# Robust $\ell_1$ Design of a Multivariable PI Controller Using a Real-Coded Genetic Algorithm*

Tramone D. Curry and Emmanuel G. Collins, Jr.
Department of Mechanical Engineering
Florida A&M University- Florida State University
Tallahassee,FL 32310
tracurry@eng.fsu.edu, ecollins@eng.fsu.edu

***Abstract*— This paper considers the use of a real-coded genetic algorithm (RCGA) in the design of a robust PI controller under the framework of multiplier theory (essentially mixed structured singular value theory (MSSV)) and $\ell_1$ theory. Using the Popov-Tsypkin multiplier, the robust control design is formulated as an $\ell_1$ optimization problem. The solution to this problem is described using an RCGA to obtain the optimal proportional and integral gains. This approach is illustrated in the design of a robust PI controller for a linear model of a jet engine. The controller is shown to exhibit good tracking performance and disturbance rejection in the presence of model uncertainty.**

## I. Introduction

The objective of the robust control problem is to obtain a feedback controller such that the closed-loop system satisfies a desired performance criteria in the presence of all possible exogenous signals and subject to all possible plant perturbations in a given set. This objective must be achieved while maintaining system stability. Methods for analyzing the robust stability and performance of the system are, in principle, based on assumptions made on the "structure" of the uncertainty. Perhaps the more well-known methods for addressing unstructured uncertainty are the small gain theory and fixed quadratic Lyapunov functions. However, these theories can be conservative based on the assumptions made. For example, even if the uncertainty is real and parametric, the small gain theorem assumes that the uncertainty is complex and unstructured with bounded magnitude. Likewise, fixed quadratic Lyapunov function theory assumes that the uncertainty is arbitrarily time-varying. A less conservative approach is one that is based on multiplier theory (essentially mixed structure singular value theory) [1], [2]. Mixed structured singular value (MSSV) theory allows real parametric uncertainty to be treated as slowly time-varying, real parametric uncertainty.

In recent years a new paradigm for optimization has emerged. The approach is adapted from traits of biological systems. Genetic algorithms (GAs) [3], [4] can efficiently find an optimal solution from complex and possibly discontinuous solution spaces without problem reformulation or evaluation of each solution candidate. Recently, publications on GAs [5] have highlighted the advantages of real representation over binary representation. Recognizing the shortcomings of binary representation, real-coded genetic algorithms (RCGAs) are becoming a more popular choice for optimization.

This paper considers the use of an RCGA in the design of a robust, discrete-time, multivariable PI controller under the framework of MSSV theory. In order to address time-domain point-wise-in-time performance, $\ell_1$ theory is used to specify the performance criterion. It is appropriate in situations involving persistent, unknown but bounded exogenous signals, such as when it is desired to minimize the worst-case tracking error while maintaining the control action below a given value in the presence of persistent, bounded noise. In instances such as this, the $H_2$ norm, which requires a bounded energy signal, or the induced 2-norm (i.e., the $H_\infty$ norm) are not the most adequate in capturing the features of this problem. To obtain the robust $\ell_1$ proportional and integral gains a real-coded genetic algorithm is utilized.

The organization of this paper is as follows. Section 2 presents the formulation of the robust control design as an $\ell_1$ optimization problem. Section 3 describes the solution to the optimization problem using an RCGA. Section 4 illustrates the design technique for a linear model of a jet engine. Section 5 gives concluding remarks.

### Nomenclature

| | |
|---|---|
| $\mathcal{R}, \mathcal{C}, \mathcal{Z}^+$ | real numbers, complex numbers, nonnegative integers |
| $\mathcal{R}^{m\times n}, \mathcal{C}^{m\times n}$ | $m\times n$ real matrices, complex matrices |
| $\mathcal{D}^n, \mathcal{N}^n, \mathcal{P}^n$ | $n\times n$ real diagonal, nonnegative definite, positive definite matrices |
| $0,\ I$ | zero matrix, identity matrix |
| tr | trace |
| $M_2 > M_1$ | $M_2 - M_1$ positive definite |
| $M_2 \geq M_1$ | $M_2 - M_1$ nonnegative definite |
| $\dim(M)$ | dimension of $M$ |
| $\|z(\cdot)\|_{\infty,2}$ | $ess\sup_{t\geq 0} \|\|z(t)\|\|_2$ |
| $\|z(\cdot)\|_{(\infty,2),[N_0,N]}$ | $ess\sup_{t\in[N_0T,NT]} \|\|z(t)\|\|_2$ |

*This work was supported in part by AFOSR under grant F49620-01-0550.

| | |
|---|---|
| $\|H_{zw}\|_1$ | $\sup_{w(\cdot)\in\ell_\infty} \frac{\|\|z\|\|_{\infty,2}}{\|\|w\|\|_{\infty,2}}$ |
| $z_{ij}$ | $(i,j)$ element of matrix $Z$ |
| $Vec(Z),\ Z \in \mathcal{R}^{m\times n}$ | $[z_{11},\ \ldots,\ z_{m1},\ z_{12},\ \ldots,\ z_{m2},\ \ldots,\ z_{mn}]^T$ |

## II. ROBUST PI CONTROLLER DESIGN

Consider the linear, uncertain discrete-time system described by

$$x(k+1) = (A+\Delta A)x(k) + (B+\Delta B)u(k) + D_{\infty,1}w_\infty(k), \quad k \in \mathcal{Z}^+ \tag{1}$$

$$y(k) = (C+\Delta C)x(k) + Du(k) + D_{\infty,2}w_\infty(k), \tag{2}$$

where $x \in \mathcal{R}^n$ is the state vector, $u \in \mathcal{R}^d$ is the control input, $y \in \mathcal{R}^p$ denotes the plant measurements, $w_\infty \in \mathcal{R}^{d_\infty}$ denotes an $\ell_\infty$ disturbance signal satisfying $\|w_\infty(\cdot)\|_{\infty,2} \leq 1$. The uncertainties $\Delta A$, $\Delta B$ and $\Delta C$ satisfy

$$\Delta A \in \mathcal{U}_A \triangleq \{\Delta A \in \mathcal{R}^{n\times n} : \Delta A = -H_A F_A G_A,\ F_A \in \mathcal{F}_A\}, \tag{3}$$

$$\Delta B \in \mathcal{U}_B \triangleq \{\Delta B \in \mathcal{R}^{n\times d} : \Delta B = -H_B F_B G_B,\ F_B \in \mathcal{F}_B\}, \tag{4}$$

$$\Delta C \in \mathcal{U}_C \triangleq \{\Delta C \in \mathcal{R}^{p\times n} : \Delta C = -H_C F_C G_C,\ F_C \in \mathcal{F}_C\}, \tag{5}$$

where

$$\mathcal{F}_A \triangleq \{F_A \in \mathcal{D}^r : M_{A,1} \leq F_A \leq M_{A,2}\}, \tag{6}$$

$$\mathcal{F}_B \triangleq \{F_B \in \mathcal{D}^s : M_{B,1} \leq F_B \leq M_{B,2}\}, \tag{7}$$

$$\mathcal{F}_C \triangleq \{F_C \in \mathcal{D}^t : M_{C,1} \leq F_C \leq M_{C,2}\}, \tag{8}$$

with $M_{A,1}, M_{A,2} \in \mathcal{D}^r$, $M_{B,1}, M_{B,2} \in \mathcal{D}^s$, $M_{C,1}, M_{C,2} \in \mathcal{D}^t$, $M_{A,2} - M_{A,1} \geq 0$, $M_{B,2} - M_{B,1} \geq 0$, and $M_{C,2} - M_{C,1} \geq 0$.

The dynamic system of (1) and (2) is controlled by a PI controller, shown in Figure 1, which generates the control signal

$$u(k) = K_P e(k) + K_I e_I(k), \tag{9}$$

where $e(k) = r(k) - y(k)$ is the tracking error between the reference input $r(k)$ and the plant output $y(k)$, $e_I(k)$ is the discrete integral of the error, and $K_P$ and $K_I$ are the $p \times p$ proportional and integral gain matrices, respectively. Substituting (9) into (1)-(2), then gives

$$x(k+1) = (A+\Delta A)x(k) + (B+\Delta B)\cdot [K_P e(k) + K_I e_I(k)] + D_{\infty,1}w_\infty(k), \tag{10}$$

$$y(k) = (C+\Delta C)x(k) + D[K_P e(k) + K_I e_I(k)] + D_{\infty,2}w_\infty(k). \tag{11}$$

The difference equation representing the integral of the error signal is

$$e_I(k+1) = e_I(k) + T_s e(k), \tag{12}$$

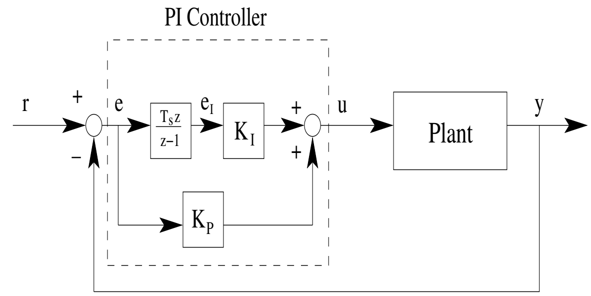


Fig. 1. Discrete-time PI controller

where $T_s$ is the sampling period. The error difference equation is

$$e(k+1) = r(k+1) - y(k+1). \tag{13}$$

It is assumed that the reference input $r(\cdot)$ is slowly varying, that is $r(k+1) \cong r(k)$. Thus, (13) can be written as

$$\begin{aligned} e(k+1) &= r(k) - y(k+1) \\ &= r(k) - (C+\Delta C)x(k+1) - D[K_P e(k+1) \\ &\quad + K_I e_I(k+1)] - D_{\infty,2}w_\infty(k). \end{aligned} \tag{14}$$

Defining $L = (I + DK_P)^{-1}$, (14) can be rewritten as

$$\begin{aligned} e(k+1) &= Lr(k) - L(C+\Delta C)x(k+1) \\ &\quad - LDK_I e_I(k+1) - LD_{\infty,2}w_\infty(k). \end{aligned} \tag{15}$$

Now, substituting (10) and (12) into (15) yields

$$\begin{aligned} e(k+1) &= Lr(k) - L(C+\Delta C)(A+\Delta A)x(k) \\ &\quad - L(C+\Delta C)(B+\Delta B)K_P e(k) \\ &\quad - L(C+\Delta C)(B+\Delta B)K_I e_I(k) \\ &\quad - L(C+\Delta C)D_{\infty,1}w_\infty(k) \\ &\quad - LDK_I e_I(k) - LDK_I T_s e(k) \\ &\quad - LD_{\infty,2}w_\infty(k). \end{aligned} \tag{16}$$

Note that the terms $L\Delta C\Delta Ax(k)$, $L\Delta C\Delta BK_P e(k)$ and $L\Delta C\Delta BK_I e_I(k)$ appear with multiplicative uncertainties. The effects of these terms are assumed to be insignificant with respect to the other uncertain terms and are therefore neglected. It should also be noted that the uncertain term $L\Delta CD_{\infty,1}w_\infty(k)$ also appears. This can be modeled as a disturbance signal as follows.

Define $L\Delta CD_{\infty,1}w_\infty(k) \triangleq D_\Delta w_\infty(k)$. From (5),

$$D_\Delta = -H_\Delta F_C G_\Delta, \tag{17}$$

where

$$H_\Delta = LH_C, \quad G_\Delta = G_C D_{\infty,1}. \tag{18}$$

Now, $D_\Delta w_\infty(k)$ can be written in the modified form

$$D_\Delta w_\infty(k) = \Delta\Phi q(k), \tag{19}$$

where

$$\Delta\Phi \triangleq \left[\begin{array}{ccc} h_{\Delta,1}g_{\Delta,1}^T & \dots & h_{\Delta,t}g_{\Delta,t}^T \end{array}\right], \quad (20)$$

$$q(k) \triangleq \left[\begin{array}{c} \delta_1 w_\infty(k) \\ \vdots \\ \delta_t w_\infty(k) \end{array}\right]. \quad (21)$$

Note that $h_{\Delta,i}$ and $g_{\Delta,i}^T$ are the $i^{th}$ column and row of $H_\Delta$ and $G_\Delta$, respectively, and $\delta_i$ is the $i^{th}$ diagonal element of $F_C$.

Combining (10)-(14) and defining the performance output $z(k) \triangleq E_p e(k)$, the closed-loop tracking system is given as

$$\begin{aligned}\tilde{x}(k+1) = (\tilde{A} + \Delta\tilde{A})\tilde{x}(k) + B_1 r(k) \\ + D_{w,1}w_\infty(k) + \Delta\Phi q(k), \quad (22)\end{aligned}$$

$$z(k) = \tilde{C}\tilde{x}(k), \quad (23)$$

where

$$\tilde{A} = \left[\begin{array}{ccc} A & BK_p & BK_i \\ -LCA & -L(CBK_p + DK_I T_s) & -L(D+CB)K_i \\ 0 & T_s I_n & I_n \end{array}\right], \quad (24)$$

$$\tilde{x}(k) = \left[\begin{array}{c} x(k) \\ e(k) \\ e_I(k) \end{array}\right], \; B_1 = \left[\begin{array}{c} 0 \\ T \\ 0 \end{array}\right], \; \tilde{C} = \left[\begin{array}{ccc} 0 & E_p & 0 \end{array}\right], \quad (25)$$

$$D_{w,1} = \left[\begin{array}{c} D_{\infty,1} \\ -L(CD_{\infty,1} + D_{\infty,2}) \\ 0 \end{array}\right]. \quad (26)$$

Furthermore, $\Delta\tilde{A}$ satisfies

$$\Delta\tilde{A} \in \mathcal{U}_{\tilde{A}} \triangleq \{\Delta\tilde{A} \in \mathcal{R}^{3n\times 3n} : \Delta\tilde{A} = -\tilde{H}\tilde{F}\tilde{G}, \;\; \tilde{F} \in \tilde{\mathcal{F}}\}, \quad (27)$$

where

$$\tilde{\mathcal{F}} \triangleq \{\tilde{F} \in \mathcal{D}^{r+s+t} : \tilde{M}_1 \leq \tilde{F} \leq \tilde{M}_2\}, \quad (28)$$

and

$$\tilde{F} = \left[\begin{array}{ccc} F_A & 0 & 0 \\ 0 & F_B & 0 \\ 0 & 0 & F_C \end{array}\right], \quad (29)$$

$$\tilde{H} = \left[\begin{array}{ccc} -H_A & -H_B & 0 \\ LCH_A & LCH_B & LH_C \\ 0 & 0 & 0 \end{array}\right], \quad (30)$$

$$\tilde{G} = \left[\begin{array}{ccc} G_A & 0 & 0 \\ 0 & G_B K_P & G_B K_I \\ G_C A & G_C BK_P & G_C BK_I \end{array}\right], \quad (31)$$

with

$$\begin{aligned}\tilde{M}_1 = diag(M_{A,1}, M_{B,1}, M_{C,1}), \\ \tilde{M}_2 = diag(M_{A,2}, M_{B,2}, M_{C,2}). \quad (32)\end{aligned}$$

Notice that all of the uncertainty now appears in the $\Delta\tilde{A}$ matrix.

Now, defining $\tilde{w} \triangleq [r^T \; w_\infty^T \; q^T]^T$, (22) can be rewritten as

$$\tilde{x}(k+1) = (\tilde{A} + \Delta\tilde{A})\tilde{x}(k) + \tilde{D}_1\tilde{w}(k), \quad (33)$$

$$z(k) = \tilde{C}\tilde{x}(k), \quad (34)$$

where

$$\tilde{D}_1 = \left[\begin{array}{ccc} B_1 & D_{w,1} & \Delta\Phi \end{array}\right]. \quad (35)$$

**The robust control problem** is to find the controller gains $K_P, K_I$ such that the closed-loop system in (33)-(34) is asymptotically stable and the cost functional

$$J(K_P, K_I) = \|H_{z\tilde{w}}\|_1^2, \quad (36)$$

is minimized, where $H_{z\tilde{w}}$ is the convolution operator from the disturbance $\tilde{w}(\cdot)$ to the $\ell_\infty$ performance variable $z(\cdot)$.

As shown in [7], direct minimization of the $\ell_1$ norm can lead to irrational compensators. However, it is shown in [8] that it is possible to characterize an upper bound $\mathcal{J}(K_P, K_I)$ on the $\ell_1$ performance such that

$$J(K_P, K_I) \leq \mathcal{J}(K_P, K_I). \quad (37)$$

Let $G(z) \in \mathcal{C}^{q\times d_\infty}$ be the transfer function representation of the system described in (33) and (34). The Popov-Tsypkin multiplier [2], [1] has the transfer function form,

$$M(z) = H + N\frac{z-1}{z}, \quad (38)$$

where $H \in \mathcal{D}^m$, $N \in \mathcal{D}^m$ with $H > 0$ and $N \geq 0$. Let $A_a$ denote the state matrix of the augmented system $M(z)G(z)$. Then, the uncertain system for robust analysis is given by [1]

$$x_a(k+1) = (A_a + \Delta A_a)x_a(k) + D_{a,\infty}\tilde{w}(k), \quad (39)$$

$$z(k) = E_a x_a(k), \quad (40)$$

where $x_a(k) = [x_m^T(k) \; x^T(k)]^T$, $x_m(k) \in \mathcal{R}^m$ denotes the states of the multiplier,

$$A_a = \left[\begin{array}{cc} 0 & 0 \\ \tilde{H} & \tilde{A} \end{array}\right], \;\; D_{a,\infty} = \left[\begin{array}{c} 0 \\ \tilde{D}_1 \end{array}\right], \quad (41)$$

$$E_a = \left[\begin{array}{cc} 0 & \tilde{C} \end{array}\right], \quad (42)$$

and

$$\begin{aligned}\Delta A_a \in \mathcal{U}_a \triangleq \{\Delta A_a \in \mathcal{R}^{m+3n} : \Delta A_a = -H_a\tilde{F}G_a, \\ \tilde{F} \in \tilde{\mathcal{F}}\}, \quad (43)\end{aligned}$$

with

$$H_a = \left[\begin{array}{c} 0 \\ \tilde{H} \end{array}\right], \;\; G_a = \left[\begin{array}{cc} 0 & \tilde{G} \end{array}\right]. \quad (44)$$

Following [8], the $\ell_1$ performance is then bounded as

$$\|H_{z\tilde{w}}\|_1^2 \leq \mathcal{J} = [\text{tr}(E_a Q_a E_a^T)^q]^{\frac{1}{q}}, \;\; \Delta A_a \in \mathcal{U}_a, \quad (45)$$

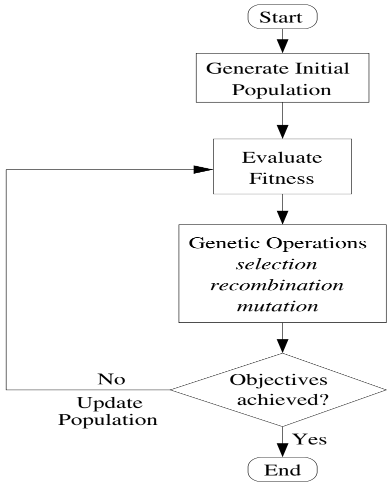


Fig. 2. Flow Chart of Single Population GA

where $Q_a$ satisfies the algebraic Riccati equation

$$\begin{aligned} Q_a &= \alpha(A_a - H_a\tilde{M}_1G_a)Q_a(A_a - H_a\tilde{M}_1G_a)^T \\ &\quad + [\sqrt{\alpha}(A_a - H_a\tilde{M}_1G_a)Q_aC_a^T - \sqrt{\alpha}B_a(H+N) \\ &\quad + S_aN][2H(\tilde{M}_2 - \tilde{M}_1)^{-1} - G_aQ_aG_a^T]^{-1}\cdot \\ &\quad [\sqrt{\alpha}(A_a - H_a\tilde{M}_1G_a)Q_aC_a^T - \sqrt{\alpha}B_a(H+N) \\ &\quad + S_aN]^T + \frac{\alpha}{\alpha-1}V_{a,\infty}, \end{aligned} \tag{46}$$

where $V_{a,\infty} \triangleq D_{a,\infty}D_{a,\infty}^T$ and $S_a \triangleq \begin{bmatrix} I \\ 0 \end{bmatrix}$, with $\dim(S_a) = \dim(H_a)$. Thus, for the robust optimization problem the $\ell_1$ cost function is given as

$$J(K_P, K_I, H, N) = [\text{tr}(E_aQ_aE_a^T)^q]^{\frac{1}{q}}. \tag{47}$$

With an enforced stability constraint, this optimization problem can be solved using an RCGA.

## III. Optimization Using a Real-Coded Genetic Algorithm

As previously discussed, the design of the robust PI controller is formulated as an optimization problem. An RCGA is used to search for a solution. GAs can efficiently search in complex and possibly discontinuous solution spaces without problem reformulation or evaluation of each solution candidate.

The RCGA begins with an arbitrarily chosen initial population within the search region. The algorithm then follows three general operations: (1) *selection*, (2) *recombination*, and (3) *mutation* [3], [5]. The flowchart for a single population RCGA is shown in Figure 2.

For the controller optimization problem the chromosome is constructed by formulating matrices $K_P$, $K_I$, $H$ and $N$ into a single vector $\Theta$ such that

$$\Theta = \left[Vec(K_P)^T\ Vec(K_I)^T\ Vec(H)^T\ Vec(N)^T\right]^T, \tag{48}$$

The search region is then defined by establishing upper and lower limits $\overline{\Theta}$ and $\underline{\Theta}$ such that

$$\underline{\theta}_{ij} \leq \theta_{ij} \leq \overline{\theta}_{ij}. \tag{49}$$

To account for the requirement of closed-loop stability, the RCGA is formulated as a constrained optimization problem. This is achieved by imposing a constraint on the cost with a penalty function. Specifically, if the stability criterion is not satisfied a penalty is added to the cost such that

$$\text{if} \begin{cases} \max[\lambda_i(A_a)] < 1, & J = J \\ \text{otherwise}, & J = J + penalty \end{cases}, \tag{50}$$

where $\lambda_i,\ i \in (1, 2, \ldots, m+3n)$ are the eigenvalues of the augmented system $A_a$. The penalty is chosen as 100 to obtain the results described below.

## IV. Illustrative Example: Robust PI Design for a Jet Engine

A numerical example is presented in this section to illustrate robust PI controller design using the Popov-Tsypkin multiplier and $\ell_1$ theory. The model used was supplied by NASA Glenn Research Center and is given as

$$\begin{aligned} x(k+1) &= (A + \Delta A)x(k) + (B + \Delta B)u(k) \\ &\quad + D_{\infty,1}w(k),\ \ k \geq 0 \end{aligned} \tag{51}$$

$$y(k) = (C + \Delta C)x(k) + Du(k) + D_{\infty,2}w(k) \tag{52}$$

where the sampling period is 0.01 sec. The elements of the state vector $x \in \mathcal{R}^3$, are

$x_1 \triangleq$ High Pressure Spool Speed (rpm)
$x_2 \triangleq$ Low Pressure Spool Speed (rpm)
$x_3 \triangleq$ High Pressure Compressor Inlet Temperature ($^\circ C$).

The elements of the control input vector $u \in \mathcal{R}^3$, are

$u_1 \triangleq$ Main Burner Fuel Flow (kg/hr)
$u_2 \triangleq$ Exhaust Nozzle Throat Area ($m^2$)
$u_3 \triangleq$ Bypass Duct Area ($m^2$).

The elements of the output vector $y \in \mathcal{R}^3$, are

$y_1 \triangleq$ Corrected High Pressure Spool Speed (rpm)
$y_2 \triangleq$ Corrected Low Pressure Spool Speed (rpm)
$y_3 \triangleq$ Corrected High Pressure Compressor Inlet Temperature ($^\circ C$).

The variable $w$ denotes a vector of disturbance signals.

Using the cost function (47) and stability constraints the controller matrices were obtained. The initial search region was defined by setting upper and lower bounds of $K_P$ and $K_I$ at $\pm 500$ and $\pm 200$, respectively. The algorithm was run for 400 generations to obtain an initial string $\Theta_0$. The value

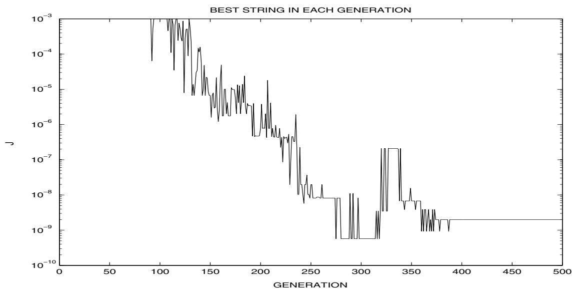


Fig. 3. Value of Cost Function $J$ During Final RCGA Optimization.

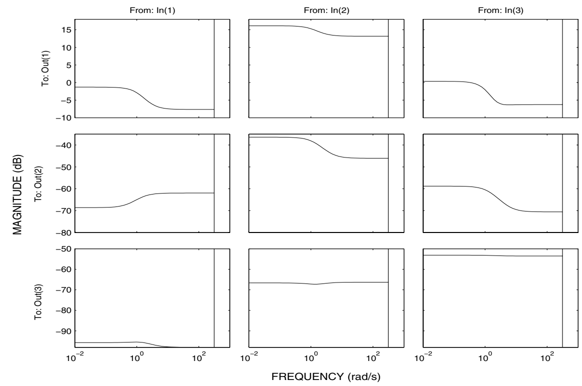


Fig. 4. Frequency Response of Open Loop System Without PI Control

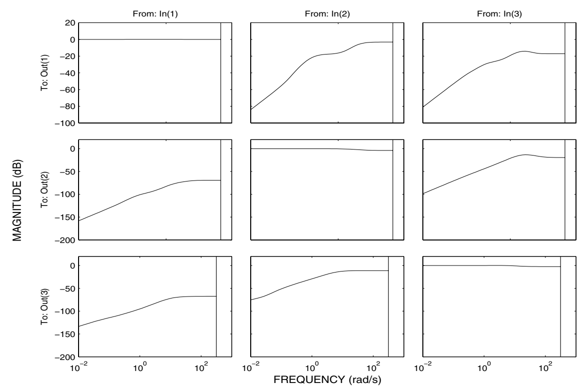


Fig. 5. Frequency Response of Closed-Loop System With PI Controller

of the achieved cost was $J = 1.6889e-6$. After the initial run the search region was refined to a region centered about the initial string. Specifically, the upper and lower bounds were defined as

$$\overline{\theta}_{ij} = \theta_{0_{ij}} + \kappa|\theta_{0_{ij}}|, \tag{53}$$

$$\underline{\theta}_{ij} = \theta_{0_{ij}} - \kappa|\theta_{0_{ij}}|, \quad \kappa \neq 1. \tag{54}$$

Setting $\kappa = 2$, the optimal matrices obtained were

$$K_P = \begin{bmatrix} -1758.2 & 18.502 & -0.07567 \\ -4.4632 & 1.1849 & -1050.7 \\ 6.0986 & 33.890 & 1.6161 \end{bmatrix}, \tag{55}$$

$$K_I = \begin{bmatrix} -116.99 & -0.00099 & 0.21174 \\ -20.536 & -362.15 & 0.07116 \\ -0.63296 & -0.17123 & -71.656 \end{bmatrix}. \tag{56}$$

The performance of the RCGA in terms of the cost function can be seen in Figure 3. It is observed that the final run of the algorithm converged to the optimal solution within approximately 400 generations. The algorithm completed the 500 generations with a final cost of $J = 1.9835e-9$ and the computational time was approximately 15 minutes and 32 seconds on a 1 GHz Pentium 3 processor.

Let $DC_{i,j}$ $i, j \in (1,2,3)$ represent the DC gain from input $i$ to output $j$. It is the aim of the multivariable PI controller to make $DC_{i,i} \cong 1$ while simultaneously reducing the influence of $DC_{i,j}$ for $i \neq j$. It should be noted that the steady-state errors approach but do not reach a zero value. This is due to the nature of the multivariable system, where each output channel is to some degree affected by each input. Figure 4 shows the frequency response of the open loop system without PI control. Figure 5 shows the frequency response of the closed-loop system with the PI controller implemented. It can be seen that each gain $DC_{i,i}$ is approximately equal to 1 while the influences of $DC_{i,j}$ have been reduced in each output channel. A final value test of the discrete-time system gives gain values of $DC_{1,1} = 0.997$, $DC_{2,2} = 0.999$ and $DC_{3,3} = 0.997$, which all give steady-state errors of less the $1\%$. It can be seen in Figure 5 that, as desired, the magnitude frequency response has these values over a wide frequency range.

In order to examine the performance of the robust controller, the step response of the closed-loop system is observed. The reference input vector $r = [3000kg/hr \;\; 250m^2 \;\; 140m^2]^T$ and random white noise signals with zero mean are added as the disturbance inputs. The variances of the disturbance inputs, $w_1$, $w_2$ and $w_3$, were 0.09, 0.02 and 0.05, respectively. In order to show the extent of robustness, uncertainty for all system matrices was considered in Figure 6. Random parameter variations were used for system uncertainty. It is observed that the closed-loop system exhibited good tracking performance and disturbance rejection.[1] Thus, the robustness of the PI controller is shown within the prescribed region of uncertainty.

## V. Conclusions

This paper considered the use of a real-coded genetic algorithm in the design of a robust PI controller under the framework of mixed structured singular value theory (MSSV). Using a Popov-Tsypkin multiplier the robust control design is formulated as an $\ell_1$ optimization problem. An RCGA is used to obtain the optimal proportional and integral gains. Using a linear model of a jet engine this approach is used to design a robust PI controller. The controller is shown to exhibit good tracking performance and disturbance rejection in the presence of model uncertainty.

[1]Simulations with other random parameter variations were also conducted with very similar results.

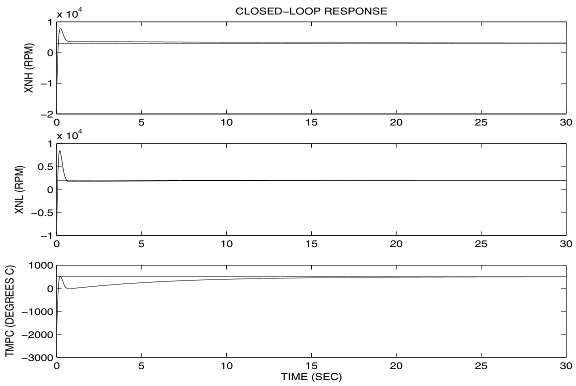


Fig. 6. Step Response Closed-Loop System With Random Uncertainty

## REFERENCES


[1] Haddad, W. M. and Kapila, V., "Discrete-time extensions of mixed-$\mu$ bounds to monotonic and odd monotonic nonlinearities," *International Journal of Control*, Vol. 61, No. 2, 1995, pp. 423–441.

[2] Collins, E. G. and Song, T., "Robust $\ell_1$ estimation using the Popov-Tsypkin multiplier with applications to robust fault detection," *International Journal of Control*, Vol. 74, No. 3, 2001, pp. 303–313.

[3] Goldberg, D., *Genetic Algorithms in Search, Optimization and Machine Learning*, Addison-Wesley, Massachusetts, 1989.

[4] Jamshidi, M., dos Santos Coelho, L., Krohling, R., and Fleming, P. J., *Robust Control Systems with Genetic Algorithms*, CRC Press LLC, Boca Raton, Florida, 2003.

[5] Wright, A. H., "Genetic algorithms for real parameter optimization," *Foundations of Genetic Algorithms*, 1991, pp. 205–218, Rawlins, G., Ed., Morgan Kaufmann Publishers, Los Altos.

[6] Janikow, C. Z. and Michalewicz, Z., "An experimental comparison of binary and floating point representation in genetic algorithms," *Proceedings of International Conference on Genetic Algorithms*, 1991, pp. 31–36.

[7] Blanchini, F. and Sznaier, M., "Rational $\ell_1$ suboptimal compensators for continuous-time systems," *IEEE Transactions on Automatic Control*, Vol. 39, 1994, pp. 1487–1492.

[8] Haddad, W. M. and Chellaboina, V.-S., "Mixed-Norm $H_2/\ell_1$ Controller Synthesis via Fixed-Order Dynamic Compensation: A Riccati Equation Approach," *Proceedings of IEEE Conference on Decision and Control*, December 1997, pp. 452–457, San Diego-California.