

# Low Frequency Vibration Suppression Shape Filter and High Frequency Vibration Suppression Shape Filter

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**Abstract**—A method is presented for generating a control command which suppresses all the resonant dynamics in a flexible dynamic system. Residual vibration of a flexible system can be significantly reduced. The experimental results for hard disk drive short seek control illustrate the effectiveness of the proposed method.

## I. INTRODUCTION

Control of flexible structures has been extensively studied in recent years. Flexible structures such as high-speed disk drive actuators require extremely precise positioning under very tight time constraints. Whenever a fast motion is commanded, residual vibration in the flexible structure is induced, which increases the settling time. One solution is to design a closed-loop controller to damp out vibrations caused by the command inputs and disturbances to the plant. However, the resulting closed-loop response may still be too slow to provide an acceptable settling time, and the closed-loop control is not able to compensate for high frequency residual vibration which occurs beyond the closed-loop bandwidth. An alternative approach is to develop an appropriate reference trajectory that is able to minimize the excitation energy imparted to the system at its natural frequencies.

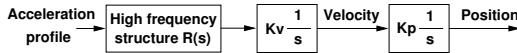


Fig. 1. A typical mechanical flexible system.

Fig. 1 shows a typical mechanical flexible system, where  $\frac{1}{s}$  is an integrator,  $K_v$  is a velocity constant gain, and  $K_p$  is a position constant gain. The high frequency modes can be described as a transfer function  $R(s) = \lim_{n \rightarrow \infty} \frac{b_n s^n + b_{n-1} s^{n-1} + \dots + b_1 s + 1}{a_n s^n + a_{n-1} s^{n-1} + \dots + a_1 s + 1}$  in which an infinite number of lightly damped resonant structures is possible. The goal of vibration suppression trajectory generation is to find a fast input trajectory, under some physical constraint, with minimum possible residual vibration.

The phrase “move time” refers to the time duration of the feed forward control input, such as acceleration, current, or voltage. Settle time means the time duration after the end of

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move time to achieve the settle criterion, for example  $\pm 5\%$  tracking error. Seek time is the sum of the move time and the settle time. Fig. 2 clearly shows the move time and the settle time for a hard disk drive arm movement with open-loop control. The top plot is the current input signal, the middle plot is the resultant position signal, and the bottom plot shows the position signal near the target track. In this instance, the move time is 2.5 milliseconds (msec). Because of the resonant structure in the flexible system, the position signal cannot settle down immediately after the move. In Fig. 2, the settle time is about 2.5 msec with  $\pm 1\%$  tracking error criterion. The objective of the vibration suppression trajectory generation is to minimize the seek time.

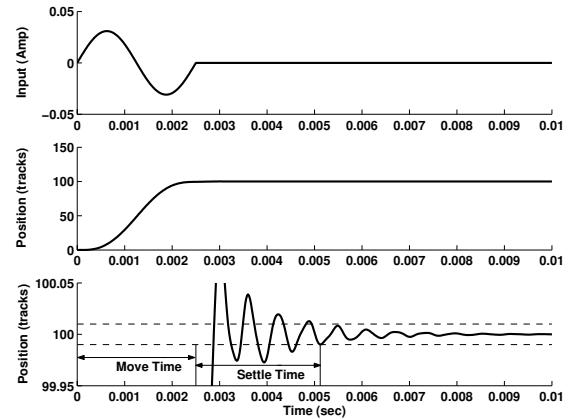


Fig. 2. Illustration of move time and settle time of an open-loop control.

The same concepts apply to closed-loop control. Fig. 3 clearly shows the move time and the settle time for a hard disk drive arm movement with closed-loop control. The top two plots are the reference position command profile and the bottom two plots show the real position signal. In this instance, the ideal reference position move time of the flexible arm is 2.5 msec. Due to the resonant structure in the flexible system, the position signal cannot settle down immediately after the move. In Fig. 3, the settle time is about 2.5 msec with  $\pm 1\%$  tracking error criterion.

The position reference input can be generated in two ways. First, it can be assumed as the integral of a vibration suppression velocity profile. In this case, since the velocity profile is a smooth trajectory starting and ending at zero, the resultant position reference is a smooth trajectory. Secondly, it can be generated from a step movement command  $s(t) = S \cdot 1(t)$ , through a finite support filter,  $f(t)$ ,  $0 \leq t \leq T$ , where  $T$  is the time duration of the finite support filter. To

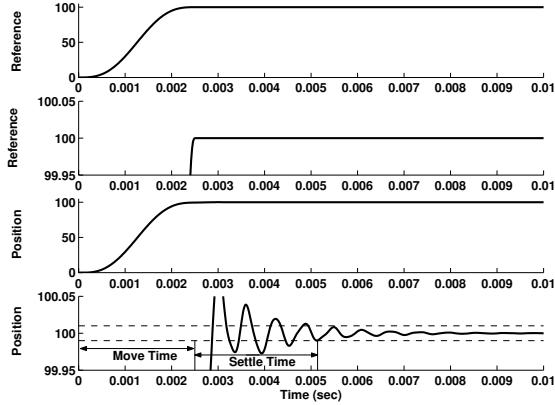


Fig. 3. Illustration of move time and settle time of a closed-loop control.

guarantee that the filtered command reaches the same set point as the step movement command, the integral of  $f(t)$  must be equal to 1, i.e.,

$$\int_0^T f(t)dt = 1. \quad (1)$$

This finite support filter  $f(t)$ ,  $0 \leq t \leq T$ , which generates a vibration suppression position reference profile is called a vibration suppression shape filter, or simply a shape filter. In the discrete-time case, if the finite impulse response shape filter is  $f[k]$ ,  $0 \leq k \leq M$ , the constraint reduces to  $\sum_{k=0}^M f[k] = 1$ .

A normalized vibration suppression velocity profile can be used as the shape filter function to generate the reference position profile. The normalization is to make the velocity profile satisfy the constraint (1). So if there is a vibration suppression velocity profile,  $v(t)$ ,  $0 \leq t \leq T$ , a vibration suppression shape filter  $f(t)$  can be generated by

$$f(t) = \frac{v(t)}{\int_0^T v(t)dt}. \quad (2)$$

The good property of this kind of shape filters is that the shape filter itself is a vibration suppression velocity profile, so it can generate a smooth vibration suppression position reference.

A shape filter does not necessarily start and end at zero. Fig. 4 shows a step position command, a typical shape filter that has non-zero values at the start and end, and the filtered position reference. Although the shape filter smoothly changes from the start to the end, the initial value at time zero and the final value at time  $T$  of the shape filter are not zero. Furthermore, a shape filter can also be a non-smooth function. Fig. 5 shows a step position command, a typical non-smooth shape filter, and the filtered position reference. It shows that the shape filter function is not smooth from the start to the end.

It is easy to understand that the vibration suppression position reference generated from a step command  $s(t) = S \cdot 1(t)$  through a shape filter,  $f(t)$ ,  $0 \leq t \leq T$ , can also be

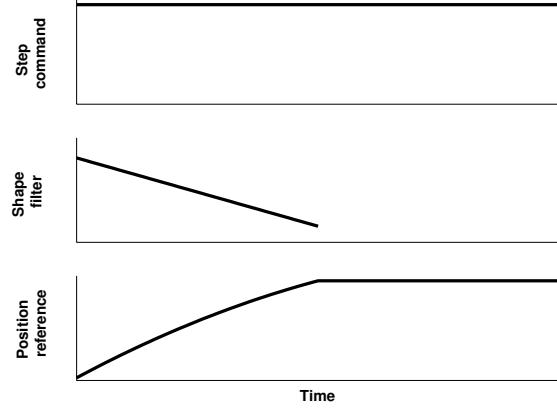


Fig. 4. A step position command, a non-zero start and end shape filter and the filtered position reference.

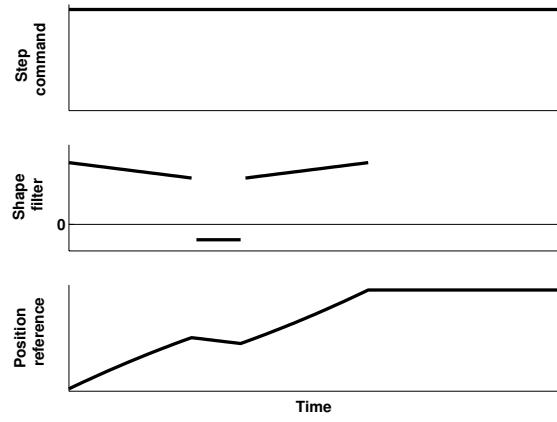


Fig. 5. A step position command, a non-smooth shape filter and the filtered position reference.

generated from the integral of a scaled shape filter  $S \cdot f(t)$ ,  $0 \leq t \leq T$ , i.e.,  $s(t) * f(t) = \int_0^t S \cdot f(\tau)d\tau$ , where  $*$  is the convolution operator.

## II. HIGH FREQUENCY VIBRATION SUPPRESSION SHAPE FILTER

In the previous study [1], [2], a velocity profile is generated which suppresses all the high frequency resonant dynamics in a flexible dynamic system. If the velocity profile is given by  $vel[k]$ ,  $k = 0, \dots, M$ , then a high frequency vibration suppression shape filter  $f_H[k]$  can be generated by

$$f_H[k] = \frac{vel[k]}{\sum_{i=0}^M vel[i]}. \quad (3)$$

Since the velocity profile  $vel[k]$  has zero initial and final values, the resultant shape filter  $f_H[k]$  also has zero initial and final values. A high frequency suppression shape filter with non-zero initial and final values will be reported separately.

### III. LOW FREQUENCY VIBRATION SUPPRESSION SHAPE FILTER

In a practical system, a lower resonance frequency mode may exist which is located far from the high frequency resonance modes as shown in Fig. 6. If the low frequency  $\Omega_1$  in Fig. 6 is chosen to be a bandwidth in [1], [2] for the vibration suppression profile generation, the time duration of the control profile is inefficiently increased. Low frequency suppression shape filter is studied in [3]. In [3], it shows that both the Input Shaping<sup>®</sup><sup>1</sup> [4], [5] and OATF [6], [7] are special cases of a non-continuous function based vibration suppression shape filter. Different from the Input Shaping<sup>®</sup> and OATF, the vibration suppression shape filter in [3] is generated from a continuous function, so it is able to suppress the unmodeled high frequency resonance modes besides canceling the low frequency resonance mode if the shape filter is designed based on a low frequency resonance mode. However, the Input Shaping<sup>®</sup> and OATF are not able to suppress the unmodeled high frequency vibrations if they are designed based on a low frequency resonance mode [3].

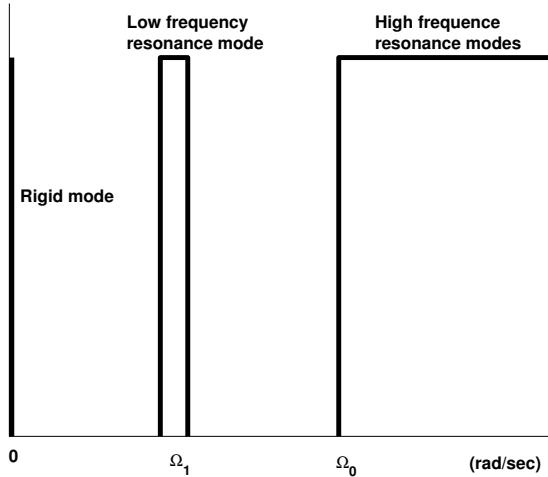


Fig. 6. Illustration of existence of a low resonance frequency mode located far from the high frequency modes in a flexible system.

### IV. VIBRATION SUPPRESSION SHAPE FILTER TO SUPPRESS ALL THE RESONANT DYNAMICS IN A FLEXIBLE SYSTEM

A vibration suppression shape filter to suppress all the resonant dynamics in a flexible system is generated in this section. If the high frequency vibration suppression shape filter is given by  $f_H[k], k = 0, 1, \dots, M$ , and the low frequency vibration suppression shape filter is given by  $f_L[k], k = 0, 1, \dots, N$ , then a vibration suppression shape filter to suppress all the resonant dynamics can be generated through a filter operation as shown in Fig. 7. The filtering operation is mathematical convolution in the time domain

<sup>1</sup>Input Shaping<sup>®</sup> is a registered trademark of Convolve, Inc. in the United States. When this technique is referred to in this report, the terms “input shaping” or “input shaper” are used.

and multiplication in the frequency domain. The resultant vibration suppression shape filter is

$$f[k] = f_H * f_L[k], \quad (4)$$

$$= \sum_{i=0}^k f_H[k-i]f_L[i], \quad k = 0, 1, \dots, M + N. \quad (5)$$

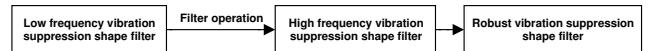


Fig. 7. Generation of a vibration suppression shape filter.

## V. EXPERIMENTAL RESULTS FOR HARD DISK DRIVE SHORT SEEK CONTROL

### A. Open-Loop Resonant Dynamics and Experiment Setup

The disk drive used for the experiment is produced by Conner. The model of the disk drive is CP3000 and the series number is E59JKA. The equipment used to set up the experiment is shown in Table I. The open-loop of the actuator arm is  $H(s) = 7.5 \times 10^9 R(s) \frac{1}{s^2}$ , where the input is the current signal in amps and the output is the position signal in  $\mu\text{m}$ .  $R(s)$  is a resonance structure. The Bode magnitude plot of a reduced order (28<sup>th</sup>)  $R(s)$  is shown in Fig. 8. The resonance modes change drastically due to variation of the mode parameters. On the Bode plot, the peaks of the frequency response may shift both in frequency and in amplitude. Fig. 9 shows the whole setup of the experiment.

Equipments For Disk Drive Research
An Open Disk Drive
Kepco Power Supply/ Amplifier
Polytec Laser Doppler Vibrometer (LDV)
DS1104 PPC Controller Board (dSPACE)
1GHz LeCroy Oscilloscope
MATLAB Real-Time Workshop
Newport Vibration Isolation Table
Pentium II 450 MHz computer

TABLE I  
EQUIPMENT FOR THE DISK DRIVE EXPERIMENT.

### B. Standard Closed-Loop Control with Step Reference Command

A position and velocity feedback control is used to control the position of the flexible arm. Fig. 10 shows the Simulink diagram of the position and velocity feedback closed-loop control structure. The position gain is chosen to be 3.162278 Amp/mm and the velocity gain is chosen to be 0.000316 Amp/(mm/sec). The command reference is a step function with amplitude 10  $\mu\text{m}$  and frequency 20 Hz. The resultant position and the control signal are shown in Fig. 11. It shows that residual vibration exists after a long period of time.

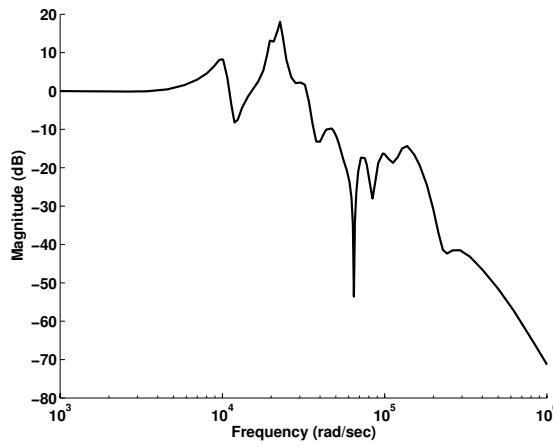


Fig. 8. Bode magnitude of the resonance structure.

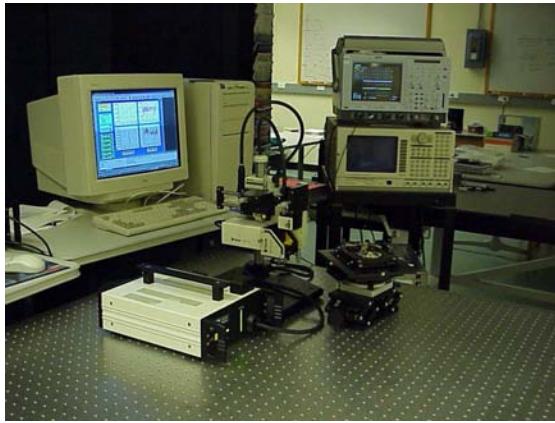


Fig. 9. The experimental setup.

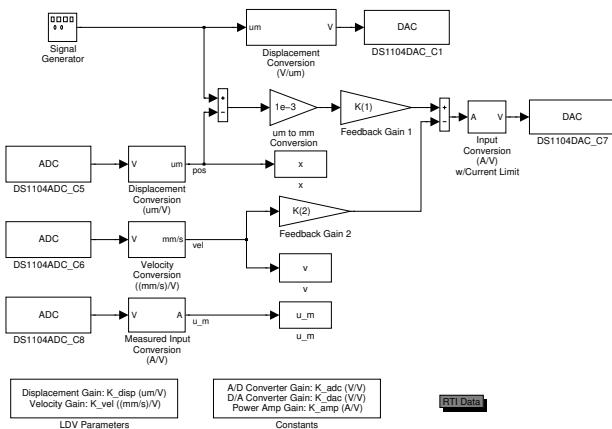


Fig. 10. Simulink diagram for the position and velocity feedback control.

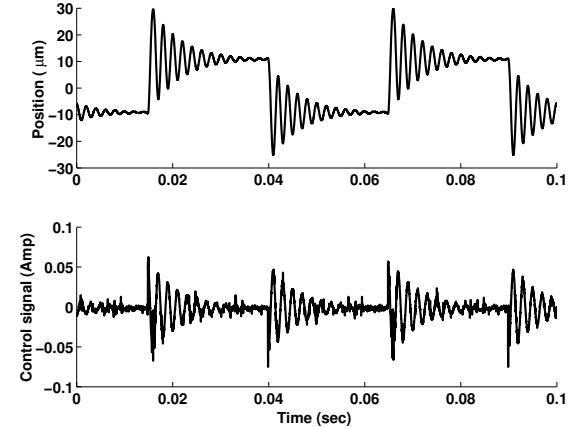


Fig. 11. Experimental result for  $20 \mu\text{m}$  move with the standard closed-loop control and step reference.

### C. Experimental Verification Between ZVD Input Shaping<sup>®</sup> Technique and Rectangle Based Shape Filter

In [3], a comparison of simulation results between the ZVD input shaper [4], [5] and rectangle based shape filter [3] is performed. The ZVD input shaper amplifies the high frequency vibrations, but the rectangle based shape filter reduces the high frequency vibrations. In this section, both ZVD input shaper and rectangle based shape filter are used to suppress the low frequency vibrations in the hard disk drive flexible arm control. The undamped natural frequency and the damping ratio of the low frequency mode are  $\omega_1 = 3.64 \times 10^3 \text{ rad/sec}$  and  $\zeta_1 = 0.425$ . Fig. 13 shows the discrete-time ZVD input shaper with the sampling period  $T_s = 10^{-5} \text{ sec}$ . Fig. 14 shows the discrete-time rectangle based shape filter with the sampling period  $T_s = 10^{-5} \text{ sec}$ . The vibration suppression shape filter is implemented as in Fig. 12. The step reference command is first input to the vibration suppression shape filter, then the shaped reference command is sent to the closed-loop control. Fig. 15 shows the experimental results with the discrete-time ZVD input shaper. Fig. 16 shows the experimental results with the discrete-time rectangle based shape filter. In both cases, the low frequency vibration is canceled. The position reference generated from step reference and discrete-time ZVD input shaper amplifies the high frequency vibrations as shown in Fig. 15. However, the position reference generated from step reference and discrete-time rectangle based shape filter suppresses all the high frequency vibrations as shown in Fig. 16. From comparison of the control signals in Fig. 15 and Fig. 16, it is also clear that the ZVD input shaper makes the control signal very aggressive since the high frequency vibrations are amplified. A vibration suppression shape filter that is able to suppress all the resonant dynamics will be generated and tested in the next section.



Fig. 12. Implementation of a vibration suppression shape filter.

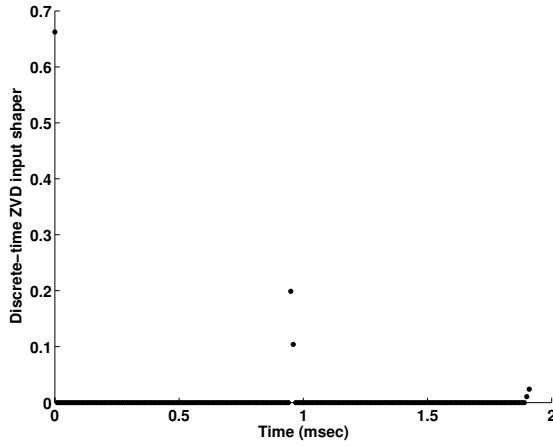


Fig. 13. Discrete-time ZVD input shaper.

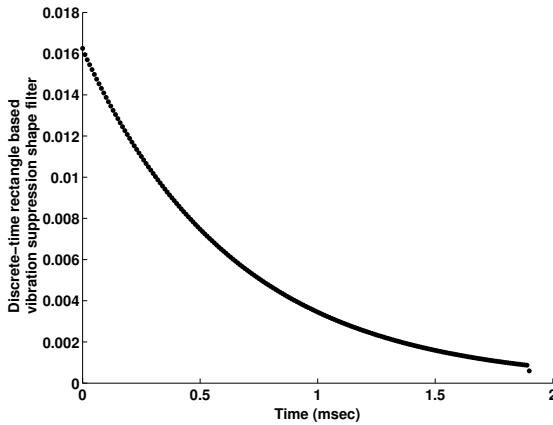


Fig. 14. Discrete-time rectangle based shape filter.

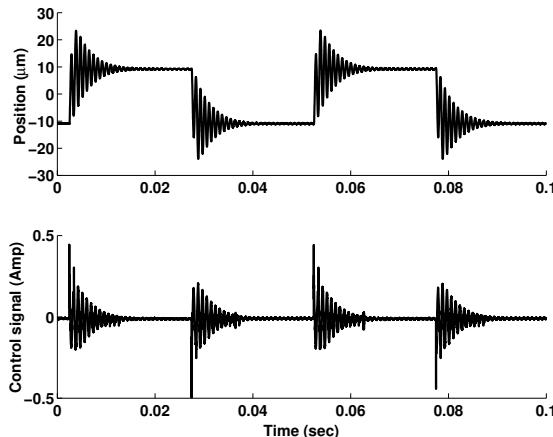


Fig. 15. Experimental results for 20  $\mu\text{m}$  move with step reference and discrete-time ZVD input shaper.

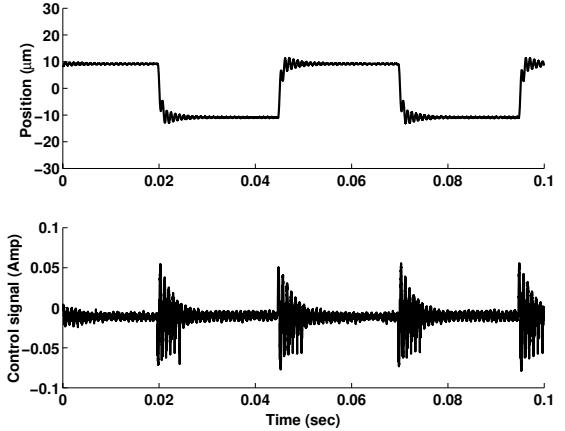


Fig. 16. Experimental results for 20  $\mu\text{m}$  move with step reference and discrete-time rectangle based shape filter.

#### D. Standard Closed-Loop Control with Vibration Suppression Shape Filter

In this section, a robust vibration suppression shape filter is generated to shape the command reference. The high frequency resonant modes in the closed-loop control approximately occur beyond  $\Omega_{h0} = 10^4$  rad/sec. The move time duration of the high frequency vibration suppression shape filter is chosen to be 1.5 msec. The prolate spheroidal wave function based high frequency vibration suppression shape filter [2] is shown in Fig. 17. Before the high frequency resonance modes, there is one low frequency resonance mode in the closed-loop transfer function. The undamped natural frequency and the damping ratio of this low frequency mode are  $\omega_{l1} = 3.64 \times 10^3$  rad/sec and  $\zeta_{l1} = 0.425$ . The rectangle window based low frequency vibration suppression shape filter [3] is shown in Fig. 18 which is the same as Fig. 14. The high frequency vibration suppression shape filter and low frequency vibration suppression shape filter can be combined together through the filtering operation in Fig. 7. The resultant vibration suppression shape filter is shown in Fig. 19.

Fig. 20 shows experimental results of the vibration suppression shape filter in Fig. 19. It shows that both the low frequency and high frequency residual vibrations are suppressed by the position reference generated from step reference and the discrete-time vibration suppression shape filter in Fig. 19.

## VI. CONCLUSIONS

In this examination, a vibration suppression shape filter is generated which is able to suppress all the resonant dynamics in a flexible system. The experimental results of the hard disk drive short seek control show the residual vibration of the actuator arm is significantly reduced. Since the smoothly changing shape filter can also be used as a vibration suppression velocity profile, drive current command can be generated. The model reference closed-loop control experimental results will be reported separately.

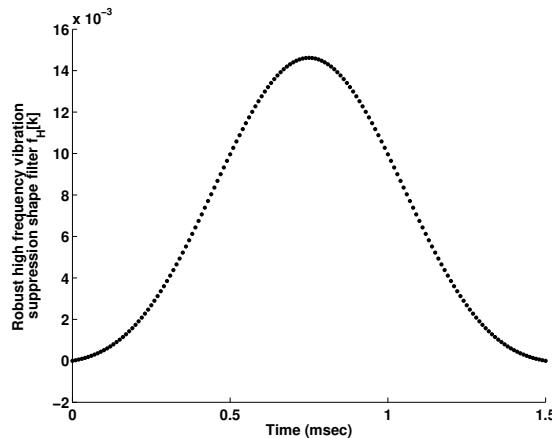


Fig. 17. High frequency vibration suppression shape filter.

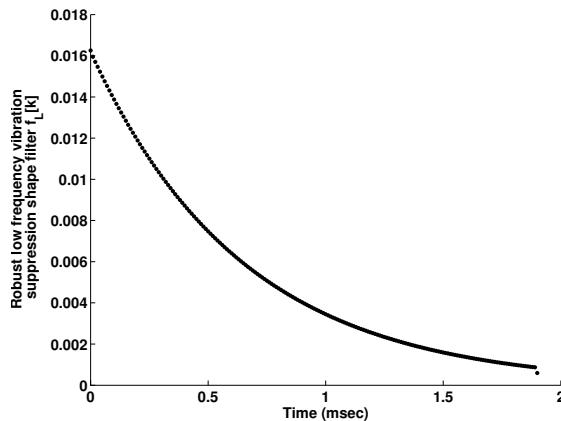


Fig. 18. Low frequency vibration suppression shape filter.

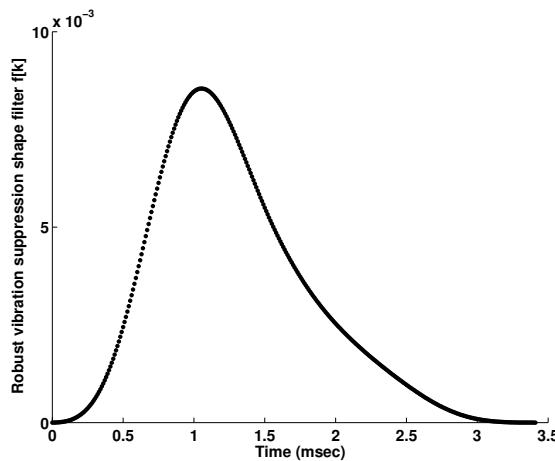


Fig. 19. Vibration suppression shape filter.

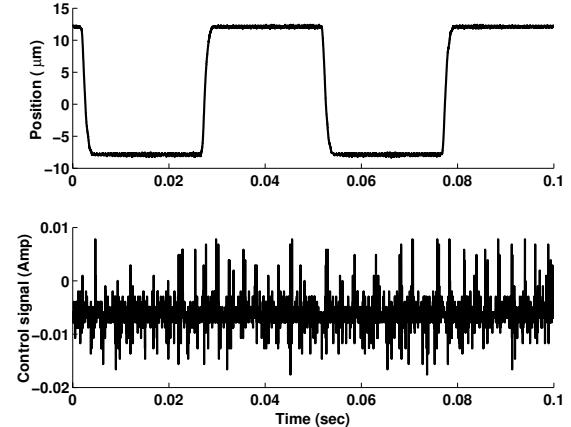


Fig. 20. Experimental result for  $20 \mu\text{m}$  move with the standard closed-loop control and shaped reference.

For hard disk drive short seek control, both acceleration (or drive current) and velocity constraints are generally not met. However, for hard disk drive long seek control, both acceleration (or drive current) and velocity constraints should be considered. The vibration suppression control profile generation with both acceleration (or drive current) and velocity constraints is studied in [8].

The methods in this paper are patented (pending). Commercial use of these methods requires written permission from the Oklahoma State University.

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