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Stabilization of nonlinear time delay systems with delay-independent feedback

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Abstract

The concept of control Lyapunov functions and its extension to control Lyapunov-Razumikhin functions (CLRF) has proven a useful tool for designing robust control laws for nonlinear systems. In particular, CLRF based domination redesign can be used to robustly stabilize nonlinear time-delay systems. In this paper we explore the possibility of using the domination redesign to obtain stabilizing control laws that do not depend on the past state values. The advantages are simpler implementation, robustness to uncertainty in the delay, and the possibility to "back-step" the stabilizing controller for a subsystem through an integrator or a chain of integrators.

1. Introduction

In general, it is desirable to control (stabilize) a delay system by a feedback that is independent of the delayed state. Such a feedback is, in general, much more robust to uncertainty in the delay. Knowledge of the delayed state up to a maximal delay "r" in the system dynamics is a part of the standard set of assumptions. In practice, however, it is difficult to assure that such an information is available to the controller at the moment it is turned on. The problem is compounded if the control is designed on the system transformed for delay dependent stability (see, for example, [8]) or if the backstepping with cancellations of the delay terms is employed as in [7]. In both cases, the control law may depend on the state values delayed by a multiple of r.

To illustrate the issue, let us consider the problem of stabilizing a nonlinear time delay system

$$\dot{x} = x(t-\tau)x^2 + u \tag{1.1}$$

to the equilibrium at the origin. As customary, the initial condition is given by $x(\theta) = \varphi(\theta)$, $\theta \in [-\tau, 0]$ where φ is a continuous vector valued function. An obvious choice for the control law is

$$u = -x(t-\tau)x^2 - \lambda x \tag{1.2}$$

with $\lambda > 0$. This control law cancels the nonlinear term with delay providing closed loop dynamics described by $\dot{x} = -\lambda x$. Hence the closed loop system is globally asymptotically stable (GAS).

There are two drawbacks to this method of control design. First, it uses cancellations and is potentially sensitive to uncertainties and unmodeled dynamics that would prevent exact cancellation. The second drawback, which is the subject of this paper, is that the control depends on past values of the state. At the very least, dependence on the past trajectory requires that the delay be relatively accurately known, and the past values be constantly stored into and retrieved from the controller memory. The standard assumption is that the values for the past τ seconds are available at time 0 (when the system and/or controller is initialized). This may not be the case in practical applications, and so the control may be running for the first τ seconds without needed information, potentially causing large deviations of regulated variable(s).

Another potential drawback of the cancellation design is that dependence of the control on delayed state may make it difficult or impossible to back-step it through an integrator. Let us explain this point by considering the system (1.1) with an integrator added at the input(ξ is the integrator's state):

$$\begin{aligned} \dot{x} &= x(t-\tau)x^2 + \xi \\ \dot{\xi} &= u \end{aligned} \tag{1.3}$$

The standard backstepping control design procedure (see [6]) is to consider ξ as the (virtual) control input to the x-subsystem. So we choose the expression in (1.2) for the virtual control and apply the change of coordinates $z_1 = x$, $z_2 = \xi + x(t-\tau)x^2 + \lambda x$ to obtain

$$\begin{aligned} \dot{z}_1 &= -\lambda z_1 + z_2 \\ \dot{z}_2 &= u + 2z_1 z_1(t-\tau)(-\lambda z_1 + z_2) + z_1^2 \dot{z}_1(t-\tau) \end{aligned} \tag{1.4}$$

In a non-delay system, the backstepping procedure could continue by using the control input u to cancel the nonlinear terms and introduce stabilizing, typically linear, feedback in z_1 and z_2. The problem with doing this in the delay case is that, under nominal assumptions, $z_1(t-\tau)$ may not be differentiable. Differentiability is assured if one assumes that the part of the past

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trajectory for $t-\tau<0$ is also generated by the system dynamics: $\dot{z}_1(t-\tau)=x(t-2\tau)x^2(t-\tau)+\xi(t-\tau)$. To cancel these terms, the state delayed by 2τ has to be assumed known. A version of backstepping for delay systems given by equation (5.5) in [4] obviously relies on these two assumptions, though they have not been explicitly stated in the paper. Both assumptions, however, are nonstandard and undesirable from conceptual, as well as practical, standpoint. The same applies to the version of adaptive backstepping presented in [7].

A better version of backstepping also considered in [4] is based on "domination redesign" in which the nonlinearities and delay terms are dominated rather than cancelled. The domination redesign is derived directly from a control Lyapunov-Razumikhin function (discussed in the next section). For the system (1.1), a control Lyapunov-Razumikhin function is $V(x)=\frac{1}{2}x^2$, and the domination redesign control law (see [4]) is given by $u=-(1-2V(x))\frac{\partial V}{\partial x}g(x)=-x-2x^3$. Not that this control law is independent of the delay state and, hence, it is robust to uncertainty in the delay. Such a control law does not produce delay-state time derivative in the case it needs to be "back-stepped" through an integrator.

The CLRF approach guarantees that the domination redesign control law for a system of the form $\dot{x}=f(x,x(t-\tau))+g(x)u$ is delay free. In this paper, we allow $g=g(x,x(t-\tau))$ (for notational simplicity we shall consider only one discrete delay in the vector field g) and show that a system such as

$$\dot{x}=x(t-\tau)x^2+(x-0.9x(t-\tau))u \tag{1.5}$$

can be stabilized by a delay-free feedback even though the sign of the term multiplying the control input depends on the delay term. Next we show that the "lower-triangular" system of the form (5.6) in [4], under the assumptions of that paper, can be stabilized by a delay-free feedback. Such a result has not been claimed in [4].

This paper is organized as follows. Section 2 reviews the definition of CLRF and the domination redesign control law. Section 3 provides the results that allows delay-state free stabilizing control for a class of systems that contains (1.5). In Section 4 we give a globally asymptotically stabilizing feedback law for delay systems in the strict feedback form [6]. The form allows the delayed states in the interconnecting terms, but they must be sign definite.

Notation

Two types of objects describe the state of the time delay system: $x(t)\in R^n$ a time dependent vector and $x_d(t):[-r,0]\mapsto R^n$ a time dependent function defined by $x_d(t)(\theta)=x_d(t+\theta)$. For the sake of simplicity we shall often omit the dependence on t in the notation; for example, we shall write $\dot{x}=f(x_d)$ instead of $\dot{x}(t)=f(x_d(t))$ and $x_d(\theta)$ instead of $x_d(t)(\theta)$. The notation $|\cdot|$ is used to denote the Euclidean 2-norm of a vector, while $||\cdot||$ denotes the norm of uniform convergence of functions, that is, for $\phi_d:[-r,0]\mapsto R^n$, $||\phi_d||=sup_{\theta\in[-r,0]}|\phi_d(\theta)|$. By $C([-r,0],R)$ we denote the space of continuous functions and by $CP([-r,0],R)$ the space of piecewise continuous functions from $[-r,0]$ into R. A continuous function α is said to belong to class $\mathcal{K}_\infty$ if it is strictly increasing, $\alpha(0)=0$, and $\alpha(s)\to\infty$ as $s\to\infty$.

2. CLRF based domination redesign

In this section we review the concept of CLRF's and the stabilization results of [4]. The starting point is a class of input affine, time delay nonlinear systems described by

$$\dot{x}(t)=f(x_d)+g(x_d)u \tag{2.1}$$

with the initial condition given by $x_d(0)(\cdot)=\phi_d$, where $\phi_d:[-r,0]\mapsto R^n$ is a continuous vector valued function. Vector fields f and g are assumed smooth functionals.

The stability theory underlying our definition of control Lyapunov-Razumikhin functions is provided by Razumikhin theorems [2, 5], which state that the equilibrium at the origin for the system

$$\dot{x}=f(x_d)$$

is globally stable[1] if there exist two $\mathcal{K}_\infty$ function β_1 and β_2, and a differentiable function $V(x)$ such that

$$\beta_1(|x|)\le V(x)\le\beta_2(|x|),$$
$$\text{and}\quad \dot{V}=L_fV\le 0$$

whenever $V(x)\ge V(x(t+\theta))$, $\theta\in[-r,0)$. The equilibrium at the origin is globally asymptotically stable if there exists a function α, $\alpha(s)>0$ for $s>0$, such that

$$\dot{V}=L_fV\le-\alpha(|x|)$$

whenever $\pi(V(x))\ge V(x(t+\theta))$, $\theta\in[-r,0)$, with the continuous nondecreasing function $\pi:R^+\mapsto R^+$ satisfying $\pi(s)>s$ for all $s>0$.

Definition 1 *(Control Lyapunov Razumikhin Function)*
A smooth function $V:R^n\to R$, that satisfies

$$\beta_1(|x|)\le V(x)\le\beta_2(|x|) \tag{2.2}$$

is a control Lyapunov-Razumikhin function (CLRF) for the system (2.1) if there exists a continuous nondecreasing function $\pi:R^+\to R^+$, $\pi(s)>s$ for $s>0$, and a

[1] We say that an equilibrium is globally stable if it is locally Lyapunov stable and all the trajectories of the system are bounded.

function $\alpha : R^+ \mapsto R^+$, $\alpha(s) > 0$ for $s > 0$, such that, for all piecewise continuous functions $\chi_d : [-r, 0] \mapsto R^n$ with $\chi_d(0) = x$,

$$L_g V(\chi_d) = 0 \;\Rightarrow\; L_f V(\chi_d) \le -\alpha(|x|)$$

whenever

$$\pi(V(x)) \ge V(\chi_d(\theta)), \quad \forall \theta \in [-r, 0) \qquad (2.3)$$

The condition (2.3) is referred to as the Razumikhin condition. □

To design an asymptotically stabilizing controller based on a CLRF, the class of time-delay systems under consideration has been restricted in [4] to

$$\begin{aligned}\dot{x} &= f(x_d) + g(x_d)\, u = f_0(x, x(t-\tau_1), \dots, x(t-\tau_l)) \\ &+ \textstyle\int_{-r}^{0} \Gamma(\theta) F(x, x(t-\tau_1), \dots, x(t-\tau_l), x(t+\theta))\, d\theta \\ &+ g(x, x(t-\tau_1), \dots, x(t-\tau_l))\, u \end{aligned} \qquad (2.4)$$

with the initial condition $x_d(0) = \phi$, where f_0, g, and $F : R^{(l+2)n} \mapsto R^{r_\Gamma}$ are smooth functions of their arguments. Without loss of generality we assume that $F(x, x(t-\tau_1), \dots, x(t-\tau_l), 0)) = 0$. The matrix $\Gamma : [-r, 0] \mapsto R^{n \times r_\Gamma}$ is assumed to be piecewise continuous (hence, integrable) and bounded. The restriction in the class of systems under consideration is needed to avoid the problems that arise due to noncompactness of closed bounded sets in the space $(C([-r, 0], R^n), ||\cdot||)$.

Several "universal formulas" exist for global stabilization of a nonlinear systems with a known CLF [1, 10]. However, because the CLRF conditions do not guarantee that $L_f V \le 0$ when $L_g V = 0$ along every trajectory, these formulas may result in infinite values of the control input and, hence, are not applicable. A formula that can be applied is given by the domination redesign control law [9]

$$u = -\gamma(V(x))(L_g V(x_d))^T \qquad (2.5)$$

where the domination function $\gamma(\cdot)$ must satisfy $\gamma(s) > 0$, and $\lim_{T \to \infty} \int_0^T \gamma(s)\, ds = \infty$.

It has been shown in [4] that if, for all x in a neighborhood of the origin,

$$\frac{L_f V(x_d)}{|L_g V(x_d)|^2} < c, \quad c > 0 \qquad (2.6)$$

whenever the Razumikhin condition (2.3) holds and $L_g V \neq 0$, then the control law (2.5) achieves global asymptotic stability. This condition is a delay-system version of the one employed in [3] to prove the global asymptotic stabilization by the CLF based domination redesign control law for non-delay systems. It was shown in [4] that the condition (2.6) is satisfied if $V_2 = x^T P x$ (where $P = \frac{1}{2}\frac{\partial^2 V}{\partial x^2}(0)$) is a CLRF for the Jacobian linearization of (2.4),

$$\dot{x} = A_0 x + \sum_{i=1}^{l} A_i x(t-\tau_i) + \int_{-r}^{0} G(\theta) x(t+\theta)\, d\theta + Bu \qquad (2.7)$$

where $A_0 := \frac{\partial f_0}{\partial x}(0)$, $A_i := \frac{\partial f_0}{\partial x_d(t-\tau_i)}(0)$, $G(\theta) := \Gamma(\theta)\frac{\partial F}{\partial x_d(\theta)}(0)$, and $B = g(0)$. The following theorem summarizes the results of [4] relevant to this paper.

Theorem 1 If $\pi(s) \ge p\, s$, $p > 1$, and V_2, the quadratic part of the CLRF V, is a CLRF for the Jacobian linearization (2.7) then there exists a smooth function γ^* such that the closed loop system (2.4), (2.5) is globally asymptotically stable for all γ with $\gamma(s) \ge \gamma^*(s)$ for all $s \ge 0$. Moreover, the time derivative of $V(t)$ satisfies

$$\dot{V} = L_f V - \gamma(V)|L_g V|^2 \le -\mu(|x|) \qquad (2.8)$$

with $\mu(s) > 0$ for all $s > 0$ and, in a neighborhood of $x = 0$, $\mu(|x|) > \nu x^2$, $\nu > 0$.

3. Stabilization by delay-independent control law

To simplify the notation let us assume that the control vector field g depends only on one discrete delay τ, that is,

$$\dot{x} = f(x_d) + g(x(t), x(t-\tau))u \qquad (3.1)$$

The case of multiple discrete delays is completely analogous, while the case of distributed delay in the control vector field g is not covered by the theory presented in [4], which is the foundation for this paper.

If the assumption of Theorem 1 are satisfied, the domination redesign control law is given by

$$\begin{aligned} u &= -\gamma(V) L_g V^T(x, x(t-\tau)) \\ &= -\gamma(V(x)) \left(\frac{\partial V(x)}{\partial x} g(x, x(t-\tau)) \right)^T \end{aligned}$$

and, hence, depends on delayed terms. As discussed in the introduction, delay-state dependence has several practical and conceptual drawbacks. The goal of this section is to introduce conditions such that the system (3.1) can be stabilized by a delay independent feedback.

Assumption 1 There exists a smooth vector valued function $\psi : R^n \to R^m$ such that $\psi(0) = 0$ and for all x and y in R^n

$$\pi(V(x)) \ge V(y) \;\;\Rightarrow\;\; |L_g V(x, y)|^2 \le L_g V(x, y)\psi(x) \qquad (3.2)$$

□

The condition (3.2) in general means that, if the Razumikhin condition $\pi(V(x)) \ge V(x(t-\tau))$ is satisfied, the sign of each component of the vector $L_gV^T(x, x(t-\tau))$ is determined by the value of x alone (and is independent of the delayed state $x(t-\tau)$). A more formal statement is available in the case $m = 1$ (scalar input u).

Proposition 1 If $m = 1$, the condition (3.2) is equivalent to the following statement: $\forall x, y_1, y_2 \in R^n$

$$\begin{aligned} &\pi(V(x)) \geq V(y_1) \text{ AND } \pi(V(x)) \geq V(y_2) \\ &\Rightarrow \quad L_gV(x, y_1)L_gV(x, y_2) \geq 0 \end{aligned} \tag{3.3}$$

Proof: First let us assume that the condition (3.3) does not hold, that is, for some x, y_1, y_2, $L_gV(x, y_1)L_gV(x, y_2) < 0$. Thus, regardless of what $\psi(x)$ is, either $L_gV(x, y_1)\psi(x) < 0$ or $L_gV(x, y_2)\psi(x) < 0$ and the condition (3.2) cannot be satisfied.

To prove the other direction, assume that the condition (3.3) is satisfied and denote set $S_V(x) = \{y \in R^n : \pi(V(x)) \geq V(y)\}$. Note that the Razumikhin condition can be rewritten as $x(t+\theta) \in S_V(x(t)), \forall \theta \in [-r, 0)$. Next, define two sets $P = \{x \in R^n : \exists y \in S_V(x))$ such that $L_gV(x, y) > 0\}$ and $N = \{x \in R^n : \exists y \in S_V(x)$ such that $L_gV(x, y) < 0\}$. By assumption (3.3), $P \bigcap N = \emptyset$ and $R^n - (P \bigcup N) = \{x : L_gV(x, y) = 0,\ \forall y\}$.

Consider the function

$$\begin{aligned} \phi(x) &= \text{ext}_{y \in S_V(x)} L_gV(x, y) \\ &= \begin{cases} \min_{y \in S_V(x)} L_gV(x, y) \text{ if } x \in N \\ \max_{y \in S_V(x)} L_gV(x, y) \text{ if } x \in P \\ 0 \ \text{ otherwise} \end{cases} \end{aligned}$$

The function $\phi(x)$ is Lipschitz continuous in N and P, and $\forall y \in S_V(x)$ $\phi(x)L_gV(x, y) \geq 0$ and by definition, $|\phi(x)| \geq |L_gV(x, y)|$. Note that $\phi(x) = 0 \Leftrightarrow L_gV(x, \cdot) = 0$ which allows us to show that the function $\phi(x)$ is differentiable at all x where $\phi(x) = 0$ (details are omitted).

Hence, one can find a smooth function $\psi(x)$ such that $\psi(x) > \phi(x)$ on P, $\psi(x) < \phi(x)$ on N, and $\psi(x) = 0 \Leftrightarrow \phi(x) = 0$.

By construction, $\psi(x)$ satisfies $\psi(x)L_gV(x, y) \geq 0$ and $|\psi(x)| \geq |L_gV(x, y)|$ $\forall y \in S(\pi(V(x)))$, which means that the condition (3.2) is satisfied. □

If the Assumption 1 is satisfied, the obvious choice for the control law is

$$u(x) = -\gamma(V(x))\psi(x) \tag{3.4}$$

Then, whenever $\pi(V(x)) \geq V(x(t+\theta), \theta \in [-r, 0)$,

$$\begin{aligned} &\dot{V}(x) = L_fV(x_d) - \gamma(V(x))L_gV(x, x(t-\tau))\psi(x) \\ &\leq L_fV(x_d) - \gamma(V(x))|L_gV(x, x(t-\tau))|^2 \leq -\mu(|x|) \end{aligned}$$

where the second inequality follows from Theorem 1. By the Razumikhin theorem, the closed loop system is globally asymptotically stable. Hence, (3.4) provides a delay-state independent control law.

With this result we return to the problem introduce earlier in the paper, that is, delay-independent stabilization of the system

$$\dot{x} = x(t-\tau)x^2 + (x - 0.9x(t-\tau))u$$

Selecting $V(x) = \frac{1}{2}x^2$ and $\pi(s) = 1.05^2 s$, we have that, whenever the Razumikhin condition holds (i.e. $|x(t-\tau)| < 1.05|x|$), the sign of $L_gV(x, x(t-\tau)) = x(x - 0.9x(t-\tau))$ is known. Thus we select $\psi(x) = 2x^2$ and note that $|L_gV|^2 = x^2(x - 0.9x(t-\tau))^2 \leq 2x^3(x - 0.9x(t-\tau))$. Straightforward computation shows that if we select $\gamma(V) = 10$ (that is, $u = -20x^2$), and the Razumikhin condition holds,

$$\dot{V} = x^3x(t-\tau) - 20x^3(x - 0.9x(t-\tau)) \leq -0.9x^4$$

Thus the closed loop system is GAS.

4. Backstepping for delay systems

With the result of the previous section, one may be able to complete a backstepping design and avoid issues related to differentiation of the delayed states discussed in the introduction. One remaining problem is that, for repeated application, it is difficult to establish conditions that can be checked a-priori. Indeed, as condition (3.2) depends on the CLRF V, at each stage of the backstepping design Assumption 1 depends on the "virtual" CLRF at that stage, which in turn depends on the virtual controls chosen in the preceding stages. While one may proceed with the design and verify Assumption 1 at each step, we prefer to give conditions that are independent of the virtual control selection. In this case we assume that $g(x, x(t-\tau)) \neq 0,\ \forall x, x(t-\tau) \in R^n$. That is, the class of systems considered is given by

$$\begin{aligned} \dot{x}_1 &= f_1(x_{1d}) + g_1(x_{1d})x_2 \\ \dot{x}_2 &= f_2(x_{1d}, x_{2d}) + g_2(x_{1d}, x_{2d})x_3 \\ &\vdots \\ \dot{x}_n &= f_n(x_d) + g_n(x_d)u \end{aligned} \tag{4.1}$$

where we have assumed that each f_i, g_i are of the form (2.4) and that $g_i(x_{1d}, \ldots, x_{id}) \neq 0$. Again, for clarity of presentation we will consider the case when the control vector field is a function of x and $x(t-\tau)$ only and show the backstepping design for $n = 2$. The cases when g depends on more than one discrete delay and $n > 2$ are straightforward extensions of the presented result.

According to the standard backstepping procedure [6], in the first stage we stabilize the first x_1-subsystem, considering x_2 as the virtual control v_1. That is, consider the system

$$\dot{x}_1 = f_1(x_{1d}) + g_1(x_1, x_1(t-\tau))v_1 \tag{4.2}$$

and the CLRF candidate $V_1(x_1) = \frac{1}{2}x_1^2$. That V_1 is indeed a CLRF follows from

$$\begin{aligned} &L_{g_1}V_1 = x_1 g_1(x_1, x_1(t, t-\tau)) = 0 \;\Leftrightarrow\; x_1 = 0 \\ &\Rightarrow\; L_{f_1}V_1 = x_1 f_1(x_d) = 0 \end{aligned}$$

The Razumikhin condition did not play a role here, but we'll need it in the subsequent derivation, so we set $\pi(s) = p\ s$, $p > 1$.

In contrast to non-delay backstepping, we shall design a virtual control for the system (4.2) in two steps. First we show that the conventional domination redesign stabilizes the system and then replace delay-state dependent feedback with a delay-state independent one. The first part consists of checking that the conditions of Theorem 1 are satisfied. Indeed, $\pi(s) = ps$, $g(0,0) \neq 0$, and $V_1(x_1)$ is quadratic, so using the same argument that showed V_1 is a CLRF for (4.2), one can show that it is a CLRF for the Jacobian linearization of (4.2). Thus, the conditions of Theorem 1 are satisfied and the control law of the form

$$u = -\gamma_1(V_1(x_1))x_1 g_1(x_1, x_1(t-\tau)) \tag{4.3}$$

results in $\dot{V}_1 \leq -\mu_1(|x_1|)$ achieving global asymptotic stability. Without loss of generality we assume that we have chosen $\gamma_1(0) > 0$ sufficiently large which guarantees that, in a neighborhood of $x_1 = 0$, $\mu_1(|x_1|) \geq \nu_1 x_1^2$ for some $\nu_1 > 0$.

Now note that the Razumikhin condition implies that $|x(t-\tau)| \leq \beta_{11}^{-1}(pV_1(x_1))$ where β_{11} is the $\mathcal{K}_\infty$ function from the CLRF definition (in this case $\beta_{11}(s) = \frac{1}{2}s^2$). As g_1 is never equal to 0, its sign, denoted by $sign(g_1)$, is a constant. Moreover, there exists a class $\mathcal{K}$ function $q_1(.)$ such that

$$\begin{aligned} &|g_1(x, x(t-\tau))| \leq q_1\left(|[x_1, x(t-\tau)]|\right) \\ &\leq q_1(|[x_1, \beta_{11}^{-1}(pV_1(x_1))]|) = \eta_1(|x_1|) \end{aligned}$$

Note that the function η_1 "dominates" $|g_1|$ only when the Razumikhin condition holds, that is, when the past values of the state were closer (in a certain "Lyapunov" sense) to the origin then the present state. The advantage of the Razumikhin theory is that this is the only time the domination is needed.

To reduce the notational complexity we combine $\eta_1(|x_1|)$ with $\gamma_1(V_1)$. The function $\eta_1(|x_1|)$ is strictly positive so we choose a smooth function $\rho_1(.)$ such that $\rho_1(V_1(x_1)) > \gamma_1(V_1(x_1))\tilde{\eta}_1(|x_1|)$, with $\gamma_1(.)$ chosen in (4.3). Now choose the control

$$v_1 = -a_1(x) = -sign(g_1)\rho_1(V_1(x_1))x_1 \tag{4.4}$$

As this control law has the same sign and larger magnitude than $L_{g_1}V_1$, it also results in

$$\dot{V}_1 = L_{f_1}V_1 + L_{g_1}V_1 a_1 \leq -\mu_1(|x_1|)$$

and the closed loop system is globally asymptotically stable.

Note that the design freedom provided by the procedure is in the domination function $\gamma_1(\cdot)$. The designer may decide to use a large, but constant gain in the operating region, or use relatively small gain in the neighborhood of the origin (operating point) and much larger gain away form it. Even though the result of Theorem 1 guarantees asymptotic stability only if the domination function is sufficiently large, there is nothing in the derivation or construction that prevents designer from choosing $v_1 = 0$ if the x_1-subsystem (4.2) is open-loop stable. These points apply to the virtual and final control design in subsequent stages.

The next step is the standard change of coordinates $z_1 = x_1$, $z_1 = x_2 - a_1(x_1)$. Note that $sign(g_1)$ is a constant, and $\rho_1(\cdot)$ is a smooth function. In other words $a_1(\cdot)$ is smooth, and we can rewrite the system dynamics as

$$\begin{aligned} \dot{z}_1 &= f_1(z_{1d}) + g_1(z_{1d})a_1(z_1) + g_1(z_{1d})z_2 \\ \dot{z}_2 &= f_2(z_d) + g_2(z_d)u - \tfrac{\partial a_1}{\partial z_1}\dot{z}_1 \end{aligned} \tag{4.5}$$

Choose the CLRF candidate $V_2(z) = V_1(z_1) + \frac{1}{2}z_2^2$ and again $\pi(s) = p\ s$. The condition $L_gV = 0$ is in this case $z_2 = 0$, and the Razumikhin condition becomes

$$pV_1(z_1) \geq V_1(z_1(t+\theta)) + \frac{1}{2}z_2^2(t+\theta) \geq V_1(z_1(t+\theta))$$

which is, actually, the Razumikhin condition for the x_1-subsystem. Thus,

$$L_fV_2|_{z_2=0} = L_{f_1}V_1 + L_{f_1}V_1 a_1 \leq -\mu_1(|z_1|)$$

implies that V_2 is the CLRF for (4.5).

Next we need that the quadratic part of V_2, which is V_2 itself, be the CLRF for the Jacobian linearization of (4.5). This follows from our design in the first stage which guarantees that $\mu_1(|x_1|) \geq \nu_1 x_1^2$ in a neighborhood of $x_1 = 0$, because this can only be accomplished through linear terms in the x_1 subsystem. Theorem 1 directly implies that the feedback of the form

$$u = -\gamma_2(V_2)g_2(z, z(t-\tau))z_2$$

achieves GAS and that $\dot{V}_2 \leq -\mu_2(|z|)$ where $\mu_2(|z|) \geq \nu_2 z^T z$ for some $\nu_2 > 0$.

Finally, we replace $g_2(z_d)$ with a function of z only that dominates g_2:

$$\begin{aligned} &|g_2(z, z(t-\tau)| \leq q_2(|(z, z(t-\tau))|) \\ &\leq q_2(|(z, \beta_{12}^{-1}(pV_2(z))|) = \eta_2(|z|) \end{aligned}$$

We combine η_2 and γ_2 into the single dominating function ρ_2 to obtain

$$u = -sign(g_2)\rho_2(V_2(z))z_2 \tag{4.6}$$

which again achieves $\dot{V}_2 \leq -\mu_2(|z|)$ and GAS.

If there are more than two states, the design process continues in the obvious way and provides a globally asymptotically stabilizing feedback which is independent of the delayed state.

From the practical point of view, it makes sense to parameterize the domination functions ρ_i, which are the only free design variables, and tune the parameters for best transient response. For example, one may choose

$$\rho_i(V) = k_{i0} + k_{i1}\rho_{i1}(V)$$

where ρ_{i1} is a fixed function, and k_1, k_2, or both are adjusted in simulations or experimentally.

5. Conclusion

It is of theoretical and practical importance to design feedback laws for delay system that are independent of delayed state. In this paper we provide such a design for nonlinear delay systems that have a control Lyapunov-Razumikhin function. We extend this result to design a delay-state independent feedback for systems in the strict feedback form using a version of the well known backstepping design. The condition that allows delay-state independent feedback is that the components of the control vector field g don't change their signs.

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