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Stable Receding-Horizon Cooperative Control of a Class of Distributed Agents

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***Abstract*— This paper develops a cooperative control scheme for a team of distributed agents. Each agent is assumed to evolve in discrete-time and exchange delayed state information with a subset of neighboring or cooperating agents. Delays can be different for each pair of cooperating agents. The control design is derived using a finite-horizon cost function that includes both the regulation and cooperation objectives. The cooperative control problem is formulated in a receding-horizon framework, where the control law can be explicitly broken up in two components: one due feedback from the local state variables and the other based on delayed information gathered from cooperating neighboring agents. Novel theoretical and constructive results are presented, concerning the choice of the cost function in order to guarantee stability of the overall team of agents, and extending sufficient conditions previously established by the authors. Simulation results are used to illustrate the effectiveness of the proposed control scheme.**

I. INTRODUCTION

Cooperative control among a team of distributed agents is an area that has attracted significant attention during the last few years. Research activity in this area can be classified into two main broad categories. The first one deals with large scale dynamic systems, where the main issue is to reduce the computational load stemming from a centralized approach [1], [2], [3]. The second category considers the problem of controlling and coordinating teams of cooperating dynamic systems (mainly Uninhabited Autonomous/Air Vehicles (UAVs)), whose aim is to fulfill a global and/or local goal. In this paper, we are particularly interested in approaches related to this second category.

In the area of cooperative control of autonomous systems (agents), one of the main trends is to match global and local targets, optimizing some suitably chosen local cost function in a model-predictive control framework, where the cost function takes into account collisions and violation of specific constraints on formation, and may reward the tracking of a certain path. For example, in [4] a two-degrees of freedom team of UAVs assigned to visit a certain number of points, is considered. Coordination of a large group of cooperating nonlinear vehicles is considered in [5], [6], where a centralized cost function is decomposed and locally minimized; stability is assured by exploiting graph theory assumptions and potential functions. Other recent works deal with agreement issues [7] and topology-independent stability criteria [8].

In [9] the authors considered the cooperative control problem for a team of agents described by discrete-time dynamic systems. In this framework, each agent is provided with a "local" control objective that depends on two different sub-objectives: (i) a control objective that depends only on the local state variables and (ii) a control objective that depends on the information exchanged with neighboring/cooperating agents, which aims to achieve a desired global cooperation behavior between the team of agents. The information exchanged between the cooperating agents is delayed and the length of the delays are different, depending on the specific agents considered. The cooperative control problem was formulated for each agent in a receding-horizon (RH) framework, where the control law can be explicitly broken up in two components: the first one is based on the feedback from the local state variables and the second one is based on information gathered from cooperating neighboring agents. The proposed scheme starts from the local formulation of the problem, then drawing conditions on the overall stability of the team, whereas works as [10] deal with the synthesis of local controllers by starting from a centralized cost minimization, imposing the suboptimal solution to take certain geometric directions.

In this paper, the results of [9] are extended by developing a set of bounding expressions for the linear control law. This allows the derivation of a relationship between the stability of the team of cooperating agents with the coupling matrices chosen in the local cost functions. Hence, the stability of the overall team of cooperating agents can be guaranteed by appropriate selection of cooperation weight matrices. Simulations results using a set of UAVs moving in a two-dimensional space are shown to illustrate the effectiveness of the proposed receding-horizon cooperative control scheme.

II. PROBLEM FORMULATION

In this section the cooperative control problem is outlined. The proposed formulation follows closely the one developed in the recent article [9], but for completeness it is also described here.

Let us consider a distributed dynamic system made of a set of M agents denoted as $\mathcal{A} \triangleq \{\mathcal{A}^i,\, i = 1, ..., M\}$. Each

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agent $\mathcal{A}^i$ is described by the LTI state equation:

$$x_{t+1}^i = A^i x_t^i + B^i u_t^i, \quad i = 1, ..., M, \tag{1}$$

where, for each $i = 1, ..., M$, $x_t^i \in \mathbb{R}^{n^i}$ denotes the state vector and $u_t^i \in \mathbb{R}^{m^i}$ denotes the control vector.

We assume that, for each $i = 1, ..., M$, the pair $[A^i, B^i]$ is stabilizable and that all M agents are synchronized. The agents are dynamically uncoupled; moreover the members of the team do not know the dynamics of their neighbors. We assume though that each agent $\mathcal{A}^i$ exchanges some information with a given set of neighboring agents $\mathcal{G}^i \triangleq \{\mathcal{A}^j, j \in G^i\}$, where G^i denotes the set of indexes identifying the agents belonging to the set $\mathcal{G}^i$. More specifically, consider a generic time–instant t; then for each $i = 1, ..., M$, agent $\mathcal{A}^i$ receives from each cooperating agent $\mathcal{A}^j \in \mathcal{G}^i$ the value of its local state vector with a delay of Δ_{ji} time steps, that is, agent $\mathcal{A}^i$ receives the vector $x_{t-\Delta_{ji}}^j$ from agent $\mathcal{A}^j \in \mathcal{G}^i$. For simplicity, in the rest of this paper we let $\bar{x}^{ij} \triangleq x_{t-\Delta_{ji}}^j$, $j \in G^i$.

For each $i = 1, ..., M$ and for a given value of x_t^i at time–instant t, the following finite–horizon (FH) cost function is introduced ($\| \cdot \|$ denotes the euclidean norm):

$$\begin{aligned} J_{FH}^i = & \sum_{k=0}^{N^i-1} \left(\|x_{t+k}^i\|_{P^i}^2 + \|u_{t+k}^i\|_{R^i}^2 \right) + \|x_{t+N^i}^i\|_{P_{N^i}^i}^2 \\ & + \sum_{k=0}^{N^i-1} \sum_{j \in G^i} \|x_{t+k}^i - \bar{x}^{ij} + d_k^{ij}\|_{S^{ij}}^2 \\ & + \sum_{j \in G^i} \|x_{t+N^i}^i - \bar{x}^{ij} + d_{N^i}^{ij}\|_{S_{N^i}^{ij}}^2 , \end{aligned} \tag{2}$$

where without loss of generality, we consider agents with the same state size, i.e. $n^i = n$. The parameters $N^i, i = 1, \ldots, M$ denote the lengths of the control horizons for each agent and $d_k^{ij} \in \mathbb{R}^{n^i}, k = 0, \ldots, N^i - 1, i = 1, \ldots, M, j \in G^i$ are given constant vectors representing the desired "distance" between the state variables of cooperating agents. The symmetric state–weighting matrices satisfy $P^i \geq 0, P_{N^i}^i \geq 0, S^{ij} \geq 0, S_{N^i}^{ij} \geq 0$ and the symmetric control weighting matrices satisfy $R^i > 0$.

The local control strategy is based on a RH framework (see [9] for more details). Hence we have the following

Problem 2.1: At every time instant $t \geq 0$ and for every agent $\mathcal{A}^i, i = 1, \ldots, M$, find the RH optimal control law $u_t^{RH^{i\circ}} = \gamma_{RH}^{i\circ}(x_t^i; x_{t-\Delta_{ji}}^j, j \in G^i)$ where $u_t^{RH^{i\circ}}$ is the first vector of the control sequence $u_t^{FH^{i\circ}}, \ldots, u_{t+N^i-1}^{FH^{i\circ}}$ that minimizes the cost (2) for the local state x_t^i and the delayed states $\bar{x}^{ij}, j \in G^i$.

The RH control law $\gamma_{RH}^{i\circ}(x_t^i; x_{t-\Delta_{ji}}^j, j \in G^i)$ is of the *feedback–feedforward* type as the control vector $u_t^{FH^{i\circ}}$ depends on the local current state x_t^i and on the delayed states $\bar{x}^{ij}, j \in G^i$ communicated to the agent $\mathcal{A}^i$ by the cooperating agents belonging to $\mathcal{G}^i$. The structure of the control objective is made of two parts: a "local" control objective aimed at minimizing the partial cost given by the terms $\sum_{k=0}^{N^i-1}(\|x_{t+k}^i\|_{P^i}^2 + \|u_{t+k}^i\|_{R^i}^2) + \|x_{t+N^i}^i\|_{P_{N^i}^i}^2$ and a "cooperation" control objective aiming at minimizing the partial cost given by the remaining terms $\sum_{k=0}^{N^i-1}\sum_{j \in G^i}\|x_{t+k}^i - x_{t-\Delta_{ji}}^j + d_k^{ij}\|_{S^{ij}}^2 + \sum_{j \in G^i}\|x_{t+N^i}^i - \bar{x}^{ij} + d_{N^i}^{ij}\|_{S_{N^i}^{ij}}^2$. Then, as the control law for each agent $\mathcal{A}^i, i = 1, \ldots, M$ takes on the form $\gamma_{RH}^{i\circ}(x_t^i; x_{t-\Delta_{ji}}^j, j \in G^i)$, then the dynamic behaviors of the agents are coupled and depend on the values of $P^i, P_{N^i}^i, S^{ij}, S_{N^i}^{ij}, R^i$ and d_k^{ij}.

III. The local RH control law

The solution of Problem 2.1 gives rise to an unconstrained minimization problem in the unknowns $u_t^i, \ldots, u_{t+N^i-1}^i$. The analytical solution can be obtained by resorting to results which are available in the literature (see, for instance, [11]). For $k = N^i - 1, N^i - 2, \ldots, 0$, we obtain the backwards difference equations:

$$\begin{aligned} q_k^i &= P^i + \sum_{j \in \mathcal{G}^i} S^{ij} + A^{i\top}[q_{k+1}^i - q_{k+1}^i B^i \\ & \quad \times (B^{i\top} q_{k+1}^i B^i + R^i)^{-1} B^{i\top} q_{k+1}^i] A^i, \end{aligned} \tag{3}$$

$$\begin{aligned} v_{t+k}^i &= [A^{i\top} - A^{i\top} q_{k+1}^i B^i (B^{i\top} q_{k+1}^i B^i + R^i)^{-1} \\ & \quad \times B^{i\top}] v_{k+1}^i + \sum_{j \in G^i} S^{ij}(\bar{x}^{ij} - d_k^{ij}), \end{aligned} \tag{4}$$

with boundary conditions $q_{N^i}^i = P_{N^i}^i + \sum_{j \in \mathcal{G}^i} S_{N^i}^{ij}$ and $v_{t+N^i}^i = \sum_{j \in \mathcal{G}^i} S_{N^i}^{ij}(\bar{x}^{ij} - d_{N^i}^{ij})$. From now on, without loss of generality and again for the sake of notational simplicity let us assume $S_{N^i}^{ij} = S^{ij}$ and $d_{N^i}^{ij} = d_k^{ij} = d^{ij}, k = 0, \ldots, N^i - 1$.

After some algebra (see again [9]), and owing to the fact that the quantities $(\bar{x}^{ij} - d^{ij})$ are constant inside the FH time window, we can write

$$v_{t+k}^i = \tilde{\Phi}_k^i \sum_{j \in G^i} S^{ij}(\bar{x}^{ij} - d^{ij}). \tag{5}$$

where $\tilde{\Phi}_k^i$ are suitable matrices which are computable *off-line*. Therefore, for $k = 0, ..., N^i - 1$, matrix gains are obtained: $K_k^{x^i} = (B^{i\top} q_{k+1}^i B^i + R^i)^{-1}(B^{i\top} q_{k+1}^i A^i)$ and $K_k^{v^i} = (B^{i\top} q_{k+1}^i B^i + R^i)^{-1} B^{i\top}$. For each k, the FH control action is then given by

$$u_{t+k}^i = -K_k^{x^i} x_{t+k}^i + K_k^{v^i} v_{t+k+1}^i, \tag{6}$$

where v_{t+k+1}^i is given by (5). The RH control law solving Problem 2.1 is thus derived from (6) setting $k = 0$, that is

$$u_t^{RH^{i\circ}} = -K_0^{x^i} x_t^i + K_0^{v^i} \tilde{\Phi}_1^i \sum_{j \in G^i} S^{ij}(\bar{x}^{ij} - d^{ij}). \tag{7}$$

IV. Stability of the team of cooperating agents

For the sake of simplicity, we will assume from now on that all the agents have the same input dimension, and also the same finite optimization horizon (i.e., we set $m^i = m$, and $N^i = N$, for each $i = 1, \dots, M$). Moreover, again without loss of generality and not to make the notation too heavy, we'll suppose that *all* the agents in the set $\mathcal{A}$ cooperate to minimize their local costs, that is $G^i = \{1, \dots, M\} \setminus \{i\}$, $i = 1, \dots, M$. According to the framework proposed in [12], we want to recast this tracking problem into an LQR problem in order to find explicit stability conditions for each agent. In the following we will sketch the steps which carry to such equivalent problem form. First of all, at time t and for each $i = 1, ..., M$ we have to introduce an *auxiliary* dynamic system described by the state equation

$$x^{i,h}_{t+k+1} = A^{i,h} x^{i,h}_{t+k},\ k = 0, \dots, N-1, \tag{8}$$

where $x^{i,h}_{t+k} \in \mathbb{R}^{n(M-1)}$ and where the initial condition is given by $x^{i,h}_t \triangleq [(\bar{x}^{i1} - d^{i1})^\top | \cdots (\bar{x}^{i,i-1} - d^{i,i-1})^\top \,|\, (\bar{x}^{i,i+1} - d^{i,i+1})^\top \cdots | (\bar{x}^{iM} - d^{iM})^\top]^\top$. Matrix $A^{i,h}$ in system (8) is an identity matrix of appropriate dimensions. Hence, the following augmented systems is obtained[1]:

$$x^{i,r}_{t+k+1} = A^{i,r} x^{i,r}_{t+k} + B^{i,r} u^{i,r}_{t+k},\ k = 0, ..., N-1, \tag{9}$$

where $x^{i,r}_{t+k} \triangleq \mathrm{col}\,[x^i_{t+k}, x^{i,h}_{t+k}]$, $u^{i,r}_{t+k} \triangleq u^i_{t+k}$,

$$A^{i,r} \triangleq \begin{bmatrix} A^i & \emptyset \\ \emptyset & A^{i,h} \end{bmatrix}, \quad B^{i,r} \triangleq [B^i\ \emptyset \cdots \emptyset]^\top.$$

Clearly, $A^{i,r} \in \mathbb{R}^{nM \times nM}$, $B^{i,r} \in \mathbb{R}^{nM \times m}$ and $\emptyset$ will denote from now on zero–matrices of appropriate dimensions. As toroughly detailed in [9] (to which we address the interested reader), an equivalent optimal regulator problem has can be easily formulated, by suitably modifying the weighting matrices in the FH cost (2); the latter can be rewritten as:

$$J^{i,r} = \sum_{k=0}^{N-1} \left(\|x^{i,r}_{t+k}\|^2_{P^{i,r}} + \|u^{i,r}_{t+k}\|^2_{R^{i,r}} \right) + \|x^{i,r}_{t+N}\|^2_{P^{i,r}_N}. \tag{10}$$

Therefore, Problem 2.1 can be restated in an equivalent form as an LQR problem with the new matrices, referred to the new augmented system (8).

Problem 4.1: At every time instant $t \geq 0$ and for every agent $\mathcal{A}^i$, $i = 1, \dots, M$, find the RH optimal control law $u^{RH^{i,r\circ}}_t = \gamma^{i,r\circ}_{RH}(x^{i,r}_t)$ where $u^{RH^{i,r\circ}}_t$ is the first vector of the control sequence $u^{FH^{i,r\circ}}_t, \dots, u^{FH^{i,r\circ}}_{t+N-1}$ that minimizes the cost (10) for the local augmented state $x^{i,r}_t$.

[1]The original dynamics of agent $\mathcal{A}^i$ is not affected by the augmentation with the auxiliary system.

Let us now consider the Fake Algebraic Riccati Equation (FARE) associated to Problem 4.1 (see, for instance [12]):

$$\begin{aligned} q^{i,r}_k &= A^{i,r\top}[q^{i,r}_k - q^{i,r}_k B^{i,r}(B^{i,r\top} q^{i,r}_{k+1} B^{i,r} \\ &\quad + R^{i,r})^{-1} B^{i,r\top} q^{i,r}_k] A^{i,r} + \bar{P}^{i,r}_k \\ &\quad k = N-1, N-2, \dots, 0, \\ q^{i,r}_N &= P^{i,r}_N, \end{aligned} \tag{11}$$

where

$$\bar{P}^{i,r}_k \triangleq P^{i,r} - (q^{i,r}_k - q^{i,r}_{k+1}). \tag{12}$$

The asymptotic stability of the local closed loop system, when the RH control law solving Problem 4.1 is applied, is related to the eigenvalues of the closed loop matrix $\bar{A}^{i,r}_{k=0}$ of system (9), where

$$\bar{A}^{i,r}_k \triangleq A^{i,r} - B^{i,r}(B^{i,r\top} q^{i,r}_{k+1} B^{i,r} + R^{i,r})^{-1} \times B^{i,r\top} q^{i,r}_{k+1} A^{i,r}. \tag{13}$$

The conditions for which the eigenvalues of matrix $\bar{A}^{i,r}_0$ are all strictly inside the unit circle can be found in the following result [12].

Theorem 4.1: Consider the FARE (11) and definition (12); if the following assumptions hold: $a)$ $R^{i,r}$ is positive definite; $b)$ $[A^{i,r}, B^{i,r}]$ is stabilizable; $c)$ $[A^{i,r}, (\bar{P}^{i,r}_0)^{\frac{1}{2}}]$ is detectable; $d)$ $\bar{P}^{i,r}_0$ is positive semidefinite. Then matrix $q^{i,r}_1$ is stabilizable, that is, the eigenvalues of matrix $\bar{A}^{i,r}_0$ are all strictly within the unit circle. □

Assumptions made in Section II imply that hypotheses a) and b) of Theorem 4.1 are fulfilled by construction. As to Assumption c), the following alternative result holds true (see [9], where several comments are also reported).

Corollary 4.1: Let us take into consideration definition (12); if the following assumptions hold: $a)$ $[A^{i,r}, B^{i,r}]$ is stabilizable and $R^{i,r}$ is positive definite; $b)$ $[A^{i,r}, (P^{i,r})^{\frac{1}{2}}]$ is detectable; $c)$ $\bar{P}^{i,r}_0 \geq P^{i,r}$; then the eigenvalues of matrix $\bar{A}^{i,r}_0$ are all strictly within the unit circle. □

The above stability results concern each single agent as an uncoupled system where the cooperation information gathered from the other agents has just the role of an external input (hence not affecting its dynamics). Let us now recall from [9] the stability results in the case where the dynamic behaviors of the agents are coupled due to the cooperation objective. Under the application of the RH optimal control law (7), the closed loop dynamics of the i-th agent can be described as

$$x^i_{t+1} = \tilde{A}^i x^i_t + \sum_{j \in G^i} F^{ij}(\bar{x}^{ij} - d^{ij}), \tag{14}$$

where $\tilde{A}^i \triangleq A^i - B^i K^{x^i}_0$ and $F^{ij} \triangleq B^i K^{v^i}_0 \tilde{\Phi}^i_1 S^{ij}$. The dynamics of the whole group of cooperating agents can be then modeled. Define the maximum delay $\Delta \triangleq \max_{i,j;\, i \neq j} \Delta_{ji}$; then introduce $M\Delta$ further state equations $\rho^{i,1}_{t+1} = x^i_t$, $\rho^{i,2}_{t+1} = \rho^{i,1}_t$, ..., $\rho^{i,\Delta}_{t+1} = \rho^{i,\Delta-1}_t$,

$i = 1,\ldots,M$. Accordingly, an augmented state vector $x_t^a \in \mathbb{R}^{nM(1+\Delta)}$ is defined in the form $x_t^a \triangleq [x_t^{1^\top} \ldots x_t^{M^\top}\ \rho_t^{1,1^\top} \cdots \rho_t^{M,1^\top} \ldots \rho_t^{1,\Delta^\top} \ldots \rho_t^{M,\Delta^\top}]^\top$. Then, the global system dynamics can be described as

$$x_{t+1}^a = A^a x_t^a + \sum_{i=1}^{M} \mathcal{F}^i d^i \,. \qquad (15)$$

The above introduced matrices definitions follow; A^a is given by:

$$A^a \triangleq \left[\begin{array}{c|cccc} \tilde{A} & \bar{F}^1 & \bar{F}^2 & \cdots & \bar{F}^\Delta \\ \hline \multicolumn{4}{c|}{I_{nM\Delta}} & \emptyset \end{array}\right], \qquad (16)$$

where $\tilde{A} \in \mathbb{R}^{nM\times nM}$, $\tilde{A} \triangleq blkdiag(\tilde{A}^1, ..., \tilde{A}^M)$, $I_{nM\Delta}$ is an $nM\Delta \times nM\Delta$ identity matrix, and $\emptyset$ denotes a zero rectangular matrix of dimension $nM\Delta \times nM$. Matrices $\bar{F}^k \in \mathbb{R}^{nM\times nM}$, $k = 1, ..., \Delta$ are block matrices for $k = 1,\ldots,M$: the i,j block is equal to F^{ij} if agent $\mathcal{A}^i$ receives the information about the state of agent $\mathcal{A}^j$ with a delay of k time–steps, otherwise is a null matrix. We address the reader to [9] for a precise definition of matrices $\mathcal{F}^i$ and vectors d^i, which are rather obvious.

Let us consider the following partition of matrix A^a:

$$A^a \triangleq \begin{bmatrix} \tilde{A} & \mathcal{F}^\Delta \\ \mathcal{I}^\Delta & \mathcal{H}^\Delta \end{bmatrix}, \qquad (17)$$

where $\mathcal{F}^\Delta \in \mathbb{R}^{nM\times nM\Delta}$, $\mathcal{I}^\Delta \in \mathbb{R}^{nM\Delta\times nM}$, and $\mathcal{H}^\Delta \in \mathbb{R}^{nM\Delta\times nM\Delta}$ are appropriately defined.

In [9], the following sufficient condition on the stability of the system (16) has been proved.

Proposition 4.1: Consider the global coupled system (15). Moreover, for each agent $\mathcal{A}^i$, $i = 1,\ldots,M$, consider the FARE (11) and definition (12) and suppose that: $a)$ $R^{i,r}$ is positive definite; $b)$ $[A^{i,r}, B^{i,r}]$ is stabilizable; $c)$ $[A^{i,r}, (\bar{P}_0^{i,r})^{\frac{1}{2}}]$ is detectable; $d)$ $\bar{P}_0^{i,r}$ is positive semidefinite. Then, there exists $0 < \alpha < 1$ such that $\|(zI - \mathcal{H}^\Delta)^{-1}\mathcal{I}^\Delta(zI - \tilde{A})^{-1}\|_\infty < \alpha$. Moreover, if the cooperation weighting matrices are chosen in such a way that $\|\mathcal{F}^\Delta\|_\infty < 1/\alpha$, then the team of agents is asymptotically stable. □

The stability properties of the team of agents depend on the terms in the FH cost function related to the cooperation objective for the team. Proposition 4.1 states as a sufficient but not constructive condition, that a suitable choice of the coupling matrices in the local cost functions can guarantee stability of the team. In the next sections, some novel results will be given, providing some more insight on the appropriate choice of the coupling weighting matrices.

V. Bounding expressions for the control law

In order to relate the stability of the team with the coupling matrices chosen in the local cost functions, some bounds on matrices F^{ij} in (14) will be given as functions of matrices S^{ij}. References for the following section are given by [13] and [14]. First of all, let us give a more explicit expression for matrices $\tilde{\Phi}_k^i$ appearing in (5). Let us rewrite (4) defining

$$\Phi_k^i \triangleq A^{i^\top} - A^{i^\top} q_k^i B^i (B^{i^\top} q_k^i B^i + R^i)^{-1} B^{i^\top}. \qquad (18)$$

As vectors $(\bar{x}^{ij} - d^{ij})$ are constant within the FH time window, (4) becomes:

$$v_{t+k}^i = [I + \Phi_{k+1}^i + \Phi_{k+2}^i\Phi_{k+1}^i + \ldots \\ \ldots + \Phi_{N^i}^i \cdots \Phi_{k+2}^i\Phi_{k+1}^i] \times \sum_{j\,\in\, G^i} S^{ij}(\bar{x}^{ij} - d^{ij}).$$

Hence, matrices $\tilde{\Phi}_k^i$ take on the form

$$\tilde{\Phi}_k^i \triangleq [I + \Phi_{k+1}^i + \Phi_{k+2}^i\Phi_{k+1}^i \cdots + \Phi_{N^i}^i \cdots \Phi_{k+2}^i\Phi_{k+1}^i], \qquad (19)$$

In the following, some bound on $\tilde{\Phi}_k^i$ will be determined. To this end, let us consider the Riccati equation (3). Recall that, for a suitably large choice of $q_{N^i}^i$, it can be shown that $q_k^i \le q_{k+1}^i$, for $k = 0, ..., N^i - 1$. Letting

$$\bar{P}_{N^i}^i \triangleq P_{N^i}^i + \sum_{j\,\in\,\mathcal{G}^i} S_{N^i}^{ij}$$

we obtain $k = 0,\ldots,N^i - 1$ the inequality [2]

$$\Phi_k^i \le A^{i^\top}\left[I - q_k^i B^i (B^{i^\top}\bar{P}_{N^i}^i B^i + R^i)^{-1} B^{i^\top}\right].$$

Hence, defining $\Theta_{\bar{P}_{N^i}^i}^i \triangleq B^i(B^{i^\top}\bar{P}_{N^i}^i B^i + R^i)^{-1}B^{i^\top} \ge 0$ it follows that

$$\Phi_k^i \le A^{i^\top}\left[I - q_k^i\,\Theta_{\bar{P}_{N^i}^i}^i\right], \quad k = 0,\ldots,N^i - 1.$$

Let us now determine a lower bound on q_k^i as a function of $\sum_{j\,\in\,\mathcal{G}^i} S_{N^i}^{ij}$. We assume that weight matrices R^i are identity matrices of appropriate dimension. Then, applying the matrix inversion lemma we can write

$$\begin{aligned} q_k^i &= A^{i^\top} q_{k+1}^i \left[I - B^i(B^{i^\top} q_{k+1}^i B^i + I)^{-1} B^{i^\top} q_{k+1}^i\right] A^i \\ &\quad + P^i + \sum_{j\,\in\,\mathcal{G}^i} S^{ij} \\ &= A^{i^\top}\left[(q_{k+1}^i)^{-1} + B^i\, B^{i^\top}\right]^{-1} A^i + P^i + \sum_{j\,\in\,\mathcal{G}^i} S^{ij}, \\ &\qquad k = N^i - 1, N^i - 2, \ldots, 0. \end{aligned}$$

Since $q_{k+1}^i \ge 0$, the first term on the right–hand side of the above equality is positive; since $P^i \ge 0$ and $S^{ij} \ge 0$ by assumption, we have

$$q_k^i \ge \underline{P}^i \triangleq P^i + \sum_{j\,\in\,\mathcal{G}^i} S^{ij}, \quad k = N^i - 1, N^i - 2, \ldots, 0.$$

and then the upper bound on Φ_k^i takes on the form

$$\Phi_k^i \le A^{i^\top}\left[I - \underline{P}^i\,\Theta_{\bar{P}_{N^i}^i}^i\right], \quad k = 0,\ldots,N^i - 1.$$

[2] We remark that, for the sake of simplicity, we are dealing with the case of cooperating agents with the same dimension of the state and control vectors, without losing the general validity of the results.

Now, define the matrix $\Psi^i_{S^{ij}} \triangleq [I - \underline{P}^i\,\Theta^i_{\bar{P}^i_{N^i}}]$, which depends on the choice of matrices S^{ij}. By substitution in (19), we draw the following upper bound for $\tilde{\Phi}^i_1$:

$$\tilde{\Phi}^i_1 \leq [I + {A^i}^\top \Psi^i_{S^{ij}} + ({A^i}^\top \Psi^i_{S^{ij}})^2 + \cdots + ({A^i}^\top \Psi^i_{S^{ij}})^{N^i}].$$

As a result, the following upper bound on matrices F^{ij} in (14) has been derived:

$$F^{ij} = B^i K_0^{v^i} \tilde{\Phi}^i_1 S^{ij} \leq \mathcal{E}^i_{S^{ij}}\, S^{ij}, \tag{20}$$

where $\mathcal{E}^i_{S^{ij}} \triangleq B^i K_0^{v^i}[I + {A^i}^\top \Psi^i_{S^{ij}} + ({A^i}^\top \Psi^i_{S^{ij}})^2 + ... + ({A^i}^\top \Psi^i_{S^{ij}})^{N^i}]$. It is worth noting that the subscript S^{ij} in matrices $\Psi^i_{S^{ij}}$ and $\mathcal{E}^i_{S^{ij}}$ emphasizes the fact that these matrices depend on the choice of sum of the cooperation weighting matrices S^{ij} for each agent $\mathcal{A}^i$.

VI. Choice of the stabilizing cost function

Now that a bound on matrices F^{ij} has been derived, we are able to proceed to the following step: find a choice of matrices S^{ij} in (2), guaranteing the stability of the team of cooperating agents. In Section IV, the overall system dynamics were described as $x^a_{t+1} = A^a x^a_t + \sum_{i=1}^M \mathcal{F}^i d^i$, where matrix A^a was partitioned as in (17). Proposition 4.1 stability was guaranteed in terms of a bound on the norm $\|\mathcal{F}^\Delta\|_\infty$. In the following, by exploiting the bounds (20) on matrices F^{ij}, we shall analyze how the cooperation weighting matrices S^{ij} in the cost function alterate the norm $\|\mathcal{F}^\Delta\|$ thus influencing the stability property of the system. More specifically, once it has been verified that $\|(zI - \mathcal{H}^\Delta)^{-1}\mathcal{I}^\Delta(zI - \tilde{A})^{-1}\|_\infty < \alpha$ for some positive scalar α, let us find some explicit condition on the coupling matrices to ensure that $\|\mathcal{F}^\Delta\| < 1/\alpha$, thus fulfilling Proposition 4.1.

Let us first recall the structure of matrix $\mathcal{F}^\Delta = \left[\bar{F}^1 \quad \bar{F}^2 \ldots \bar{F}^\Delta\right]$. Assuming again that all the agents have the same state dimension, and if the communication between the agents is not replicated over the time frame Δ, it follows that

$$\|\mathcal{F}^\Delta\|_\infty = \max_{i=1,..,M} \|\sum_{j \in \mathcal{G}^i} F^{ij}\|_\infty .$$

Supposing that all the coupling matrices S^{ij} are diagonal, from (20), we obtain immediately

$$\sum_{j\,\in\,\mathcal{G}^i} F^{ij} \leq \mathcal{E}^i_{S^{ij}} \sum_{j\,\in\,\mathcal{G}^i} S^{ij}$$

and thus

$$\begin{aligned} \|\mathcal{F}^\Delta\|_\infty &= \max_{i=1,..,M} \|\sum_{j\in\mathcal{G}^i} F^{ij}\|_\infty \\ &\leq \max_{i=1,...,M} \|\mathcal{E}^i_{S^{ij}} \sum_{j\,\in\,\mathcal{G}^i} S^{ij}\|_\infty \\ &\leq \max_{i=1,...,M} \|\mathcal{E}^i_{S^{ij}}\|_\infty \|\sum_{j\,\in\,\mathcal{G}^i} S^{ij}\|_\infty . \end{aligned} \tag{21}$$

Summing up, we get the following result:

Proposition 6.1: Let us take into account the global coupled system (15). Moreover, for each agent $\mathcal{A}^i$, $i = 1, \ldots, M$, consider the FARE (11) and definition (12) and suppose that: *a)* $R^{i,r}$ is positive definite; *b)* $[A^{i,r}, B^{i,r}]$ is stabilizable; *c)* $[A^{i,r}, (\bar{P}_0^{i,r})^{\frac{1}{2}}]$ is detectable; *d)* $\bar{P}_0^{i,r}$ is positive semidefinite. Then, there exists $0 < \alpha < 1$ such that $\|(zI - \mathcal{H}^\Delta)^{-1}\mathcal{I}^\Delta(zI - \tilde{A})^{-1}\|_\infty < \alpha$. Moreover, if the cooperation weighting matrices S^{ij} are chosen in such a way that

$$\max_{i=1,...,M} \|\mathcal{E}^i_{S^{ij}}\|_\infty \|\sum_{j\,\in\,\mathcal{G}^i} S^{ij}\|_\infty < \frac{1}{\alpha}$$

then the team of agents is asymptotically stable. □

It is worth noting that, once the other parameters of the local system and the cost function are set, and asymptotic stability of matrix $\tilde{A}$ is ensured, inequality (21) can be tested adapting suitably $\sum_{j\,\in\,\mathcal{G}^i} S^{ij}$; the choice of the single matrices S^{ij} can thus be interpreted as a tuning parameter in the design of the cooperative control scheme.

VII. Simulation results

The cooperation of a set of UAVs moving in $\mathbb{R}^2$ has been studied, by applying the proposed decentralized RH control framework. The objective of the cooperative controller is to reach a certain “formation” around the origin, maintaining the formation during the entire trajectory. The state of the vehicles is given by the two independent space position and velocity components, plus the uncoupled orientation angle and its rate. The discretization has been carried out with a sampling time $\delta = 0.1$; the physical parameters have been derived from [15], even though our model is highly simplified.

For the local cost function (2), we set $P^i = blkdiag(300, 1, 300, 1, 300, 1)$, $P^i_N = 5\,P^i$ and $R^i = blkdiag(1, 1, 1)$. As to the cooperation weight matrices S^{ij}, several values have been considered in order to test the effectiveness of the algorithm and of the proposed bounds on the $\sum_{j\,\in\,\mathcal{G}^i} S^{ij}$, for each i . The initial condition is a set of matrices that causes instability of the team, with all the matrices S^{ij} equal to $S^{ij}_0 = 30 * blkdiag(750, 50, 750, 50, 200, 10)$. A first bound has been found by imposing that the eigenvalues of the system matrix in (16) lie strictly inside the unit disk. This first method guarantees stability of the team, and a good cooperative behavior; the desired formation is to form a line of 45° crossing the origin, keeping a distance of $1.5\,m$ both in the x and y coordinates and showing the same orientation angle of 0°. For $M = 5$, by setting all the coupling matrices as $S^{ij}_b = blkdiag(1526, 101, 1526, 101, 407, 20)$, for each i, j, (which is the bound on the sum multiplied for a factor $1/M$), the performance shows a satisfactory respect of the desired formation (see Fig. 1: red–agent 1, blue–agent 2, green–agent 3, magenta–agent 4, cyan–agent 5). The algorithm is slow due to the computation of the eigenvalues at each step, and could not be implemented

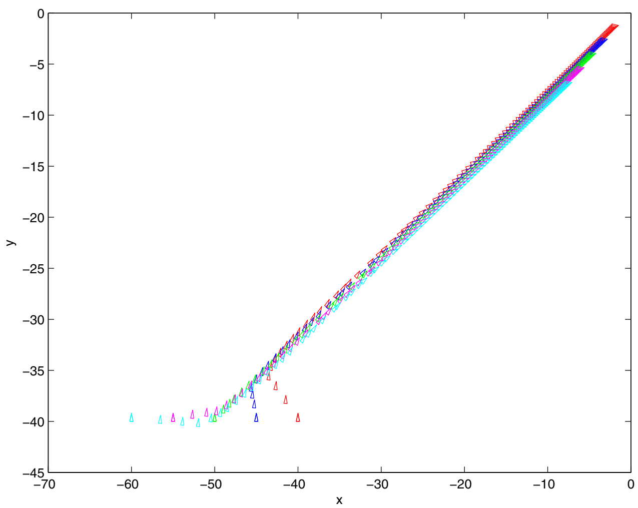

Fig. 1. Trajectories of the UAVs

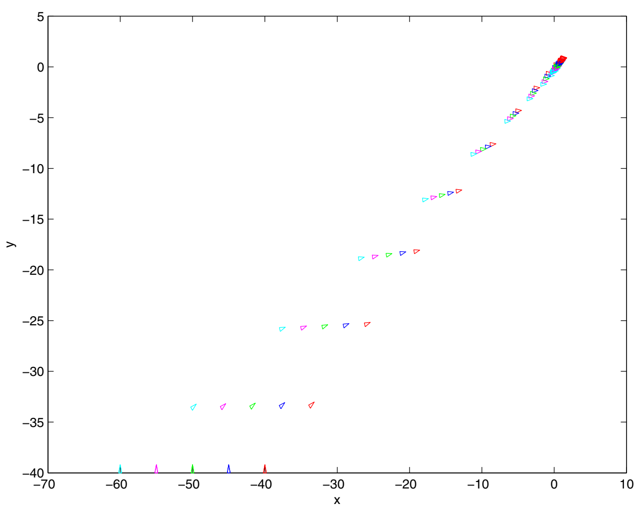

Fig. 2. Trajectories of the UAVs with tighter bounds of cooperation

on-line. The distances among the agents reach the desired value of $\sqrt{2\,(1.5)^2}\,m$ with an error of at most $14\% - 15\%$ and vehicles never hit each other. Some simulations have been run tuning the S^{ij} with a law of the form $S^{ij}(t) = S_b^{ij}(1 - \exp\left[-\beta\, d_t^{ij}\right])$, where $d_t^{ij} = \|x_t^i - x_{t-\Delta_j i}^j + d^{ij}\|$, but the results have not given significant improvement of the performance other than letting the cooperation cost expense be lower when the target distance is attained.

Simulations have also been run tuning the bound according to the sufficient condition (21), which is very restrictive; the applied value of cooperation matrices has been set as $S_b^{ij} = blkdiag(22.28, 1.48, 22.28, 1.48, 5.94, 0.29)$, for each i, j. The results on this test system (see Fig. 2) and show that cooperation among the agents is lowered but still present; computations are much faster than the first eigenvalue-based criterion. For space reasons we only report the trajectory behavior of the team, but the distance plots showed that collisions are always avoided. The simulation results have a strong dependence of the control performances (like, for example, reaching of desired final configurations) on the choice of the initial set of cooperation matrices and on the FH length N. No input constraints have been imposed.

VIII. Conclusions

This paper presented a receding–horizon framework for designing and analyzing cooperative control algorithms for a distributed team of agents, deriving sufficient conditions for stability. The resulting cooperative control algorithm can be viewed as a feedback-feedforward controller, where the feedback component is based on the regulation problem for the local state variables and the feedforward controller depends on the exchange of delayed information between neighboring/cooperative agents. Further investigation will focus on the extension of the presented framework to the case of a team of nonlinear agents (some early results have been obtained very recently), and the attempt to consider asynchronous members of the team.

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