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Adaptive Robust H_∞ Dynamic Output Feedback for Continuous Time-Varying Uncertain Systems

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Abstract— **The problem of designing an adaptive robust H_∞ dynamic output feedback controller for uncertain linear systems is considered. The uncertainties are assumed to be time-varying, unknown, but bounded, which appear affinely in the matrices of system model. Based on the online estimations of uncertain parameters, a robust dynamic output feedback controller with variable gains is constructed to compensate the effect of uncertainty on systems. An adaptive mechanism is introduced to estimate uncertain parameters according to the designed adaptive laws and to enhance system performance. New sufficient conditions with less conservativeness than those of traditional robust controllers are also derived to guarantee the stability and H_∞ performance of the closed-loop systems. A numerical example is given to illustrate the effectiveness of the proposed method.**

Key words: Robust H_∞ control; dynamic output feedback; linear matrix inequalities (LMIs); indirect adaptive control.

I. Introduction

In many applications, modeling errors and system uncertainties in the plant model are inevitable. For preciseness, a design technique must accommodate these errors and uncertainties to be practically feasible. In the past few decades, much research works have been focused on the robust control of linear systems with parameter uncertainties. A particular uncertainty representation is called norm bounded uncertainty, where the mathematical model of the uncertain system explicitly exhibits a nominal model located at the center of the hyper ellipsoid of uncertainty in the parameter space. Riccati equation approaches have been proposed for linear systems subject to norm-bounded parameter uncertainty in the state-space model [17], [18] [23] and [24]. Another uncertain representation is convex polytopic uncertainty [7]. Recently, system with polytopic-type parameter uncertainty have been treated in [3], [12], [15], [16] using linear matrix inequality (LMI) methodologies, which are computationally simple and numerically reliable for solving convex optimization problems [2], [4]-[6], [9] and [20]. However, most of the above-mentioned works are about robust state feedback control of uncertain linear systems, where the system state is assumed to be measurable. Thus, a severe restriction is imposed on the class of systems to which they are applicable.

Recently, some attempts have been made to design robust dynamic output feedback controllers [10], [11], [13], [21], [22], [24] either for norm bounded uncertainty or polytopic uncertainty. For continuous-time linear systems with polytopic uncertainty, the resultant necessary and sufficient conditions are generally represented in terms of bilinear matrix inequalities (BLMIs) and are nonconvex for most design objectives. Only sufficient conditions can be derived for computing dynamic output feedback control laws using convex optimization method. In [10], a locally optimal dynamic output feedback controller is proposed based on iterative algorithm. An initial feasible robust output-feedback controller is obtained by a two-step procedure, which proposes a convex optimation design method for traditional robust output feedback controller with fixed gain.

Adaptive method is one of the effective method to deal with parameter uncertainty [1] and [8]. They rely on the potential of adjustments of uncertain parameters to assure stability of closed-loop systems. Most of the results in adaptive robust control are based on model reference adaptive control (MRAC)[14], [19], [25], where the outputs of closed-loop systems can track the pre-described referent outputs. Unfortunately, this adaptive method is not easily extended to treat performance tasks such as H_2/H_∞ indexes when the external disturbance exists.

In this paper, we proposed a novel robust H_∞ dynamic output feedback controller design method for uncertain linear continuous-time systems. The uncertainties are assumed to be time-varying, unknown, but bounded, which appear affinely in the matrices of system model. By the introduction of adaptive mechanism, the designed controller gains are variable and online adjusting based on estimation of uncertain parameters. Due to the successful combination between indirect adaptive method and LMI approach, sufficient conditions with less conservativeness than those of traditional robust controller are derived. The effectiveness of the proposed approach is demonstrated on a numerical example.

This paper is organized as follows. Section 2 introduces the problem and some preliminaries. It is followed by the adaptive robust H_∞ dynamic output feedback controller design method in Section 3. An illustrative example is given

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in Section 4 to demonstrate the proposed method. Finally, Section 5 concludes the paper.

II. Problem Statement and Preliminaries

A. Problem Statement

Consider a linear uncertain model described by

$$\begin{aligned} \dot{x}(t) &= A(\delta(t))x(t) + B(\delta(t))u(t) + B_\omega\omega(t) \\ z(t) &= C_1x(t) + D_{12}u(t) \\ y(t) &= C_2x(t) + D_{21}\omega(t) \end{aligned} \quad (1)$$

where $x(t) \in R^n$ is the state, $u(t) \in R^m$ is the control input, $y(t) \in R^p$ is the measured output and $z(t) \in R^q$ is the regulated output,respectively. $\omega(t) \in L_2[0,\infty)$ is the exogenous disturbance. And

$$A(\delta(t)) = A_0 + \sum_{i=1}^{N_0}\delta_i(t)A_i, \quad B(\delta(t)) = B_0 + \sum_{i=1}^{N_0}\delta_i(t)B_i.$$

$A_0, A_i, B_0, B_i, B_\omega, D_{12}, C_1, C_2$ and D_{21} are known constant matrices of appropriate dimensions. $\delta_i(t), i = 1\cdots N_0$ are unknown time-varying uncertainty, which satisfy $\underline{\delta}_i \le \delta_i(t) \le \bar{\delta}_i$. Here $\underline{\delta}_i$ and $\bar{\delta}_i$ are known lower and upper bounds of $\delta_i(t)$, respectively. Since $C_2 \in R^{p\times n}$ and $\text{rank}(C_2) = p_1 \le p$, then there exists a matrix $T_c \in R^{p_1\times p}$ such that $\text{rank}(T_cC_2) = p_1$. Furthermore, there exists a matrix C_{cn} such that rank $\begin{bmatrix} T_cC_2 \\ C_{cn}\end{bmatrix} = n$. Denote $T_{cn} = \begin{bmatrix} T_cC_2 \\ C_{cn}\end{bmatrix}^{-1}$.
For traditional robust control, the following dynamic output feedback controller is usually used.

$$\begin{aligned} \dot{\xi}_1(t) &= A_{Kf}\xi_1(t) + B_{Kf}y(t) \\ u(t) &= C_{Kf}\xi_1(t) \end{aligned} \quad (2)$$

then combing (2) with (1), it follows

$$\begin{aligned} \dot{x}_{e1}(t) &= A_{e1}x_{e1}(t) + B_{e1}\omega(t) \\ z(t) &= C_{e1}x_{e1}(t) \end{aligned} \quad (3)$$

where $x_{e1}(t) = [x^T(t)\ \xi_1^T(t)]^T$, and

$$A_{e1} = \begin{bmatrix} A(\delta) & B(\delta)C_{Kf} \\ B_{Kf}C_2 & A_{Kf}\end{bmatrix}, \quad B_e = \begin{bmatrix} B_\omega \\ B_{Kf}D_{21}\end{bmatrix},$$

$$C_{e1} = [C_1 \quad D_{12}C_{Kf}].$$

In this paper, the following dynamic output feedback controller with variable gains is considered.

$$\begin{aligned} \dot{\xi}(t) &= A_K(\hat{\delta}(t))\xi(t) + B_K(\hat{\delta}(t))y(t) \\ u(t) &= C_K(\hat{\delta}(t))\xi(t) \end{aligned} \quad (4)$$

where $\hat{\delta}_i(t)(i = 1\cdots N_0)$ are the estimations of $\delta_i(t)$, which are obtained according to the introduced adaptive mechanism. $A_K(\hat{\delta}) \in R^{n\times n}$, $B_K(\hat{\delta}) \in R^{n\times p}$ and $C_K(\hat{\delta}) \in R^{m\times n}$ have the following forms, that is

$$A_K(\hat{\delta}) = A_{K0} + \sum_{i=1}^{N_0}\hat{\delta}_iA_{Ki}, \quad B_K(\hat{\delta}) = B_{K0} + \sum_{i=1}^{N_0}\hat{\delta}_iB_{Ki},$$

$$C_K(\hat{\delta}) = C_{K0} + \sum_{i=1}^{N_0}\hat{\delta}_iC_{Ki}$$

where A_{K0}, A_{Ki}, B_{K0}, B_{Ki}, C_{K0}, C_{Ki} are fixed parameter matrices to be designed.
Applying the dynamic output feedback controller (4) to the system (1), it follows

$$\begin{aligned} \dot{x}_e(t) &= A_ex_e(t) + B_e\omega(t) \\ z(t) &= C_ex_e(t) \end{aligned} \quad (5)$$

where $x_e(t) = [x^T(t)\ \xi^T(t)]^T$,

$$A_e = \begin{bmatrix} A(\delta) & B(\delta)C_K(\hat{\delta}) \\ B_K(\hat{\delta})C_2 & A_K(\hat{\delta})\end{bmatrix},$$

$$B_e = \begin{bmatrix} B_\omega \\ B_K(\hat{\delta})D_{21}\end{bmatrix}, \quad C_e = [C_1 \quad D_{12}C_K(\hat{\delta})]$$

Control objective: Find an adaptive robust H_∞ controller (4) via dynamic output feedback such that the closed-loop system with the above-mentioned time-varying uncertainty is robustly stable and its H_∞ disturbance attenuation index is minimized.

B. Preliminaries

The following lemma presents a condition for the system (3) to have robust H_∞ performance bound.

Lemma 1: Consider the system described by (3), and let $\gamma > 0$ be given constant. Then the following statements are equivalent:
(i) there exist a symmetric matrix $X > 0$ and a dynamic output feedback controller **K** described by (2) such that

$$A_{e1}^TX + XA_{e1} + \frac{1}{\gamma^2}XB_{e1}B_{e1}^TX + C_{e1}^TC_{e1} < 0 \quad (6)$$

holds for $\delta_i \in [\underline{\delta_i}, \bar{\delta_i}]$
(ii) there exist symmetric matrices $0 < N < Y$, and a dynamic output feedback controller described by (2) with $A_{Kf} = A_{Ke1}$, $B_{Kf} = B_{Ke1}$ and $C_{Kf} = C_{Ke1}$ such that

$$V_a = \begin{bmatrix} V_{11} & V_{12} & YB_\omega - NB_{Ke1}D_{21} & C_1^T \\ * & V_{22} & -NB_\omega + NB_{Ke1}D_{21} & C_{Ke1}^TD_{12}^T \\ * & * & -\gamma^2I & 0 \\ * & * & * & -I\end{bmatrix} < 0 \quad (7)$$

holds for $\delta_i \in [\underline{\delta_i}, \bar{\delta_i}]$ where

$$\begin{aligned} V_{11} &= YA(\delta) - NB_{Ke1}C_2 + (YA(\delta) - NB_{Ke1}C_2)^T \\ V_{12} &= YB(\delta)C_{Ke1} - NA_{Ke1} - A^T(\delta)N + C_2^TB_{Ke1}^TN^T \\ V_{22} &= -NB(\delta)C_{Ke1} + NA_{Ke1} \\ &\quad + [-NB(\delta)C_{Ke1} + NA_{Ke1}]^T \end{aligned}$$

Proof: Due to the limitation of space, the proof is omitted. □

Remark 1: It should be noted that conditions (7) are not convex. But when C_{Kf} is given, and NA_{Kf} and NB_{Kf} are defined as new variables, they become LMIs.

Algorithm 1: Let γ denotes the robust H_∞ performance bound of the closed-loop system (3). Then γ is minimized by
Step 1.

$$\min \eta \quad \text{s.t.} \quad X > 0$$

$$\begin{bmatrix} \Gamma & B_\omega & C_1X + D_{12}Y_0 \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0 \quad (8)$$

where $\Gamma = A(\delta)X + B(\delta)Y_0 + (A(\delta)X + B(\delta)Y_0)^T$. Here condition (8) is the classical robust H_∞ control results via state feedback [2]. The optimal solutions are denoted as X_{opt} and Y_{0opt}. Let $C_{Kf} = Y_{0opt}X_{opt}^{-1}$.
Step 2. Let $NA_{Kf} = \bar{A}_{Kf}$ and $NB_{Kf} = \bar{B}_{Kf}$.

$$\min \eta \text{ s.t. } 0 < N < Y \quad (7)$$

where $\eta = \gamma^2$. Then the resultant controller gains are $A_{Kf} = \bar{A}_{Kf}N^{-1}$, $B_{Kf} = \bar{B}_{Kf}N^{-1}$, $C_{Kf} = Y_{0opt}X_{opt}^{-1}$.
Remark 2: Algorithm 1 gives a method for the traditional robust dynamic output controller design by two-step optimizations. Step 1 is performed to find a C_{Kf}, which solves the corresponding design problem via state feedback. With the C_{Kf} fixed, controller parameter matrices A_{Kf} and B_{Kf} can be obtained by performing Step 2. Such a two-step procedure is also used in [10] to obtain an initial feasible robust output feedback controller for beginning the proposed iterative algorithm.

III. Dynamic Output Feedback

In this section, the problem of designing an adaptive robust H_∞ controller via dynamic output feedback for system (1) is studied. An adaptive mechanism is introduced to reduce the conservativeness compared with traditional robust control.

Theorem 1: The closed-loop system (5) is stable and H_∞ disturbance attenuation is no large than γ, if there exist matrices $0 < N < Y, A_{K0}, A_{Ki}, B_{K0}, B_{Ki}, C_{K0}, C_{Ki}$, $i = 1 \cdots N_0$ such that for $\delta_i(t), \hat{\delta}_i(t) \in [\underline{\delta}_i, \bar{\delta}_i]$ the following matrix inequalities hold:

$$\begin{bmatrix} T_1 + T_1^T & T_2 & T_4 & C_1^T \\ * & T_3 + T_3^T & T_5 & C_K^T(\hat{\delta})D_{12}^T \\ * & * & -\gamma^2 I + T_6 & 0 \\ * & * & * & -I \end{bmatrix} < 0 \quad (9)$$

with

$$T_1 = YA(\delta) - NB_K(\delta)C_2 - \sum_{i=1}^{N_0}(\hat{\delta}_i - \delta_i)N_3^T NB_{Ki}C_2$$

$$T_2 = YM_1 - NA_K(\delta) - A^T(\delta)N + C_2^T B_K^T(\delta)N + \sum_{i=1}^{N_0}(\hat{\delta}_i - \delta_i)N_3^T(YB_0C_{Ki} - NA_{Ki})$$

$$T_3 = -NM_1 + NA_K(\delta),$$

$$T_4 = \sum_{i=1}^{N_0}(\hat{\delta}_i - \delta_i)[C_2^T B_{Ki}^T NN_2 + NB_{Ki}D_{21} - N_3^T NB_{Ki}D_{21}] + YB_\omega - NB_K(\hat{\delta})D_{21}$$

$$T_5 = \sum_{i=1}^{N_0}(\hat{\delta}_i - \delta_i)[-(YB_0C_{Ki} - NA_{Ki})^T N_2 - NB_{Ki}D_{21}] - NB_\omega + NB_K(\hat{\delta})D_{21}$$

$$T_6 = \sum_{i=1}^{N_0}(\hat{\delta}_i - \delta_i)[N_2^T NB_{Ki}D_{21} + (N_2^T NB_{Ki}D_{21})^T]$$

$$N_1 = T_{cn}\begin{bmatrix} T_c \\ 0 \end{bmatrix}, N_2 = T_{cn}\begin{bmatrix} T_cD_{21} \\ 0 \end{bmatrix}, N_3 = T_{cn}\begin{bmatrix} 0 \\ C_{cn} \end{bmatrix},$$

$$A_K(\delta) = A_{K0} + \sum_{i=1}^{N_0}\delta_i A_{Ki}, \quad B_K(\delta) = B_{K0} + \sum_{i=1}^{N_0}\delta_i B_{Ki}$$

$$M_1 = B_0C_K(\delta) + \sum_{i=1}^{N_0}B_i\delta_iC_K(\hat{\delta}),$$

and also $\hat{\delta}_i(t)$ is determined according to the adaptive law

$$\hat{\delta}_i = \begin{cases} \bar{\delta}_i, & \text{if } M_{2i} < 0 \\ \underline{\delta}_i, & \text{if } M_{2i} \geq 0 \end{cases}, \quad i = 1 \cdots N_0 \quad (10)$$

$$M_{2i} = \xi^T(-NB_0C_{Ki} + NA_{Ki})\xi + \xi^T NB_{Ki}y - y^T N_1^T NB_{Ki}y + y^T N_1^T(YB_0C_{Ki} - NA_{Ki})\xi$$

Then the dynamic output feedback controller gains of the form (4) are given by $A_{K0}, A_{Ki},, B_{K0}, B_{Ki}, C_{K0}, C_{Ki}, i = 1 \cdots N_0$
Proof: Due to the limitation of space, the proof is omitted here. □
Remark 3: Theorem 1 presents sufficient conditions for adaptive robust H_∞ controller design via dynamic output feedback. Generally, (9) is not LMIs. But when C_{K0} is given, and NA_{K0}, NA_{Ki}, NB_{K0} and NB_{Ki} are defined as new variables, (9) becomes LMIs and linearly depends on δ_i and $\hat{\delta}_i$.
For the comparison between Theorem 1 and Lemma 1, we have the following theorem
Theorem 2: If the conditions in Lemma 1 hold for the closed-loop system (3) with traditional robust dynamic output feedback controller (2), then the conditions in Theorem 1 hold for the closed-loop system (5) with adaptive robust dynamic output feedback controller (4).
Proof: Due to the limitation of space, the proof is omitted here. □

Remark 4: Theorem 2 shows that the adaptive robust H_∞ controller design method given in Theorem 1 is less conservative than that given in Lemma 1 for the traditional robust H_∞ controller design method.

The following algorithm is to optimize the robust H_∞ performance of the closed-loop systems (5).
Algorithm 2: Let γ denotes the robust H_∞ performance bound of the closed-loop system (5). Then γ is minimized by
Step 1. Choose $C_{K0} = C_{Kf}, C_{Ki} = 0$ with C_{Kf} being a solution to the problem of traditional robust dynamic output controller design via Algorithm 1.
Step 2.

$$\min \eta \quad \text{s.t.} \quad 0 < N < Y \text{ and } (9),$$

where $\eta = \gamma^2$. Then the resultant controller gains are $A_{K0} = \bar{A}_{K0}N^{-1}$, $A_{Ki} = \bar{A}_{Ki}N^{-1}$, $B_{K0} = \bar{B}_{K0}N^{-1}$, $B_{Ki} = \bar{B}_{Ki}N^{-1}$, $C_{K0} = C_{Kf}, C_{Ki} = 0$, $i = 1 \cdots N_0$.

In order to obtain $C_{K0}, C_{Ki}, i = 1 \cdots N_0$ via adaptive robust state feedback H_∞ controller, next we derive the corresponding conditions for adaptive robust state feedback H_∞ controller.

The corresponding adaptive state feedback controller structure is chosen as

$$u(t) = K(\hat{\delta}) = (K_0 + \sum_{i=1}^{N} \hat{\delta}_i(t)K_i)x(t) \tag{11}$$

Then the closed-loop system is given by

$$\begin{aligned} \dot{x}(t) &= (A(\delta(t)) + B(\delta(t))K(\hat{\delta}))x(t) + B_\omega \omega(t) \\ z(t) &= (C_1 + D_{12}K(\hat{\delta}))x(t) \end{aligned} \tag{12}$$

Lemma 2: For all $\delta_i \in [\underline{\delta}_i \ \bar{\delta}_i]$, the closed-loop system (12) is stable and H_∞ performance index is no large than a given constant γ, if there exist matrices $X > 0, Y_0, Y_i, i = 1 \cdots N$ such that for all $\delta_i(t), \hat{\delta}_i(t) \in \{\underline{\delta}_i, \bar{\delta}_i\}$

$$\begin{bmatrix} M_0 + M_0^T & B_\omega & C_1X + D_{12}Y(\hat{\delta}) \\ * & -\gamma^2 I & 0 \\ * & * & -I \end{bmatrix} < 0 \tag{13}$$

where

$$\begin{aligned} M_0 = {} & (A_0 + \sum_{i=1}^{N} \delta_i(t)A_i)X + (B_0 + \sum_{i=1}^{N} \delta_i(t)B_i)Y_0 \\ & + B_0 \sum_{i=1}^{N} \delta_i(t)Y_i + \sum_{i=1}^{N} \delta_i(t)B_i \sum_{i=1}^{N} \hat{\delta}_i(t)Y_i \end{aligned}$$

and also $\hat{\delta}_i(t)$ is determined according to the adaptive law for $i = 1 \cdots N_0$

$$\hat{\delta}_i = \begin{cases} \bar{\delta}_i, & \text{if } x^T P B_0 K_i x \le 0 \\ \underline{\delta}_i, & \text{if } x^T P B_0 K_i x > 0 \end{cases} \tag{14}$$

where $P = X^{-1}, K_0 = Y_0X^{-1}, K_i = Y_iX^{-1}, i = 1 \cdots N$. Then the controller gain is given by

$$K(\hat{\delta}) = Y_0X^{-1} + \sum_{i=1}^{N} \hat{\delta}_i Y_i X^{-1}.$$

Proof: Due to the limitation of space, the proof is omitted. □

Remark 5: It is easy to see if the condition (8) is feasible, then the condition (13) in Lemma 2 is feasible with $X = X_0, Y_0 = Y_{00}$ and $Y_i = 0, i = 1 \cdots N$.

Another algorithm can also be proposed to design adaptive robust H_∞ controller according to Lemma 2 via state feedback, that is

Algorithm 3: Let γ denotes the robust H_∞ performance bound of the closed-loop system (5). Then γ is minimized by

Step 1. Choose $C_{K0} = K_0, C_{Ki} = K_i$ with K_0, K_i being a solution to the problem of adaptive robust controller design via state feedback, i.e., Lemma 2.

Step 2.

$$\min \eta \quad \text{s.t.} \quad 0 < N < Y \text{ and } (9),$$

where $\eta = \gamma^2$. Then the resultant controller gains are $A_{K0} = \bar{A}_{K0}N^{-1}$, $A_{Ki} = \bar{A}_{Ki}N^{-1}$, $B_{K0} = \bar{B}_{K0}N^{-1}$, $B_{Ki} = \bar{B}_{Ki}N^{-1}$, $C_{K0} = K_0, C_{Ki} = K_i$, $i = 1 \cdots N_0$.

Remark 6: Similar to Algorithm 1, Algorithm 2 and Algorithm 3 are also composed of two-step optimizations, where the purpose of Step 1 is to determine state feedback gain C_{K0}, C_{Ki}. When we choose $C_{K0} = C_{Kf}, C_{Ki} = 0$ with C_{Kf} being a solution to the problem of robust dynamic output controller design via Algorithm 1, then by Theorem 2, it follows that Algorithm 2 can give less conservative design than Algorithm 1. Due to different C_{K0} is chosen in Algorithm 1 and Algorithm 3, it is difficult to conclude Algorithm 3 can get less conservativeness results in theory. However, in the numerical example of next section, the resultant H_∞ performance indices in Algorithm 2 and Algorithm 3 are both smaller than that in Algorithm 1.

IV. Numerical Example

Consider a linear system (1) with time-varying uncertainty satisfying

$$A(\delta(t)) = \begin{bmatrix} -5 & 2 \\ 1 & -2 \end{bmatrix} + \delta_1(t)\begin{bmatrix} 1 & 0.2 \\ 0 & -1 \end{bmatrix} + \delta_2(t)\begin{bmatrix} -1 & 0.5 \\ 0.6 & 0.1 \end{bmatrix},$$

$$B(\delta(t)) = \begin{bmatrix} -1 & 3 \\ 0 & 2 \end{bmatrix} + \delta_1(t)\begin{bmatrix} 0.2 & 0.5 \\ 0 & 3 \end{bmatrix} + \delta_2(t)\begin{bmatrix} 0 & 0 \\ -0.5 & 1 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} 2 & 0 \\ 0 & 4 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \ D_{12} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.5 & 0 \\ 0 & 1 \end{bmatrix}, \ B_\omega = \begin{bmatrix} 0 & -1 \\ 0 & 1 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0 & 2 \\ 0 & 1 \end{bmatrix}, \quad D_{21} = \begin{bmatrix} 3 & 0 \\ 1 & 0 \end{bmatrix}, \quad x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

with $\delta_1(t) = 0.5\text{sin(t)}$ and $\delta_2(t) = \text{cos(t)}$.

Using Matlab LMI tool box [5], Algorithm 1 and Algorithm 2, we get the H_∞ performance index is 6.6616 with the adaptive robust controller while that of traditional robust controller is 8.1946. Just as the theory has proved the adaptive robust H_∞ controller design method is less conservative than the traditional robust controller design method. Moreover, the corresponding H_∞ performance index is 6.4416 obtained by Algorithm 3.

In order to see the effectiveness of our method more clearly, some simulation results are also given. Here the disturbance $\omega(t) = \begin{bmatrix} \omega_1(t) & \omega_2(t) \end{bmatrix}^T$ that used is

$$\omega_1(t) = \omega_2(t) = \begin{cases} 3, & 2 \le t \le 3 \text{ (seconds)} \\ 0 & \text{otherwise} \end{cases}$$

Figure 1 and Figure 2 are the responses curves with adaptive robust H_∞ controller base on Algorithm 2 and traditional robust H_∞ controller base on Algorithm 1, respectively. It is easy to see our adaptive robust H_∞ controller has more disturbance attenuation ability than that of the traditional robust controller as theory has proved. While

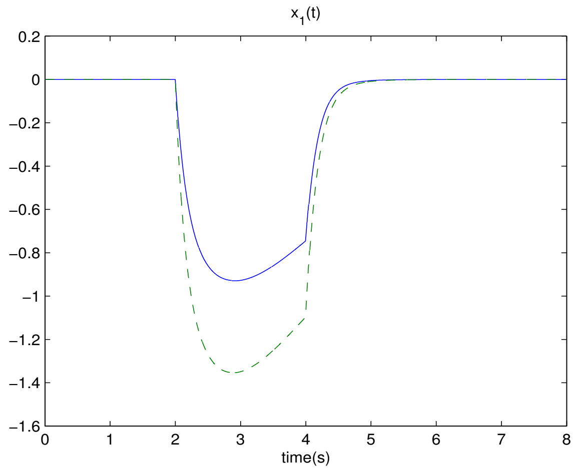

Fig. 1. Response curve of the first state with adaptive robust controller based on Algorithm 2 (solid) and traditional robust controller based on Algorithm 1 (dashed).

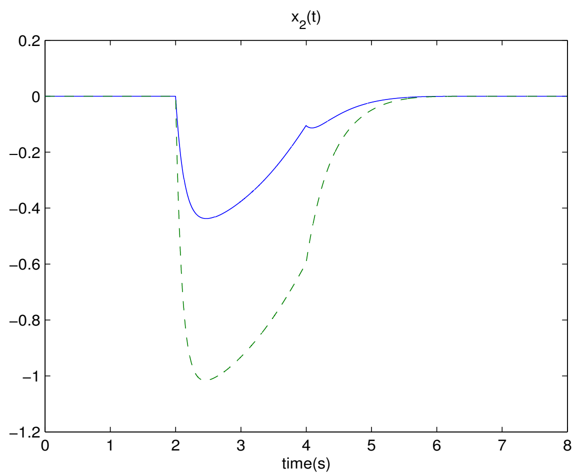

Fig. 2. Response curve of the second state with adaptive robust controller on Algorithm 2 (solid) and traditional robust controller based on Algorithm 1 (dashed).

Figure 3 and Figure 4 are the responses curves with adaptive robust H_∞ controller base on Algorithm 3 and traditional robust H_∞ controller base on Algorithm 1, respectively. In this example, the adaptive robust controller designed by Algorithm 3 performs better than the traditional robust controller.

V. Conclusions

In this paper, we deal with the robust H_∞ controller design problem via dynamic output feedback for uncertain linear systems. The uncertainties are assumed to be time-varying, unknown, but bounded, which appear affinely in the matrices of system model. An adaptive mechanism is introduced to construct a robust H_∞ controller with variable gain and to reduce the conservativeness inherent in traditional robust H_∞ controller design. The proposed controller gains are adjustable and updated automatically according to the online estimations of uncertain parameters. More relaxed sufficient conditions than those of traditional robust H_∞ controller are given in the framework of LMIs. A numerical example is also given to illustrate the effectiveness of the proposed method.

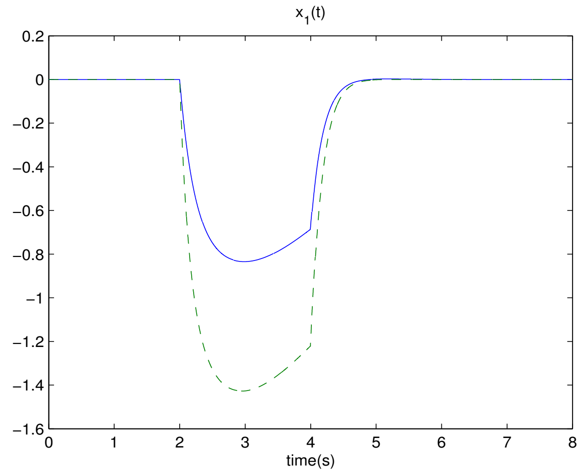

Fig. 3. Response curve of the first state with adaptive robust controller based on Algorithm 3 (solid) and traditional robust controller based on Algorithm 1 (dashed).

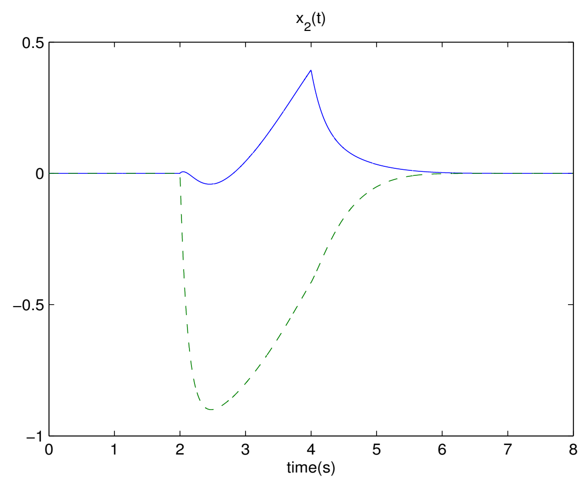

Fig. 4. Response curve of the second state with adaptive robust controller based on Algorithm 3 (solid) and traditional robust controller based on Algorithm 1 (dashed).

References

[1] K. J. Astrom and B. Wittenmark, *Adaptive Control*, Pearson Education, Inc., 1995.
[2] S. Boyd, L. El Ghaoui, E. Feron and V. Balakrishan, *Linear Matrix Inequalities in Systems and Control Theory*, Philadelphia, PA: SIAM, 1994.
[3] M. Chilali, P. Gahinet and P. Apkarian, "Robust pole placement in LMI regions", *IEEE Transactions on Automatic Control*, vol. 44 no. 12, pp. 2257-2269, 1999.
[4] G. E. Dullerud and F. Paganini, *A course in robust control theorył a convex approach*, New York: Springer, 2000.
[5] P. Gahinet, A. Nemirovski, A. Laub and M. Chilali, *LMI control toolbox users guide*, Natick: MathWorks, Inc. P. Gahinet, 1995.
[6] G. Garcia, G. Daafouz and J. Bernussou, "The infinite time near optimal decentralized regulator problem for singularly perturbed systems: a convex optimization approach", *Automatica*, vol. 38, pp. 1397-1406, 2002.

[7] J. C. Geromel, P. L. D. Peres, and J. Bernussou, "On a convex parameter space method for linear control design of uncertain systems," *SIAM J. Contr. Optimiz.*, vol. 29, no. 2, pp. 381-402, 1991.
[8] P. A. Ioannou and J. Sun, *Robust Adaptive Control,* Prentice-Hall International, Inc, 1996.
[9] T. Iwasaki and R. E. Skelton, "All controllers for the general H_∞ control problem: LMI existence conditions and state space formulas," *Automatica*, vol. 30, pp. 1307-1317, 1994.
[10] S. Kanev, C. Scherer, M. Verhaegen and B. De Schutter, "Robust output-feedback controller design via local BMI optimization", *Automatica*, vol. 40, pp. 1115-1127, 2004.
[11] I. E. Kose and F. Jabbari, "Robust control of linear systems with real parametric uncertainty", *Automatica*, vol. 35, pp. 679-687, 1999.
[12] M. V., Kothare, V. Balakrishnan, and M. Morari, "Robust constrained model predictive control using linear matrix inequalities", *Automatica*, vol. 32, pp. 1361-1379, 1996.
[13] M. S. Mahmoud and L. H. Xie, "Positive real analysis and synthesis of uncertain discrete-time systems", *IEEE Transactions on Circuits and SystemsłPart I: Fundamental Theory and Applications*, vo. 47, pp. 403-406, 2000.
[14] B. M. Mirkin and P. Gutman, "Output feedback model reference adaptive control for multi-inputCmulti-output plants with state delay", *Systems & Control Letters*, vol. 54, pp. 961-972, 2005.
[15] R. M. Palhares, D. C. W. Ramos and P. L. D. Peres, "Alternative LMIs characterization of H_2 and central H_∞ discrete-time controllers, " In *Proceedings of the 35th IEEE conference on decision and control*, Kobe, Japan, pp. 1459-1496, 1996.
[16] R. M. Palhares, R. H. C. Takahashi and P. L. D. Peres, "H_2 and H_∞ guaranteed costs computation for uncertain linear systems", *International Journal of Systems Science*, vol. 28, no. 2, pp. 183-188, 1997.
[17] I. R. Petersen, "A stabilisation algorithm for a class of uncertain linear systems", *Syst. Control Lett.*, vol. 8, pp.351-357, 1987.
[18] I. R. Petersen, "Guaranteed cost LQG control of uncertain linear systems", *IEE Proc., Control Theory Appl.*, vol. 142, no. 2, pp. 95-102.
[19] Z. H. Qu, J. F. Dorsey and D. M. Dawson, " Model reference robust control of a class of SISO systems", *IEEE Transactions on Automatic Control*, vol. 39, pp. 2219-2234, 1994.
[20] C. Scherer, P. Gahinet and M. Chilali, "Multiobjective output feedback control via LMI optimization." *IEEE Trans. on Automatic Control*, vol. 42, no. 7, pp. 896-911, 1997.
[21] V. Suplin and U. Shaked, "Robust H_∞ output-feedback control of linear discrete-time systems", *Systems & Control Letters*, vol. 54, pp. 799-808, 2005.
[22] J. G. VanAntwerp and R. D. Braatz, "A tutorial on linear and bilinear matrix inequalities", *Journal of Process Control*, vol. 10, pp. 363-385, 2000.
[23] L. Xie and C. E. de Souza, "Robust H_∞ control for linear time0invariant systems with norm-bound uncertainty in the input matrix", *Syst. Control Lett.,*, vol. 14, pp. 389-396, 1990.
[24] L. Xie, M. Fu and C. E. de Souza, "H_∞ control and quadratic stabilization of systems with parameter uncertainty via output feedback", *IEEE Transactions on Automatic Control*, vol. 37, pp. 1253-1256, 1992.
[25] F. Zheng, Q. G. Wang and T. H. Lee, "Adaptive robust control of uncertain time delay systems", *Automatica*, vol. 41, pp. 1375-1383, 2005.