2008 American Control Conference
Westin Seattle Hotel, Seattle, Washington, USA
June 11-13, 2008

High Gain Observer-based Fault Estimation for Nonlinear Networked Control Systems

Zehui Mao, Bin Jiang, Peng Shi, and Vincent Cocquempot

***Abstract*— In this paper, an observer-based fault estimation (FE) method is presented for a class of nonlinear networked control systems (NCSs) with Markov transfer delays. First, the nonlinear NCSs are modelled by nonlinear discrete Takagi-Sugeno (T-S) fuzzy model with modelling uncertainty. Under some geometric conditions, the proposed nonlinear T-S model can be transformed into two subsystems with one of them having backstepping form. Then, the discrete nonlinear observer is designed to provide the estimation of unmeasurable state and the modelling uncertainty, which is used to construct a fault estimation algorithm. Finally, an example is included to show the efficiency of the proposed method.**

***Index Terms*— Nonlinear networked control systems, Takagi-Sugeno fuzzy model, fault estimation, observer.**

I. Introduction

Fault detection and identification algorithms and their applications to a wide range of industrial processes have been the subjects of intensive research over the past three decades. Fruitful results can be found in several books and many papers, e.g. [1]-[2]. As the rapid development of communication networks, recently, a great amount of effort has been made on fault detection problem of networked control systems (NCSs).

In networked control systems, a controller and spatially distributed sensors/actuators are grouped into network nodes and communicate by exchanging packet-based messages via a network. NCSs have several advantages over the classical control systems, such as low cost, simple installation and maintenance, increased system agility and reduced system wiring. However, NCSs require novel control design method due to network's presence in the closed loop. Modelling, analysis, and design of NCSs have received increasing attention in recent years, see [3]-[5]. For some representative works on fault detection (FD) of NCSs, to name a few, we refer the readers to [6], [7] and references therein. However, to the best of our knowledge, until now, few results have been reported about fault estimation (or fault identification) for nonlinear NCSs.

This work is partially supported by National Natural Science Foundation of China (60574083), National "863" program of China (2006AA12A108) and Natural Science Foundation of Jiangsu Province (BK2007195).

Z. H. Mao is with College of Automation Engineering (CAE), Nanjing University of Aeronautics and Astronautics (NUAA), 29 YuDao Street, Nanjing, 210016, China. zehuimao@nuaa.edu.cn.

B. Jiang is with CAE, NUAA, China. binjiang@nuaa.edu.cn. Tel: +86 25 85214846, Fax: +86 25 84892300.

P. Shi is with Institute for Logistics and Supply Chain Management, School of Computer Science and Mathematics, Victoria University, PO Box 14428, Melbourne, VIC 8001 Australia. peng.shi@vu.edu.au.

V. Cocquempot is with LAGIS-CNRS, UMR 8146, USTL, France. vincent.cocquempot@univ-lille1.fr.

The challenges of observer-based FE for nonlinear NCSs are twofold: 1) There is a lack of appropriate model for nonlinear NCSs. It is difficult for nonlinear NCSs fault estimation to find a model with enough accuracy, under the conditions of network-induced delays and packets loss; 2) Most of the NCSs are modelled as discrete-time systems with delays. However, some control theories, such as stability analysis using Lyapunov function or adaptive observer design for discrete-time systems are not so matured yet as that for continuous-time systems.

Our objective in this work is to propose an observer-based FE method for nonlinear networked control systems. Motivated by the work in [4] and using Euler approximate method, we present a Takagi-Sugeno (T-S) fuzzy model for nonlinear NCSs with Markov transfer delays. Under geometric conditions, each mode of discrete T-S model is transformed into a new form which is more convenient for adaptive observer design. A novel observer is designed for each mode to estimate the unmeasurable state and the modelling uncertainty, then a fault estimation algorithm is designed.

The rest of this paper is organized as follows. Section 2 describes the system model and presents some NCSs modelling techniques. The geometric conditions are derived in Section 3 for model transformation, and an observer-based fault estimation method is proposed in sections 4. In Section 5, an application example is given to illustrate the potential of the proposed method, followed by some concluding remarks in Section 6.

II. System description

A. Notations

The notations used in this paper are fairly standard. $\Re$ denotes the field of real numbers, $\Re^r$ denotes r-dimensional real vector space. $|\cdot|$ is the Euclidean norm. $L_g h$ denotes Lie derivative of h along a vector field g, $L_g h \triangleq dh \cdot g$, $L_g^n h = L_g(L_g^{n-1} h)$, where $dh = (\frac{\partial h}{\partial x_1} \cdots \frac{\partial h}{\partial x_n})$ is the differential of a smooth function, $[g_1, g_2] = \frac{\partial g_1}{\partial x} \cdot g_2 - \frac{\partial g_2}{\partial x} \cdot g_1$. $\circ$ is the composition, and define $\triangle_g h \triangleq h \circ g$, then $\triangle_g^n h \triangleq \triangle_g(\triangle_g^{n-1} h)$.

B. T-S fuzzy model for nonlinear NCSs

Consider the NCSs as shown in Fig. 1. The continuous-time state-space model of the nonlinear time-invariant plant dynamics can be described by the following standard form:

$$\dot{x} = g_0(x) + g(x)u + e(x)f \quad (1)$$

$$y = h(x) \quad (2)$$

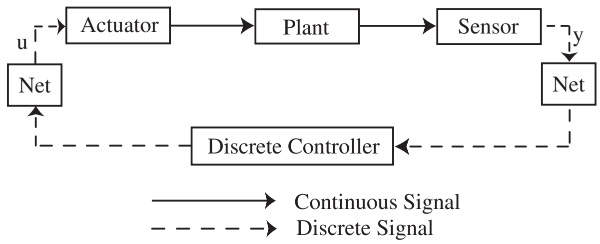

Fig. 1. The block of the networked control system with FTC

where $x \in \Re^n$ denotes the state vector, $u \in \Re^m$ is the control input vector, $y \in \Re^r$ is the measurable output vector. $f \in \Re^q$ represents the fault signal that could be constant or time-varying, while $e(x)$ represents the fault distribution function.

The sampling period of the NCS is T, sensors are time-driven, controller and actuator are event-driven. The network channel load and limited communication bandwidth can cause transfer delays, which include the sensor-controller delay τ_{sc} and controller-actuator delay τ_{ca}. Then, the overall network delay, which is also the transfer delay, can be computed by $\tau = \tau_{sc} + \tau_{ca}$.

Assumption 1 [4] : The transfer delay of the data packet, which is received by the actuator at the instant kT, is $\tau_k(\in \mathcal{N})$ periods and $\max(\tau_k) = n$.

Remark 1 : At the instant kT the actuators receive the sensor data packet that is sent out at the instant $(k-i)T$. Therefore, τ_{k+1} is only affected by τ_k and is irrelevant to $\tau_1, \ldots,$ and τ_{k-1}. That is, the sequence $\{\tau_1, \tau_2, \ldots, \tau_k, \ldots\}$ constructs a Markov chain [4].

Under Assumption 1, considering the sampling period and delays τ_k, in one sampling period, system (1) - (2) can be transformed into:

$$x(k+1) = x(k) + \int_{kT}^{(k+1)T} g_0(x(s)) + g(x(s))u(k-\tau_k) + e(x(s))f(k)ds \quad (3)$$

$$y(k) = h(x(k)) \quad (4)$$

where $x(k) = x(kT)$, $y(k) = y(kT)$ and $f(k) = f(kT)$. Then Eq. (3) can be simplified as in [8]:

$$x(k+1) = x(k) + T\Big(g_0(x) + g(x)u(k-\tau_k) + e(x)f\Big) + \mathcal{O}_1(T)$$

where $\mathcal{O}_1(T)$ is the higher order terms greater than 2 of the second term in Eq. (3). According to [10], we can obtain the expression of the term $\mathcal{O}_1(T)$, which depends on the related state, input and fault. To guarantee the accuracy of the NCS model and make the model not too complex, an approximate term $\phi(x(k), u(k))\theta(k)$ is considered with $\phi(x(k), u(k))$ a known function determined from the higher order terms and $\theta(k)$ a unknown vector to replace the term $\mathcal{O}_1(T)$.

Further, as in [4], the global model is the fuzzy fusion of the local ones. IF-THEN rules provides the relationship between the local models and the NCS global model. As τ_k has n different values, and the number of fuzzy rules is also n, the NCS model is regarded as the blending of n local nonlinear models.

Rule $i(i = 1, 2, \ldots, n-1, n)$: if τ_k is i, then the NCS model is

$$x(k+1) = x(k) + T\Big(g_0(x(k)) + g(x(k))u(k-i) + \phi(x(k), u(k))\theta(k) + e(x(k))f(k)\Big)$$
$$y(k) = h(x(k))$$

Therefore, the global model of NCS can be written as in [4]:

$$x(k+1) = \sum_{i=1}^{n} \mu_i(k)\Big\{x(k) + T\Big(g_0(x(k)) + g(x(k))u(k-i) + \phi(x(k), u(k))\theta(k) + e(x(k))f(k)\Big)\Big\} \quad (5)$$

$$y(k) = \sum_{i=1}^{n} \mu_i(k)h(x(k)) \quad (6)$$

where $\mu_i(k)$ is the membership function, representing the probability of $\tau_k = i$, i.e., $\mu_i(k) = Prob(\tau_k = i)$. It satisfies $\sum_{i=1}^{n} \mu_i(k) = 1$, $0 \leq \mu_i(k) \leq 1$, $\forall i = 1, 2, \ldots, n$. A method to obtain the membership function $\mu_i(k)$ is presented in [4].

Denote

$$x(k) = x_k, \quad y(k) = y_k, \quad u(k-i) = u_{k-i},$$
$$f(k) = f_k, \quad \theta(k) = \theta_k, \quad \mu_i(k) = \mu_k^i$$
$$\bar{g}_0(x_k) = x(k) + Tg_0(x(k)), \quad \bar{g}(x_k) = Tg(x(k)),$$
$$\bar{e}(x_k) = Te(x(k)), \quad \bar{\phi}(x_k, u_k) = T\phi(x(k), u(k)).$$

Then system (5) - (6) can be written as:

$$x_{k+1} = \sum_{i=1}^{n} \mu_k^i \Big\{\bar{g}_0(x_k) + \bar{g}(x_k)u_{k-i} + \bar{\phi}(x_k, u_k)\theta_k + \bar{e}(x_k)f_k\Big\} \quad (7)$$

$$y_k = \sum_{i=1}^{n} \mu_k^i h(x_k) \quad (8)$$

Remark 2 : It should be noted that some sufficient conditions for sampling time are given in [9], under which the Euler approximate model (without the high order terms) is equivalent to the accurate discrete-time model. Then, the term $\mathcal{O}_1(T)$, which is considered as the modelling uncertainty in this paper, can be omitted in practice.

III. Geometric transformation

The purpose of this section is to transform the global system (7) - (8) into a new form, which is more suitable for observer-based design.

Let us first consider one local model of (7) - (8):

$$x_{k+1} = \bar{g}_0(x_k) + \bar{g}(x_k)u_{k-i} + \bar{\phi}(x_k, u_k)\theta_k + \bar{e}(x_k)f_k \quad (9)$$

$$y_k = h(x_k) \quad (10)$$

Some definitions are given based on the above system (9) - (10) as follows:

Definition 1 : The relative degree ρ_i of system (9) - (10) is defined as:

$$\frac{\partial h_i((\bar{g}_0+\bar{g}_j u_j)^k)}{\partial u_j}=0,\quad 0\le k\le \rho_i-1,\quad 1\le i\le r,\quad \forall j:1\le j\le m;$$

$$\frac{\partial h_i((\bar{g}_0+\bar{g}_j u_j)^{\rho_i})}{\partial u_j}\neq 0,\quad 1\le i\le r,\quad \exists j:1\le j\le m.$$

where $h(\cdot)=[h_1,\cdots,h_r]^T$, $(\bar{g}_0+\bar{g}_j u_j)^s$, $\bar{g}=[\bar{g}_1,\cdots,\bar{g}_m]$, $(\bar{g}_0+\bar{g}_j u_j)^s$, $s\in\mathcal{Z}^+$ is recursively defined as follows:

$$(\bar{g}_0+\bar{g}_j u_j)^s=\underbrace{(\bar{g}_0+\bar{g}_j u_j)\circ\cdots\circ(\bar{g}_0+\bar{g}_j u_j)}_{s}$$

Definition 2 : *The fault diagnosis feedback form* of system (9)-(10) is

$$z^1_{k+1}=Az^1_k+\gamma_1(z^1_k,y_k)u_{k-i}+\gamma_2(z^1_k,y_k)+\psi_1(z^1_k,u_k,y_k)\theta_k \quad (11)$$

$$y^1_k=Cz^1_k \quad (12)$$

$$z^2_{k+1}=\psi_0(z_k)+\gamma_3(z^2_k,y_k)u_{k-i}+\bar{\bar{e}}(z_k)f_k+\psi_2(z_k,u_k)\theta_k \quad (13)$$

$$y^2_k=z^2_k \quad (14)$$

where $z=[(z^1)^\top,(z^2)^\top]^\top$, $z^1=[(\xi^1)^\top,\ldots,(\xi^{m-q})^\top]^\top\in\Re^{n-q}$, $z^2=[(\xi^{m-q+1})^\top,\ldots,(\xi^m)^\top]^\top\in\Re^q$ are the states of system (11)-(14), with $\xi=[(\xi^1)^\top,(\xi^2)^\top,\ldots,(\xi^m)^\top]^\top\in\Re^n$, $\xi_i\in\Re^{\varrho_i}=[\xi^{i1},\ldots,\xi^{i\varrho_i}]^\top$. $y=[(y^1)^\top,(y^2)^\top]^\top$ with $y^1\in\Re^{m-q}$, $y^2\in\Re^q$. Moreover, $A=\mathrm{diag}[A_1,\ldots,A_{m-q}]\in\Re^{(n-q)\times(n-q)}$, $C=\mathrm{diag}[C_1,\ldots,C_{m-q}]\in\Re^{(m-q)\times(n-q)}$, with

$$A_i\in\Re^{\varrho_i\times\varrho_i}=\begin{bmatrix}0&1&\cdots&0\\ \vdots&\vdots&\ddots&\vdots\\ 0&0&\cdots&1\\ 0&0&\cdots&0\end{bmatrix},\quad 1\le i\le m-q.$$

$$C_i\in\Re^{1\times\varrho_i}=\begin{bmatrix}1,0,\ldots,0\end{bmatrix},\quad 1\le i\le m-q$$

$\gamma_1(z^1_k,y_k)=[(\bar{\gamma}^1_k)^\top,(\bar{\gamma}^2_k)^\top,\ldots,(\bar{\gamma}^{m-q}_k)^\top]^\top$ with

$$\bar{\gamma}^i_k\in\Re^{\varrho_i\times p}=\begin{bmatrix}\bar{\gamma}^{i1}_k(\xi^1_k,\ldots,\xi^{i-1}_k,\xi^{i1}_k,y^{i+1}_k,\ldots,y^m_k)\\ \bar{\gamma}^{i2}_k(\xi^1_k,\ldots,\xi^{i-1}_k,\xi^{i1}_k,\xi^{i2}_k,y^{i+1}_k,\ldots,y^m_k)\\ \vdots\\ \bar{\gamma}^{i\varrho_i}_k(\xi^1,\ldots,\xi^{i-1},\xi^i,y^{i+1},\ldots,y^m)\end{bmatrix},\quad 1\le i\le m-q$$

and $\gamma_2(z^1_k,y_k)=[(\bar{\bar{\gamma}}^1_k)^\top,(\bar{\bar{\gamma}}^2_k)^\top,\ldots,(\bar{\bar{\gamma}}^{m-q}_k)^\top]^\top$ with

$$\bar{\bar{\gamma}}^i_k\in\Re^{\varrho_i}=[\underbrace{0,\ 0,\ \ldots,\ 0}_{\varrho_i-1\ \text{orders}},\triangle^{\varrho_i}_{\bar{g}_0}h_i(\xi^1_k,\ldots,\xi^i_k,y^{i+1}_k,\ldots,y^m_k)]^\top,\quad 1\le i\le m-q \quad (15)$$

Assumption 2 : There exists a set of integer numbers $\{\varrho_1,\varrho_2,\ldots,\varrho_m\}$ such that $\sum_{i=1}^m\varrho_i=n$ and $\xi_k=T(x_k)\in\Re^n$ is a diffeomorphism where

$$T(x_k)=[h_1(x_k),\triangle_{\bar{g}_0(x_k)}h_1(x_k),\ldots,\triangle^{\varrho_1-1}_{\bar{g}_0(x_k)}h_1(x_k),h_2(x_k),\ldots,\triangle^{\varrho_2-1}_{\bar{g}_0(x_k)}h_2(x_k),\ldots,\triangle^{\varrho_m-1}_{\bar{g}_0(x_k)}h_m(x_k)]^\top \quad (16)$$

Moreover, the relative degree of y_r, denoted as ρ_r, is such that $\rho_r=\varrho_r=1$, $m-q+1\le r\le m$. ♢

Under Assumption 2, $dT(x_k)$ is invertible $\forall x\in\Re^n$. Let $r_i(x_k)$ be the ith column of $[dT(x_k)]^{-1}$ and $\mathcal{R}(i)_j:=\mathrm{span}\{r_{v_i-j},\ldots,r_{v_i}\}$, where $v_i=\sum_{j=1}^i\varrho_j$.

The following lemma gives geometric conditions to transform the system (9)-(10) into (11)-(14).

Lemma 1 : Under Assumption 1, the diffeomorphism $\xi_k=T(x_k)$ can transform the system (9)-(10) into (11)-(14) if and only if

I $[\bar{g}_0(x_k),\mathcal{R}(i)_{\varrho_i-2}]\subset\mathcal{R}(i)_{\varrho_i-1}+\mathcal{R}(i+1)_{\varrho_{i+1}-1}+\cdots+\mathcal{R}(m-q)_{\varrho_{m-q}-1}$, for $2\le i\le m-q$.
II $[\bar{g}_0(x_k),\mathcal{R}(i)_j]\subset\mathcal{R}(i)_j+\mathcal{R}(i+1)_{\varrho_{i+1}-1}+\cdots+\mathcal{R}(m-q)_{\varrho_{m-q}-1}$, for $1\le i\le m-q$, $0\le j\le\varrho_i-2$.
III $[\bar{g}_0(x_k),\mathcal{R}(\bar{i})_{\varrho_{\bar{i}}-2}]\subset\mathcal{R}(1)_{\varrho_1-2}+\mathcal{R}(2)_{\varrho_2-2}+\cdots+\mathcal{R}(m-q)_{\varrho_{m-q}-2}$, for $m-q+1\le i\le m$, $1\le\bar{i}\le m-q$.
IV $L_{\bar{e}}L^s_{\bar{g}}h_i=0$, for $1\le s\le\varrho_i-1$, $1\le i\le m-q$.

The proof of Lemma 1 is modified from the reference [11] and [12]. Due to the page limit, it is omitted.

It is clear that after transformation, subsystem (11) - (12) is not affected by faults, an observer can be designed for this subsystem to provide the estimates of z^1_k and θ_k for estimating the faults in the subsystem (13) - (14).

Remark 3 : In Lemma 1, the proposed transformation method is for T-S local model. Since the fuzzy weight coefficient only relates with the control u_{k-i}, then Lemma 1 is suitable for the global model. Further the fault diagnosis block strict feedback form of global model (7) - (8) is

$$z^1_{k+1}=\sum_{i=1}^n\mu^i_k\Big\{Az^1_k+\gamma_1(z^1_k,y_k)u_{k-i}+\gamma_2(z^1_k,y_k)+\psi_1(z^1_k,u_k,y_k)\theta_k\Big\} \quad (17)$$

$$y^1_k=\sum_{i=1}^n\mu^i_kCz^1_k \quad (18)$$

$$z^2_{k+1}=\sum_{i=1}^n\mu^i_k\Big\{\psi_0(z_k)+\gamma_3(z^2_k,y_k)u_{k-i}+\bar{\bar{e}}(z_k)f_k+\psi_2(z_k,u_k)\theta_k\Big\} \quad (19)$$

$$y^2_k=\sum_{i=1}^n\mu^i_kz^2_k \quad (20)$$

IV. Fault diagnosis scheme

A. Observer design

An high gain observer as in [12] and [13] for continuous system can be developed for continuous system with form of (11) - (12) to estimate z_1 and θ. However, little work has been done on the high gain observer for discrete nonlinear systems. Combining the methods in [14] and [13], we extend them to the modelling uncertainty case.

Considering the form of model (17) - (20), we can firstly design the observer for local system (11) - (14), then extend it to the global one as that of Lemma 1.

In order to design the observer, let consider the following steps :

Step 1 : $H_i = \begin{bmatrix} C_i \\ C_i A_i \\ \vdots \\ C_i A_i^{\varrho_i - 1} \end{bmatrix}$

Step 2 : Define $\bar{W}_i$ for $1 \le i \le m-q$ as

$$\bar{W}_i(z_k^1, u_{k-i}, y_k) = \begin{bmatrix} C_i \\ C_i F_i(z_k^1, u_{k-i}, y_k) \\ \vdots \\ C_i F_i^{\varrho_i - 1}(z_k^1, u_{k-i}, y_k) \end{bmatrix}$$

where $F_i(z_k^1, u_{k-i}, y_k) = A_i + G_{ij}(z_k^1, y_k)u_{k-i}$, with $G_{ij}(z_k^1, y_k) = \partial(\bar{\gamma}_k^i u_{k-i} + \bar{\bar{\gamma}}_k^i)/\partial \xi_k^j$ for $1 \le j \le m-q$. Further, define $M_{i0}(z_k^1, u_{k-i}) = H_i^{-1} W_i(z_k^1, u_{k-i})$.

Step 3 : Let
$Q_i(z_k^1, u_{k-i}) = M_{i0}(z_k^1, u_{k-i}) F_i(z_k^1, u_{k-i}) M_{i0}^{-1}(z_k^1, u_{k-i}) - A_i$,

It can be shown that $Q_i(z_k^1, u_{k-i})$ has the following form:

$$Q_i(z_k^1, u_{k-i}) = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & & \vdots \\ 0 & \cdots & 0 \\ Q_{i1}(z_k^1, u_{k-i}) & \cdots & Q_{i\varrho_i}(z_k^1, u_{k-i}) \end{bmatrix}$$

Step 4 : Define $\bar{A}_i(z_k^1, u_{k-i}) = A_i + L_i(z_k^1, u_{k-i})C_i$, where $L_i = \text{col}[Q_{i\varrho_i}(z_k^1, u_{k-i}), \ \cdots \ Q_{i1}(z_k^1, u_{k-i})]$, with Q_i's the entries of the matrix $Q_i(z_k^1, u_{k-i})$.

Step 5 : Compute $\tilde{W}_i(z_k^1, u_{k-i}) = \begin{bmatrix} C_i \\ C_i \bar{A}_i(z_k^1, u_{k-i}) \\ \vdots \\ \bar{A}_i^{\varrho_i - 1}(z_k^1, u_{k-i}) \end{bmatrix}$,

it can again be checked that $\tilde{W}_i(z_k^1, u_{k-i}) = H_i \tilde{M}_{i0}(z_k^1, u_{k-i})$.

Step 6 : Define $M_i(z_k^1, u_{k-i})$ as
$M_i(z_k^1, u_{k-i}) = \tilde{W}_i^{-1}(z_k^1, u_{k-i}) W_i(z_k^1, u_{k-i})$
$= \tilde{M}_i^{-1}(z_k^1, u_{k-i}) M_i(z_k^1, u_{k-i})$.

From the above procedure, we can obtain the following special properties of matrix $M_i(z_k^1, u_{k-i})$ [14]:

i) $\text{diag}[M_{i0}(z_k^1, u_{k-i})] = \text{diag}[M_i(z_k^1, u_{k-i})] = (1, \ \cdots, \ 1)$. In other word, $M_i(z_k^1, u_{k-i})$ is nonsingular for all z_k^1, u_{k-i}.

ii) $M_i(z_k^1, u_{k-i}) F_i(z_k^1, u_{k-i}) M_i^{-1}(z_k^1, u_{k-i}) = A_i + L_i(z_k^1, u_{k-i}) C_i$ and $C_i M_i^{-1}(z_k^1, u_{k-i}) = C_i M_i(z_k^1, u_{k-i}) = C_i$.

iii) $M_i(z_k^1, u_{k-i}) M_i^{-1}(z_{k-1}^1, u_{k-i-1}) = I + R_{i0}(z_k^1, u_{k-i})$, where $R_{i0}(z_k^1, u_{k-i})$ is a lower triangular matrix with $\text{diag}[R_{i0}(z_k^1, u_{k-i})] = (0, \ \cdots, \ 0)$.

Assumption 3 :

3.1 The partial derivatives of $\bar{\gamma}_k^i$ w.r.t. z_k^1 and their respective time derivatives are bounded.

3.2 There exists a function $B(z_k^1, u_k, y_k) \in \Re^{(n-q)\times(m-q)}$ such that $\psi_1(z_k^1, u_k, y_k) = B\bar{\psi}_1(z_k^1, u_k, y_k)$, where B is Lipschitz w.r.t. z_k^1, $|B| \le b_0$, and $|\bar{\psi}_1| \le \bar{q}(z_k^1, u_k, y_k) \le q_0$ for a function q and numbers $b_0, q_0 > 0$.

3.3 There exist matrices $P = P^\top \in \Re^{n\times n} = \text{diag}[P_1, \ldots, P_{m-q}]$ with $P_i = P_i^\top \in \Re^{\varrho_i \times \varrho_i}$

$$\epsilon_i^2 (A_i - K_i C_i)^\top P_i (A_i - K_i C_i) - P_i = -Q_i$$

where $\Delta_\epsilon = \text{diag}[\Delta_{\epsilon_1}, \ldots, \Delta_{\epsilon_{m-q}}]$, $\Delta_{\epsilon_i} = \text{diag}[1/\epsilon_i, \ldots, 1/\epsilon_i^{\varrho_i}]$, with ϵ a design parameter. $M = \text{diag}[M_1, \ldots, M_{m-q}]$, $Q_i = Q_i^\top > 0$, $K_i \in \Re^{\varrho_i \times 1}$ are such that $(A_i - K_i C_i)$ is stable. $\Diamond$

Remark 4 : Conditions 3.1 and 3.2 are taken instead of Lipschitz conditions on γ_1, γ_2 and ψ_1, that are classically considered. Condition 3.1 is similar to Assumption 1 in [13]. Condition 3.2 relaxes the *Lipschitz extension* condition in [12] where only y is allowed in $\bar{q}$. Condition for chosen P_i is given in 3.3.

The observer is constructed as

$$\begin{aligned} \hat{z}_{k+1}^1 = & A\hat{z}_{k+1}^1 + \gamma_1(\hat{z}_k^1, y_k)u_{k-i} + \gamma_2(\hat{z}_k^1, y_k) \\ & + \psi_1(\hat{z}_k^1, y_k, u_k)\hat{\theta} \\ & + M^{-1}(\hat{z}_k^1)[L(\hat{z}_k^1, y_k) + \Delta_\epsilon^{-1} K](y_k^1 - \hat{y}_k^1) \\ & + \underbrace{\hat{B} sgn(\hat{R}^\top)[\theta_0(q_0 + \bar{q}(\hat{z}_k^1, y_k)) sgn(y_k^1 - \hat{y}_k^1)]}_{\Upsilon} \end{aligned} \quad (21)$$

$$\hat{y}_k^1 = C\hat{z}_k^1 \quad (22)$$

$$\begin{aligned} \hat{\theta}_{k+1} = & \hat{\theta}_k \\ & + \Gamma \bar{\psi}_1^\top(\hat{z}_k^1, u_k, y_k) R^\top(\hat{z}_k^1, u_k, y_k)(y_k^1 - \hat{y}_k^1) \end{aligned} \quad (23)$$

where $L_k = \text{diag}[L_k^1, \ldots, L_k^{m-q}]$, and $K = \text{diag}[K_1, \ldots, K_{m-q}]$. The weighting matrix $\Gamma = \Gamma^\top > 0$. The design method of $R(\hat{z}_k^1, u_k, y_k)$ will be given later. sgn represents the canonical sign function.

Denote $e_k^{z^1} = [(e_k^1)^\top, \ldots, (e_k^{m-q})^\top]^\top$ with $e_k^i = \xi_k^i - \hat{\xi}_k^i$, $1 \le i \le m-q$, $e_k^\theta = \theta_k - \hat{\theta}_k$.

Theorem 1 : Under Assumption 3, the observer described by (21) - (22) together with the adaptive algorithm (23) can realize $\lim_{k\to\infty} e_k^{z^1} = 0$ and $\lim_{k\to\infty} e_k^\theta = 0$.

Proof : The proof of the theorem follows the recursive way. Consider the first subsystem of (11) and (21), we have

$$\begin{aligned} e_{k+1}^1 = & A_1 e_k^1 + (\bar{\gamma}_k^1 - \hat{\bar{\gamma}}_k^1)u_{k-i} + (\bar{\bar{\gamma}}_k^1 - \hat{\bar{\bar{\gamma}}}_k^1) \\ & - (\hat{M}_k^1)^{-1}(\hat{L}_k^1 + \Delta_{\epsilon_1}^{-1} K_1) C_1 e_k^1 \\ & + \underbrace{\psi_{11}\theta_k - \hat{\psi}_{11}\hat{\theta}_k - \Upsilon_1}_{\Psi_1} \end{aligned} \quad (24)$$

where $\Upsilon = [\Upsilon_1^\top, \ldots, \Upsilon_{m-q}^\top]^\top \triangleq Bsgn(R^\top)[\theta_0(q_0 + \bar{q}(\hat{z}_1, y)) sgn(y_k^1 - \hat{y}_k^1)]$. $\psi_1 = [\psi_{11}^\top, \ldots, \psi_{1(m-q)}^\top]^\top$.

According to [14], consider a transformation $\tilde{e}_k^1 \triangleq \Delta_{\epsilon_1} \hat{M}_{k-1}^1 e_k^1$, then (24) can be changed into

$$\tilde{e}_{k+1}^1 = \epsilon_1 (A_1 - K_1 C_1)\tilde{e}_k^1 + T_{11}\tilde{e}_k^1 + \Delta_{\epsilon_1} \hat{M}_{k-1}^1 \Psi_1$$

with $T_{11} = \Delta_{\epsilon_1}(A_1 \hat{R}_{10} + \hat{M}_{k-1}^1 r_k^1 (\hat{M}_{k-1}^1)^{-1})\Delta_{\epsilon_1}^1$, $(\bar{\gamma}_k^1 - \hat{\bar{\gamma}}_k^1)u_{k-i} + (\bar{\bar{\gamma}}_k^1 - \hat{\bar{\bar{\gamma}}}_k^1) = G_{11} e_k^1 + r_k^1 e_k^1$.

Choose a Lyapunov candidate function $V_k^1 = (\tilde{e}_k^1)^T P_1 \tilde{e}_k^1$, it can be shown that the time derivative of V_k^1 along (24) satisfies

$$\begin{aligned}\Delta V_{k+1}^1 \le & -\lambda_{\max}(Q_1)|\tilde{e}_k^1|^2 + \mu_{11}|\tilde{e}_k^1|^2 \\ & +\Psi_1^T(\hat{M}_{k-1}^1)^T \Delta_{\epsilon_1}^T P_1 \Delta_{\epsilon_1} \hat{M}_{k-1}^1 \Psi_1 \\ & +2\epsilon_1(\tilde{e}_k^1)^T[(A_1 - K_1C_1) + T_{11}]^T \Delta_{\epsilon_1}\hat{M}_{k-1}^1\Psi_1\end{aligned}$$

where $\mu_{11} > 0$.

Next, consider the last subsystems of (11) and (21), and following the same way, we can get

$$\begin{aligned}\Delta V_{k+1}^i \le & -\lambda_{\min}(Q_i)|\tilde{e}_k^i|^2 + \sum_{j=1}^{i}\mu_{ij}|\tilde{e}_k^i|^2 + \sum_{j=1}^{i-1}\mu_{ji}|\tilde{e}_k^j|^2 \\ & +\Psi_i^T(\hat{M}_{k-1}^i)^T\Delta_{\epsilon_i}^T P_i \Delta_{\epsilon_i}\hat{M}_{k-1}^i\Psi_i \\ & +2\epsilon_i(\tilde{e}_k^i)^T(A_i - K_iC_i)^T\Delta_{\epsilon_i}\hat{M}_{k-1}^i\Psi_i \\ & +2\epsilon_i\sum_{n=1}^{i}\left((\tilde{e}_k^n)^T T_{ni}^T\right)\Delta_{\epsilon_i}\hat{M}_{k-1}^i\Psi_i \end{aligned} \quad (25)$$

Now, consider the Lyapunov candidate function as $W(e_k^{z^1}, e_k^\theta) = V_{e_k^{z^1}} + V_{e_k^\theta}$ for the overall system, where $V_{e_k^{z^1}} = \sum_{i=1}^{m-q} V_k^i$, $V_{e_k^\theta} = (e_k^\theta)^\top \Gamma^{-1} e_k^\theta$. Denote $\tilde{e}_k = [(\tilde{e}_k^1)^\top, \ldots, (\tilde{e}_k^{m-q})^\top]^\top$. The time derivative of W along (11), (21) and (23) is

$$\begin{aligned}\Delta W_{k+1} \le & \sum_{i=1}^{m-q}\left((-\lambda_{\min}(Q_i) + \sum_{j=1}^{m-q}\mu_{ij})|\tilde{e}_k^i|^2\right) \\ & +\Psi^T(\hat{M}_{k-1})^T\Delta_\epsilon^T P\Delta_\epsilon\hat{M}_{k-1}\Psi \\ & +2\epsilon(\tilde{e}_k^{z^1})^T[(A-KC)+T]^T\Delta_{\epsilon_1}\hat{M}_{k-1}\Psi \\ & -2\tilde{\theta}_k^T\bar{\psi}_1 RCe_k^{z^1} + (e_k^{z^1})^T C^T R^T \bar{\psi}_1^T \Gamma\bar{\psi}_1 RCe_k^{z^1}\end{aligned}$$

where $T = [\sum_{i=1}^{m-q} T_{1i} \ \sum_{i=2}^{m-q} T_{2i} \ \cdots \ \sum_{i=m-q}^{m-q} T_{(m-q)i}]$.

Further, from Assumption 3.2, we can obtain that if $R(\hat{z}_k^1, u_k, y_k)$ is chosen such that

$$\epsilon \hat{M}_{k-1}^T \Delta_\epsilon^T[(A-KC)+T]^T\Delta_\epsilon\hat{M}_{k-1}B - C^T R \le 0 \quad (26)$$

Then, we further have

$$\Delta W_{k+1} \le -\eta|\tilde{e}_k^{z^1}|^2 \quad (27)$$

where ϵ_i, $1 \le i \le m-p$, is chosen such that $\eta > 0$.

Since M and Δ_ϵ are all bounded and nonsingular, inequality (27) implies the stability of the origin $e^{z^1} = 0$, $\tilde{\theta} = 0$ and the uniform boundedness of e^{z^1} and $\tilde{\theta}$. According to Barbalat's Lemma [16], one can get $\lim_{k\to\infty} e_k^{z^1} = 0$. From (23) and the uniform boundedness of $\tilde{\theta}$, one can conclude that $\lim_{k\to\infty}\tilde{\theta}_k = 0$. This completes the proof. □

Remark 5 : Similar to the transformation in Section 3, the observer for global T-S model can be obtained, which is omitted here.

B. fault estimation

Supposed that $\bar{\bar{e}}(z_k)$ is invertible, which implies that the effect of faults on outputs y_k^2 is independent. The fault estimates can be obtained from (13) as

$$\hat{f}_k = \hat{\bar{\bar{e}}}^{-1}[y_{k+1}^2 - \hat{\psi}_0 - \hat{\psi}_2\hat{\theta}_k - \gamma_3 u_{k-i}] \quad (28)$$

with $\hat{\bar{\bar{e}}} = \bar{\bar{e}}(\hat{z}_k)$ then, we have

$$\hat{\bar{\bar{e}}}\hat{f}_k - \bar{\bar{e}} f_k = (\hat{\bar{\bar{e}}} - \bar{\bar{e}})\hat{f}_k + \bar{\bar{e}}(\hat{f}_k - f_k) = (\psi_0 - \hat{\psi}_0) + (\psi_2\theta_k - \hat{\psi}_2\hat{\theta}_k)$$

Since $\lim_{k\to\infty} e_z = 0$, $\lim_{k\to\infty}\tilde{\theta}_k = 0$, due to the continuity of $\bar{\bar{e}}, \psi_0$ and ψ_2, there always exist two numbers $k_z, k_\theta > 0$ such that for all bounded z^k, $\hat{z}^k$, if $|e^z|$ and $|\tilde{\theta}|$ are sufficiently small (which can be achieved by observer), then the following inequality holds

$$|\bar{\bar{e}}(\hat{f}_k - f_k)| \le k_z|e_z| + k_\theta|\tilde{\theta}| \quad (29)$$

Moreover, we have $\lim_{k\to\infty}|\bar{\bar{e}}(\hat{f}_k - f_k)| = 0$, i.e. $\lim_{k\to\infty}|\hat{f}_k - f_k| = 0$.

Remark 6 : From Eq. (28), it can be seen that the faulty signal at time instant k can be estimated only after the measurements from time instant $(k+1)$ become available. It means that there is a one step delay in the fault estimation, whose effect on the dynamic response can be neglected for practical application [2]. On the other hand, we can avoid such problem via setting a new vector containing the $y_2(k)$, as in [15]. ◊

V. AN ILLUSTRATIVE EXAMPLE

To show the effectiveness of the proposed design scheme, we provide a PM synchronous motor whose analytic model can be written as [17]:

$$\begin{aligned}\dot{\omega}_r &= \frac{3}{2}\frac{p^2}{J}\lambda_m i_{qs} - \frac{B}{J}\omega_r - \frac{p}{J}T_L \\ \dot{i}_{qs} &= -\frac{R_s}{L_s}i_{qs} - \omega_r i_{ds} - \frac{\lambda_m}{L_s}\omega_r + \frac{1}{L_s}v_{qs} \\ \dot{i}_{ds} &= -\frac{R_s}{L_s}i_{ds} + \omega_r i_{qs} + \frac{1}{L_s}v_{ds}\end{aligned}$$

where R_s is the stator resistance, L_s is the stator inductance, ω_r is the electrical rotor angular velocity, λ_m is the flux linkage established by the permanent magnet. J is the moment of inertia of the rotor and its attached load, B is the viscous friction coefficient, p is the number of pole pairs, and T_L is the load torque. The rated power, speed and torque are 400 W, 3000 rpm and 1.27 Nm, the number of poles is 4, magnetic flux is 0.153 Wb, stator resistance and inductance are 3.0 Ω and 10.5 mH, moment of inertia is 1.75×10^{-4} $\text{Nm}\cdot\text{s}^2$.

Assume that sampling time $T = 0.1s$, $n = \max(d_k) = 3$, $\mu_0 = 0.1826$, $\mu_1 = 0.2071$, $\mu_2 = 0.4776$, $\mu_3 = 0.1227$. After discretizing, we obtain a T-S model: Rule $i (i = 0, 1, 2, 3)$: if τ_k is i, then the NCS model is

$$\begin{bmatrix}\omega_r(k+1)\\ i_{qs}(k+1)\\ i_{ds}(k+1)\end{bmatrix} = \begin{bmatrix}\omega_r(k) + \frac{0.3}{2}\frac{p^2}{J}\lambda_m i_{qs}(k) - \frac{0.1B}{J}\omega_r(k) - \frac{0.1p}{J}T_L \\ i_{qs}(k) - \frac{0.1R_s}{L_s}i_{qs}(k) - 0.1\omega_r(k)i_{ds}(k) - \frac{0.1\lambda_m}{L_s}\omega_r(k) \\ i_{ds}(k) - \frac{0.1R_s}{L_s}i_{ds}(k) + 0.1\omega_r(k)i_{qs}(k)\end{bmatrix} + \begin{bmatrix}0\\ \frac{0.1}{L_s}v_{qs}(k-i)\\ \frac{0.1}{L_s}v_{ds}(k-i)\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 0.1x_1(k)x_3(k)\end{bmatrix} f(k)$$

$$+\begin{bmatrix} 0.006 \\ 0.01(x_1(k)-x_3(k))^2 \\ 0.01x_1(k) \end{bmatrix}\theta(k)$$
$$y_1(k)=\omega_r(k),\ \ y_2(k)=i_{ds}(k)$$

It can be checked that Assumption 3 is satisfied. Indeed, according to Lemma 1, a diffeomorphism is chosen as $\xi_1(k)=\omega_r(k)$, $\xi_2(k)=\omega_r(k)+\frac{0.3}{2}\frac{p^2}{J}\lambda_m i_{qs}(k)-\frac{0.1B}{J}\omega_r(k)-\frac{0.1p}{J}T_L$, $\xi_3(k)=x_3(k)$. Then the model in ξ-coordinate can be represented as

$$\begin{aligned}
\xi_1(k+1)&=\xi_2(k)+0.006\theta(k)\\
\xi_2(k+1)&=(1-\frac{0.1B}{J})(\xi_2(k)+0.006\theta(k))\\
&\quad+\frac{0.3}{2}\frac{p^2}{J}\lambda_m\Bigg[\left(1-\frac{0.1R_s}{L_s}\right)\Bigg(\xi_2(k)-y_1(k)\\
&\quad+\frac{0.1B}{J}y_1(k)+\frac{0.1p}{J}T_L+\frac{2}{0.3}\frac{J}{p^2\lambda_m}\Bigg)\\
&\quad-0.1y_1(k)y_2(k)-\frac{0.1\lambda_m}{L_s}y_1(k)+\frac{0.1}{L_s}v_{qs}(k-i)\\
&\quad+0.01(y_1(k)-y_2(k))^2\theta(k)\Bigg]-\frac{0.1p}{J}T_L\\
y_1(k)&=\xi_1(k)\\
\xi_3(k+1)&=y_2(k)-\frac{0.1R_s}{L_s}y_2(k)+0.1y_1(k)\Bigg[\xi_2(k)-y_1(k)\\
&\quad+\frac{0.1B}{J}y_1(k)+\frac{0.1p}{J}T_L+\frac{2}{0.3}\frac{J}{p^2\lambda_m}\Bigg]+\frac{0.1}{L_s}v_{ds}(k-i)\\
&\quad+0.1y_1(k)y_2(k)f(k)+0.01y_1(k)\theta(k)\\
y_2(k)&=\xi_3(k)
\end{aligned}$$

It can be checked that Assumption 4 holds. One further have from Section 4.1 that

$$L(k)=\begin{bmatrix} -1-\frac{2\xi_2(k)}{y_1(k)}+0.01(y_2(k)-y_1(k))^2 \\ 1+\frac{2\xi_2(k)}{y_2(k)}+0.01(y_2(k)-y_1(k))^2 \end{bmatrix}$$

$$M(k)=\begin{bmatrix} 2 & 0 \\ -1-\frac{2\xi_2(k)}{y_2(k)}+0.01(y_2(k)-y_1(k))^2 & 1 \end{bmatrix}$$

Choose $\Gamma=1.182$, $\epsilon_1=5$, $K=[3\quad 1]^T$, $R=60$, the observer (21) is applied with $\hat{\theta}(0)=0$. Fig. 2 shows the estimation results of the system uncertainty and the fault.

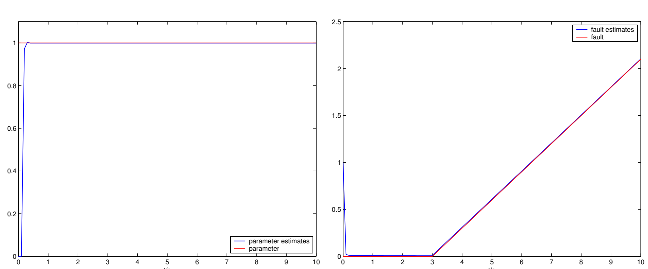

Fig. 2. Estimation performance

VI. Conclusion

In this paper, we discussed the FE problem for a class of nonlinear networked control systems with transfer delays. The Euler approximate method and T-S model are combined to model the nonlinear NCSs, which is suitable to design the observer. Furthermore, Markovian jump system modelling method is another effective way for modelling nonlinear NCSs. Then, the proposed method can be extended to this model is our future work.

References

[1] M. Staroswiecki and G. Comtet-Varga, Analytical redundancy relations for fault detection and isolation in algebraic dynamic systems, *Automatica*, 2001, Vol. 37, No. 5, pp. 687-699.
[2] B. Jiang, and F. N. Chowdhuery, Fault estimation and accommodation for linear MIMO discrete-time systems, *IEEE Trans. on Control System technology,* 2005, Vol. 13, No. 3, pp. 493-499.
[3] W. Zhang, M. S. Branicky and Philips, S. M., Stability of networked control systems, *IEEE Control System Magazine*, 2001, Vol. 21, No. 1, pp. 84-99.
[4] Y. Zheng, H. J. Fang and Wang, H. O., Takagi-Sugeno fuzzy-model-based fault detection for networked control systems with markov delays, *IEEE Trans. on Systems, Man, and Cybernetics - Part B: Cybernetics*, 2006, Vol. 36, No. 4, pp. 924-929.
[5] D. Nešić and A. R. Teel, Input-to-state stability of networked control systems, *Automatica*, 2004, Vol. 40, No. 12, pp. 2121-2128.
[6] P. Zhang and S. X. Ding, Fault detection of networked control systems with limited communication, *Proc. of the 6th IFAC Safeprocess*, Beijing, 2006, pp. 1135-1140.
[7] Z. H. Mao, B. Jiang and P. Shi, H_∞ fault detection filter design for networked control systems modelled by discrete Markovian jump systems, *IEE Proc. Control Theory and Application*, 2007, Vol. 1, No. 5, pp. 1336-1343.
[8] D. Nešić and A. R. Teel, Stabilization of sampled-data nonlinear systems via backstepping on their Euler approximate model, *Automatic*, 2006, Vol. 42, No. 10, pp. 1801-1808.
[9] D. Nešić, A. R. Teel and P. V. Kokotović, Sufficient conditions for stabilization of sampled-data nonlinear systems via discrete-time approximations, *Systems and Control letters*, 1999, Vol. 38, No. 4, pp. 259-270.
[10] C. Califano, S. Monacoa and D. Normand-Cyrot, On the observer design in discrete-time, *System and Control letters*, 2003, Vol. 49, pp. 255-265.
[11] R. Marino and Tomei, P., *Nonlinear Control Design–Geometric, Adaptive and Robust*, London: Prentice Hall, 1995.
[12] H. Shim, Y. I. Son and J. H. Seo, Semi-global observer for multi-output nonlinear systems, *Systems and control letters*, 2001, Vol. 42, No. 3, pp. 233-244.
[13] S. Lee and M. Park, State observer for MIMO nonlinear systems, *IEE Proceedings: Control Theory and Applications*, 2003, Vol. 150, No. 4, pp. 421-426.
[14] K. Busawon, M. Saif and J. De. Leon-Morales, Estimation and control of a class of Euler discretized nonlinear systems, *Proc. of the 1999 American Control Conference*, San Diego, Califomia, USA, 1999, pp. 3579-3583.
[15] B. Jiang, J. L. Wang and Y. C. Soh, Robust fault diagnosis for a class of bilinear systems with uncertainty, *Proc. of the 38th IEEE Conference on Decision and Control*, Phoenix, USA, 1999, Vol. 5, pp. 4499-4504.
[16] H. K. Khalil, *Nonlinear Systems,* 3rd edition, Upper Saddle River, NJ: Prentice-Hall, 2002.
[17] K. H. Kim and M. Y. Youn, A nonlinear speed control for a PM synchronous motor using a simple disturbance estimation technique, *IEEE Trans. on Industrial Electrontics*, Vol.49, No. 3, 524-535, 2002.