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Observer Design for Discrete Time-Delay Singular Systems with Unknown Inputs

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***Abstract*— The observer design problem for linear discrete multiple delays rectangular singular systems with unknown inputs is discussed. Under a series of equivalent transformation, the problem is transformed into the observer design problem for standard state-space linear discrete systems with multiple delays. In terms of LMI, the sufficient condition for the existence of a full-order observer is derived, and the state and unknown inputs are all estimated very well. An numerical example is provided to show efficiency of the proposed method.**

I. INTRODUCTION

Recently, the observer design problem for linear systems with unknown inputs has been paid much attention. Since states and disturbance inputs can always not be measured, the research for this problem is important both in theoretical and practical. The observer design for linear singular systems, there is lots of excellent results [1-5], in [5], the state and unknown inputs are all estimated. The observer design for time-delay systems, there is some results [6,7], but the observer design for time-delay singular systems with unknown inputs, there is few result [8]. In [8], the multiple delays singular systems is enlarged singular systems with no delay, it leads to the dimensions of the observer is high, and the unknown inputs is not estimated in [8].

In this paper, the observer design problem for linear discrete multiple delays rectangular singular systems with unknown inputs is discussed. Under a series of equivalent transformation, the problem is transformed into the observer design problem for standard state-space linear discrete systems with multiple delays. In terms of LMI, the sufficient condition for the existence of a full-order observer is derived, dimensions of the observer are same as state dimensions of the singular system is given, and the state and unknown inputs are all estimated very well. An numerical example is provided to show efficiency of the proposed method.

II. DESCRIPTION OF PROBLEM

Consider the discrete time-delays singular system described by

$$\begin{cases} Ex(k+1)=\sum\limits_{i=0}^{d}A_ix(k-i)+\sum\limits_{j=0}^{s}B_ju(k-j)+Hf, \\ y(k)=\sum\limits_{i=0}^{h}C_ix(k-i)+\sum\limits_{j=0}^{g}D_ju(k-j)+Ff, \end{cases} \tag{1}$$

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where $x(k)\in R^n$ is the state variable, $u(k)\in R^p$ is the control input, $y(k)\in R^q$ is the output, $f\in R^t$ is the unknown disturbance input. The matrix $E\in R^{m\times n}$ is singular, rank$E=r<\min\{m,n\}$, d,s,h,g are positive integers, $A_i, i=0,1,\cdots,d$, $B_j, j=0,1,\cdots,s$, $C_i, i=0,1,\cdots,h$, $D_j, j=0,1,\cdots,g$, D, F are constant matrices. Without loss of generality, let $d=h$, $s=g$.

In this paper, the aim is to design a n dimensions observer as following

$$\begin{cases} z(k+1)=\sum\limits_{i=0}^{d}\tilde{A}_iz(k-i)+\sum\limits_{j=0}^{s}\tilde{B}_ju(k-j)+Jy(k), \\ \tilde{x}(k)=\sum\limits_{i=0}^{d}T_{1i}z(k-i)+\sum\limits_{j=0}^{s}S_{1j}u(k-j)+J_1y(k), \\ \tilde{f}=\sum\limits_{i=0}^{d}T_{2i}z(k-i)+\sum\limits_{j=0}^{s}S_{2j}u(k-j)+J_2y(k), \end{cases} \tag{2}$$

such that

$$\lim_{k\to\infty}(x(k)-\tilde{x}(k))=0,\quad \lim_{k\to\infty}(f-\tilde{f})=0, \tag{3}$$

with any admissible initial values.

To this end, the following assumptions are needed.

Assumption 1:

$$\text{rank}\begin{bmatrix} 0 & E \\ E & A_0 \\ 0 & C_0 \end{bmatrix}=n+r. \tag{4}$$

Assumption 2:

$$\text{rank}\begin{bmatrix} 0 & E & 0 \\ E & A_0 & H \\ 0 & C_0 & F \end{bmatrix}=n+r+t. \tag{5}$$

Remark 1: If the system (E,A) is regular, then Assumption 1 the definition of the system (E,A_0,C_0) being Y-observable [9], so Assumptions 1 is generalized for the system (1) and it is the necessary condition which guarantees the observer design problem is solvable.

III. OBSERVER DESIGN

Since rank$E=r$, there exist two nonsingular matrices $M\in R^{m\times m}$, $N\in R^{n\times n}$, such that

$$MEN=\begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \tag{6}$$

accordingly, denote

$$\left\{\begin{array}{l} MA_iN=\left[\begin{array}{cc} A_{i1} & A_{i2}\\ A_{i3} & A_{i4}\end{array}\right],\quad MB_j=\left[\begin{array}{c} B_{j1}\\ B_{j2}\end{array}\right],\\ MH=\left[\begin{array}{c} H_1\\ H_2\end{array}\right],\quad C_iN=[C_{i1}\ \ C_{i2}],\\ x(k)=N\left[\begin{array}{c} x_1(k)\\ x_2(k)\end{array}\right], i=0,1,\cdots,d,\ j=0,1,\cdots,s,\end{array}\right. \tag{7}$$

where $x_1(k)\in R^r,\ x_2(k)\in R^{n-r}$. So the system (1) is restricted system equivalent to the following system

$$\left\{\begin{array}{rl} x_1(k+1)= & \sum_{i=0}^{d}A_{i1}x_1(k-i)+\sum_{i=0}^{d}A_{i2}x_2(k-i)\\ & +\sum_{j=0}^{s}B_{j1}u(k-j)+H_1f,\\ 0= & \sum_{i=0}^{d}A_{i3}x_1(k-i)+\sum_{i=0}^{d}A_{i4}x_2(k-i)\\ & +\sum_{j=0}^{s}B_{j2}u(k-j)+H_2f,\\ y(k)= & \sum_{i=0}^{d}C_{i1}x_1(k-i)+\sum_{i=0}^{d}C_{i2}x_2(k-i)\\ & +\sum_{j=0}^{s}D_ju(k-j)+Ff.\end{array}\right. \tag{8}$$

It is easy to prove that Assumption 1 is equivalent to that the matrix $\left[\begin{array}{c} A_{04}\\ C_{02}\end{array}\right]$ has full column rank [9], so there exist nonsingular matrix $P\in R^{(m-r+q)\times(m-r+q)}$, such that

$$P\left[\begin{array}{c} A_{04}\\ C_{02}\end{array}\right]=\left[\begin{array}{c} I_{n-r}\\ 0\end{array}\right], \tag{9}$$

accordingly, denote

$$\left\{\begin{array}{ll} P\left[\begin{array}{c} A_{i3}\\ C_{i1}\end{array}\right]=\left[\begin{array}{c} \bar{A}_{i3}\\ \bar{C}_{i1}\end{array}\right], & i=0,1,\cdots,d,\\ P\left[\begin{array}{c} A_{i4}\\ C_{i2}\end{array}\right]=\left[\begin{array}{c} \bar{A}_{i4}\\ \bar{C}_{i2}\end{array}\right], & i=1,\cdots,d,\\ P\left[\begin{array}{c} B_{j2}\\ D_j\end{array}\right]=\left[\begin{array}{c} \bar{B}_{j2}\\ \bar{D}_j\end{array}\right], & j=0,1,\cdots,s,\\ P\left[\begin{array}{c} H_2\\ F\end{array}\right]=\left[\begin{array}{c} \bar{H}_2\\ \bar{F}\end{array}\right], & \\ P\left[\begin{array}{c} 0\\ y(k)\end{array}\right]=\left[\begin{array}{c} y_1(k)\\ y_2(k)\end{array}\right], & \end{array}\right. \tag{10}$$

let $P=\left[\begin{array}{cc} P_{11} & P_{12}\\ P_{21} & P_{22}\end{array}\right]$, $P_{11}\in R^{(n-r)\times(m-r)}$, $P_{12}\in R^{(n-r)\times q}$, $P_{21}\in R^{(m-n+q)\times(m-r)}$, $P_{22}\in R^{(m-n+q)\times q}$, then

$$y_1(k)=P_{12}y(k),\quad y_2(k)=P_{22}y(k). \tag{11}$$

By the transformation (9), (10), the system (8) is transformed into

$$\left\{\begin{array}{l} x_1(k+1)=\sum_{i=0}^{d}\bar{A}_{i1}x_1(k-i)\\ \qquad+\sum_{i=0}^{d-1}\bar{A}_{(i+1)2}x_2(k-1-i)\\ \qquad+\sum_{j=0}^{s}\bar{B}_{j1}u(k-j)+\bar{H}_1f+A_{02}y_1(k),\\ x_2(k)=-\sum_{i=0}^{d}\bar{A}_{i3}x_1(k-i)\\ \qquad-\sum_{i=0}^{d-1}\bar{A}_{(i+1)4}x_2(k-1-i)\\ \qquad-\sum_{j=0}^{s}\bar{B}_{j2}u(k-j)-\bar{H}_2f+y_1(k),\\ y_2(k)=\sum_{i=0}^{d}\bar{C}_{i1}x_1(k-i)+\sum_{i=0}^{d-1}\bar{C}_{(i+1)2}x_2(k-1-i)\\ \qquad+\sum_{j=0}^{s}\bar{D}_ju(k-j)+\bar{F}f,\end{array}\right. \tag{12}$$

where

$$\left\{\begin{array}{l} \bar{A}_{i1}=A_{i1}-A_{02}\bar{A}_{i3}, \bar{A}_{(i+1)2}=A_{(i+1)2}-A_{02}\bar{A}_{(i+1)4},\\ \bar{B}_{j1}=B_{j1}-A_{02}\bar{B}_{j2},\quad \bar{H}_1=H_1-A_{02}\bar{H}_2,\end{array}\right. \tag{13}$$

it is easy to prove that the expression of system (12) is independence of the choice of the matrix P.

Lemma 1: If Assumption 1 holds, then Assumption 2 holds if and only if

$$\mathrm{rank}\bar{F}=t(\text{full column rank}). \tag{14}$$

Proof: Since

$$\begin{array}{l} n+r+t=\mathrm{rank}\left[\begin{array}{ccc} 0 & E & 0\\ E & A_0 & H\\ 0 & C_0 & F\end{array}\right]\\ =\mathrm{rank}\left[\begin{array}{ccc} M & & \\ & M & \\ & & I_q\end{array}\right]\left[\begin{array}{ccc} 0 & E & 0\\ E & A_0 & H\\ 0 & C_0 & F\end{array}\right]\left[\begin{array}{ccc} N & & \\ & N & \\ & & I_t\end{array}\right]\\ =\mathrm{rank}\left[\begin{array}{ccccc} 0 & 0 & I_r & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ I_r & 0 & A_{01} & A_{02} & H_1\\ 0 & 0 & A_{03} & A_{04} & H_2\\ 0 & 0 & C_{01} & C_{02} & F\end{array}\right]\\ =\mathrm{rank}\left[\begin{array}{cc} A_{04} & H_2\\ C_{02} & F\end{array}\right]+2r=\mathrm{rank}P\left[\begin{array}{cc} A_{04} & H_2\\ C_{02} & F\end{array}\right]+2r\\ =\mathrm{rank}\left[\begin{array}{cc} I_{n-r} & \bar{H}_2\\ 0 & \bar{F}\end{array}\right]+2r=n+r+\mathrm{rank}\bar{F},\end{array}$$

the conclusion is obtained.

According to Lemma 1, from the third formula of the system (12), yields

$$\begin{array}{l} f=\bar{F}^{+}y_2(k)-\sum_{i=0}^{d}\bar{F}^{+}\bar{C}_{i1}x_1(k-i)\\ \quad-\sum_{i=0}^{d-1}\bar{F}^{+}\bar{C}_{(i+1)2}x_2(k-1-i)-\sum_{j=0}^{s}\bar{F}^{+}\bar{D}_ju(k-j),\end{array} \tag{15}$$

where

$$\bar{F}^{+}=(\bar{F}^T\bar{F})^{-1}\bar{F}^T \tag{16}$$

is the Penrose-Moore inverse of matrix $\bar{F}$. Let

$$\hat{y}(k) = (I_q - \bar{F}\bar{F}^{+})y_2(k), \tag{17}$$

the system (12) is transformed into

$$\begin{cases} x_1(k+1) = \sum_{i=0}^{d} \hat{A}_{i1}x_1(k-i) \\ \qquad + \sum_{i=0}^{d-1} \hat{A}_{(i+1)2}x_2(k-1-i) \\ \qquad + \sum_{j=0}^{s} \hat{B}_{j1}u(k-j) + G_1 y_(k), \\ x_2(k) = \sum_{i=0}^{d} \hat{A}_{i3}x_1(k-i) \\ \qquad + \sum_{i=0}^{d-1} \hat{A}_{(i+1)4}x_2(k-1-i) \\ \qquad + \sum_{j=0}^{s} \hat{B}_{j2}u(k-j) + G_2 y_(k), \\ \hat{y}(k) = \sum_{i=0}^{d} \hat{C}_{i1}x_1(k-i) \\ \qquad + \sum_{i=0}^{d-1} \hat{C}_{(i+1)2}x_2(k-1-i) \\ \qquad + \sum_{j=0}^{s} \hat{D}_j u(k-j), \end{cases} \tag{18a}$$

where

$$\begin{cases} \hat{A}_{i1} = \bar{A}_{i1} - \bar{H}_1\bar{F}^{+}\bar{C}_{i1},\ \hat{A}_{i3} = -\bar{A}_{i3} + \bar{H}_2\bar{F}^{+}\bar{C}_{i1}, \\ \hat{A}_{(i+1)2} = \bar{A}_{(i+1)2} - \bar{H}_1\bar{F}^{+}\bar{C}_{(i+1)2}, \\ \hat{A}_{(i+1)4} = -\bar{A}_{(i+1)4} + \bar{H}_2\bar{F}^{+}\bar{C}_{(i+1)2}, \\ \hat{B}_{j1} = \bar{B}_{j1} - \bar{H}_1\bar{F}^{+}\bar{D}_j,\ \hat{B}_{j2} = -\bar{B}_{j2} + \bar{H}_2\bar{F}^{+}\bar{D}_j, \\ \hat{C}_{(i+1)2} = (I_q - \bar{F}\bar{F}^{+})\bar{C}_{(i+1)2}, \\ \hat{C}_{i1} = (I_q - \bar{F}\bar{F}^{+})\bar{C}_{i1},\ \hat{D}_j = (I_q - \bar{F}\bar{F}^{+})\bar{D}_j, \\ G_1 = A_{02}P_{12} + \bar{H}_1\bar{F}^{+}P_{22},\ G_2 = P_{12} - \bar{H}_2\bar{F}^{+}P_{22}, \end{cases} \tag{18b}$$

denote by

$$\begin{cases} \hat{x}(k+1) = \begin{bmatrix} x_1(k+1) \\ x_2(k) \end{bmatrix}, \hat{A}_i = \begin{bmatrix} \hat{A}_{i1} & \hat{A}_{(i+1)2} \\ \hat{A}_{i3} & \hat{A}_{(i+1)4} \end{bmatrix}, \\ \hat{A}_d = \begin{bmatrix} \hat{A}_{d1} & 0 \\ \hat{A}_{d3} & 0 \end{bmatrix},\ \hat{B}_j = \begin{bmatrix} \hat{B}_{j1} \\ \hat{B}_{j2} \end{bmatrix}, \\ \hat{C}_i = \begin{bmatrix} \hat{C}_{i1} & \hat{C}_{(i+1)2} \end{bmatrix},\ \hat{C}_d = \begin{bmatrix} \hat{C}_{d1} & 0 \end{bmatrix}, \\ G = \begin{bmatrix} G_1 \\ G_2 \end{bmatrix},\ i = 0, 1, \cdots, d-1,\ j = 0, 1, \cdots, s, \end{cases} \tag{19}$$

and rewrite the system (18a) as

$$\begin{cases} \hat{x}(k+1) = \sum_{i=0}^{d} \hat{A}_i\hat{x}(k-i) + \sum_{j=0}^{s} \hat{B}_j u(k-j) + Gy(k), \\ \hat{y}(k) = \sum_{i=0}^{d} \hat{C}_i\hat{x}(k-i) + \sum_{j=0}^{s} \hat{D}_j u(k-j). \end{cases} \tag{20}$$

Remark 2: Under the transformations (9), (10), (15), (17), the system (8) is transformed into the system (20), the system (8) is a singular system with delays and unknown inputs, but the system (20) is a standard state-space linear system with delays. Notice that the transformation (9), (10) is only equivalent transformation for the output formula of system (8), so if the state of the system (20) is estimated , then the state of the system (8) can be estimated also.

Design the state observer with n dimensions for the system (20)

$$z(k+1) = \sum_{i=0}^{d} \hat{A}_i z(k-i) + \sum_{j=0}^{s} \hat{B}_j u(k-j) + Gy(k) \\ + K(\hat{y}(k) - \sum_{i=0}^{d} \hat{C}_i z(k-i) - \sum_{j=0}^{s} \hat{D}_j u(k-j)), \tag{21}$$

Theorem 1: If there exist matrices $P_i > 0$, $i = 0, 1, \cdots, d$ and W such that following LMI hold,

$$\begin{bmatrix} -P_0 & \Phi_1 \\ \Phi_1^T & \Theta \end{bmatrix} < 0, \tag{22a}$$

where

$$\begin{cases} \Phi_1 = \begin{bmatrix} \Phi_{10} & \Phi_{11} & \cdots & \Phi_{1(d-1)} & \Phi_{1d} \end{bmatrix}, \\ \Phi_{1i} = P_0\hat{A}_i - W\hat{C}_i,\ i = 0, 1, \cdots, d, \\ \Theta = \mathrm{diag}\{P_1 - P_0, P_2 - P_1, \cdots, P_d - P_{d-1}, -P_d\}, \end{cases} \tag{22b}$$

then the observer of form (21) for the system (20) exists, and the matrix

$$K = P_0^{-1}W. \tag{23}$$

In order to prove Theorem 1, the following lemma is useful.

Lemma 2 [10]: The discrete system $x(k+1) = Ax(k)$ is stable, if and only if there exists matrix $P > 0$ such that

$$A^T PA - P < 0.$$

Proof of Theorem 1: Let

$$e(k+1) = \hat{x}(k+1) - z(k+1), \tag{24}$$

then by (20), (21), it is obtained that

$$e(k+1) = \sum_{i=0}^{d} (\hat{A}_i - K\hat{C}_i)e(k-i), \tag{25}$$

by introducing new state

$$\hat{e}(k) = \begin{bmatrix} e^T(k) & e^T(k-1) & \cdots & e^T(k-d) \end{bmatrix}^T, \tag{26}$$

(25) takes the form

$$\hat{e}(k+1) = A_e\hat{e}(k), \tag{27}$$

where

$$\begin{cases} A_e = \begin{bmatrix} A_{e0} & A_{e1} & \cdots & A_{e(d-1)} & A_{ed} \\ I_n & 0 & \cdots & 0 & 0 \\ 0 & I_n & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & I_n & 0 \end{bmatrix}, \\ A_{ei} = \hat{A}_i - K\hat{C}_i,\ i = 0, 1, \cdots, d. \end{cases} \tag{28}$$

Next, Prove the stability of the system (27). Since LMI (22) holds with matrices W and $P_i > 0$, $i = 0, 1, \cdots, d$, and (23) holds, the LMI (22) is equivalent to

$$\begin{bmatrix} -P & \Phi_2 \\ \Phi_2^T & \Theta \end{bmatrix} < 0, \tag{29a}$$

where

$$\begin{cases} P = \mathrm{diag}\{P_0, P_1, \cdots, P_d\}, \\ \Theta = \mathrm{diag}\{-P_0 + P_1, \cdots, -P_{d-1} + P_d, -P_d\}, \\ \Phi_2 = \begin{bmatrix} P_0 A_{e0} & \cdots & P_0 A_{e(d-1)} & P_0 A_{ed} \\ 0 & \cdots & 0 & 0 \\ \vdots & \cdots & \vdots & \vdots \\ 0 & \cdots & 0 & 0 \end{bmatrix}. \end{cases} \tag{29b}$$

Take the matrix with $(d+1)n \times (d+1)n$ dimensions as

$$\bar{T} = \begin{bmatrix} 0 & -I_n & \cdots & 0 \\ & \ddots & \ddots & \vdots \\ & & \ddots & -I_n \\ & & & 0 \end{bmatrix}, \tag{30}$$

$$T = \begin{bmatrix} I_{(d+1)n} & 0 \\ \bar{T} & I_{(d+1)n} \end{bmatrix},$$

Pre- and postmultiply the inequality (29a) by T and T^T, respectively, yields

$$\begin{bmatrix} -P & \Phi_3 \\ \Phi_3^T & -P \end{bmatrix} < 0, \tag{31a}$$

where

$$\Phi_3 = \begin{bmatrix} P_0 A_{e0} & P_1 A_{e1} & \cdots & P_0 A_{e(d-1)} & P_0 A_{ed} \\ P_1 & 0 & \cdots & 0 & 0 \\ 0 & P_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & P_d & 0 \end{bmatrix}. \tag{31b}$$

The inequality (31a) is the inequality

$$\begin{bmatrix} -P & PA_e \\ A_e^T P & -P \end{bmatrix} < 0, \tag{32}$$

(32) is equivalent to

$$A_e^T P A_e - P < 0, \quad P > 0. \tag{33}$$

From Lemma 2, the system (27) is stable, then the system (25) is stable. This completes the proof.

In the following, we consider the observer for the system (1).

Theorem 2: If there exist matrices $P_i > 0, i = 0, 1, \cdots, d$ and W, such that LMI (22) holds, then the observer of form (2) for the system (1) exists, and

$$\begin{cases} \tilde{A}_i = \hat{A}_i - K\hat{C}_i, \ i = 0, 1, \cdots, d, \ \tilde{B}_j = \hat{B}_j - K\hat{D}_j, \\ J = G + K(I_q - \bar{F}\bar{F}^+)P_{22}, T_{10} = N \begin{bmatrix} I_r & 0 \\ \hat{A}_{03} & \hat{A}_{14} \end{bmatrix}, \\ T_{1d} = N \begin{bmatrix} 0 & 0 \\ \hat{A}_{d3} & 0 \end{bmatrix}, \ S_{1j} = N \begin{bmatrix} 0 \\ \hat{B}_{j2} \end{bmatrix}, \\ T_{1i} = N \begin{bmatrix} 0 & 0 \\ \hat{A}_{i3} & \hat{A}_{(i+1)4} \end{bmatrix}, \ i = 1, \cdots, d-1, \\ J_1 = \begin{bmatrix} 0 \\ G_2 \end{bmatrix}, \ T_{2d} = -\begin{bmatrix} \bar{F}^+\bar{C}_{d1} & 0 \end{bmatrix}, \\ T_{2i} = -\begin{bmatrix} \bar{F}^+\bar{C}_{i1} & \bar{F}^+\bar{C}_{(i+1)2} \end{bmatrix}, \ i = 0, 1, \cdots, d-1, \\ S_{2j} = \bar{F}^+\bar{D}_j, J_2 = \bar{F}^+P_{22}, \ j = 0, 1, \cdots, s. \end{cases} \tag{34}$$

Proof: According to Theorem 1 and formulas (27), (15), (18a) and transformation (7), it is easy to know that

$$\lim_{k\to\infty}(x(k) - \tilde{x}(k)) = 0, \quad \lim_{k\to\infty}(f - \tilde{f}) = 0.$$

this completes the proof.

IV. EXAMPLE

Consider the following time-delay discrete-time singular system Σ

$$\begin{cases} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} \\ \quad + \begin{bmatrix} 0.5 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k-2) \\ x_2(k-2) \end{bmatrix} \\ \quad + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u(k) + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} f, \\ y(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + f, \end{cases}$$

the system Σ satisfies Assumptions 1, 2. Take the matrix $P = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & 0 & 0 \end{bmatrix}$, by (9), (10), (14), (15), (20) yields

$$f = -0.5(x_1(k) + x_1(k-2) - x_2(k-2) - u(k) - y(k)),$$

$$\begin{cases} \begin{bmatrix} x_1(k+1) \\ x_2(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.5 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k-1) \end{bmatrix} \\ \quad + \begin{bmatrix} 0 & 1 \\ 0 & -0.5 \end{bmatrix} \begin{bmatrix} x_1(k-1) \\ x_2(k-2) \end{bmatrix} \\ \quad + \begin{bmatrix} 0.5 & 0 \\ 0.5 & 0 \end{bmatrix} \begin{bmatrix} x_1(k-2) \\ x_2(k-3) \end{bmatrix} \\ \quad + \begin{bmatrix} 1 \\ -0.5 \end{bmatrix} u(k) + \begin{bmatrix} 1 \\ 0.5 \end{bmatrix} y(k), \\ \hat{y}(k) = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k-1) \end{bmatrix} \\ \quad + \begin{bmatrix} 0 & 0.5 \\ 0 & 0.5 \end{bmatrix} \begin{bmatrix} x_1(k-1) \\ x_2(k-2) \end{bmatrix} \\ \quad + \begin{bmatrix} 0.5 & 0 \\ 0.5 & 0 \end{bmatrix} \begin{bmatrix} x_1(k-2) \\ x_2(k-3) \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} u(k). \end{cases}$$

Solve LMI (22), it is obtained that

$$P_0 = \begin{bmatrix} 4.4612 & 0.5939 \\ 0.5939 & 2.0465 \end{bmatrix}, \ P_1 = \begin{bmatrix} 2.7646 & 0.3882 \\ 0.3882 & 1.7668 \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 1.9920 & 0.2154 \\ 0.2154 & 0.4666 \end{bmatrix},$$

$$W = \begin{bmatrix} 413.8070 & -405.9186 \\ 109.3318 & -107.9139 \end{bmatrix},$$

so $K = \begin{bmatrix} 89.0860 & -87.3427 \\ 27.5713 & -27.3844 \end{bmatrix}$, by (34), (2), it is obtained that the observer of the system Σ is

$$\begin{cases} z(k+1) = \begin{bmatrix} 0.1284 & 0 \\ 0.4066 & 0 \end{bmatrix} z(k) \\ \quad + \begin{bmatrix} 0 & 0.1284 \\ 0 & -0.5935 \end{bmatrix} z(k-1) \\ \quad + \begin{bmatrix} -0.3717 & 0 \\ 0.4066 & 0 \end{bmatrix} z(k-2) \\ \quad + \begin{bmatrix} 0.1284 \\ -0.5935 \end{bmatrix} u(k) + \begin{bmatrix} 0.1284 \\ 0.4066 \end{bmatrix} y(k), \\ \begin{bmatrix} \tilde{x}_1(k) \\ \tilde{x}_2(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.5 & 0 \end{bmatrix} z(k) \\ \quad + \begin{bmatrix} 0 & 0 \\ 0 & -0.5 \end{bmatrix} z(k-1) \\ \quad + \begin{bmatrix} 0 & 0 \\ 0.5 & 0 \end{bmatrix} z(k-2) \\ \quad + \begin{bmatrix} 0 \\ -0.5 \end{bmatrix} u(k) + \begin{bmatrix} 0 \\ 0.5 \end{bmatrix} y(k), \\ \tilde{f} = \begin{bmatrix} -0.5 & 0 \end{bmatrix} z(k) + \begin{bmatrix} 0 & 0.5 \end{bmatrix} z(k-1) \\ \quad + \begin{bmatrix} -0.5 & 0 \end{bmatrix} z(k-2) + 0.5u(k) + 0.5y(k). \end{cases}$$

V. CONCLUSIONS

In this paper, the observer design problem for linear discrete multiple delays rectangular singular systems with unknown inputs is discussed. It is pointed out that the problem can be transformed into the observer design problem for standard state-space linear discrete systems with multiple delays by discussion. In terms of LMI, the sufficient condition for the existence of a full-order observer is derived, dimensions of the observer are same as state dimensions of the singular system is given, and the state and unknown inputs are all estimated very well. An numerical example is provided to show efficiency of the proposed method. The observer given in this paper is a nonsingular system, and observer dimensions is lower than observer dimensions of the enlarged singular systems with no delay based on the given discrete multiple delays singular systems , the LMI can be solved by Matlab.

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