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Modeling and Analysis of Steady-State Torque Characteristics for a Miniature Electromagnetic Retarder

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Abstract—**An exponential type function has been utilized to model the torque vs. speed characteristics of a miniature electromagnetic retarder. The model is based on the results from a detailed electromagnetic analysis of the eddy current machines with rotor thickness as a design variable. The model parameters are then adjusted for optimal retarding performance. Model of the electromagnetic retarder with an inertia load and adjusted parameters has been used to simulate a series of torque curves with peaks shifting over a range of rotor speeds. The simulation results show that the peaks of the torque curves should be designed to occur within a specific range of speeds for optimal retarding performance**.

I. Introduction

EDDY current machines have long been used in retarding the motion of a number of devices including automobiles, dump trucks, and so on. Modeling of an eddy current retarder has been the subject of a few publications for last couple of decades. Simeu and Georges [2] reported the modeling and control of a class of eddy current brakes which assumes that the braking force or torque varies linearly with speed. A theoretical polynomial-n-control and state-affine model for an eddy current brake system was presented in the paper. This model was based on Wouterse's [5] experimental results. Lee and Park [3] presented a paper on the optimal robust control of an eddy current brake system. Ryoo et al [4] presented a design and analysis of an eddy current brake for a high-speed railway train with constant torque control. Anwar [1] proposed a parametric model for an eddy current retarder for automotive braking applications. This model included a quadratic term in speed as well.

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In this paper we discuss the modeling and analysis of the steady state torque characteristics of a miniature eddy current retarder for possible application in retarding the motion of small motors, used in disc drives for example. Eddy current machines generally provide non-contact braking of any moving part thus extending the overall device life. The proposed model is based on a detailed electromagnetic analysis by Rastogi [7] and Stevenson & Li [8] of an eddy current device. The results of this analysis indicate that an exponential type model of the steady-state torque characteristics would be more accurate. Simulation results are presented based on the proposed exponential model for a number of model parameter variations in order to obtain an optimal range of torque peak points of the torque vs. speed curves.

II. Modeling of a Miniature Eddy Current Retarder

Electromagnetic retarders including miniature eddy current machines (MECM) follow the basic principles of electromagnetic induction. For one type of retarder topology, an eddy current machine has an iron core, which is a field-wound stator. The stator windings induce currents in a rotor element, which is typically a featureless metal ring. A torque is generated according to the Lorentz equation [6]. That the torque is retarding, and does not average to zero, is due to the fact that the induced eddy currents generate power loss through Joule heating. Simeu and Georges [2] proposed an eddy current brake model, assuming that the torque was a function of excitation current and the rotor speed. However, the assumption that the retarding force or torque is a linear function of the rotor speed results in a less accurate eddy current machine model, according Anwar [1]. As explained in references [1], [7], & [8], the departure from the linear behavior for the braking torque with respect to angular speed can be attributed to the eddy current induced magnetic flux which causes the reduction of the net magnetic flux passing through the pole projection area (when temperature is held constant). The induced magnetic flux is smaller than the magnetic flux generated by the electromagnet in the low angular speed range. As a result, the braking torque increases linearly with angular speed in this speed range. However, as the angular speed increases, the induced magnetic flux increases, which causes the net magnetic flux to decrease with speed. As a result, the rate of braking torque increase does not keep pace with the rate of increase in the angular speed. This torque characteristics, in turn, results in the nonlinear braking torque vs. angular speed relationship. At micro-scale level, the above analysis still hold true. An investigation of the macro-level machines' torque characteristics [7, 8] show that the torque produced can be modeled as a double-exponential function. In this paper, we propose a similar model with necessary modification of the model parameter characteristics. In a typical micro-scale braking application, the intermittent usage of the eddy current brake would not cause a sustained temperature rise. Hence, it is assumed that the effect of temperature on retarder torque characteristics is ignorable.

In developing a model for a miniature eddy current machine to capture the steady state torque-speed characteristics, it was observed in the electro-magnetic analysis [7, 8] that the eddy current retarder torque peaks at a particular rotor speed (Figure 1) based on a number of design parameters (size, rotor thickness, etc.). It can be readily observed that the torque peaks for these curves can be shifted over the rotor speed axis by varying the rotor thickness. The torque peak shifts to the left as the rotor thickness increases. It then becomes very important to design the torque-speed characteristics (via optimal selection of design parameters such as rotor thickness) such that the measure of retarding performance (e.g. stopping time / travel) is optimized. Therefore it is desirable that the peak torque RPM (rotor speed at which the steady state MECM torque peaks out) be optimized based on the retarding performance measure.

According to the above observation, we studied a number of parametric functions to model the torque-speed relationship of such a miniature eddy current machine. The result was a double-exponential function with three parameters dependent on the design variables (e.g. rotor thickness) and excitation current.

Hence, we propose a double-exponential function $T(\omega)$ to model the torque-speed characteristics for a class of miniature eddy current machines. The following equation describes the model:

$$T(\omega) = \gamma(e^{-\beta\omega} - e^{-\alpha\omega}) \qquad (1)$$

where T(ω) describes the MECM steady state torque characteristics., N-mm, ω = Rotor Speed, RPM, and α, β = shape parameters, and γ = parameter associated with excitation current.

Since the retarder torque peaks at a particular rotational speed ω_P, the specification of ω_P for control should be based on the optimization of a performance objective. For our analysis, we choose to minimize stopping travel.

$T(\omega)$ captures the torque saturation characteristics of the retarder reasonably well for a wide speed ranges. At very low speeds, the accuracy of torque estimation for the retarder is somewhat less than that at higher speeds. $T(\omega)$ represents the steady state relationship between MECM torque and rotor speed with variations in shape and excitation current parameters. It is assumed here that the torque response with respect to the feedback current is instantaneous.

It is very important to ensure the fastest possible torque response from the eddy current machines. A simple example of the controller that will ensure fast torque response for such a brake system is an open loop control strategy that derives the current command from equation (1). The equation captures the retarder torque characteristics as a function of rotor speed and excitation current parameter (a function of γ). Assuming that the measured current embedded in γ is same as the commanded current (this assumption is good for the relatively short time constants of the present design), one may solve the exponential equation for the commanded current, given a desired torque command. This scheme will represent an open loop control strategy for the eddy current retarder to produce the desired retarding torque for a disk drive, for example.

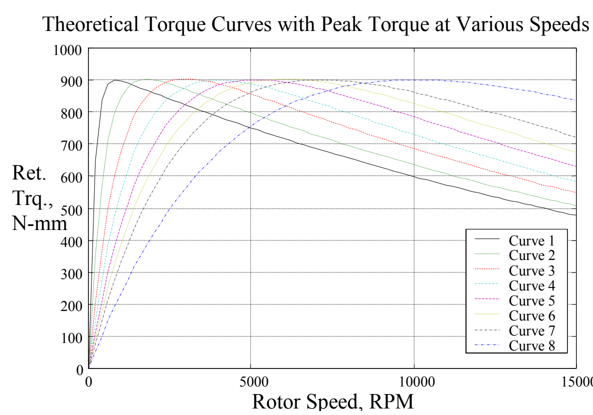

Figure 1: Theoretical Torque Curves for a Miniature Eddy Current Machine (MECM).

III. Simulation Results

The torque-speed curves based on the model presented in section II were plotted for a number of parameter values shown in table 1. These torque-speed curves are shown in figure 1. The shape of these curves is very similar to that obtained by Rastogi [7] and Stevenson & Li [8].

We now present simulation results for the a particular application where the MECM torque model was incorporated in a rotating machine model with inertia, mass, and friction characteristics that are similar to the characteristics of a disc drive system. When the eddy current machine is energized with excitation current input, the retarding torque follows the output of the model. This retarding torque is then used to compute the travel for the disc to come to a full stop. It is assumed that the drive motor does not provide any torque during this time. However, a viscous friction type damping was included to model the bearing friction. The mass moment of inertia and the damping coefficient of the disc drive system were assumed to be 4.6E-04 Kg-m^2 and 0.075 N-m-sec/rad respectively. Simulation of the combined disc drive model with the eddy current retarder model was performed as follows:

1. The disc was sped up to a speed of 10000 RPM.
2. The MECM was energized from zero to maximum current in 1 millisecond.
3. The following outputs from the model were recorded: travel for the stop, disc speed, disc deceleration, and MECM torque.

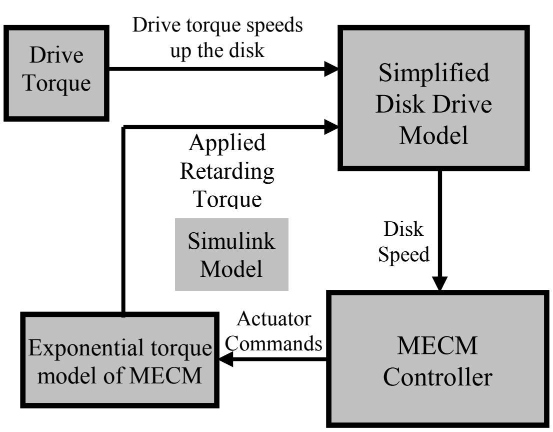

Figure 2 Block diagram representation of the simulation model.

A block diagram representation of the simulation model is shown in figure 2. MATLAB/SIMULIN^{TM} from MathWorks, Inc. was used for simulation the MECM system. Simulation results presented some interesting features.

Table 1 also shows the travel for the stop corresponding to each torque curve in figure 1. The stop travel versus peak torque RPM (i.e. rotor RPM at which retarder produces max torque) plot is shown in figure 3. It is clear from figure 3 that the optimum peak torque RPM for the retarder lies in the range of 3000-4000 RPM.

Table 1: MECM model parameter values for different peak torque RPMs and corresponding stopping travels.

Curve No.	Parameter α, (RPM)$^{-1}$	Parameter β, (RPM)$^{-1}$	Parameter γ, N-mm	Rotor RPM at Peak Torque	Stopping Travel, deg
1	0.005972	0.00004515	940.99	1000	58.95
2	0.0022395	0.00004515	1004.11	2000	58.33
3	0.00111975	0.00004515	1090.17	3000	57.73
4	0.0007465	0.00004515	1147.55	4000	57.72
5	0.00052255	0.00004515	1243.94	5000	57.77
6	0.000410575	0.00004515	1334.37	6000	57.995
7	0.000320995	0.00004515	1445.91	7500	58.45
8	0.000186625	0.00004515	1870.51	10000	59.91

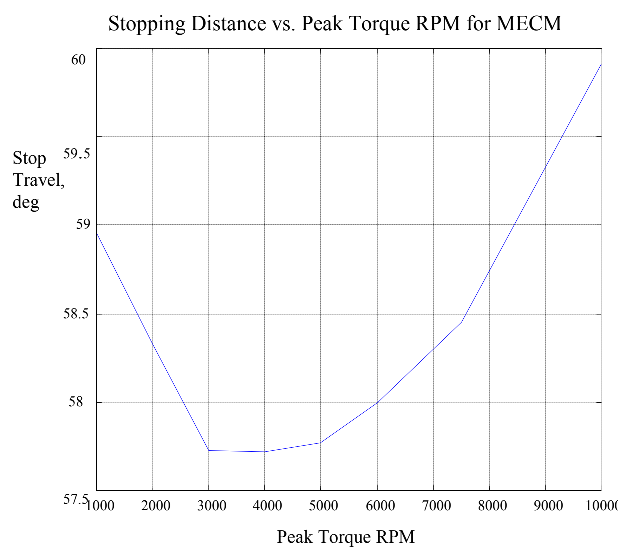

Figure 3 Stop Travel vs. Peak torque RPM the MECM.

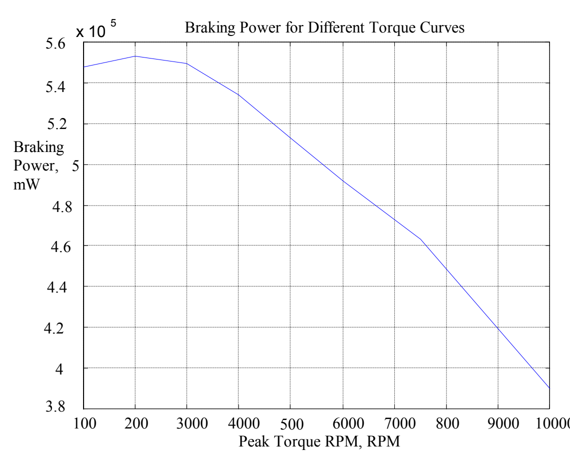

Figure 4: Braking power for different torque curves in Figure 1.

$$P = \int_0^{\omega_{\max}} T(\omega)d\omega = \int_0^{\omega_{\max}} \gamma(e^{-\beta\omega} - e^{-\alpha\omega})d\omega \qquad (2)$$

$$P = \gamma\left\{\frac{e^{-\alpha\omega_{\max}} - 1}{\alpha} - \frac{e^{-\beta\omega_{\max}} - 1}{\beta}\right\}$$

The optimum range of the peak torque RPM is explained as follows. The disc speed at the start of braking is 10000

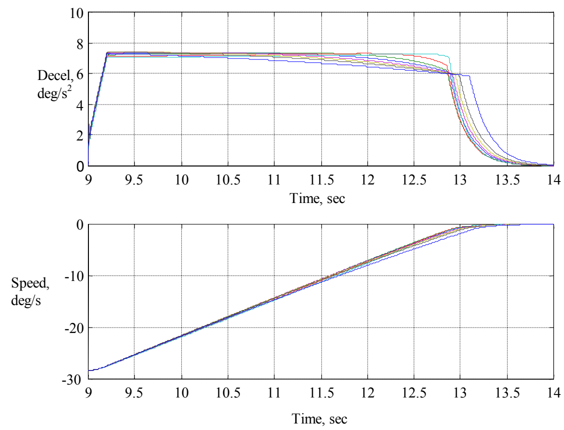

Figure 5 Rotor deceleration and speed corresponding to torque curves in figure 1.

RPM. Now the area under the torque curves from rotor speed 0 to 10000 RPM in figure 1 would provide the total braking power applied by the MECM. The braking power can be approximated by using equation (2) above. The approximate braking power P for different torque curves shown in figure 1 is plotted in figure 4. It is reading observed from figure 4 that the braking power is maximized within a peak torque RPM range of 2000-3000 RPM.

Figures 5 and 6 show the plots of rotor deceleration, speed, and braking torque for each torque curve in figure 1.

IV. Conclusions

An exponential model describing the steady state torque characteristics of a miniature eddy current machine has been presented in this paper. The model is based on a detailed electromagnetic analysis of an eddy current machine. It has been observed that the speed value at the peak torque point must be designed in a specific way in order to obtain optimal retarding performance from these eddy current machines.

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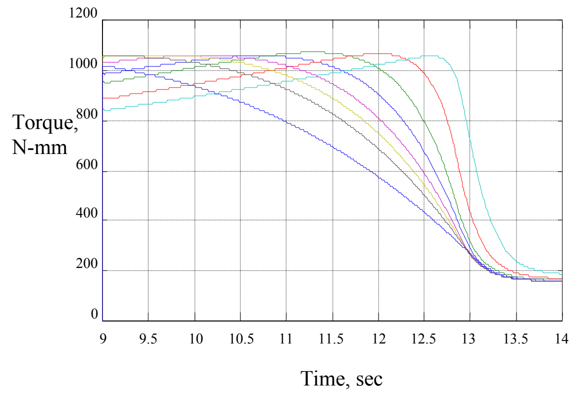

Figure 6: MECM torques corresponding to the torque curves in figure 1.

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