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Output-Feedback Adaptive Stabilization for Nonlinear Systems with Unknown Direction Control Coefficients

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***Abstract*— This paper investigates the problem of output-feedback adaptive stabilization control design for a class of nonlinear systems with unknown direction control coefficients. By introducing novel input scaling transformations, the unknown control coefficients can be lumped together such that the original system becomes a new system and make the output feedback control design possible. After the introduction of an observer and estimator for state and parameter estimates, respectively, a constructive design procedure is given for the output-feedback adaptive stabilization control based on integrator backstepping. It is shown that, under some conditions, the controller designed ensures the state and the estimation error of the closed-loop system asymptotically converge to zero, while other closed-loop signals are uniformly bounded.**

I. INTRODUCTION

Ever since the introduction of backstepping design, extensive research has been investigated on control design for nonlinear systems in strict-feedback form [1], [3], [4], [5], [6], [7], [8], [9], [10], [12].

The control design problems of nonlinear systems with unknown control coefficients (both magnitude and sign, so-called direction, are unknown) has received intense investigation recently [11], [13], [14], [15], [16], [17]. During the past two decades, a general methodology has been developed to cope with such control problems, which is based on the **Nussbaum function** introduced first by R. D. Nussbaum in his classical paper [18]. The key point of this methodology is by using a Nussbaum function [13], [18] to estimate the signs of the control coefficients. However, most existing research works on this topics concentrate on the control problems via state-feedback [11], [15], [16], [19], the output-feedback controls are studied rarely. Recently, output-feedback control was investigated for several special classes of nonlinear systems [14], [17], [19]. For example, in [14], [17], there is only one unknown sign coefficient which appears in the equation where the control appears. For output-feedback case, the unknown signs of control coefficients causes great difficulty in observer design, though it is key for output feedback control design.

In this paper, we will consider the output-feedback stabilization adaptive control problem of the more general nonlinear systems, whose state-feedback counter part has been considered completely in [11], [15], [16]. The objective of this paper is to investigate the output-feedback adaptive stabilization control of this nonlinear systems. First, we introduce a series of novel input scaling transformations, through which the unknown control coefficients distributing in the each subsystem can be lumped, thus leading to a new nonlinear system with only one unknown direction control coefficient in the first subsystem. Then, starting with this new nonlinear system, an observer and an estimator can be designed for state and parameter estimates, respectively. Thus, by using the integrator backstepping with tuning functions approach and the observer and parameter estimator designed, a constructive design procedure for output-feedback adaptive stabilization control is given. Our main result shows that, under some conditions, the controller designed ensures the state and the estimation error of the closed-loop system asymptotically converge to zero while all other signals are uniformly bounded.

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II. SYSTEM MODEL AND CONTROL PROBLEM

Throughout this paper, $\mathcal{C}^i(\mathbb{R}_1;\mathbb{R}_2)$ denotes the set of all functions from $\mathbb{R}_1$ to $\mathbb{R}_2$ with ith continuous derivative, $\mathcal{C}^\infty(\mathbb{R}_1;\mathbb{R}_2)$ denotes the set of all smooth functions from $\mathbb{R}_1$ to $\mathbb{R}_2$; for any $x\in\mathbb{R}^n$, x_i denotes its ith element, $\bar{x}_i$ denotes the column vector consisting of the first i elements of x in the original order, i.e., $\bar{x}_i=[x_1,\ldots,x_i]^T$.

A. System model

Consider the nonlinear system in the following form:

$$\begin{aligned} \dot{x}_i &= g_1 x_{i+1} + \theta_i^T\phi_i(y), \quad i=1,\ldots,n-1,\\ \dot{x}_n &= g_n u + \theta_n^T\phi_n(y),\\ y &= x_1, \end{aligned} \tag{1}$$

where $x=[x_1\ x_2\ \cdots\ x_n]^T\in I\!R^n$, $u\in\mathbb{R}$, $y\in\mathbb{R}$ are the state variables, system input and output, respectively; the control coefficients g_i, $i=1,\ldots,n$ are nonzero unknown constants; $\theta_i\in\mathbb{R}^{r_i}$, $i=1,\ldots,n$ are unknown time-invariant parameters and their estimations are denoted by $\hat{\theta}_i$; $\phi_i(\cdot)\in\mathbb{R}^{r_i}$, $i=1,\ldots,n$ are known regressor vector-valued functions depending on system output y only.

It is clear that the system (1) is in strict-feedback form. This suffices to using integrator backstepping approach to control design. When g_i's equal to one, the system (1) will become the canonical form of strict-feedback nonlinear control systems, which have been intensely investigated in the past decade. However, when g_i's are unknown, especially when their signs are unknown, the problems

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of such control systems will become much challenging and difficult. It has been shown that the method based on Nussbaum function is the most effective tool up to now, and many excellent results have been obtained, but most are in full-state feedback. Systems (1) is very general, and includes that in [14] as a special case where the only one unknown control coefficient appears in the last subsystem x_r.

Assumption 1: The nonlinear functions $\phi_i(y), i = 1, \ldots, n$, are smooth, and vanish at the origin; i.e. $\phi_i(0) = 0$.

Assumption 2: Unknown parameters θ_i, $i = 1, 2, \ldots, n$ are time-invariant.

Assumption 3: The signs of nonzero control coefficients g_i, $i = 1, \ldots, n$ are unknown.

Remark 1: Assumption 1 implies that the origin 0 is the equilibrium point of the open-loop system. From Assumption 2, we know that θ_i, $i = 1, \ldots, n$ are constant vectors and their time-derivatives are zero. Assumption 3 means that g_i's are either positive or negative but not zero, to satisfy the controllability condition of the system.

B. Control objective

The objective of this paper is to search for an output-feedback adaptive stabilization control in the following form:

$$\dot{\hat{\xi}} = \alpha(y, \hat{\xi}, \hat{\vartheta}), \quad \dot{\hat{\vartheta}} = \sigma(y, \hat{\xi}, \hat{\vartheta}), \quad u = \rho(y, \hat{\xi}, \hat{\vartheta}), \tag{2}$$

such that the state x of the resulting closed-loop system asymptotically converge to zero while all other signals are uniformly bounded, where $\alpha, \sigma, \rho \in \mathcal{C}^1(\mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^{r_1+1})$.

III. Output-Feedback Adaptive Stabilization Control Design

In the conventional framework of dynamic output-feedback control design, one observer is constructed to rebuild the unknown system states first, and then based on this observer, the desired control can be designed to guarantee the closed-loop system satisfies certain desired properties. If the control coefficients g_i's are known and available for feedback design, it is much easy to design observer and control. Due to the existence of unknown control coefficients, the observer design becomes much difficult and thus new method need to be developed.

In this section, we shall design the output-feedback adaptive stabilization control law in three steps. First, we introduce a series of input scaling transformations to lump all unknown control coefficients together and lead to a new system with one unknown direction control coefficient in its first subsystem, which subsequently makes the observer design possible as in [14]. With the observer dynamics in the loop, we obtain the entire system with lower triangular structure which is amenable to the application of integrator backstepping methodology. Then, we give the control design procedure, and present the performance analysis of the closed-loop systems.

A. Input Scaling Transformations

The first input scaling transformation is given as follows:

$$X_{1,1} = \frac{x_1}{g_1} \quad X_{1,2} = \frac{x_2}{g_2} \quad \cdots, \quad X_{1,n-1} = \frac{x_n}{g_n}, \tag{3}$$

Thus, $X_1 \triangleq [X_{1,1}, \ldots, X_{1,n}]^T = \text{diag}[g_1^{-1}, g_2^{-1}, \ldots, g_n^{-1}]x \triangleq G_1 x$, where G_1 is called as scaling matrix. Note that, by Assumption 3, G_1 is an unknown and nonsingular matrix. For clarity, we define the unknown and nonsingular matrices G_i's as follows:

$$G_i = \text{diag}[g_i^{-1}, \ldots, g_n^{-1}, 1, \ldots, 1], \; i = 1, \ldots, n,$$

Under the input scaling transformation (3), the system (1) becomes

$$\begin{aligned} \dot{X}_{1,i} &= g_{i+1} X_{1,i+1} + \Theta_{1,i}^T \phi_i(y), \quad i = 1, \ldots, n-1, \\ \dot{X}_{1,n} &= u + \Theta_{1,n}^T \phi_n(y), \end{aligned} \tag{4}$$

where $\Theta_{1,i} \triangleq \theta_i / g_i$, $i = 1, \ldots, n$.

From (4), we can see that no unknown control coefficient in the last subequation. If $g_2, \ldots, g_n$ are known, then we can begin the control design procedure with (4), otherwise, we have to take again input scaling transformation for system (4) until no more unknown control coefficient in the transformed system.

Generally, for $i = 2, \ldots, n$, the ith input scaling transformation is defined as follows:

$$\begin{aligned} &X_{i,1} = \frac{X_{i-1,1}}{g_i}, \cdots, X_{i,n-i} = \frac{X_{i-1,n-i}}{g_n}, \\ &X_{i,n-i+1} = X_{i-1,n-i+1}, \cdots, X_{i,n} = X_{i-1,n}, \end{aligned} \tag{5}$$

which can be written into

$$X_i = \text{diag}[g_i^{-1}, \ldots, g_n^{-1}, 1, \ldots, 1] X_{i-1} \triangleq G_i X_{i-1}.$$

Clearly, $G_i \in \mathbb{R}^{n \times n}$ is an unknown and nonsingular matrix.

Under the i-th input scaling transformation (5), system (4) can be transformed into the following system:

$$\begin{aligned} \dot{X}_{i,j} &= g_{i+j} X_{i,j+1} + \Theta_{i,j}^T \phi_j(y), \; j = 1, \ldots, n-i, \\ \dot{X}_{i,k} &= X_{i,k+1} + \Theta_{i,k}^T \phi_k(y), \; k = n-i+1, \ldots, n-1, \\ \dot{X}_{i,n} &= u + \Theta_{i,n}^T \phi_n(y), \end{aligned} \tag{6}$$

where $\Theta_{i,j} \triangleq \Theta_{i-1,j} / g_{i+j-1}, j = 1, \ldots, n-i$, $\Theta_{i,j} \triangleq \Theta_{i-1,j}$, $j = n-i+1, \ldots, n$.

Therefore, taking $i = n$ in (6) and $\xi \triangleq [\xi_1, \xi_2, \ldots, \xi_n]^T = X_n$, we obtain

$$\begin{aligned} \dot{\xi}_i &= \xi_{i+1} + \Theta_{n,i}^T \phi_i(y), \quad i = 1, \cdots, n-1 \\ \dot{\xi}_n &= u + \Theta_{n,n}^T \phi_n(y). \end{aligned} \tag{7}$$

The system (7), whose all control coefficients equal to 1, is in strict-feedback form and with uncertainties $\Theta_{n,i}^T \phi_i(y)$, $i = 1, \ldots, n$ depending only on system output y. Starting with this system, we can design the output-feedback adaptive stabilization control. However, it should be pointed out that all of states ξ_i, $i = 1, \ldots, n$ of system (7) are unmeasurable and hence a full-order observer is needed to rebuild them for control design.

Remark 2: The inherent connection existing between the system (7) and the original system (1) can be exactly described by the following equation:

$$\xi = Gx, \tag{8}$$

where $G \triangleq \prod_{i=1}^{n} G_i$. By the expressions of G_i's, and Assumption 3, we know that G is an unknown, nonsingular and diagonal matrix. The equation (8) defines a linear diffeomorphism, and if the state ξ of the system (7) converges to zero, so does the state x of the system (1). Thus, we need only to consider the stabilization control problem of the system (7).

The equation (8) can be rewritten into the following scalar form:

$$x_i = \frac{\xi_i}{\prod_{j=i}^{n} g_i}, \quad i = 1, \ldots, n, \tag{9}$$

For the convenience of control design and performance analysis, by replacing subsystem ξ_1 in (7) by subsystem x_1 and noting that $y = x_1$, we have

$$\begin{aligned} \dot{y} &= \overline{g}\xi_2 + \theta_1^T \phi_1(y), \\ \dot{\xi}_i &= \xi_{i+1} + \Theta_{n,i}^T \phi_i(y), \quad i = 2, \ldots, n-1, \\ \dot{\xi}_n &= u + \Theta_{n,n}^T \phi_n(y), \end{aligned} \tag{10}$$

where $\overline{g} \triangleq \frac{g_1}{\prod_{i=2}^{n} g_i}$ is some nonzero unknown constant, whose value and sign are unknown.

Remark 3: Through the input scaling transformations, the unknown control coefficients distributed in each subsystem has been lumped together and then the original system becomes a new system (10) with only one unknown direction control coefficient.

The directions and values of constants $g_1, \ldots, g_n$ are unknown, and so are the direction and value of $\overline{g}$. In addition, from Assumption 3, we have

$$\overline{g}_{\min} \triangleq \frac{\min\{|l_1^-|, |l_1^+|\}}{\prod_{i=2}^{n} \max\{|l_i^-|, |l_i^+|\}} \leq |\overline{g}| \leq \overline{g}_{\max} \triangleq \frac{\max\{|l_1^-|, |l_1^+|\}}{\prod_{i=2}^{n} \min\{|l_i^-|, |l_i^+|\}}.$$

Note that $\overline{g}_{\min}$ and $\overline{g}_{\max}$ are unknown and thus cannot be used in control design.

B. Observer Design

For system (10), only output $y = x_1$ is measurable and the rest of the states are not available for feedback design, we need to design observer to estimate $\xi_1, \ldots, \xi_n$ to construct feedback control.

We design the following observer associated with the system (7) as in [14]:

$$\begin{aligned} \dot{\hat{\xi}}_i &= \hat{\xi}_{i+1} - k_i\hat{\xi}_i, \; i = 1, \ldots, n-1, \\ \dot{\hat{\xi}}_n &= u - k_n\hat{\xi}_1, \end{aligned} \tag{11}$$

where $k_1, \ldots, k_n$ are constants to be determined by the designer.

Let $\hat{\xi} = [\hat{\xi}_1, \ldots, \hat{\xi}_n]^T$. The estimation error $\tilde{\xi} = \xi - \hat{\xi}$ satisfies the following dynamical equations:

$$\begin{aligned} \dot{\tilde{\xi}}_i &= \tilde{\xi}_{i+1} - k_i\tilde{\xi}_1 + \Theta_{n,i}^T \phi_i(y) + k_i\xi_1, \\ \dot{\tilde{\xi}}_n &= -k_n\tilde{\xi}_1 + \Theta_{n,n}^T \phi_n(y) + k_n\xi_1. \end{aligned} \tag{12}$$

where $i = 1, \cdots, n-1$. In order to obtain this equation, we have added and subtracted $-k_i\xi_1$'s on the right-hand side of the subequations, respectively.

The differential equations (12) can be rewritten into the following compact form:

$$\begin{aligned} \dot{\tilde{\xi}} &= \begin{bmatrix} -k_1 & & \\ \cdots & I_{n-1} & \\ -k_n & 0 \; \cdots & 0 \end{bmatrix} \tilde{\xi} + \begin{bmatrix} \Theta_{n,1}^T\phi_1 \\ \cdots \\ \Theta_{n,n}^T\phi_n \end{bmatrix} + \begin{bmatrix} k_1 \\ \cdots \\ k_n \end{bmatrix} \xi_1 \\ &\triangleq A\tilde{\xi} + \Theta\Phi(y) + K\xi_1. \end{aligned} \tag{13}$$

where $\Phi = \left[\phi_1^T, \phi_2^T, \ldots, \phi_n^T\right]^T$, $K = [k_1, k_2, \ldots, k_n]^T$, and $\Theta = \text{diag}[\Theta_{ij}]$, $\Theta_{ij} = \Theta_{n,j}^T$.

Remark 4: The matrix A is in the controllable canonical form and hence there exists an appropriate selection of design parameters $k_1, \ldots, k_n$ such that all the eigenvalues of A has any pre-given negative real parts, say A being **Hurwitz** and then there exists a positive definite matrix P satisfying:

$$A^T P + PA = -I.$$

Remark 5: Due to the unavailability of ξ_1, we cannot design an asymptotical observer for the reconstruction of ξ driven by $\xi_1 - \hat{\xi}_1$ and independent of control in the conventional framework od observer design. However, if a suitable control can be designed such that "$\Theta\Phi(y) + K\xi_1$" asymptotically converges to zero, then, in view of connection (8) between ξ and x, the estimation error $\tilde{\xi}$ satisfying (13) will asymptotically converges to zero and hence an asymptotical observer is obtained.

Remark 6: Clearly, $\Theta \in \mathbb{R}^{n \times r}$, $r = \sum_{i=1}^{n} r_i$, and for some constant r_{Θ_n}, it follows that $\|\Theta\| \leq r_{\Theta_n}$, say that Θ belongs to a compact set Ω_Θ with center 0 and radius r_Θ; from Assumption 1, it follows that $\Phi(y)$ is known, smooth and vanishes at the origin 0. We have the following lemma for estimation error equation (13).

Lemma 1: There always exists a Lyapunov function $V_0(\tilde{\xi})$, a positive smooth known function $\Omega_0(y)$, a positive design parameter $c_0 < 1$ and a positive unknown constant ν, such that

$$\dot{V}_0 \leq -c_0\|\tilde{\xi}\|^2 + \nu\Omega_0(y)y^2. \tag{14}$$

Proof: Choose $V_0(\tilde{\xi}, \tilde{\vartheta}) = \tilde{\xi}^T P\tilde{\xi}$ where P is the solution of the Lyapunov equation: $A^T P + PA = -I_n$. Then, taking time-derivative of V_0 along the solution of (13), we have

$$\dot{V}_0 = -\|\tilde{\xi}\|^2 + 2\tilde{\xi}^T P \left(\Theta\Phi(y) + K\xi_1\right). \tag{15}$$

By Assumption 1, it is easy to see that there exists a smooth vector-valued function $\overline{\Phi}(y) \triangleq \int_0^1 \frac{\partial\Phi(s)}{\partial s}\Big|_{s=ay} da$ such that $\Phi(y) = y\overline{\Phi}(y)$. Therefore, Noting Remark 6, we have

$$\begin{aligned} 2\tilde{\xi}^T P\Theta\Phi(y) &= 2\tilde{\xi}^T P\Theta\overline{\Phi}(y)y \leq 2r_{\Theta_n}\|P\|\|\tilde{\xi}\| \left\|\overline{\Phi}(y)y\right\| \\ &\leq \frac{\varepsilon_0}{2}\|\tilde{\xi}\|^2 + \frac{2\|P\|^2 r_{\Theta_n}^2}{\varepsilon_0} y^2 \left\|\overline{\Phi}(y)\right\|^2, \end{aligned}$$

where and whereafter ε_0 and ε_1 are positive design parameters less than 1 and to be determined shortly.

In addition, from (9) and Assumption 3, it follows that

$$2\tilde{\xi}^T PK\xi_1 \leq \frac{\varepsilon_0}{2}\|\tilde{\xi}\|^2 + \frac{2\|PK\|^2 \prod_{i=1}^{n} \max\{|l_i^-|, |l_i^+|\}}{\varepsilon_{01}} y^2.$$

Let $c_0 \triangleq 1 - \varepsilon_0$, $\Omega_0 \triangleq 1 + \left\|\overline{\Phi}(y)\right\|^2$ and $\nu \triangleq 2\max\left\{\frac{\|P\|^2 r_{\Theta_n}^2}{\varepsilon_0}, \frac{\overline{g}_{\max}^2}{4\varepsilon_1}, \frac{\|PK\|^2 \prod_{i=1}^{n} \max\{|l_i^-|, |l_i^+|\}}{\varepsilon_0}\right\}$. Clearly, $c_0 > 0$, $\Omega_0(\cdot)$ is smooth and ν is unknown.

Thus, it follows from (15) that this lemma is true. □

C. Output-feedback adaptive stabilization control design

In this subsection, we will begin the design procedure of the output-feedback adaptive stabilization control with the following *entire* system:

$$\begin{aligned} \dot{\tilde{\xi}} &= A\tilde{\xi} + \Theta\Phi(y), \\ \dot{y} &= \overline{g}(\hat{\xi}_2 + \tilde{\xi}_2) + \theta_1^T \phi_1(y), \\ \dot{\hat{\xi}}_i &= \hat{\xi}_{i+1} - k_i\hat{\xi}_1,\ i = 2, \ldots, n-1, \\ \dot{\hat{\xi}}_n &= u - k_n\hat{\xi}_1, \end{aligned} \tag{16}$$

which is called as the entire system is because that only from it, the system state reconstruction and the control objective of this paper stated previously can be completely realized: If one has designed a control such that all signals in the system (16) are asymptotically stable or bounded, so are all signals in the system (6), then with the help of the scaling transformations presented in subsection III-A, we can easily see that all signals of the original system (1) are asymptotically stable or bounded as well.

The design procedure will be carried out step-by-step using integrator backstepping.

Step 1: Let $z_1 = y$ and $z_2 = \hat{\xi}_2 - \alpha_1(y, \zeta, \hat{\vartheta})$, where $\alpha_1(\cdot)$ is a sufficiently smooth function. Let $\vartheta = [\nu, \overline{g}, \theta_1^T]^T$. Use $\hat{\vartheta}$ and $\tilde{\vartheta}$ to denote the estimation value and estimation error of ϑ, respectively.

Choose Lyapunov function $V_1 = V_0 + \frac{1}{2}\tilde{\vartheta}^T\Gamma_\vartheta^{-1}\tilde{\vartheta} + \frac{1}{2}z_1^2$ for this step, where Γ_ϑ is a positive definite matrix determined prior by the designer. Then, taking the time-derivative of V_1 along the solution of the subsystem y, saying z_1 of (16), using inequality (14) and noting that $\dot{\tilde{\vartheta}} = -\dot{\hat{\vartheta}}$, we have

$$\begin{aligned} \dot{V}_1 \leq& -c_0\|\tilde{\xi}\|^2 + \nu\Omega_0(y)y^2 - \tilde{\vartheta}^T\Gamma_\vartheta^{-1}\dot{\hat{\vartheta}} \\ &+ z_1\left(\overline{g}(\alpha_1 + z_2 + \tilde{\xi}_2) + \theta_1^T\phi_1(y)\right) \\ \leq& -(c_0 - \varepsilon_1)\|\tilde{\xi}\|^2 - \tilde{\vartheta}^T\Gamma_\vartheta^{-1}\dot{\hat{\vartheta}} \\ &+ z_1\left(\overline{g}(\alpha_1 + \vartheta^T\Psi_1(y)\right) + \overline{g}z_1z_2, \end{aligned} \tag{17}$$

where $\Psi_1(y) = [\Omega_0(y)y + 0.5y,\ 0,\ \phi_1^T(y)]^T$, which is sufficiently smooth. In order to obtain the last inequality of (17), we have used the following inequality:

$$\overline{g}z_1\tilde{\xi}_2 \leq \varepsilon_1\tilde{\xi}_2^2 + \frac{\overline{g}^2}{4\varepsilon_1}z_1^2 \leq \varepsilon_1\|\tilde{\xi}\|^2 + \frac{\overline{g}_{\max}^2}{4\varepsilon_1}z_1^2.$$

To construct virtual controller α_1, we need the following dynamical equation:

$$\dot{\zeta} = z_1\eta, \tag{18}$$

where

$$\eta \triangleq \beta_1 z_1 + \hat{\vartheta}^T\Psi_1(z_1). \tag{19}$$

where and hereafter, $\beta_1, \beta_2, \ldots, \beta_n$ are positive design parameters to be determined later.

Thus, we can choose the virtual controller α_1 as follows:

$$\alpha_1 = N(\zeta)\eta, \tag{20}$$

where function $N(\cdot)$ is a sufficient smooth even Nussbaum function, chosen as $e^{\zeta^2}\cos(\pi\zeta/2)$ throughout this paper.

Substituting (20) and (19) into (17), and after some simple calculations, we obtain

$$\begin{aligned} \dot{V}_1 \leq& -c_1\|\tilde{\xi}\|^2 - \tilde{\vartheta}^T\Gamma_\vartheta^{-1}(\dot{\hat{\vartheta}} - \tau_1) \\ &- \beta_1 z_1^2 + \overline{g}N(\zeta)\dot{\zeta} + \dot{\zeta} + \overline{g}z_1z_2, \end{aligned} \tag{21}$$

where $c_1 \triangleq c_0 - \varepsilon_1$ is positive constant, $\tau_1 \triangleq \Gamma_\vartheta\Psi_1 z_1$ and $\tau_i (i = 2, \ldots, n-1)$ in the coming steps are tuning functions [6] used to avoid overparameterization.

Since $z_1 = y$, the virtual controller α_1 can be easily re-expressed as the function of $(y, \zeta, \hat{\vartheta})$ and vanishes when $y = 0$, $\zeta = 0$ and $\hat{\vartheta} = 0$. The last term "$\overline{g}z_1z_2$" on the right-hand-side of inequality (21) will be tackled in the next step.

Step 2: Define $z_3 = \hat{\xi}_3 - \alpha_2(y, \bar{\hat{\xi}}_2, \zeta, \hat{\vartheta})$. Like the preceding statement, $\alpha_2(\cdot)$ is a sufficient smooth function called as virtual control to be determined later.

Then, by differential equations (16), z_2 satisfies the following dynamical equation:

$$\begin{aligned} \dot{z}_2 =& z_3 + \alpha_2 - k_2\hat{\xi}_1 - \frac{\partial\alpha_1}{\partial y}\Big(\overline{g}(\hat{\xi}_2 + \tilde{\xi}_2) \\ &+ \theta_1^T\phi_1(y)\Big) - \frac{\partial\alpha_1}{\partial\zeta}\dot{\zeta} - \frac{\partial\alpha_1}{\partial\hat{\vartheta}}\dot{\hat{\vartheta}}. \end{aligned} \tag{22}$$

Choose Lyapunov functions $V_2 = V_1 + \frac{1}{2}z_2^2$ for this step. Then, taking time-derivative of V_2 along the solution of (22) and taking (21) into account, we have

$$\begin{aligned} \dot{V}_2 \leq& -c_1\|\tilde{\xi}\|^2 - \tilde{\vartheta}^T\Gamma_\vartheta^{-1}(\dot{\hat{\vartheta}} - \tau_1) \\ &- \beta_1 z_1^2 + \overline{g}N(\zeta)\dot{\zeta} + \dot{\zeta} + \overline{g}z_1z_2 \\ &+ z_2\Big(z_3 + \alpha_2 - k_2\hat{\xi}_1 - \frac{\partial\alpha_1}{\partial y}\Big(\overline{g}(\hat{\xi}_2 + \tilde{\xi}_2) \\ &+ \theta_1^T\phi_1(y)\Big) - \frac{\partial\alpha_1}{\partial\zeta}\dot{\zeta} - \frac{\partial\alpha_1}{\partial\hat{\vartheta}}\dot{\hat{\vartheta}}\Big). \end{aligned} \tag{23}$$

Note that the following inequality:

$$z_2\frac{\partial\alpha_1}{\partial y}\overline{g}\tilde{\xi}_2 \leq \varepsilon_1\|\tilde{\xi}\|^2 + \frac{\overline{g}_{\max}^2}{4\varepsilon_1}\left(\frac{\partial\alpha_1}{\partial y}\right)^2 z_2^2.$$

We can define $\Psi_2 \triangleq \left(\frac{1}{2}\left(\frac{\partial\alpha_1}{\partial y}\right)^2 z_2,\ z_1 - \frac{\partial\alpha_1}{\partial y}\hat{\xi}_2,\ -\frac{\partial\alpha_1}{\partial y}\phi_1^T(y)\right)^T$, which is sufficiently smooth. Then, from (23), it follows that

$$\begin{aligned} \dot{V}_2 \leq& -(c_1 - \varepsilon_1)\|\tilde{\xi}\|^2 - \tilde{\vartheta}^T\Gamma_\vartheta^{-1}(\dot{\hat{\vartheta}} - \tau_1) - \beta_1 z_1^2 + \overline{g}N(\zeta)\dot{\zeta} \\ &+ \dot{\zeta} + z_2\Big(z_3 + \alpha_2 - k_2\hat{\xi}_1 + \vartheta^T\Psi_2 - \frac{\partial\alpha_1}{\partial\zeta}\dot{\zeta} - \frac{\partial\alpha_1}{\partial\hat{\vartheta}}\dot{\hat{\vartheta}}\Big). \end{aligned} \tag{24}$$

Thus, we can choose the second virtual controller α_2 as follows:

$$\alpha_2 = -\beta_2 z_2 + k_2\hat{\xi}_1 - \hat{\vartheta}^T\Psi_2 + \frac{\partial\alpha_1}{\partial\zeta}\dot{\zeta} - \frac{\partial\alpha_1}{\partial\hat{\vartheta}^T}\tau_2, \tag{25}$$

where $\tau_2 = \tau_1 + \Gamma_\vartheta\Psi_2 z_2$. Similarly, from the expression of (25), by $z_1 = y$ and $z_2 = \hat{\xi}_2 - \alpha_1(y, \zeta, \hat{\vartheta})$ α_2 can be easily transformed into the function of $y, \bar{\hat{\xi}}_2$, ζ and $\hat{\vartheta}$, and vanishes when $y = 0, \bar{\hat{\xi}}_2 = 0$, $\zeta = 0$ and $\hat{\vartheta} = 0$.

Substituting (25) into (24) and after some simple calculations, we have

$$\begin{aligned} \dot{V}_2 \leq& -c_2\|\tilde{\xi}\|^2 - \left(\tilde{\vartheta} + \Gamma_\vartheta\frac{\partial\alpha_1}{\partial\hat{\vartheta}^T}z_2\right)^T\Gamma_\vartheta^{-1}(\dot{\hat{\vartheta}} - \tau_2) \\ &- \beta_1 z_1^2 - \beta_2 z_2^2 + \overline{g}N(\zeta)\dot{\zeta} + \dot{\zeta} + z_2z_3, \end{aligned} \tag{26}$$

where $c_2 \triangleq c_1 - \varepsilon_1$.

Step i ($i = 3, \ldots, n-1$). Assume that Lyapunov functions V_i, $i = 3, \ldots, n-1$ correspond to Step i, respectively, and V_{i-1} satisfy the following inequalities:

$$\dot{V}_{i-1} \le -c_{i-1}\|\tilde{\xi}\|^2 - \Big(\tilde{\vartheta} + \Gamma_\vartheta \sum_{j=2}^{i-1} \frac{\partial \alpha_{j-1}}{\partial \hat{\vartheta}^T} z_j\Big)^T \Gamma_\vartheta^{-1} \cdot \Big(\dot{\hat{\vartheta}} - \tau_{i-1}\Big) - \sum_{j=1}^{i-1} \beta_j z_j^2 + \overline{g}N(\zeta)\dot{\zeta} + \dot{\zeta} + z_{i-1}z_i. \quad (27)$$

Define $z_{i+1} = \hat{\xi}_{i+1} - \alpha_i(y, \bar{\hat{\xi}}_i, \zeta, \hat{\vartheta})$. Here α_i is the ith virtual controller and will be designed constructively in this step.

From equations (16), we know that z_i satisfies the following dynamical equation:

$$\dot{z}_i = z_{i+1} + \alpha_i - k_i\hat{\xi}_1 - \frac{\partial \alpha_{i-1}}{\partial y}\overline{g}\,(\hat{\xi}_2 + \tilde{\xi}_2) - \frac{\partial \alpha_{i-1}}{\partial y}\theta_1^T\phi_1(y) - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\xi}_j}\Big(\hat{\xi}_{j+1} - k_j\hat{\xi}_1\Big) - \frac{\partial \alpha_{i-1}}{\partial \zeta}\dot{\zeta} - \frac{\partial \alpha_{i-1}}{\partial \hat{\vartheta}}\dot{\hat{\vartheta}}. \quad (28)$$

Choose the Lyapunov function $V_i = V_{i-1} + \frac{1}{2}z_i^2$ for this step. Then, taking the time-derivative of V_i along the solution of (28), we have

$$\dot{V}_i = \dot{V}_{i-1} + z_i\Big(z_{i+1} + \alpha_i - k_i\hat{\xi}_1 - \frac{\partial \alpha_{i-1}}{\partial y}\overline{g}\,(\hat{\xi}_2 + \tilde{\xi}_2) - \frac{\partial \alpha_{i-1}}{\partial y} \cdot \theta_1^T\phi_1 - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\xi}_j}\Big(\hat{\xi}_{j+1} - k_j\hat{\xi}_1\Big) - \frac{\partial \alpha_{i-1}}{\partial \zeta}\dot{\zeta} - \frac{\partial \alpha_{i-1}}{\partial \hat{\vartheta}}\dot{\hat{\vartheta}}\Big). \quad (29)$$

Let $\Psi_i \triangleq \Big(\frac{1}{2}\Big(\frac{\partial \alpha_{i-1}}{\partial y}\Big)^2 z_i,\ z_{i-1} - \frac{\partial \alpha_{i-1}}{\partial y}\hat{\xi}_2,\ -\frac{\partial \alpha_{i-1}}{\partial y}\phi_1^T(y)\Big)^T$. Noting the following inequality:

$$-z_i\frac{\partial \alpha_{i-1}}{\partial y}\overline{g}\tilde{\xi}_2 \le \varepsilon_1\|\tilde{\xi}\|^2 + \frac{\overline{g}^2_{\max}}{4\varepsilon_1}\Big(\frac{\partial \alpha_{i-1}}{\partial y}\Big)^2 z_i^2,$$

and substituting which and (27) into (29), and after some simple calculations, we obtain

$$\begin{aligned}\dot{V}_i \le& -(c_{i-1} - \varepsilon_1)\|\tilde{\xi}\|^2 - \Big(\tilde{\vartheta} + \Gamma_\vartheta \sum_{j=2}^{i-1} \frac{\partial \alpha_{j-1}}{\partial \hat{\vartheta}^T} z_j\Big)^T \\ &\cdot \Gamma_\vartheta^{-1}(\dot{\hat{\vartheta}} - \tau_{i-1}) - \sum_{j=1}^{i-1} \beta_j z_j^2 + \overline{g}N(\zeta)\dot{\zeta} + \dot{\zeta} \\ &+ z_i\Big(\alpha_i + z_{i-1} - k_i\hat{\xi}_1 + \vartheta^T\Psi_i - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\xi}_j} \\ &\cdot \Big(\hat{\xi}_{j+1} - k_j\hat{\xi}_1\Big) - \frac{\partial \alpha_{i-1}}{\partial \zeta}\dot{\zeta} - \frac{\partial \alpha_{i-1}}{\partial \hat{\vartheta}}\dot{\hat{\vartheta}}\Big) + z_i z_{i+1}. \quad (30)\end{aligned}$$

Then, we can choose the ith virtual controller α_i as follows:

$$\begin{aligned}\alpha_i =& -\beta_i z_i - z_{i-1} + k_i\hat{\xi}_1 + \hat{\vartheta}^T\Psi_i + \sum_{j=2}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\xi}_j}(\hat{\xi}_{j+1} \\ &- k_j\hat{\xi}_1) + \frac{\partial \alpha_{i-1}}{\partial \zeta}\dot{\zeta} - \sum_{j=2}^{i} \frac{\partial \alpha_{j-1}}{\partial \hat{\vartheta}} z_j\Psi_i - \frac{\partial \alpha_{i-1}}{\partial \hat{\vartheta}^T}\tau_i, \quad (31)\end{aligned}$$

where $\tau_i \triangleq \tau_{i-1} + \Gamma_\vartheta\Psi_i z_i$. Clearly, α_i can be re-expressed as the function of $y, \hat{\xi}_{[i]}$, ζ and $\hat{\vartheta}$, and vanishes when $y = 0, \hat{\xi}_{[i]} = 0$, $\zeta = 0$ and $\hat{\vartheta} = 0$.

Thus, substituting (31) into (30), we have

$$\begin{aligned}\dot{V}_i \le& -c_i\|\tilde{\xi}\|^2 - \Big(\tilde{\vartheta} + \Gamma_\vartheta \sum_{j=2}^{i} \frac{\partial \alpha_{j-1}}{\partial \hat{\vartheta}^T} z_j\Big)^T \Gamma_\vartheta^{-1}(\dot{\hat{\vartheta}} - \tau_i) \\ &- \sum_{j=1}^{i} \beta_j z_j^2 + \overline{g}N(\zeta)\dot{\zeta} + \dot{\zeta} + z_i z_{i+1}, \quad (32)\end{aligned}$$

where $c_i \triangleq c_{i-1} - \varepsilon_1$.

Step n. It is easy to see that the results of Step $i (i \ge 3)$ hold true also for $i = n$. Thus, we choose the actual controller u and adaptive law of ϑ as follows, respectively:

$$u(y, \hat{\xi}, \zeta, \hat{\vartheta}) = \alpha_n(y, \hat{\xi}, \zeta, \hat{\vartheta}), \qquad \dot{\hat{\vartheta}} = \tau_n. \quad (33)$$

Note that in this case $i = n$, all functions $V_n, \tau_n, \alpha_n, c_n$ can be defined similarly and $z_{n+1} = 0$.

Thus, the time-derivative of Lyapunov function V_n satisfies

$$\dot{V}_n \le -c_n\|\tilde{\xi}\|^2 - \sum_{i=1}^{n} \beta_i z_i^2 + \overline{g}N(\zeta)\dot{\zeta} + \dot{\zeta}. \quad (34)$$

D. Main Results

In the previous subsection, we have given the output-feedback adaptive control design procedure using integrator backstepping. We are now ready to summarized the main results as follows.

Lemma 2: The observer described by (11), the adaptive law and the control u determined by (33) can be rewritten into the following more compact form:

$$\begin{aligned}\dot{\hat{\xi}} &= \alpha(y, \hat{\xi}_{[n]}, \zeta, \hat{\vartheta}), \\ \dot{\hat{\vartheta}} &= \sigma(y, \hat{\xi}_{[n]}, \zeta, \hat{\vartheta}), \\ u &= \rho(y, \hat{\xi}_{[n]}, \zeta, \hat{\vartheta}),\end{aligned}$$

where $\alpha, \sigma, \rho \in \mathcal{C}^1$.

Proof. It is apparent from the design procedure in the previous subsection. □

Theorem 1: Consider the system (1) under the Assumptions 1, 2 and 3. If the design parameters ε_0, ε_1 and β_i, $i = 1, \ldots, n$ are chosen such that

$$1 - \varepsilon_0 - (n-1)\varepsilon_1 > 0, \quad \beta_1 > 0, \beta_2 > 0, \ldots, \beta_n > 0. \quad (35)$$

Then, the adaptive stabilization output-feedback controller given by (33) guarantees that

(i) all signals in the closed-loop system are uniformly bounded; and

(ii) z and x asymptotically converge to zero.

Proof: In view of the selection of the design parameters, we see that $W(\tilde{\xi}, z) \triangleq -c_n\|\tilde{\xi}\|^2 - \sum_{j=1}^{n}\beta_j z_j^2$ is negatively definite. Then, by using (34), Lemma A.1 and Remark A.1, it is easy to see that $\zeta(t), V(t)$ are bounded on $[0, \infty)$. This implies that $V(t) \le \infty$, for all $t \ge 0$. Thus,

$$\sup_{t \ge 0}\|\tilde{\xi}\| < \infty, \quad \sup_{t \ge 0}\sum_{i=1}^{n} z_i^2 < \infty, \quad \sup_{t \ge 0}\|\tilde{\vartheta}\| < \infty, \quad (36)$$

and

$$\int_0^\infty \|\tilde{\xi}\|^2 dt < \infty, \qquad \sum_{i=1}^{n}\int_0^\infty z_i^2 dt < \infty. \quad (37)$$

From (36) and the boundedness of ϑ, we know that $\hat{\vartheta}$ is uniformly bounded. From (37) it follows that the first derivatives of $\tilde{\xi}$ and z_i ($i = 1, 2, \ldots, n$) are uniformly bounded. This implies that $\tilde{\xi}$ and z_i ($i = 1, 2, \ldots, n$) are uniformly continuous, and further, together with (37) and Barbalat's Lemma [21], leads to

$$\lim_{t\to\infty} \tilde{\xi} = 0, \qquad \lim_{t\to\infty} z_i = 0, \quad \forall i = 1, 2, \ldots, n. \tag{38}$$

We will show that $[y, \hat{\xi}_2, \ldots, \hat{\xi}_n]^T$ is asymptotically stable, and then together (8) and (38), $[\xi_1\ \xi_2, \ldots, \xi_n]^T$ is asymptotically stable as well.

From $z_1 = y = x_1$, it follows that y, and hence x_1 is asymptotically stable. By the boundedness of ζ and $\hat{\vartheta}$, the smoothness of α_1, the expressions (20), (19) of α_1, η, respectively, and Assumption 1 on ϕ_i's, we know that $\lim_{y\to\infty} \alpha_1(y, \zeta, \hat{\vartheta}) = 0$. This together with $\hat{\xi}_2 = z_2 + \alpha_1(y, \zeta, \hat{\vartheta})$ and (38) implies that $\hat{\xi}_2$ is asymptotically stable. Suppose that $[\hat{\xi}_2, \ldots, \hat{\xi}_{k-1}]^T$ is asymptotically stable for any k ($k = 3, \ldots, n$). Then, by $\hat{\xi}_k = z_k + \alpha_{k-1}(y, \hat{\xi}_2, \ldots, \hat{\xi}_{k-1}, \zeta, \hat{\vartheta})$, the boundedness of ζ and $\hat{\vartheta}$, the smoothness and expression of α_{k-1}, Assumption 1 and (38) we know that $\hat{\xi}_k$, and hence $[y, \hat{\xi}_2, \ldots, \hat{\xi}_k]^T$, is asymptotically stable. Therefore, by induction, $[y, \hat{\xi}_2, \ldots, \hat{\xi}_n]^T$ is asymptotically stable. □

Remark 7: From Theorem 1, we know that the design parameters ε_0 and ε_1 should be positive and sufficiently small so that inequalities (35) holds. However, the smaller the design parameters ε_0 and ε_1, the more the control effort.

IV. CONCLUDING REMARKS

This paper studied the output-feedback adaptive stabilization control design for nonlinear systems with unknown control coefficients. By using integrator backstepping approach, the control law is designed constructively. The control law designed in this paper can guarantee the boundedness of all signals in the closed-loop system.

APPENDIX

Lemma A.1: [15] Let $V(\cdot)$ and $\zeta(\cdot)$ be smooth functions defined on $[0, t_f)$ with $V(\cdot): [0, t_f) \to \mathbb{R}_+$, and $N(\cdot)$ be an even Nussbaum- function. If the following inequality holds:

$$V(t) \le C + \int_0^t (gN(\zeta(v)) + 1)d\zeta(v), \ \forall\, t \in [0, t_f), \tag{A.1}$$

where g is a nonzero constant and C represents some suitable constant, then $\zeta(t)$, $V(t)$ and $\int_0^t N(\zeta(v))d\zeta(v)$ are bounded on $[0, t_f)$, and the terminal time t_f can be maximized to $+\infty$.

Remark A.1: As a alterative form of Lemma A.1, if the time-derivative of Lyapunov function $V(t)$ satisfies

$$\dot{V}(t) \le W(t) + (gN(\zeta) + 1)\dot{\zeta}, \qquad \forall\, [0, t_f], \tag{A.2}$$

where $W(t)$ is negative or integrable on $[0, \infty)$, then using Lemma A.1, the boundedness of $\zeta(t)$, $V(t)$ and $\int_0^t N(\zeta(v))d\zeta(v)$ can be guaranteed, and the terminal time t_f can be maximized to ∞. Actually, (A.2) can be used more directly to control design than that by (A.1).

Lemma A.1 can be extended into the following lemma which can be used to deal with the case where g is time-varying, but Lemma A.1 cannot.

Lemma A.2: Let $V(\cdot)$ be a Lyapunov function, $\zeta(\cdot)$ be a smooth function defined on $[0, t_f)$, $g(\cdot)$ be either a positive or negative function and $N(\cdot)$ be an even Nussbaum function. If the following inequality holds:

$$V(t) \le \mathit{const} + \int_0^t (g(v)N(\zeta(v)) + 1)d\zeta(v), \ \forall t \in [0, t_f),$$

where *"const"* represents some suitable constant, and

$$\liminf_{s\to\infty} \frac{1}{s} \int_0^s g(v)N(\zeta(v))d\zeta(v) = -\infty, \tag{A.3}$$

then $\zeta(t)$, $V(t)$ and $\int_0^t g(v)N(\zeta(v))d\zeta(v)$ must be bounded on $[0, t_f)$.

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