta}{\partial p}\tilde{\xi}. \tag{38}$$

Compared with (30), it suggests that we let

$$\partial\eta/\partial p \triangleq -\mathbf{\Gamma}\phi\left(p,\hat{\xi},q\right), \tag{39}$$

which gives

$$\dot{\tilde{\theta}} = -\mathbf{\Gamma}\phi\left(p,\hat{\xi},q\right)\tilde{\xi}. \tag{40}$$

Now, $\eta\left(\cdot\right)$ can be found by integrating (39):

$$\begin{aligned} \eta\left(p,\hat{\xi},q\right) &= -\int \mathbf{\Gamma}\phi\left(\bar{p},\hat{\xi},q\right)d\bar{p}\\ &= \mathbf{\Gamma}\begin{bmatrix} \frac{1}{2}p^2\\ -\left(\hat{\xi}+\frac{l_1}{2}p\right)p\\ \frac{1}{4l_1}\left(\hat{\xi}+l_1 p\right)^4\\ -qp\\ q\left(\hat{\xi}+\frac{l_1}{2}p\right)p\\ -p \end{bmatrix} \end{aligned} \tag{41}$$

The resulting partial derivatives become

$$\frac{\partial\eta}{\partial p} = \mathbf{\Gamma}\phi\left(p,\hat{\xi},q\right) = \mathbf{\Gamma}\begin{bmatrix} p\\ -\left(\hat{\xi}+l_1 p\right)\\ \left(\hat{\xi}+l_1 p\right)^3\\ -q\\ q\left(\hat{\xi}+l_1 p\right)\\ -1 \end{bmatrix} \tag{42}$$

$$\frac{\partial\eta}{\partial\hat{\xi}} = \mathbf{\Gamma}\begin{bmatrix} 0\\ -p\\ \frac{1}{l_1}\left(\hat{\xi}+l_1 p\right)^3\\ 0\\ qp\\ 0 \end{bmatrix} \tag{43}$$

$$\frac{\partial\eta}{\partial q} = \mathbf{\Gamma}\begin{bmatrix} 0\\ 0\\ 0\\ -p\\ \left(\hat{\xi}+\frac{l_1}{2}p\right)p\\ 0 \end{bmatrix}. \tag{44}$$

IV. Backstepping Controller Design

The system (10)-(12) based on the observer (17) and (18) is rewritten as

$$\begin{aligned} \dot{p} &= \hat{w} + \tilde{\xi}\\ \dot{\hat{w}} &= -\hat{a}_1 p + \hat{h}_0\hat{w} - \hat{h}_1\hat{w}^3 + \hat{g}_0 q - \hat{g}_1 q\hat{w} + \hat{f}_1 + l_1\tilde{\xi}\\ \dot{q} &= -\frac{1}{\tau}q + \frac{1}{\tau}\delta. \end{aligned} \tag{45}$$

In this section we design stabilizing controllers using backstepping technique. Thus, we iteratively look for a change of coordinates in the form

$$\begin{aligned} z_1 &= p - p_{ref} && (46)\\ z_2 &= \hat{w} - \alpha_1 && (47)\\ z_3 &= q - \alpha_2, && (48)\end{aligned}$$

and an accompanying Lyapunov function. The functions α_1 and α_2 are virtual controls to be determined.
Step 1 — virtual control law α_1
From (45), (46), and (47), we obtain that

$$\dot{z}_1 = \alpha_1 + z_2 + \tilde{\xi}. \tag{49}$$

Consider the CLF, $U_1 = \frac{1}{2}z_1^2 + kU$, we see that by choosing the virtual control law

$$\alpha_1 = -\left(C_1 + k_1\right)z_1, \tag{50}$$

the time-derivative of U_1 becomes

$$\begin{aligned} \dot{U}_1 &\leq -C_1 z_1^2 + z_1 z_2 - k_1 z_1^2 + z_1\tilde{\xi}\\ &\quad -k\left(l_1 - \left(h_0 - g_1 q\right)\right)\tilde{\xi}^2\\ &\leq z_1 z_2 - C_1 z_1^2 - k\left(l_1 - \left(h_0 - g_1 q\right) - \frac{1}{4k_1 k}\right)\tilde{\xi}^2. \end{aligned} \tag{51}$$

Step 2 — virtual control law α_2
Start by computing the derivative of z_2 from (45), (47), and (48)

$$\begin{aligned}\dot{z}_2 &= (\hat{g}_0 - \hat{g}_1\hat{w})\,\alpha_2 - \hat{a}_1 p + \hat{h}_0\hat{w} - \hat{h}_1\hat{w}^3 + \hat{f}_1 \\ &\quad + (\hat{g}_0 - \hat{g}_1\hat{w})\,z_3 + (C_1+k_1)(\hat{w}+\tilde{\xi}) + l_1\tilde{\xi}. \end{aligned} \tag{52}$$

Assume $\hat{g}_0 - \hat{g}_1\hat{w} \neq 0$. We choose the virtual control law α_2 as

$$\begin{aligned}\alpha_2 &= \frac{1}{(\hat{g}_0 - \hat{g}_1\hat{w})}\Big(- \big(C_2 + k_2(l_1+C_1+k_1)^2\big)\,z_2 - z_1 \\ &\quad -(C_1+k_1)\hat{w} + \hat{a}_1 p - \hat{f}_1 - \hat{h}_0\hat{w} + \hat{h}_1\hat{w}^3\Big). \end{aligned} \tag{53}$$

Consider the CLF $U_2 = U_1 + \frac{1}{2}z_2^2$ with (31), (51), (52) and (53), the time-derivative of U_2 becomes

$$\begin{aligned}\dot{U}_2 &\leq -C_1z_1^2 - C_2z_2^2 + (\hat{g}_0 - \hat{g}_1\hat{w})\,z_2z_3 \\ &\quad -k_2(l_1+C_1+k_1)^2z_2^2 + (l_1+C_1+k_1)\tilde{\xi}z_2 \\ &\quad -k\left(l_1 - (h_0 - g_1q) - \frac{1}{4kk_1}\right)\tilde{\xi}^2 \\ &\leq -C_1z_1^2 - C_2z_2^2 + (\hat{g}_0 - \hat{g}_1\hat{w})\,z_2z_3 \\ &\quad -k\left(l_1 - (h_0 - g_1q) - \frac{1}{4kk_1} - \frac{1}{4kk_2}\right)\tilde{\xi}^2. \end{aligned} \tag{54}$$

Step 3 — Final control law
We obtain the time-derivative of z_3 from (45) and (48)

$$\begin{aligned}\dot{z}_3 &= -\frac{1}{\tau}q + \frac{1}{\tau}\delta - \dot{\alpha}_2 \\ &= \frac{1}{\tau}\delta - \frac{1}{\tau}q - \frac{\partial\alpha_2}{\partial p}\dot{\hat{w}} - \frac{\partial\alpha_2}{\partial\hat{w}}\hat{\theta}^T\phi \\ &\quad -\left(\frac{\partial\alpha_2}{\partial p} + l_1\frac{\partial\alpha_2}{\partial\hat{w}} + \frac{\partial\alpha_2}{\partial\hat{\theta}}\Gamma\phi\right)\tilde{\xi}. \end{aligned} \tag{55}$$

The resulting control law δ is designed as

$$\begin{aligned}\delta &= \tau\Big(- C_3z_3 - (\hat{g}_0 - \hat{g}_1\hat{w})\,z_2 + \frac{\partial\alpha_2}{\partial p}\hat{w} + \frac{\partial\alpha_2}{\partial\hat{w}}\hat{\theta}^T\phi \\ &\quad + \frac{1}{\tau}q - k_3 \parallel \frac{\partial\alpha_2}{\partial p} + l_1\frac{\partial\alpha_2}{\partial\hat{w}} + \frac{\partial\alpha_2}{\partial\hat{\theta}}\Gamma\phi \parallel^2 z_3\Big). \end{aligned} \tag{56}$$

The derivative of the control Lyapunov function

$$U_3 = U_2 + \frac{1}{2}z_3^2 = \sum_{i=1}^{3}\frac{1}{2}z_i^2 + \frac{k}{2}\tilde{\xi}^2 + \frac{k}{2}\tilde{\theta}^T\mathbf{\Gamma}^{-1}\tilde{\theta} \tag{57}$$

along (54), (55) and (56) is

$$\begin{aligned}\dot{U}_3 &\leq -C_1z_1^2 - C_2z_2^2 - C_3z_3^2 \\ &\quad -\left(\frac{\partial\alpha_2}{\partial p} + l_1\frac{\partial\alpha_2}{\partial\hat{w}} + \frac{\partial\alpha_2}{\partial\hat{\theta}}\Gamma\phi\right)\tilde{\xi}z_3 \\ &\quad -k_3 \parallel (\frac{\partial\alpha_2}{\partial p} + l_1\frac{\partial\alpha_2}{\partial\hat{w}} + \frac{\partial\alpha_2}{\partial\hat{\theta}}\Gamma\phi) \parallel^2 z_3^2 \\ &\quad -k\left(l_1 - (h_0 - g_1q) - \frac{1}{4kk_1} - \frac{1}{4kk_2}\right)\tilde{\xi}^2 \\ &\leq -C_1z_1^2 - C_2z_2^2 - C_3z_3^2 \\ &\quad -k\left(l_1 - h_0 - \frac{1}{4kk_1} - \frac{1}{4kk_2} - \frac{1}{4kk_3}\right)\tilde{\xi}^2, \end{aligned} \tag{58}$$

where Young's inequality was used and $q > 0$. Let $\epsilon_l > 0$ and select l_1 satisfying

$$l_1 = \epsilon_l + h_0 + \frac{1}{4kk_1} + \frac{1}{4kk_2} + \frac{1}{4kk_3}. \tag{59}$$

Let

$$D = \{(p, \hat{w}, q)\,|q \geq 0\}, \tag{60}$$

U_3 is positive definite in D and

$$\dot{U}_3 \leq -C_1z_1^2 - C_2z_2^2 - C_3z_3^2 - \epsilon_l\tilde{\xi}^2 \tag{61}$$

is negative semidefinite definite in D, which proves that the system is stable. From the LaSalle-Yoshizawa Theorem, it further follows that $z_1, z_2, z_3, \tilde{\xi} \to 0$ as $t \to \infty$.
Since U_3 is non-increasing, we have

$$\begin{aligned}\parallel z_1 \parallel_2^2 &= \int_0^\infty |z_1(\tau)|^2 d\tau \\ &\leq \frac{1}{C_1}(U_3(0) - U_3(\infty)) \leq \frac{1}{C_1}U_3(0). \end{aligned} \tag{62}$$

Thus we have

$$\parallel z_1 \parallel_2 \leq \frac{1}{\sqrt{C_1}}\sqrt{U_3(0)}. \tag{63}$$

Theorem *1:* With the application of the adaptive nonlinear observer (T.4)-(T.5), the control law (T.6)-(T.8), and the parameter update law (T.9) in Table 1, the following statements hold for solutions in the set

$$\mathcal{A} = \{(p, \hat{w}, q)\,|q \geq 0\}, \tag{64}$$

the following statements hold:
• The resulting closed loop system is stable.
• The asymptotic tracking is achieved, i.e.,

$$\lim_{t\to\infty}[p - p_{ref}] = 0. \tag{65}$$

• The transient tracking error performance is given by

$$\parallel p(t) - p_{ref} \parallel_2 \leq \frac{1}{\sqrt{C_1}}\sqrt{U_3(0)}. \tag{66}$$

Remark *1:* The following conclusions can be obtained:
• The transient tracking performance for $\parallel p(t) - p_{ref} \parallel_2$ depends on the initial states and initial estimations.
• The transient tracking error performance can be improved by increasing the design parameter C_1.

V. Simulation Results

In this section we test our proposed backstepping controller on model (10)-(12). For simulation studies, the following values are selected as "true" parameters for the system: $h_0 = 1$, $h_1 = 50$, $g_0 = 0.125$, $g_1 = 5$, $a_1 = 0.025$, $b_0 = 3.5$, $\tau = 0.1$, which are not needed to be known in the controller design. The design objective is to stabilize p at the desired set point $p_{ref} = 3.8$.
With the proposed backstepping controller, we take the following set of design parameters: $C_1 = 0.8, C_2 = 0.8, C_3 = 0.2, k_1 = k_2 = k_3 = 0.01, l_1 = 0.8$ and $\Gamma = diag\{0.01, 10, 0.1, 0.1, 0.1, 0.5\}$. The initials are set as $p(0) = w(0) = q(0) = 0, \hat{\xi}(0) = 0$, and $\hat{\theta}(0) =$

Table 1: Adaptive Backstepping Control Scheme

Coordinate transformation:

$$z_1 = p - p_{ref} \quad \text{(T.1)}$$
$$z_2 = \hat{w} - \alpha_1 \quad \text{(T.2)}$$
$$z_3 = q - \alpha_2 \quad \text{(T.3)}$$

Adaptive nonlinear observer:

$$\hat{w} = \hat{\xi} + l_1 p, \quad \text{(T.4)}$$
$$\dot{\hat{\xi}} = -\hat{a}_1 p + \hat{h}_0 \hat{w} - \hat{h}_1 \hat{w}^3 + \hat{g}_0 q - \hat{g}_1 q \hat{w} + \hat{f}_1 - l_1 \hat{w} \quad \text{(T.5)}$$

Control law:

$$\delta = \tau\Big(- C_3 z_3 - (\hat{g}_0 - \hat{g}_1 \hat{w})\, z_2 + \frac{1}{\tau} q + \frac{\partial \alpha_2}{\partial \hat{w}} \hat{\theta}^T \phi + \frac{\partial \alpha_2}{\partial p} \hat{w} - k_3 \parallel \frac{\partial \alpha_2}{\partial p} + l_1 \frac{\partial \alpha_2}{\partial \hat{w}} + \frac{\partial \alpha_2}{\partial \hat{\theta}} \Gamma \phi \parallel^2 z_3 \Big) \quad \text{(T.6)}$$

with

$$\alpha_1 = -\left(C_1 + k_1\right) z_1 \quad \text{(T.7)}$$

$$\alpha_2 = \frac{1}{(\hat{g}_0 - \hat{g}_1 \hat{w})} \Big(- \left(C_2 + k_2 (l_1 + C_1 + k_1)^2\right) z_2 - z_1 - (C_1 + k_1)\hat{w} + \hat{a}_1 p - \hat{f}_1 - \hat{h}_0 \hat{w} + \hat{h}_1 \hat{w}^3 \Big) \quad \text{(T.8)}$$

Parameter update law:

$$\hat{\theta} = \hat{\sigma} - \eta\left(p, \hat{\xi}, q\right) \quad \text{(T.9)}$$
$$\dot{\hat{\sigma}} = \frac{\partial \eta}{\partial p} \hat{w} + \frac{\partial \eta}{\partial \hat{\xi}} \dot{\hat{\xi}} + \frac{\partial \eta}{\partial q} \dot{q} \quad \text{(T.10)}$$
$$\eta\left(p, \hat{\xi}, q\right) = \mathbf{\Gamma} \begin{bmatrix} \frac{1}{2}p^2 \\ -\left(\hat{\xi} + \frac{l_1}{2} p\right) p \\ \frac{1}{4 l_1}\left(\hat{\xi} + l_1 p\right)^4 \\ -qp \\ q\left(\hat{\xi} + \frac{l_1}{2} p\right) p \\ -p \end{bmatrix} \quad \text{(T.11)}$$

$[0, h0 * 1.2, h1 * 0.5, g0, g1 * 1.2, f1 * 0.5]$, respectively. Figure 4 illustrates the backstepping controller applied for stabilization at reference pressure $p_{ref} = 3.8$, where u is the choke opening and $\delta = e^{-d_1 u}, d_1 = 10$. The initial is $u_0 = 0.10$ and the actual control is applied from $t = 100$. The simulation results verify our theoretical findings.

VI. Conclusions

This paper presents a nonlinear adaptive observer control applied to stabilize riser induced slugging. A simple empirical model is developed that describes the qualitative behavior of the downhole pressure in case of severe riser slugging. A nonlinear adaptive observer is developed for state

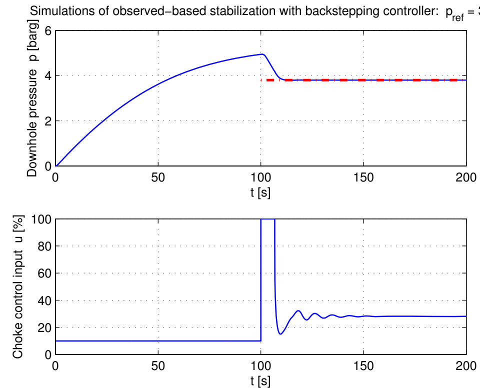

Fig. 4. Simulations illustrating stabilization using proposed control.

estimations. To obtain such an observer, a new parametrization of the state observer for the plant is proposed and a new parameter adaptation law is developed. The controller is designed by an integrator backstepping approach to stabilize the downhole pressure and eliminate riser slugging. The stabilization in the unstable region is demonstrated by the proposed control. For the design and implementation of the controller, no knowledge is assumed on the unknown system parameters. It is shown that the proposed controller not only can guarantee asymptotic stability, but also transient performance.

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