2005 American Control Conference
June 8-10, 2005. Portland, OR, USA

Output Feedback Stabilization of Nonlinear Systems with Delayed Output

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Abstract: It is proposed a novel and simple design scheme of output feedback controllers for a class of nonlinear systems with delayed output. The designed controllers have a very simple structure and do not involve any saturations or recursive computations. Moreover, the nonlinear systems considered here are more general than feedforward systems, and they could be viewed as generalized feedforward systems. By constructing appropriate Lyapunov-Krasovskii functional (LKF) and solving linear matrix inequalities(LMIs), the delay-dependent controller making the closed-loop system globally asymptotically stable (GAS) is explicitly constructed. A simulation example is given to demonstrate the effectiveness of the proposed design procedure.

Key words: Feedforward nonlinear systems; time-delay systems; output feedback; Lyapunov-Krasovskii functionals; linear matrix inequalities

I. INTRODUCTION

In many engineering applications a process to be controlled, or simply monitored is located far from the computing unit and the measured date are transmitted through a low-rate communication system(e.g.in aerospace application). In the above cases the measured outputs are available for computations after a non negligible time delay. In some applications(e.g. in biochemical reactors), the measurement process intrinsically provides an out-of-date output. In [1], the disturbance decoupling problem for nonlinear systems with delayed output is considered. In [2], it is presented a design approach for the construction of a state observer for nonlinear systems when the output measurements are available for computations after a non negligible time delay.

On the other hand, feedforward systems are nonlinear systems described by equations having a specific triangular structure. The problem of the asymptotic stabilization by state feedback of these triangular equations in the absence of delay has been studied by many researchers (see [4],[5],[13]) during the last decade mainly for two reasons: First, the problem of the global asymptotic stabilization of these systems is challenging from a theoretical point of view. Note in particular that they are in general not feedback linearizable and cant not be stabilized by applying the back-stepping method. Second, a number of physical devices, such as the system ball and beam with a friction term (see [12]), the TORA system (see [12]), are described, after a preliminary change of feedback, by equations with the feedforward structure. In recent paper [11], the problem of globally uniformly asymptotically and locally exponentially stabilizing a family of nonlinear feedforward systems when there is a delay in the input is solved.

Up to now, few paper has considered the problem of output feedback controllers design for nonlinear feedforward systems with delayed output. Motivated by [6] and [7], in this paper, by constructing the appropriate Lyapunov-Krasovskii functionals (LKF), we investigate the output feedback controllers design for a class of nonlinear systems with delayed output. Indeed, the nonlinear systems considered in this paper are more general than feedforward systems (upper-triangular form). Our method is simpler because the most work of our design procedure can be completed by MATLAB toolbox. Our stabilizing controller is delay-dependent, and hence the proposed controller is less conservative.

This work was supported by Project 973 of China under Grant G1998020300, the NSF of China under Grant 30271564 and the NSF of Shandong Province under Grant Y2004A01

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II. PRELIMINARIES

We consider the nonlinear systems of dimension $n \geq 2$ described by the equations:

$$\begin{array}{rcl} \dot{x}_1(t) &=& x_2(t) + \phi_1(t, x(t), u(t)) \\ \dot{x}_2(t) &=& x_3(t) + \phi_2(t, x(t), u(t)) \\ &\vdots& \\ \dot{x}_{n-2}(t) &=& x_{n-1}(t) + \phi_{n-2}(t, x(t), u(t)) \\ \dot{x}_{n-1}(t) &=& x_n(t) + \phi_{n-1}(t, x(t), u(t)) \\ \dot{x}_n(t) &=& u(t) \\ y(t) &=& x_1(t-d) \end{array} \qquad (1)$$

where $x = [x_1, \cdots, x_n]^{\mathrm{T}} \in R^n$ is the state, $u \in R$ is the input, $y \in R$ is the output, constant $d \geq 0$ is the delay. The argument of the functions will be omitted or simplified whenever no confusion can arise from the context. For example, we may denote $x_i(t)$ by x_i. The mappings $\phi_i : R \times R^n \times R \longrightarrow R, \quad i = 1, 2, \cdots, n-1$, are continuous and satisfy the following growth condition:

Assumption 1. For $k = 1, 2, \cdots, n-1, \quad l = 1, 2, \cdots, N_k, \ j = 1, 2, \cdots, n+1$, there exist constants $c_{k,l} \geq 0$ and $\alpha_{j,k,l} \geq 0$ satisfying $\sum_{j=1}^{n+1} \alpha_{j,k,l} = 1$, such

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that

$$\begin{array}{rl} & |\phi_k(t,x,u)| \\ \le & \sum_{l=1}^{N_k} c_{k,l}|x_1|^{\alpha_{1,k,l}}\cdots|x_n|^{\alpha_{n,k,l}}|u|^{\alpha_{n+1,k,l}} \end{array} \quad (2)$$

with $c_{k,l}=0$ for $\sum_{i=1}^{n+1} i\cdot\alpha_{i,k,l}<k+2$, where $N_k(k=1,2,\cdots,n-1)$ are positive integers.

Remark 1: When some $\alpha_{i,k,l}=0$, it can be viewed that there is no x_i in the term $c_{k,l}|x_1|^{\alpha_{1,k,l}}\cdots|x_n|^{\alpha_{n,k,l}}|u|^{\alpha_{n+1,k,l}}$. When $k=n-1$, we will get $\alpha_{1,n-1,l}=\cdots=\alpha_{n,n-1,l}=0, \quad \alpha_{n+1,n-1,l}=1$, and consequently we have

$$|\phi_{n-1}(t,x,u)|\le\left(\sum_{l=1}^{N_{n-1}} c_{n-1,l}\right)|u|.$$

Comparing the condition (2) with the following linear growth condition:

$$\begin{array}{rcl} |\phi_i(t,x(t),u(t))| & \le & c(|x_{i+2}|+\cdots+|x_n|+|u(t)|) \\ & & i=1,2,\cdots,n-2, \\ |\phi_{n-1}(t,x(t),u(t))| & \le & c|u(t)|, \end{array} \quad (2^*)$$

where $c\ge 0$ is a constant. It is easy to see that the system (1) satisfying condition (2^*) which is indeed a feedforward nonlinear system obviously satisfies the condition (2).

Now we give an example to illustrate that the nonlinear systems satisfying condition (2) are more general than feedforward (upper-triangular form) systems, and hence we can view the systems satisfying condition (2) as generalized feedforward systems. See system

$$\begin{array}{rcl} \dot{x}_1 & = & x_2-x_3\sin x_1 \\ \dot{x}_2 & = & x_3-\ln(1+\ln(1+|x_1u^3|)) \\ \dot{x}_3 & = & x_4+\frac{1}{4+4e^{-t}}u \\ \dot{x}_4 & = & x_5+\ln(1+u^2) \\ \dot{x}_5 & = & u \\ y(t) & = & x_1(t-1). \end{array} \quad (3)$$

Since there is a x_1 in the right side of the second equation of the system (3), the system (3) is not a feedforward systems. Using the inequality $\ln(1+s^2)\le s$ for $s\ge 0$, it is easy to see that the system (3) satisfies the condition (2). Up to now, to my best knowledge, there is no efficient method to design the output feedback controller for this system.

In this paper, for the vector $\xi=(\xi_1,\xi_2,\cdots,\xi_n)$(or matrix $M=[m_{ij}]_{m\times n}$), we denote vector $(|\xi_1|,|\xi_2|,\cdots,|\xi_n|)$ (or matrix$[|m_{ij}|]_{m\times n}$) by $|\xi|$ (or $|M|$). The matrix $M=[m_{ij}]_{m\times n}$ is said to be a nonnegative matrix if $m_{ij}\ge 0$, for $1\le i\le m, 1\le j\le n$. The property about the nonnegative matrix can be found in [9]. we let $\|\cdot\|$ denote the Euclidean norm for vector, or the induced Euclidean norm for matrix. For any real matrix A, A^{T} denotes the transpose, I is used to represent an identity matrix of appropriate dimension.

The following lemma is useful in the proof of our main result.

Lemma 1 (see [14]) For any $y_i\ge 0\,(i=1,2,\cdots,k)$ and $r_i>0\,(i=1,2,\cdots,k)$ satisfying $\sum_{i=1}^{k} r_i=1$, the following inequality holds:

$$y_1^{r_1}y_2^{r_2}\cdots y_k^{r_k}\le r_1y_1+r_2y_2+\cdots+r_ky_k\,.$$

III. OUTPUT FEEDBACK CONTROLLER

In practice, when some state variables are unavailable, the output variables are needed to control the system. Under the Assumption 1, we will design the linear output dynamic compensator

$$\begin{array}{rcl} \dot{z}(t) & = & Mz(t)+\tilde{M}z(t-d)+Ny(t)\,, \\ & & \quad M\in R^{n\times n}, \tilde{M}\in R^{n\times n}, N\in R^{n\times 1}, \\ u(t) & = & Fz(t), \qquad\qquad F\in R^{1\times n}, \end{array} \quad (4)$$

such that the closed-loop system (1) and (4) is GAS at the equilibrium $(x,z)=(0,0)$.

Theorem 1. Under the Assumption 1, for any given delay $d\ge 0$, there exists a linear output feedback controller of the form (4), which is dependent on the delay d, such that the closed-loop system (1) and (4) is globally asymptotically stable (GAS).

Proof :

We begin with by designing the following linear observer of the system (1)

$$\begin{array}{rcl} \dot{z}_1 & = & z_2+\frac{a_1}{L}(y(t)-z_1(t-d)) \\ \dot{z}_2 & = & z_3+\frac{a_2}{L^2}(y(t)-z_1(t-d)) \\ & \vdots & \\ \dot{z}_{n-1} & = & z_n+\frac{a_{n-1}}{L^{n-1}}(y(t)-z_1(t-d)) \\ \dot{z}_n & = & u(t)+\frac{a_n}{L^n}(y(t)-z_1(t-d)), \end{array} \quad (5)$$

where $L>1$ is a gain parameter to be determined later, and $a_j>0\,(j=1,2,\cdots,n)$ are coefficients of the Hurwitz polynomial

$$p(s)=s^n+a_1s^{n-1}+\cdots+a_{n-1}s+a_n.$$

Now we define

$$\varepsilon_i=L^{i-1}(x_i-z_i), \quad \bar{x}_i=L^{i-1}x_i, \quad \bar{z}_i=L^{i-1}z_i, \quad i=1,2,\cdots,n\,.$$

It is easy to find that

$$\begin{array}{rl} & \varepsilon_1(t-d) \\ = & \varepsilon_1(t)-\int_{t-d}^{t}\dot{\varepsilon}_1(s)ds \\ = & \varepsilon_1(t)-\int_{t-d}^{t}\left(\frac{1}{L}\varepsilon_2(s)+\phi_1(s)-\frac{a_1}{L}\varepsilon_1(s-d)\right)ds. \end{array}$$

From (1) and (5), a simple calculation gives

$$\begin{array}{rcl} \dot{\varepsilon} & = & \frac{1}{L}A\varepsilon+\Phi \\ & & +\frac{1}{L}A_1\int_{t-d}^{t}\left(\frac{1}{L}\varepsilon_2(s)+\phi_1(s)-\frac{a_1}{L}\varepsilon_1(s-d)\right)ds, \end{array} \quad (6)$$

$$\dot{\bar{z}} = \frac{1}{L}J_0\bar{z}+L^{n-1}Gu(t)+\frac{1}{L}A_1\varepsilon_1(t-d)\,, \quad (7)$$

where

$$\varepsilon=[\varepsilon_1,\varepsilon_2,\cdots,\varepsilon_n]^{\mathrm{T}}, \qquad \bar{z}=[\bar{z}_1,\bar{z}_2,\cdots,\bar{z}_n]^{\mathrm{T}},$$

$$G=[0,0,\cdots,0,1]^{\mathrm{T}},$$

$$A = \begin{pmatrix} -a_1 & 1 & 0 & \cdots & 0 \\ -a_2 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -a_{n-1} & 0 & 0 & \cdots & 1 \\ -a_n & 0 & 0 & \cdots & 0 \end{pmatrix}, \ A_1 = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_{n-1} \\ a_n \end{pmatrix},$$

$$J_0 = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix}, \ \Phi = \begin{pmatrix} \phi_1 \\ L\phi_2 \\ \vdots \\ L^{n-2}\phi_{n-1} \\ 0 \end{pmatrix}.$$

Let $b_j > 0\,(j = 1, 2, \cdots, n)$ are coefficients of the Hurwitz polynomial

$$q(s) = s^n + b_n s^{n-1} + \cdots + b_2 s + b_1.$$

Next we will choose an appreciate $L > 1$, such that the trivial solution of the closed-loop system (6),(7) and

$$u \;=\; -\tfrac{1}{L^n}\left(b_1\bar{z}_1 + b_2\bar{z}_2 + \cdots + b_n\bar{z}_n\right) \qquad (8)$$

is asymptotically stable.

From the definitions of $\bar{z}_i$ $(i = 1, 2, \cdots, n)$ and (8), we can get

$$u \;=\; -\tfrac{1}{L^n}\left(b_1 z_1 + b_2 L z_2 + \cdots + b_n L^{n-1} z_n\right). \qquad (9)$$

From (7) and (8), we have

$$\dot{\bar{z}} \;=\; \tfrac{1}{L}B\bar{z} + \tfrac{1}{L}A_1\varepsilon_1(t-d) \qquad (10)$$

where

$$B = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -b_1 & -b_2 & -b_3 & \cdots & -b_n \end{pmatrix}.$$

Since A is a stable matrix, there exists a positive definite matrix P such that

$$PA + A^{\mathrm{T}}P = -I.$$

Choosing $V_1 = \varepsilon^{\mathrm{T}}P\varepsilon$, we have

$$\begin{aligned} & \dot{V}_1|_{(6)} \\ = \; & [\tfrac{1}{L}A\varepsilon + \Phi \\ & + \tfrac{A_1}{L}\int_{t-d}^{t}\left(\tfrac{1}{L}\varepsilon_2(s) + \phi_1(s) - \tfrac{a_1}{L}\varepsilon_1(s-d)\right)ds]^{\mathrm{T}}P\varepsilon \\ & + \varepsilon^{\mathrm{T}}P[\tfrac{1}{L}A\varepsilon + \Phi \\ & + \tfrac{1}{L}A_1\int_{t-d}^{t}\left(\tfrac{1}{L}\varepsilon_2(s) + \phi_1(s) - \tfrac{a_1}{L}\varepsilon_1(s-d)\right)ds] \\ = \; & -\tfrac{1}{L}\|\varepsilon\|^2 + 2\varepsilon^{\mathrm{T}}P\Phi + \tfrac{2}{L}\varepsilon^{\mathrm{T}}PA_1 \\ & \cdot\int_{t-d}^{t}\left(\tfrac{1}{L}\varepsilon_2(s) + \phi_1(s) - \tfrac{a_1}{L}\varepsilon_1(s-d)\right)ds. \end{aligned} \qquad (11)$$

Noticing Assumption 1 (i.e. $c_{k,l} = 0$ for $\sum_{i=1}^{n+1} i\cdot\alpha_{i,k,l} < k+2$), the definitions of ε_i and $\bar{z}_i$, $L > 1$ and Lemma 1, we can get

$$\begin{aligned} & \left|L^{k-1}\phi_k\right| \\ \le \; & L^{k-1}\textstyle\sum_{l=1}^{N_k}(c_{k,l}|x_1|^{\alpha_{1,k,l}}|x_2|^{\alpha_{2,k,l}}\cdots|x_n|^{\alpha_{n,k,l}} \\ & \cdot|u(t)|^{\alpha_{n+1,k,l}}) \\ = \; & L^{k-1}\textstyle\sum_{l=1}^{N_k}(c_{k,l}|\bar{x}_1|^{\alpha_{1,k,l}}|\frac{\bar{x}_2}{L}|^{\alpha_{2,k,l}}\cdots|\frac{\bar{x}_n}{L^{n-1}}|^{\alpha_{n,k,l}} \\ & \cdot|u(t)|^{\alpha_{n+1,k,l}}) \\ \le \; & \textstyle\sum_{l=1}^{N_k}(L^{-2}c_{k,l}|\bar{x}_1|^{\alpha_{1,k,l}}\cdots|\bar{x}_n|^{\alpha_{n,k,l}} \\ & \cdot|(b_1, b_2, \cdots, b_n)\bar{z}(t)|^{\alpha_{n+1,k,l}}) \\ \le \; & \tfrac{1}{L^2}(t_{k,1}|\bar{x}_1| + t_{k,2}|\bar{x}_2| + \cdots + t_{k,n}|\bar{x}_n| \\ & + t_{k,n+1}|(b_1, b_2, \cdots, b_n)\bar{z}(t)|) \\ & \qquad\qquad k = 1, 2, \cdots, n-2, \end{aligned}$$

$$\begin{aligned} \left|L^{n-2}(\phi_{n-1})\right| \;&\le\; L^{n-2}\left(\textstyle\sum_{l=1}^{N_{n-1}} c_{n-1,l}|u(t)|\right) \\ &\le\; \tfrac{1}{L^2}t_{n-1,n+1}(b_1, b_2, \cdots, b_n)|\bar{z}(t)|, \end{aligned}$$

where $t_{k,j}$ $(k = 1, 2, \cdots, n-1,\ j = 1, 2, \cdots, n+1)$ are nonnegative real numbers dependent on $c_{k,l}, \alpha_{j,k,l}$ and N_k.

So we have

$$\begin{aligned} 2\varepsilon^{\mathrm{T}}P\Phi \;&\le\; \tfrac{2}{L^2}|\varepsilon|^{\mathrm{T}}|P|(T_0|\bar{x}| + T_1|\bar{z}(t)|) \\ &\le\; \tfrac{2}{L^2}|\varepsilon|^{\mathrm{T}}|P|T_0|\bar{x}| + \tfrac{2}{L^2}|\varepsilon|^{\mathrm{T}}|P|T_1|\bar{z}(t)|, \end{aligned} \qquad (12)$$

$$\begin{aligned} & \tfrac{2}{L}\varepsilon^{\mathrm{T}}PA_1 \\ & \cdot\int_{t-d}^{t}\left(\tfrac{1}{L}\varepsilon_2(s) + \phi_1(s) - \tfrac{a_1}{L}\varepsilon_1(s-d)\right)ds \\ \le\; & \tfrac{d}{L^2\kappa_1}\varepsilon^{\mathrm{T}}PA_1A_1^TP\varepsilon \\ & + \tfrac{\kappa_1}{dL^2}\int_{t-d}^{t}\varepsilon_2(s)ds\int_{t-d}^{t}\varepsilon_2(s)ds \\ & + \tfrac{d}{L^2\kappa_2}\varepsilon^{\mathrm{T}}PA_1A_1^TP\varepsilon \\ & + \tfrac{\kappa_2}{d}\int_{t-d}^{t}\phi_1(s)ds\int_{t-d}^{t}\phi_1(s)ds \\ & + \tfrac{da_1}{L^2\kappa_3}\varepsilon^{\mathrm{T}}PA_1A_1^TP\varepsilon \\ & + \tfrac{a_1\kappa_3}{L^2d}\int_{t-d}^{t}\varepsilon_1(s-d)ds\int_{t-d}^{t}\varepsilon_1(s-d)ds \\ \le\; & \tfrac{d}{L^2\kappa_1}\varepsilon^{\mathrm{T}}PA_1A_1^TP\varepsilon + \tfrac{\kappa_1}{L^2}\int_{t-d}^{t}\varepsilon_2^2(s)ds \\ & + \tfrac{d}{L^2\kappa_2}\varepsilon^{\mathrm{T}}PA_1A_1^TP\varepsilon + \kappa_2\int_{t-d}^{t}\phi_1^2(s)ds \\ & + \tfrac{da_1}{L^2\kappa_3}\varepsilon^{\mathrm{T}}PA_1A_1^TP\varepsilon + \tfrac{\kappa_3 a_1}{L^2}\int_{t-d}^{t}\varepsilon_1^2(s-d)ds \end{aligned} \qquad (13)$$

where

$$T_0 = \begin{pmatrix} t_{1,1} & t_{1,2} & \cdots & t_{1,n} \\ t_{2,1} & t_{2,2} & \cdots & t_{2,n} \\ \vdots & \vdots & \ddots & \vdots \\ t_{n-2,1} & t_{n-2,2} & \cdots & t_{n-2,n} \\ 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \end{pmatrix},$$

$$T_1 = \begin{pmatrix} t_{1,n+1} \\ t_{2,n+1} \\ \vdots \\ t_{n-2,n+1} \\ t_{n-1,n+1} \\ 0 \end{pmatrix}(b_1, \cdots, b_n),$$

κ_1, κ_2 and κ_3 are positive numbers to be determined later.

From (11), (12) and (13), we can get

$$\begin{aligned}\dot{V}_1|_{(6)} \quad \leq \quad & -\frac{1}{L}\|\varepsilon\|^2 + \frac{2}{L^2}|\varepsilon|^{\mathrm{T}}|P|T_0|\bar{x}| \\ & + \frac{2}{L^2}|\varepsilon|^{\mathrm{T}}|P|T_1|\bar{z}(t)| + \frac{d}{\kappa_1 L^2}\varepsilon^{\mathrm{T}}PA_1A_1^TP\varepsilon \\ & + \frac{\kappa_1}{L^2}\int_{t-d}^{t}\varepsilon_2^2(s)ds + \frac{d}{\kappa_2 L^2}\varepsilon^{\mathrm{T}}PA_1A_1^TP\varepsilon \\ & + \kappa_2\int_{t-d}^{t}\phi_1^2(s)ds + \frac{d\,a_1}{\kappa_3 L^2}\varepsilon^{\mathrm{T}}PA_1A_1^TP\varepsilon \\ & + \frac{\kappa_3 a_1}{L^2}\int_{t-d}^{t}\varepsilon_1^2(s-d)ds.\end{aligned}$$

Choosing LKF

$$\begin{aligned}\widetilde{V}_1 \quad = \quad & \varepsilon^{\mathrm{T}}P\varepsilon + \frac{\kappa_1}{L^2}\int_{-d}^{0}\int_{\theta+t}^{t}\varepsilon_2^2(s)dsd\theta \\ & + \kappa_2\int_{-d}^{0}\int_{\theta+t}^{t}\phi_1^2(s)dsd\theta \\ & + \frac{\kappa_3\,a_1}{L^2}\int_{-d}^{0}\int_{\theta+t-d}^{t}\varepsilon_1^2(s)dsd\theta,\end{aligned}$$

and noticing that $L > 1$, we have

$$\begin{aligned}\dot{\widetilde{V}}_1|_{(6)} \quad \leq \quad & -\frac{1}{L}\|\varepsilon\|^2 + \frac{2}{L^2}|\varepsilon|^{\mathrm{T}}|P|T_0|\bar{x}| \\ & + \frac{2}{L^2}|\varepsilon|^{\mathrm{T}}|P|T_1|\bar{z}(t)| + \frac{d}{\kappa_1 L^2}\varepsilon^{\mathrm{T}}PA_1A_1^TP\varepsilon \\ & + \frac{\kappa_1\,d}{L^2}\varepsilon_2^2(t) + \frac{d}{\kappa_2 L^2}\varepsilon^{\mathrm{T}}PA_1A_1^TP\varepsilon + \kappa_2\,d\,\phi_1^2(t) \\ & + \frac{d\,a_1}{\kappa_3 L^2}\varepsilon^{\mathrm{T}}PA_1A_1^TP\varepsilon + \frac{d\,\kappa_3\,a_1}{L^2}\varepsilon_1^2(t) \\ \leq \quad & -\frac{1}{L}\|\varepsilon\|^2 + \frac{2}{L^2}|\varepsilon|^{\mathrm{T}}|P|T_0|\bar{x}| \\ & + \frac{2}{L^2}|\varepsilon|^{\mathrm{T}}|P|T_1|\bar{z}(t)| \\ & + \left(\frac{d}{\kappa_1 L^2} + \frac{d}{\kappa_2 L^2} + \frac{d\,a_1}{\kappa_3 L^2}\right)\varepsilon^{\mathrm{T}}PA_1A_1^TP\varepsilon \\ & + \frac{\kappa_1\,d}{L^2}\varepsilon_2^2(t) + \frac{d\,\kappa_3\,a_1}{L^2}L^2\varepsilon_1^2(t) \\ & + \frac{\kappa_2\,d}{L^2}\left(|\bar{x}|^TT_2^TT_2|\bar{x}| + |\bar{z}|^TT_3^TT_3|\bar{z}|\right. \\ & \left. + 2|\bar{x}|^TT_2^TT_3|\bar{z}|\right),\end{aligned}$$

where

$$T_2 = (t_{1,1},\, t_{1,2},\, \cdots,\, t_{1,n}),$$

$$T_3 = t_{1,n+1}(b_1,\, b_2,\, \cdots,\, b_n).$$

Because B is a stable matrix, there exists a $Q > 0$ such that

$$QB + B^{\mathrm{T}}Q = -I.$$

Choosing $V_2 = \bar{z}^{\mathrm{T}}Q\bar{z}$, we have

$$\begin{aligned}\dot{V}_2|_{(10)} \quad = \quad & \left(\frac{1}{L}B\bar{z} + \frac{1}{L}A_1\varepsilon_1(t-d)\right)^{\mathrm{T}}Q\bar{z} \\ & + \bar{z}^{\mathrm{T}}Q\left(\frac{1}{L}B\bar{z} + \frac{1}{L}A_1\varepsilon_1(t-d)\right) \\ \leq \quad & -\frac{1}{L}\|\bar{z}\|^2 + \frac{2}{L}\bar{z}^{\mathrm{T}}QA_1\varepsilon_1(t-d) \\ \leq \quad & -\frac{1}{L}\|\bar{z}\|^2 + \frac{\lambda}{L\kappa_4}\bar{z}^{\mathrm{T}}QA_1A_1^TQ\bar{z} \\ & + \frac{\kappa_4}{\lambda L}\varepsilon_1^2(t-d),\end{aligned}$$

where κ_4 and λ are positive numbers to be determined later.

Choosing

$$V = \widetilde{V}_1 + \frac{\lambda}{L}\left(V_2 + \frac{\kappa_4}{\lambda L}\int_{t-d}^{t}\varepsilon_1^2(s)ds\right)$$

and letting

$$\delta_1 + \delta_2 < L, \qquad \delta_3 + \delta_4 < \lambda,$$

where δ_i $(i = 1, 2, 3, 4)$ are positive numbers to be determined later, we can get

$$\begin{aligned} & \dot{V}|_{(6)(10)} \\ \leq \quad & -\frac{1}{L}\|\varepsilon\|^2 + \frac{2}{L^2}|\varepsilon|^{\mathrm{T}}|P|T_0(|\varepsilon| + |\bar{z}|) \\ & + \frac{2}{L^2}|\varepsilon|^{\mathrm{T}}|P|T_1|\bar{z}(t)| \\ & + \left(\frac{d}{\kappa_1 L^2} + \frac{d}{\kappa_2 L^2} + \frac{d\,a_1}{\kappa_3 L^2}\right)\varepsilon^{\mathrm{T}}PA_1A_1^TP\varepsilon \\ & + \frac{\kappa_1\,d}{L^2}\varepsilon_2^2(t) + \frac{\kappa_3\,d\,a_1}{L^2}\varepsilon_1^2(t) \\ & + \frac{\kappa_2\,d}{L^2}\left((|\varepsilon| + |\bar{z}|)^TT_2^TT_2(|\varepsilon| + |\bar{z}|)\right. \\ & \left. + |\bar{z}|^TT_3^TT_3|\bar{z}| + 2(|\varepsilon| + |\bar{z}|)^TT_2^TT_3|\bar{z}|\right) \\ & - \frac{\lambda}{L^2}\|\bar{z}\|^2 + \frac{\lambda^2}{L^2\kappa_4}\bar{z}^{\mathrm{T}}QA_1A_1^TQ\bar{z} + \frac{\kappa_4}{L^2}\varepsilon_1^2 \\ \leq \quad & \frac{1}{L^2}(\varepsilon^{\mathrm{T}},\, \bar{z}^{\mathrm{T}},\, |\varepsilon|^{\mathrm{T}},\, |\bar{z}|^{\mathrm{T}})\Omega(\varepsilon^{\mathrm{T}},\, \bar{z}^{\mathrm{T}},\, |\varepsilon|^{\mathrm{T}},\, |\bar{z}|^{\mathrm{T}})^T\end{aligned}$$

where

$$\Omega = \begin{pmatrix} W_{11} & 0 & 0 & 0 \\ 0 & W_{22} & 0 & 0 \\ 0 & 0 & W_{33} & W_{34} \\ 0 & 0 & W_{34}^T & W_{44} \end{pmatrix},$$

$$\begin{aligned}W_{11} \quad = \quad & -\delta_1 I + d\left(\frac{1}{\kappa_1} + \frac{1}{\kappa_2} + \frac{a_1}{\kappa_3}\right)PA_1A_1^TP \\ & + d\kappa_1 J_1 + d\kappa_3 a_1 J_2 + \kappa_4 J_2\,,\end{aligned}$$

$$W_{22} = -\delta_3 I + \frac{\lambda^2}{\kappa_4}QA_1A_1^TQ,$$

$$W_{33} = -\delta_2 I + d\kappa_2 T_2^TT_2 + |P|T_0 + T_0^T|P|\,,$$

$$W_{34} = |P|T_1 + |P|T_0 + d\kappa_2 T_2^T(T_2 + T_3)\,,$$

$$W_{44} = -\delta_4 I + d\kappa_2(T_2 + T_3)^T(T_2 + T_3)\,,$$

$$J_1 = (0,\ 1,\ 0, \cdots, 0)^T(0,\ 1,\ 0, \cdots, 0),$$

$$J_2 = (1,\ 0,\ 0, \cdots, 0)^T(1,\ 0,\ 0, \cdots, 0).$$

If we choose $L > 1$ satisfying $\Omega < 0$, we shall get $\dot{V}|_{(6)(10)} < 0$ which indicate that (6)(10) is asymptotically stable at $\varepsilon = 0$ and $\bar{z} = 0$ (see[10]). The trivial solution of the closed-loop system (6), (7) and (8) is also asymptotically stable. Consequently, the closed-loop system (1), (5) and (9) is also asymptotically stable at $x = 0$ and $z = 0$. Therefore, we can conclude that (5)(9) is the linear output dynamic compensator of the system (1).

Since the inequality $\Omega < 0$ could not be solved by MATLAB, we give the following consideration. From Schur complements, we know that finding $L > 1$ (as small as possible) satisfying $\Omega < 0$ is equivalent to solving the following optimization problem with linear matrix inequalities (LMIs) constraints:

$$\min_{\kappa_1,\kappa_2,\kappa_3,\kappa_4,\delta_1,\delta_2,\delta_3,\delta_4,\lambda} L$$

$$s.t. \qquad L > 1,\ \ \delta_1 + \delta_2 < L,\ \ \delta_3 + \delta_4 < \lambda,$$

$$\text{and} \quad \begin{pmatrix} \Psi_{11} & 0 & \Psi_{13} \\ 0 & \Psi_{22} & 0 \\ \Psi_{13}^{\mathrm{T}} & 0 & \Psi_{33} \end{pmatrix} < 0, \qquad (14)$$

where

$$\Psi_{11} = \begin{pmatrix} -\delta_1 I + d\kappa_1 J_1 + d\kappa_3 a_1 J_2 + \kappa_4 J_2 & 0 \\ 0 & -\delta_3 I \end{pmatrix},$$

$$\Psi_{22} = \begin{pmatrix} W_{33} & W_{34} \\ W_{34}^T & W_{44} \end{pmatrix},$$

$$\Psi_{13} = \begin{pmatrix} \sqrt{d}PA_1 & \sqrt{d}PA_1 & \sqrt{da_1}PA_1 & 0 \\ 0 & 0 & 0 & \lambda QA_1 \end{pmatrix},$$

$$\Psi_{33} = \mathrm{diag}[-\kappa_1,\ -\kappa_2,\ -\kappa_3,\ -\kappa_4\ ].$$

LMI (14) can be easily solved by LMI toolbox in MATLAB. Our LMI (14) always has a solution, and our design method is always efficient for the system (1) satisfying the condition (2), and hence our design method proposed here is indeed a constructive method. However, the LMIs in [3] [8] maybe have no solution, and when the LMIs have no solution, the design method proposed there will not work. Hence our design method is better.

The proof of the Theorem is completed .

Remark 2: For any given delay $d > 0$, we can always find a constant L satisfying (14). The purpose of finding a small $L > 1$ satisfying (14) is that we would like get a bigger control gain. There may be many sets $a_j,\ b_j\,(j = 1,\ 2,\ \cdots,\ n)$ satisfying our design requirements, but we always choose those such that $L > 1$ is as small as possible. Consequently the gain of the controller is as big as possible and the time of stabilizing system will be as short as possible.

Remark 3: The control gain we get is dependent on the delay d. From a theoretical point of view, for any delay $d > 0$, we can always find a stabilizing controller. But when the delay $d > 0$ is bigger, the control gain of system will be much less. The time of stabilizing system will be very long, and hence this will lower efficacy of the controller in practical. So, when we design a stabilizing controller, we should decrease the delay as far as possible.

Remark 4: Our controller has good robust performance, that is, a stabilizing controller designed for the system (1) with delay $d > 0$, will be also efficient for the system (1) with delay $\tilde{d}$ satisfying $\tilde{d} < d$.

IV. EXAMPLE

Example Consider the following nonlinear system:

$$\begin{array}{rcl} \dot{x}_1 & = & x_2 + \frac{1}{8}\ln(1 + |x_2 u|) \\ \dot{x}_2 & = & x_3 - \frac{1}{10+10e^{-t}}u \\ \dot{x}_3 & = & u \\ y(t) & = & x_1(t-d). \end{array} \qquad (15)$$

For the system (15) satisfying Assumption 1, it is easy to see that $t_{1,2} = \frac{1}{16}, t_{1,4} = \frac{1}{16},\ \ t_{1,1} = t_{1,3} = t_{2,1} = t_{2,2} = t_{2,3} = 0,\ \ t_{2,4} = \frac{1}{10}$. We can construct the output feedback controller for the system (15) with $d = 1$ by the following procedures.

Choose

$$a_1 = 1.0,\ a_2 = 0.8,\ a_3 = 0.10\,;$$

$$b_1 = 0.28,\ b_2 = 1.2,\ b_3 = 0.65\,.$$

Solving the Lyapunov equations

$$PA + A^{\mathrm{T}}P = -I \ \text{ and } \ QB + B^{\mathrm{T}}Q = -I,$$

we can get

$$P = \begin{pmatrix} 1.1071 & -0.5000 & -2.0714 \\ -0.5000 & 2.0714 & -0.5000 \\ -2.0714 & -0.5000 & 19.7143 \end{pmatrix} > 0,$$

$$Q = \begin{pmatrix} 2.8258 & 2.4267 & 1.7857 \\ 2.4267 & 5.2253 & 2.4389 \\ 1.7857 & 2.4389 & 4.5214 \end{pmatrix} > 0.$$

By solving (14), we can get $L = 26.8799$. So the output feedback controller of the system (15) is

$$u(t) = -\frac{1}{L^3}\left(b_1 z_1 + b_2 L z_2 + b_3 L^2 z_3\right), \qquad (16)$$

where z_1, z_2 and z_3 are the states of the following system

$$\begin{array}{rcl} \dot{z}_1 & = & z_2 + \frac{a_1}{L}(y(t) - z_1(t-d)) \\ \dot{z}_2 & = & z_3 + \frac{a_2}{L^2}(y(t) - z_1(t-d)) \\ \dot{z}_3 & = & -\frac{1}{L^3}\left(b_1 z_1 + b_2 L z_2 + b_3 L^2 z_3\right) \\ & & +\frac{a_3}{L^3}(y(t) - z_1(t-d))\,. \end{array} \qquad (17)$$

Fig.1, Fig.2 and Fig.3 show the state response of the closed-loop system (15), (16) and (17) with the initial condition

$$[x_1(t),\ x_2(t),\ x_3(t)] = [1300,\ 15,\ 1],$$

$$[z_1(t),\ z_2(t),\ z_3(t)] = [0,\ 0,\ 0],$$

$$\text{for} \quad t \in [-1,\ 0].$$

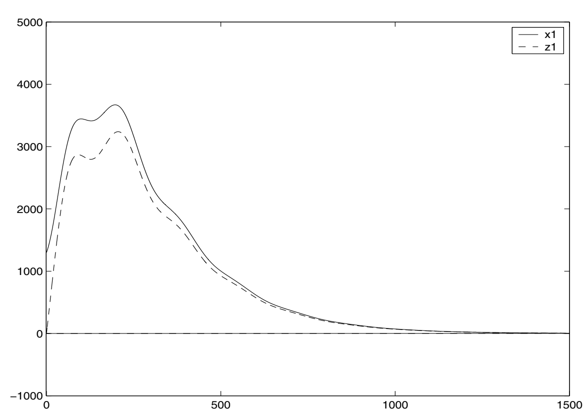

Fig.1: Trajectories of x_1 andz_1.

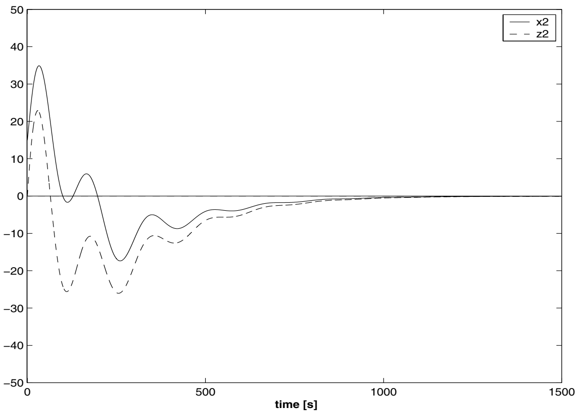

Fig.2: Trajectories of x_2 and z_2.

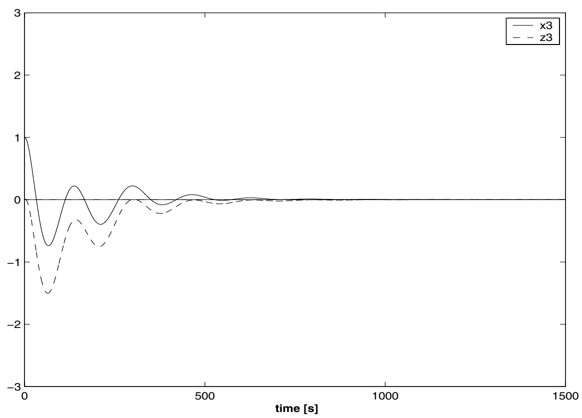

Fig.3: Trajectories of x_3 and z_3.

Remark 5: From this example, it is easy to find that the time of stabilizing the system (15) is much longer, this is mainly because the system (15) has higher order and nonlinear terms. Indeed, the gains of stabilizing controller for the strict feedforward system are, in general, lower (see [5] [11] [13]), and the time of stabilizing such system will be longer. On the other hand, the gains of stabilizing controller for the strict feedback (lower-triangular form) system are, in general, very high (see[6] [7] [15]).

V. CONCLUSION

In this paper, we have studied the problem of global stabilization by output feedback for a class of nonlinear system with delayed output. The method used here is maybe feasible for the more general nonlinear delay systems.

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