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High Performance Motion Control of DC Motors Using Wavelet Networks

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Abstract— **Based on the multi-resolution analysis and the wavelet transform, a wavelet network is presented for the control of DC motors. One of the basic advantages of wavelet network is that training is done using the recursive least square method which is suitable for online training usually required for adaptive control. The wavelet network is used to design adaptive speed controllers for a DC motor to achieve high performance speed control even if the motor model is unknown, the load characteristics are also unknown function of speed and the load torque changes online. Simulation and experimental results are presented to validate the proposed controllers.**

I. INTRODUCTION

Wavelet transform is a relatively new signal processing tool. It is based on representing any signal as a weighted summation of wavelet basis functions. Wavelet basis functions are dilated and translated versions of certain function called the *mother wavelet.* For certain function to be a valid mother wavelet, it must satisfy certain admissibility conditions [1]. Since its introduction as a specialized field in the mid 1980, wavelet transform has found many applications in many different fields. Combining both the wavelet transform and the basic ideas of neural networks results in a new network called wavelet network (WN) [2]. The objective of such network was to use the wavelet transform to overcome the problems arising in feedforward neural network especially the computationally heavy training and the application dependent structure. Actually, the network proposed in [2] was essentially an ordinary radial basis function network with wavelet functions used in the hidden units. In [3], a wavelet network which depends on multi-resolution analysis and wavelet transform is proposed.

The wavelet networks can be classified into orthogonal and non-orthogonal networks depending on the properties of the wavelet function used to construct the network. Orthogonal wavelet networks depend on generating orthonormal basis using the wavelet function. However, in order to generate an orthonormal basis, the wavelet function has to satisfy some restrictions [4]. The training of the orthonormal wavelet network is fast and the construction is easier. On the other hand, the non orthogonal wavelet network uses the so called *wavelet frame* [5]. The orthogonal wavelet network has received more interest especially in control applications where the emphasis is on the fast training required in online training. The authors of [6] have proposed a general method for the use of orthogonal wavelet networks in nonlinear system identification.

High performance electric drive systems are increasingly used in modern applications. Conventional controllers usually have poor performance due to their inability to capture the unknown load characteristics over wide operating region. The adaptive control could have better performance. The motor could be identified using a linear parametric model; for instance an ARMA model. But, the characteristics of the load are usually nonlinear. Hence, it is required to identify the motor based on nonlinear model. Neural networks have been used to control DC motors [7] with good results. But the main disadvantage of using neural networks is the back propagation training algorithm which requires a heavy computation load and thus not suitable for online training. In [8] the dynamic back propagation algorithm was used to improve the identification, however, the computational load is still heavy.

In this paper, we use orthogonal wavelet network to control the DC motor with unknown parameters and investigate the use of the forgetting factor in the least square algorithm for the online training of the wavelet network.

II. WAVELET TRANSFORM AND MULTI RESOLUTION ANALYSIS (MRA)

Essentially the MRA represents the successive approximation of a function in a sequence of nested subspaces of linear vector space [1]. Wavelet transform appears naturally in the context of the MRA. MRA based on orthogonal wavelet function is done by defining two function, scaling and wavelet functions. Scaling function $\{\varphi_{j,n}\}$ forms an orthonormal basis for a sequence of nested spaces such that:

$$... \subset V_{-1} \subset V_0 \subset V_1 \subset ... \tag{1}$$

$$\bigcap_{j\in Z} V_j = \{0\} \tag{2}$$

$$\bigcup_{j\in Z} V_j = L^2(R) \tag{3}$$

where L^2 *(R)* is the space of square integrable functions and $j \in Z$ where Z is the set of integers, and $\varphi_{j,n} \in L^2(R)$, $\varphi_{j,n} = \varphi(2^{-j} t - n)$, n is the translation parameter and j is the resolution (dilation) parameters. In particular, the above

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representation means that a function $f(t)$ in the $L^2(R)$ space could be approximated with different accuracies depending on the resolution of the space at which the function is approximated. That is:

$$f_j(t) = \sum_{l=-\infty}^{\infty} \mu(j,l)\varphi(2^{-j}t - l) \tag{4}$$

where the function $f_j(t)$ denotes the approximation of the function $f(t)$ at resolution j and $\mu(j,l)$ are the coordinates of the scaling function at this sub-space. The details added at each approximation are located in other subspaces [1]. These new subspaces W_i – which contains the details – are orthonormal and have an orthonormal basis which are the wavelet orthonormal basis $\psi_{j,n} = \psi(2^{-j}t - n)$ where $j,n \in Z$. The function ψ is the wavelet function which must have the orthonormal properties. In this paper, Meyer scaling and wavelet functions are used. Also it could be proved that [1]

$$V_{j-1} = V_j \oplus W_j \tag{5}$$

where $\oplus$ denotes the direct sum of the two spaces. Repeating this equation successively we reach the following equation:

$$L^2(R) = ... \oplus W_{-1} \oplus W_0 \oplus W_1 \oplus ...... \tag{6}$$

That is, the orthonormal wavelet basis generates an orthogonal decomposition of the L^2 space. It is noted in Equation (4) that, as the parameter of resolution – j – decreases, the approximation gets finer. Thus for a given constant $\varepsilon > 0$ there exists an integer j_0 and a function

$$\hat{f}\,(t, j_0) = \sum_{l=-\infty}^{\infty} \mu(j_0,l)\varphi(2^{-j_0}t - l) \tag{7}$$

where $\left\| f(t) - \hat{f}(t, j_0) \right\| < \varepsilon$ and $\varphi(2^{-j_0}t - l)$ denotes the scaling function with certain resolution j_0 [6] and the functional norm is the L^2 norm. Moreover if the function $f(t)$ is defined over a small region, then we can truncate the above summation and write (9) as

$$\hat{f}\,(t, j_o) = \sum_{l=L}^{U} \mu(j_0,l)\varphi(2^{-j_0}t - l) \text{ for some } U\ ,L \in Z \tag{8}$$

The choice of the parameters U, L depends on the region over which the function is defined. In other words, noting that the distance between two successive functions in the above series equals 2^{j_0} and denoting the period over which we try to approximate the function f as $[X_1, X_2]$ then,

$$L \approx \frac{X_1}{2^{j_0}} \qquad \text{and} \qquad U \approx \frac{X_2}{2^{j_0}} \tag{9}$$

Equation (8) represents a *wavelet network* which provides an approximation of a given function in single resolution j_0. Fig.(1) shows a structural representation of equation (8). The problem of approximating a single dimension function at certain predetermined resolution is reduced to the problem of finding the constants $\mu(j_0,l)$ of equation (8) by iterative method using the input and output data only. Note that the parameters $\mu(j_0,l)$ appear linearly in equation (8). Thus the problem of finding the best constants $\mu(j_0,l)$ – which corresponds to the training of the network – could be solved easily using recursive least square algorithms. The extension of the wavelet network to the multi-dimensional case is straightforward. This is achieved by defining multi-dimensional scaling functions as follows

$$\Phi(x_1, x_2, ..., x_n) = \varphi(\|X\|) \tag{10}$$

where $\varphi(.)$ is the scaling function in one dimension and $\|X\|$ denotes the Euclidean norm [5]. Thus, after defining the multidimensional scaling or wavelet functions, the process of approximating a multidimensional function is typical to the case of single dimension function as discussed in last section. However, the approximation of multi-dimensional functions is more difficult due to the curse of dimensionality problem [4].

III. SYSTEM IDENTIFICATION USING WAVELET NETWORK

For certain dynamical system, let $y(t)$ and $u(t)$ denote the output and input of a given system respectively at time t. Collecting the values of input and output at discrete instances, one should have the following data

$$\phi(t) = [\,y(t-1),..., y(t-a), u(t-1),..., u(t-b)] \tag{11}$$

where a, b are positive integers. In the identification setup, we are looking for a model which would map the past data $\phi(t)$ to the next output of the form

$$\hat{y} = f(\phi(t)) \tag{12}$$

The non-linear mapping $f(t)$ is a function from R^d to R, where $d = a + b$, represents the number of elements of the $\phi(t)$ vector. The choice of a and b is application dependent and any prior information should be utilized to determine a and b. The unknown function $f(t)$ could be approximated by the wavelet network of equation (8). The training of the network is carried out, for example, by injecting a random signal to the system and the input-output pairs are used by the least square algorithm to improve the parameters of the network. Fig.(2) shows a schematic block diagram representing the training process.

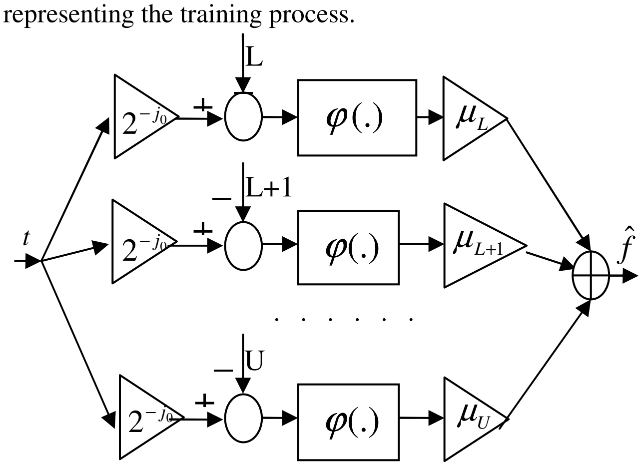

Fig.(1) Wavelet network

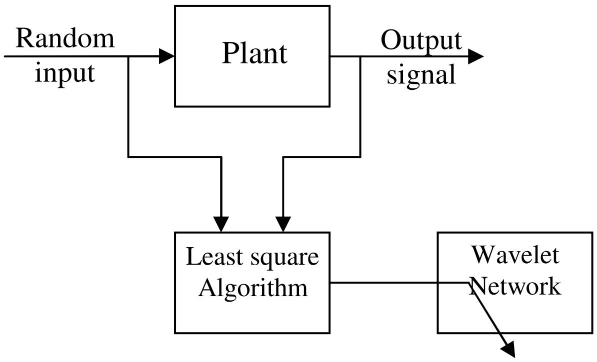

Fig.(2) Block diagram of the training process

IV. ADAPTIVE SPEED CONTROL OF DC MOTORS USING WAVELET NETWORKS

DC motors are characterized by stable and straight forward characteristics and hence they are used to test and implement advanced control algorithms. From the control point of view, the DC motor can be considered as SISO plant, thereby eliminating the complications associated with multi-input drive systems [7]. The dynamics of a separately excited DC motor is described by the following equations:

$$K\omega(t) = -R_a i_a(t) - L_a \frac{di_a(t)}{dt} + V(t)$$
$$Ki_a(t) = J\frac{d\omega(t)}{dt} + D\omega(t) + T_L(t) \quad (13)$$

$$T_L(t) = H(\omega) \quad (14)$$

where

ω : rotor speed in rad/s, $V(t)$: terminal voltage.
$i_a(t)$: armature current. K : torque and back emf constant.
D : damping constant. R_a : rotor resistance.
L_a : rotor inductance. J : total inertia of motor.
T_L : load torque.

The form of the function $H(\omega)$ depends on the nature of the load. The discrete model of the motor is obtained by first combining equations (13) and then replacing the continuous differentials with finite differences [9]:

$$\omega(k+1) = \alpha\omega(k) + \beta\omega(k-1) + \gamma T_L(k) + \delta T_L(k-1) + \zeta V(k) \quad (15)$$

where $\alpha, \beta, \gamma, \delta, \zeta$ are constants depending on the motor parameters K, D, R_a, L_a, J and the sampling period T and k denotes the k^{th} time step. Note that in the model of (15) if T_L is nonlinear function of speed, then the whole model will be nonlinear. Two controllers are designed based on wavelet network for the speed control of DC motor as discussed below:

A. One step Ahead Controller

In this control scheme the control voltage is assumed to affect the plant linearly. The model of equation (15) satisfies this requirement. However, the value of ζ must be known to calculate the control voltage. As mentioned above the value of ζ depends on motor parameters only and does not depend on the load. Thus, it is fair enough to assume that the value of ζ is known and is constant. However, in section (VI.B) we shall introduce an algorithm to identify ζ directly from motor measurements. Thus the model of the motor given in equation (15) could be written as follows:

$$\omega(k+1) = g(\omega(k), \omega(k-1)) + \zeta V(k) \quad (16)$$

Last equation represents a model which consists of two parts; an unknown function *g(.)* which is a function of the present and previous speed only and a linear term which is function of the input control voltage.

A wavelet network is used to emulate the unknown function *g(.)*. Now assume that it is required to make the speed of the motor follow certain reference *r(k)*. Then, the one step ahead control voltage could be calculated as:

$$V(k) = \frac{r(k+1) - \hat{g}(\omega(k), \omega(k-1))}{\zeta} \quad (17)$$

where $\hat{g}(.)$ denotes the output of the wavelet network and *r(k+1)* is assumed to be known at instant *k*. If the function $\hat{g}(.)$ accurately approximates the unknown function *g(.)*, then the speed $\omega(k+1)$ will follow the reference *r(k+1)*.

B. Inverse Controller

In this scheme the motor parameters are assumed to be unknown. Thus the value of ζ is unknown. Hence, the model given in equation (15) could be written as:

$$\omega(k+1) = h(\omega(k), \omega(k-1), V(k)) \quad (18)$$

To apply the inverse control, the inverse model of the plant should be obtained. Thus a wavelet network is trained to approximate the inverse of the unknown function *h(.)*, that is

$$V(k) = \hat{h}(\omega(k+1), \omega(k), \omega(k-1)) \quad (19)$$

where $\hat{h}(.)$ denotes the output of the wavelet network. Now assume that it is required to make the speed of the motor follow certain reference *r(k)*. Then, the control voltage could be calculated using the following control law:

$$V(k) = \hat{h}(r(k+1), \omega(k), \omega(k-1)) \quad (20)$$

where again the value of *r(k+1)* is known at instant *k*.

V. SIMULATION RESULTS

A separately excited DC motor with name plate rating of 1 hp, 220 V, 550 rpm is used in the following simulation. The parameter values associated with the motor are [7]:

J = 0.068 Kg m^2 , K = 3.475 NmA^{-1}, R_a = 7.56 Ω,
L_a = 0.055 H, D = 0.03475 Nm s

The sampling period is 40 ms and the load torque is:

$$T_L(t) = \mu\, \omega^2(t)\, sign(\omega).$$

The value of μ is 0.0039.

Using the above constants and equation (15), the motor model is given by:

$$\omega(k+1) = 0.3436\omega(k) - 0.1534\omega(k-1) - 0.586\mu\, sign(\omega(k))\omega(k)^2$$
$$+ 0.09024\mu\, sign(\omega(k))\omega(k-1)^2 + 0.228V(k)$$

The range of motor speed and voltage are defined as follows:

$$-30 \le \omega \le 30 \quad rad/s$$
$$-100 \le V(k) \le 100 \quad Volt$$

Experiment 1

A random input signal in the range of [-100,100] is used as the training input. Then, the corresponding motor speed is mapped between -2 and 2. These training samples are used once to train a two-input one output network to emulate the unknown function $\hat{g}(.)$ in control law (17), and once to train a three input one output network to emulate the unknown function $\hat{h}(.)$ in control law (19). The two controllers are used online and the load torque is increased after 150 samples by 10 times. The online training algorithm is the least square algorithm with forgetting factor equal to 0.98. Figures (3,4) show the results

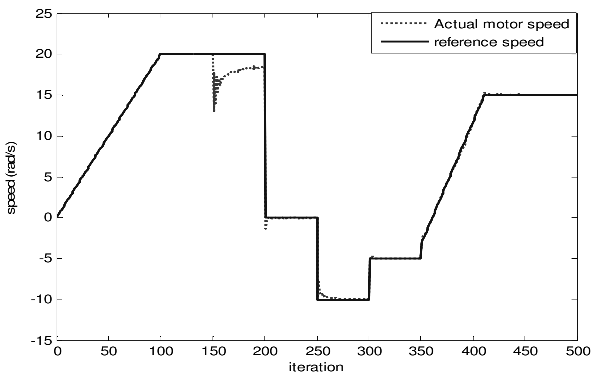

Fig.(3) response of the one step ahead controller Experiment 1.

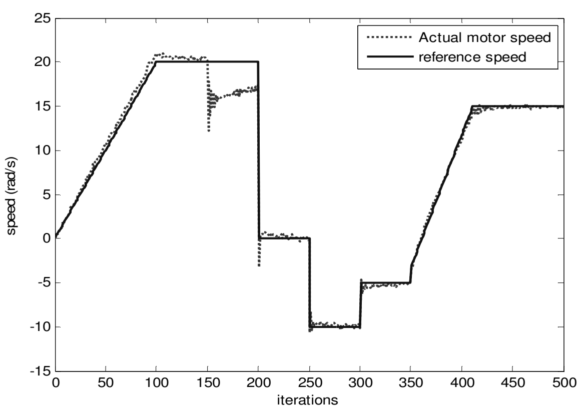

Fig.(4) response of the inverse controller Experiment 1

VI. EXPERIMENTAL IMPLEMENTATION

In this section the experimental implementation of the adaptive control of DC motor using wavelet network is presented. The experimental installation employs the laboratory Feedback® PC based analog and digital trainer kit shown in Fig.(5), while Fig.(6) shows the mechanical unit.

The motor used in this setup is a permanent magnet DC motor. Permanent magnet DC motors – ideally – have linear characteristics. Also the load is a brake magnet which produces a torque linearly proportional with speed. But, practically there exists a nonlinearity produced by the belt. Thus, the load torque-speed characteristic is nonlinear as shown in Fig.(7). In the identification procedure we consider motor voltage as the input and the tachogenerator voltage as the output. The constant of the tachogenerator is known to be 2.5 volt /1000 rpm. Motor nameplate indicates that the range of input voltage is -10 to 10 volt. Corresponding to this range, the reading of the tachogenerator ranges from -7 to 7 volts.

Fig.(5) The experimental setup

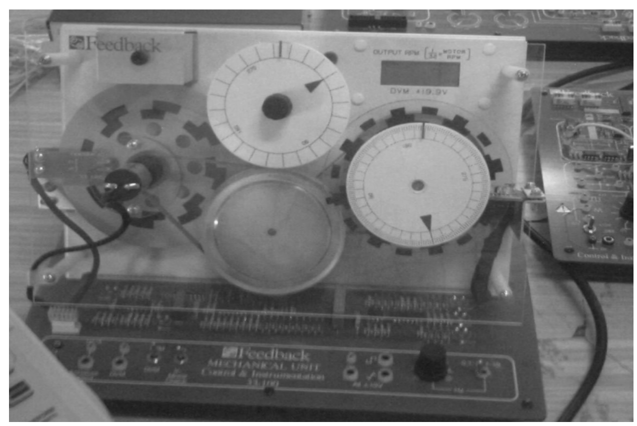

Fig.(6) The mechanical unit

The time constant of the system equals 0.5 seconds and the sampling time of 0.2 seconds gives acceptable results. The training signal shown in Fig.(8) is applied to the motor and the measured tachogenerator voltage is recorded. The brake magnet was removed during training, that is training happens at no-load. The measured tachogenerator voltage is normalized between -0.5 and 0.5 by dividing the tachogenerator voltage by 20. The recorded values of the input voltage and normalized tachogenerator output are used as the training examples. Three controllers are implemented to control the motor as discussed below:

A. Linear Controller

In this control scheme, motor dynamics are assumed to be linear and thus the motor model is taken as:

$$\omega(k+1) = a_1\omega(k) + a_2\omega(k-1) + a_3V(k) \qquad (21)$$

which could be rewritten as:

$$V(k) = b_1\omega(k+1) + b_2\omega(k) + b_3\omega(k-1) \qquad (22)$$

Training examples generated before are used here to identify the parameters b_1, b_2 and b_3 using the recursive least square algorithm. The resulting model is:

$$V(k) = 1.6187\omega(k+1) - 0.4936\omega(k) + 0.0381\omega(k-1) \quad (23)$$

Now, if the objective is to make motor speed follows certain reference trajectory $r(k+1)$, then use this control law:

$$V(k) = 1.6187r(k+1) - 0.4936\omega(k) + 0.0381\omega(k-1) \quad (24)$$

where $V(k)$ is the control voltage at time step k.

B. *One step ahead Controller*

Instead of measuring motor parameters to calculate ζ, we identify the value of ζ directly as follows:

a) Set T_L to zero in (15) results in

$$\omega(k+1) = \alpha\omega(k) + \beta\omega(k-1) + \zeta V(t) \quad (25)$$

b) Identify the parameter ζ from equation (25) using the training examples and recursive least square algorithm.

The resulting value of ζ was found to be 0.6178 (see comments in section VII).

A wavelet network is used to emulate the unknown function $g(.)$ of equation (16). Thus a two-input one output wavelet network is used to cover the region of [-0.5, 0.5] in two dimensions. And the control law (17) is used.

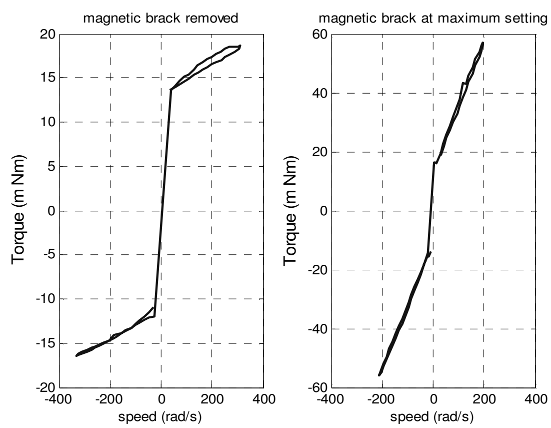

Fig.(7) Torque-speed characteristics of the load.

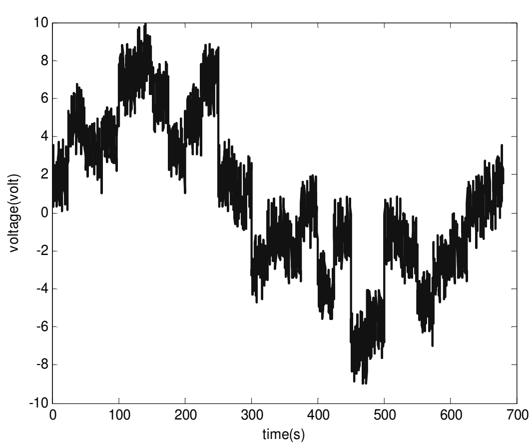

Fig.(8) training input voltage

C. *Inverse Controller*

A three-input one output wavelet network is used to emulate the function $h(.)$ of equation (18) and cover the region of [-0.5, 0.5] in the three dimensions and the control law (19) is used.

VII. EXPERIMENTAL RESULTS

The three controllers discussed above are used online to control the motor. The controllers are built using SIMULINK/MATLAB and then run at real time using the Real Time Workshop. The analog output of the tachogenerator is converted using analog to digital converter and then the speed is plotted using the SIMULINK. It is noted in the following experiments that the speed appears noise-free. This is due to the analog to digital conversion process.

Experiment2:

In this experiment the online training was stopped and the brake magnet load was removed – this is the same situation at which the model was identified -. Figures (9,10, 11) show the results. The reference signal is taken as:

$$r(k) = 1200\ \sin(\frac{2\pi t}{14}) + 400\ \sin(\frac{2\pi t}{10})\ \text{rpm}$$

Experiment 3:

In this experiment the motor starts with no-load, then after 20 seconds the load is changed to maximum value. Then after 200 seconds – from start – the motor is returned to no-load case. The forgetting factor of the recursive least square algorithm is 0.98 for the linear and the inverse controller and is 1 for the one step ahead controller. Figures (12,13,14) show the results.

Comments on the experimental results:

The performance of the one step ahead controller is not so good. This could be explained as follows: Although, the value of ζ was identified with the machine unloaded, there still be a load torque presented by the belt system as shown in Fig.(7). Unfortunately, this load torque can not be removed as long as the belt system is part of the experimental setup. Also, the value of ζ is expected to change online due to the variations in motor parameters specially the damping constant D. Thus it is not justified enough to assume a constant value for ζ.

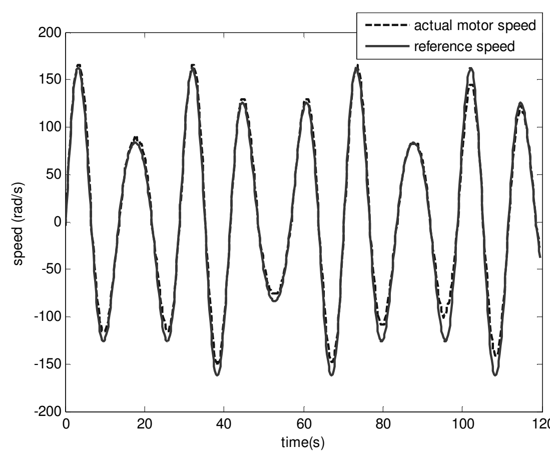

Fig.(9) Performance of the linear controller for Experiment 2

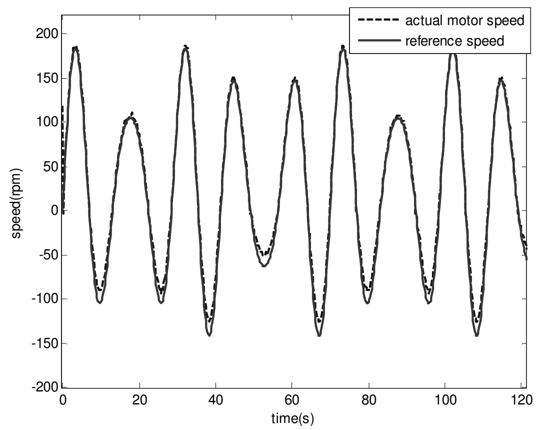

Fig.(10) Performance of the one step ahead controller for Experiment 2

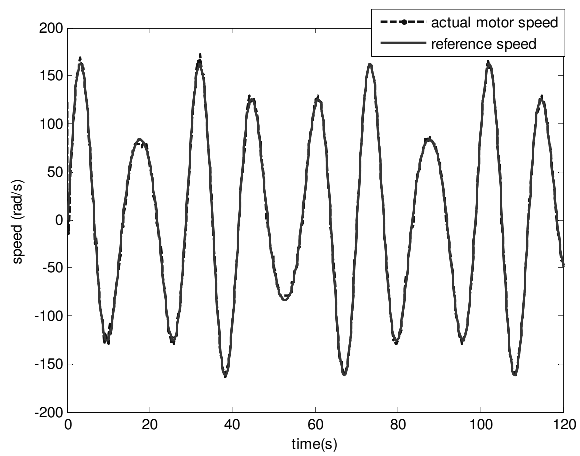

Fig.(11) Performance of the inverse controller for Experiment 2

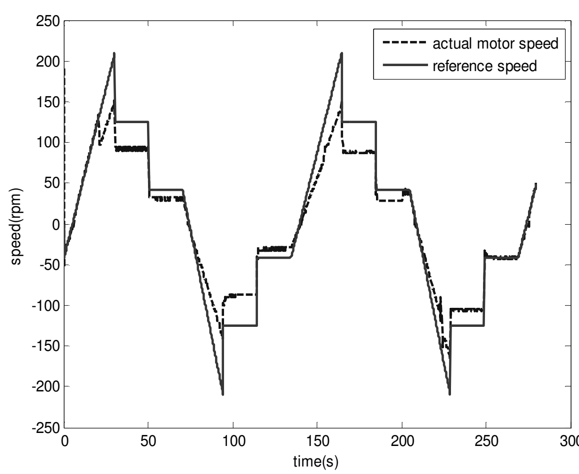

Fig.(12) Performance of the linear controller for Experiment 3

VIII. CONCLUSIONS

The results reported in this paper assure that high performance adaptive control of DC motors could be achieved by designing controllers based on wavelet networks even if motor parameters are unknown and load characteristics are also unknown function of speed and load torque changes online. The experimental study demonstrates the feasibility of the proposed techniques for practical applications.

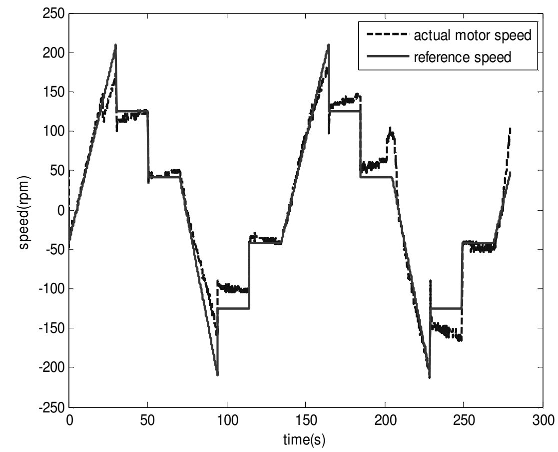

Fig.(13) Performance of the one step ahead controller for Experiment 3

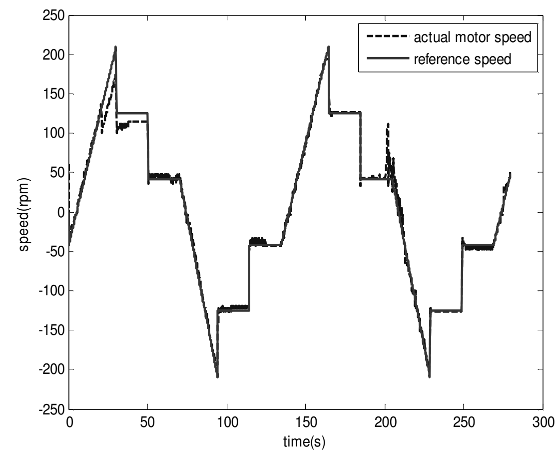

Fig.(14) Performance of the inverse controller for Experiment 3

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