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NEW STABILITY CONDITIONS FOR DISCRETE POLYNOMIALS

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Abstract: The geometry of stable discrete polynomials using their coefficients and reflection coefficients is investigated. Some simple necessary stability conditions in terms of unions of polytopes are obtained by splitting the unit hypercube of reflection coefficients. Sufficient stability conditions in terms of linear covers of reflection vectors of stable polynomials is proposed.

Keywords: discrete-time systems, stability, polynomials

I. Introduction

The stability of systems is a critical design criterion and can be investigated by root placement of the characteristic polynomial $a(z)$. For a given polynomial $a(z)$ many tests may be used to check its stability. In the case of a family of polynomials however, these tests require the testing of a set of inequalities [1]. The problem was elegantly solved in the continuous-time case by the celebrated Kharitonov's theorem [2]. To date such a solution does not exist for the discrete-time case, although partial results are available for special cases.

In this paper we investigate the geometry of stable discrete polynomials using their coefficients and their reflection coefficients . It is well known that in parameter space the set of stable polynomials is not convex. The simplex generated by vertices of the stability region is called the barycentric stability simplex [3], i.e. the barycentric simplex gives us a simple but very conservative necessary stability condition. Our aim is to improve this necessary stability condition by the use of reflection coefficients.

A simple but very conservative sufficient stability condition is given by Cohn [3]. Making use of so-called reflection vectors [4] we improve this sufficient stability condition considerably .

The paper is organized as follows. First, a characterization of the stability region will be given by reflection coefficients. Second, to find a less conservative necessary stability condition we split the reflection coefficient stability hypercube into several hyperrectangles. Third, some sufficient stability conditions are obtained via linear covers of reflection vectors of special stable polynomials.

II. Stability region and reflection coefficients

The problem of checking the stability of a linear discrete-time system reduces to the determination of weather or not the roots of the characteristic polynomial of the system lie inside the unit circle or not. A polynomial $a(z)$ of degree n with real coefficients $a_i \in \mathcal{R}$, $i = 0, ..., n$

$$a(z) = a_n z^n + ... + a_1 z + a_0$$

is said to be Schur stable if all its roots are placed inside the unit circle.

Besides the unit circle criterion some other criteria are known for checking the stability of a linear discrete-time system. It is interesting to mention that the well-known Jury's stability test leads precisely to the stability hypercube of reflection coefficients .

Let us recall the recursive definition of reflection coefficients $k_i \in \mathcal{R}$ of a polynomial $a(z)$ [5]:

$$k_i = -a_i^{(i)}, \tag{1}$$

$$a_i^{(n)} = \frac{a_{n-i}}{a_n}, \qquad i = 1, ..., n; \tag{2}$$

$$a_j^{(i-1)} = \frac{a_j^{(i)} + k_i a_{i-j}^{(i)}}{1 - k_i^2}, j = 1, ..., i-1. \tag{3}$$

The stability criterion via reflection coefficient is as follows [5]:

a polynomial $a(z)$ will be Schur stable if and only if its reflection coefficients $k_i, i = 1, ..., n$ lie within the interval $-1 < k_i < 1$.

A polynomial $a(z)$ lies on the stability boundary if some $k_i = \pm 1, i = 1, ..., n$. For monic Schur polynomials, $a_n = 1$, there is a one-to-one correspondence between the vectors $a = (a_0, ..., a_{n-1})^T$ and $k = (k_1, ..., k_n)^T$.

The transformation from reflection coefficients k_i to polynomial coefficients $a_{i-1}, i = 1, ..., n$ is multilinear. For monic polynomials we obtain from (1)-(3)

$$\begin{array}{rl} a_i = & a_{n-i}^{(n)}, \\ a_i^{(i)} = & -k_i, \\ a_j^{(i)} = & a_j^{(i-1)} - k_i a_{i-j}^{(i-1)}, \\ & i = 1, ..., n; j = 1, ..., i-1 \end{array} \tag{4}$$

or in the matrix form

$$a(k) = rR_n(k_n) \begin{bmatrix} 0^T \\ R_{n-1}(k_{n-1}) \end{bmatrix} ... \\ ... \begin{bmatrix} 0^T \\ R_1(k_1) \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{5}$$

where

$$R_j(k_j) = I_{j+1} + k_j E_{j+1},$$

I_n is a $n \times n$ unit matrix , E_n is a unit Hankel matrix $E_n = \begin{bmatrix} 0 & ... & 1 \\ . & . & . \\ 1 & ... & 0 \end{bmatrix}$ and 0^T is a row vector of zeros.

Because of the multilinearity of relations (4) the following lemma holds [6].

Lemma 1. Through an arbitrary stable point $a = [a_0, a_1, ..., a_{n-1}]$ with reflection coefficients $k_i^a \in (-1, 1)$, $i = 1, ..., n$ you can draw n stable line segments

$$a^i(\pm 1) = conv\{a | k_i^a = \pm 1\}$$

where $conv\{a | k_i^a = \pm 1\}$ denotes the convex hull obtained by varying the reflection coefficient k_i^a between -1 and 1.

The reflection vectors are defined as follows [4]:

$$a^i(1) = (a | k_i = 1), i = 1, ..., n$$

are called positive reflection vectors and

$$a^i(-1) = (a | k_i = -1), i = 1, ..., n$$

are called negative reflection vectors of a monic polynomial $a(z)$.

It means, reflection vectors are the extreme points of the Schur stable line segment $a^i(\pm 1)$ through the point a defined by Lemma 1. The following assertions hold:

1) every Schur polynomial has $2n$ reflection vectors $a^i(1)$ and $a^i(-1)$, $i = 1, ..., n$;

2) all the reflection vectors lie on the stability boundary ($k_i = \pm 1$);

3) the line segments between reflection vectors $a^i(1)$ and $a^i(-1)$ are Schur stable.

III. NECESSARY STABILITY CONDITIONS

A polynomial $a(z)$ can be expressed in the basis of vectors

$$B_i(z) = (z+1)^i (z-1)^{n-i}, \qquad i = 0, ..., n$$

as follows

$$a(z) = b_0 B_n(z) + b_1 B_{n-1}(z) + ... + b_n B_0(z)$$

where $b_0, b_1, ..., b_n$ are called the barycentric coordinates of the polynomial $a(z)$.

Let us start from a simple but very conservative necessary stability condition given by barycentric stability simplex $conv\{B_0, ..., B_n\}$ where B_i are the vectors of coefficients of the polynomials $B_i(z)$ [3]:
all the barycentric coordinates of a stable polynomial $a(z)$ must be positive

$$b_j(a) > 0, \quad j = 0, ..., n$$

or the stability region $\mathcal{A}$ is contained in the barycentric simplex

$$a \in \mathcal{A} \subset conv\{B_0, ..., B_n\}.$$

It is easy to show that the set of vertex polynomials $\mathcal{K}^\nu$ of the hypercube $\mathcal{K} = \{k_i \in (-1, 1), i = 1, ..., n\}$ will be transformed by mapping ϕ (4) to the vertices of the barycentric stability simplex $\phi(\mathcal{K}^\nu) = \{B_0, ..., B_n\}$. Because the transformation (4) is multilinear we can find the convex hull of the set of of polynomial coefficients $\mathcal{A}$ by Mapping Theorem [7] as a simple polygon whose vertices $\mathcal{A}^\nu$ are obtained from the set of vertices $\mathcal{K}^\nu$ of the box of reflection coefficients

$$\mathcal{A} = \phi(\mathcal{K}) \subseteq conv[\phi(\mathcal{K}^\nu)] = conv[\mathcal{A}^\nu].$$

Every vertex from the set $\mathcal{K}^\nu = \{k_1^\nu, ..., k_{2^n}^\nu\}$ will be transformed by (4) into a single member of the set $\mathcal{A}^\nu = \{a_1^\nu, ..., a_{n+1}^\nu\}$. But to a vertex $a_j^\nu, j = 1, ..., n+1$ may correspond several verteces of the set $\mathcal{K}^\nu$ because $2^n \geq n+1, n > 0$.

Let us now split the unit hypercube of reflection coefficients $\mathcal{K} = \{k_i \in (-1, 1), i = 1, ..., n\}$ into two hyperrectangles $\mathcal{K}_1(k_i)$ and $\mathcal{K}_2(k_i)$ by an hyperplane $k_i = k_i^*, k_i^* \in (-1, 1)$. And let the mapping (4) transforms the vertex sets $\mathcal{K}_1^\nu(k_i)$ and $\mathcal{K}_2^\nu(k_i)$ of these hyperrectangles into vertex sets $\mathcal{A}_1^\nu(k_i)$ and $\mathcal{A}_2^\nu(k_i)$ of polynomial coefficients a respectively. Then

$$\phi[\mathcal{K}_1(k_i)] \subseteq conv[\mathcal{A}_1^\nu(k_i)],$$

$$\phi[\mathcal{K}_2(k_i)] \subseteq conv[\mathcal{A}_2^\nu(k_i)]$$

and

$$\begin{aligned} \mathcal{A} = \quad & \phi(\mathcal{K}) = \phi\{[\mathcal{K}_1(k_i)] \cup [\mathcal{K}_2(k_i)]\} \subseteq \\ & \subseteq conv[\mathcal{A}_1^\nu(k_i)] \cup conv[\mathcal{A}_2^\nu(k_i)]. \end{aligned}$$

In case of splitting the unit hypercube $\mathcal{K}$ by another hyperplane $k_j = k_j^*, i \neq j; i, j = 1, ..., n$ we have

$$\mathcal{A} = \phi(\mathcal{K}) \subseteq conv[\mathcal{A}_1^\nu(k_j] \cup conv[\mathcal{A}_2^\nu(k_j)].$$

In general, we can split the unit hypercube $\mathcal{K}$ by several hyperplanes $k_i = k_{i_m}^*, k_{i_m}^* \in (-1, 1)$ for every coordinate $i = 1, ..., n; m = 1, ..., N_i$.

The question is: by what hyperplanes it is reasonable to split the hypercube of reflection coefficients ? The splitting of the hypercube $\mathcal{K}$ by an hyperplane $k_i = k_i^*, k_i^* \in (-1, 1)$ is not reasonable if:

1) $\phi[\mathcal{K}^\nu(k_i = 1)] = \phi[\mathcal{K}^\nu(k_i = -1)]$ or
2) $\phi[\mathcal{K}^\nu(k_i = 1)] = \mathcal{A}^\nu$ or
3) $\phi[\mathcal{K}^\nu(k_i = -1)] = \mathcal{A}^\nu$.

A straightforward implementation of (4) gives for $k_i = \pm 1$ that the first two conditions are not valid for any $i = 1, ..., n$ but the third condition is valid for several i. Indeed

$$\phi[\mathcal{K}^\nu(k_i = -1)] = \mathcal{A}^\nu$$

if

$$\begin{cases} i = n - 2j, & j = 1, ..., \frac{n}{2} - 1 \quad \text{for } n \text{ even}, \\ i = n - 2j + 1, & j = 1, ..., \frac{n-1}{2} \quad \text{for } n \text{ odd}. \end{cases}$$

So we have proven the following theorem.

Theorem 1. The Schur stability region $\mathcal{A}$ of polynomials $a(z)$ is contained in the intersection of the following unions of polyhedra

$$\mathcal{A} \subseteq \bigcap_i \mathcal{A}_i,$$

$$\mathcal{A}_i = \bigcup_{m=1}^{N_i+1} conv[\mathcal{A}_m^{\nu}(k_i)]$$

$$i \in \{1, ..., n\},$$

$$\begin{cases} i \neq n-2j, & j = 1, ..., \frac{n}{2}-1 \quad \text{for } n \text{ even} \\ i \neq n-2j+1, & j = 1, ..., \frac{n-1}{2} \quad \text{for } n \text{ odd} \end{cases}$$

where N_i is the number of splitting hyperplanes $k_i = k^*_{i_m}, m = 1, ..., N_i; i = 1, ..., n$ and $\mathcal{A}_m^{\nu}(k_i)$ is the vertex set of polynomials $a(z)$ corresponding to the hyperrectangle $\mathcal{K}(k^*_{i_m} \leq k_i \leq k^*_{i_{m+1}}, k_j = \pm 1, j \neq i)$.

Obviously, the necessary stability conditions will be less conservative if we increase the number N_i of different splitting hyperplanes. If $N_i \to \infty$ for $i = 1, ..., n$ then $\mathcal{A} = \cap_i \mathcal{A}_i$.

Example 1. Let $n = 3$. The barycentric stability simplex is the tetrahedron $ABCD$: $A = B_0 = (1, 3, 3), B = B_1 = (-1, -1, 1), C = B_2 = (1, -1, -1), D = B_3 = (-1, 3, -3)$. According to Theorem 1 the splitting of $\mathcal{K}$ is not reasonable for $i = 2$.

First, splitting the unit cube of reflection coefficients $\mathcal{K}$ by $k^*_1 = 0$ gives the unions of two polyhedra $ABCF$ and $BCDE$ where $E = (1, 1, 1)$ and $F = (-1, 1, -1)$. Second, splitting $\mathcal{K}$ by $k^*_3 = 0$ gives the unions of two polyhedra $ABCH$ and $BCDG$ where $G = (0, 1, 2)^T$ and $H = (0, 1, -2,)^T$. By Theorem 1 we have

$$\begin{aligned} \mathcal{A} \subset \quad & \{conv \begin{pmatrix} 1 & -1 & 1 & -1 \\ 3 & -1 & -1 & 1 \\ 3 & 1 & -1 & -1 \end{pmatrix} \cup \\ & \cup conv \begin{pmatrix} -1 & 1 & -1 & 1 \\ -1 & -1 & 3 & 1 \\ 1 & -1 & -3 & 1 \end{pmatrix} \} \cap \\ & \cap \{conv \begin{pmatrix} 1 & -1 & 1 & 0 \\ 3 & -1 & -1 & 1 \\ 3 & 1 & -1 & -2 \end{pmatrix} \cup \\ & \cup conv \begin{pmatrix} -1 & 1 & -1 & 0 \\ -1 & -1 & 3 & 1 \\ 1 & -1 & -3 & 2 \end{pmatrix} \}. \end{aligned}$$

IV. Sufficient stability conditions

In this section we derive some new sufficient stability conditions via reflection vectors of polynomials.

Let us start with the well-known but very conservative Cohn stability condition [3]:

a monic polynomial $a(z)$ is Schur stable if

$$\sum_{i=0}^{n-1} |a_i| < 1. \qquad (6)$$

It is easy to see that the linear cover of the reflection vectors of the origin coincides with the Cohn stability condition (6). Indeed, from (4) we obtain for $a = 0$

$$0^i(\pm 1) = [\underbrace{0, ..., 0,}_{i-1} \quad \pm 1, 0, ..., 0]$$

and for the inner points of the polytope

$$S^0 = conv\{0^i(1), 0^i(-1), i = 1, ..., n\}$$

the condition (6) holds.

The question is: is it possible to relax the condition for initial points $a \in \mathcal{A}$ so that the sufficient condition will be less conservative. The answer is given by the following two theorems .

Theorem 2. Let the reflection coefficients k_i of a monic polynomial $a(z)$ are as follows $k_1 = ... = k_{n-1} = 0$, $k_n \in (-1, 1)$. Then the inner points of the linear cover of reflection vectors of the point a are Schur stable

$$S^{a(n)} = conv\{a^i(1), a^i(-1), i = 1, ..., n\} \subset \mathcal{A}.$$

Proof . To prove this theorem we have to show that all the edges of the polytope $S^{a(n)}$ are stable.

First, let us mention that by Lemma 1 all the line segments between the reflection vectors $a^i(1)$ and $a^i(-1)$

$$E_{ii}(a) = conv\{a^i(1), a^i(-1), i = 1, ..., n\}$$

are stable.

Second, let us consider the edges

$$E_{ni}(a) = conv\{a^n(\pm 1), a^i(\pm 1), i = 1, ..., n-1\}.$$

By Cohn's condition (6) the line segments

$$E^{++}_{ni}(0) = conv\{0^n(1), 0^i(1), i = 1, ..., n-1\}$$

are stable. By Lemma 1 the line segments

$$conv\{a^{ni}(1, 0), a^{ni}(1, 1), i = 1, ..., n-1\}$$

$$conv\{a^{ni}(0, 1), a^{ni}(1, 1), i = 1, ..., n-1\}$$

are placed on the stability boundary. So the inner points of the triangle

$$conv\{a^{ni}(1, 0), a^{ni}(1, 1), a^{ni}(0, 1), i = 1, ..., n-1\}$$

and the edge $E^{++}_{ni}(a)$ are stable. Similarly we can prove the stability of edges $E^{--}_{ni}(a)$.

The edges $E^{+-}_{ni}(a)$ and $E^{-+}_{ni}(a)$ will be stable because the line segments

$$conv\{0^n(1), 0^{ni}(-1, \pm 1), i = 1, ..., n-1\},$$

$$conv\{0^n(-1), 0^{ni}(1, \pm 1), i = 1, ..., n-1\},$$

are stable or on the stability boundary (easy to check by straightforward calculations).

So the inner points of the triangles

$$conv\{a^{ni}(-1, 0), a^{ni}(1, 1), a^{ni}(0, 1), i = 1, ..., n-1\},$$

$$conv\{a^{ni}(1,0), a^{ni}(-1,1), a^{ni}(0,1), i=1,...,n-1\}$$

and the edges $E_{ni}^{+-}(a)$, $E_{ni}^{-+}(a)$ are stable.

Third, let us consider the edges

$$E_{ij}(a) = conv\{a^i(\pm 1), a^j(\pm 1); i,j=1,...,n-1\}.$$

We can find an arbitrary point $\bar{a}$ of the edge $E_{ij}(a)$, $\bar{a} \in E_{ij}(a)$ by the linear transformation $R_{n+1}(k_n \in (-1,1))$ from a point $\tilde{a}$ with $k_n^{\tilde{a}} = 0$, $k_i^{\tilde{a}} = k_i^a, i=1,...,n-1$ of the line segment $E_{ij}(0)$, $\tilde{a} \in E_{ij}(0)$

$$\bar{a} = R_{n+1}(k_n)\tilde{a}.$$

By Cohn's condition (6), the line segments

$$E_{ij}(0) = conv\{0^i(\pm 1), 0^j(\pm 1); i,j=1,...,n-1\}$$

are stable and by (5) the linear transformation $R_{n+1}(k_n \in (-1,1))$ does not change any of the reflection coefficients $k_i, i=1,...,n-1$. So the edges $E_{ij}(a)$ will be stable if $k_i^a \in (-1,1)$ and $k_n^a \in (-1,1)$.

By edge theorem [8] the inner points of the polytope $S^{a(n)}$ are stable.

Theorem 3. Let the reflection coefficients k_i of a monic polynomial $a(z)$ are as follows $k_1 \in (-1,1)$, $k_2 = ... = k_n = 0$. Then the inner points of the linear cover of reflection vectors of the point a are Schur stable

$$S^{a(1)} = conv\{a^i(1), a^i(-1), i=1,...,n\} \subset \mathcal{A}.$$

Proof . By (4) we obtain the the reflection vectors of a with $k_1 \in (-1,1)$, $k_2 = ... = k_n = 0$ as follows

$$\begin{array}{llllllll} a^1(-1) & =[& 0 & ... & 0 & 0 & 0 & 1 &], \\ a^1(1) & =[& 0 & ... & 0 & 0 & 0 & -1 &], \\ a^2(-1) & =[& 0 & .. & 0. & 0 & 1 & -2k_1 &], \\ a^2(1) & =[& 0 & ... & 0 & 0 & -1 & 0 &], \\ a^3(-1) & =[& 0 & ... & 0 & 1 & -k_1 & -k_1 &], \\ a^3(1) & =[& 0 & ... & 0 & -1 & k_1 & -k_1 &], \\ . & . & . & . & . & . & . & . & . \\ a^n(-1) & =[& 1 & -k_1 & 0 & ... & 0 & -k_1 &], \\ a^n(1) & =[& -1 & k_1 & 0 & ... & 0 & -k_1 &]. \end{array} \quad (7)$$

Let now for some n the polytope $S^{a(1)}(n)$ is stable. We have to prove that the polytope $S^{a(1)}(n+1)$ will be stable.

Obviously,

$$a^i(n+1) = [0, a^i(n)], \qquad i=1,...,n$$

and

$$a^{n+1}(n+1) = [\pm 1, \mp k_1, 0, ..., 0, -k_1].$$

The n-dimensional polytope generated by points $a^i(n+1)$, $i=1,...,n$ will be stable because the polynomials

$$a^i(z, n+1) = z a^i(z,n)$$

will be stable if only the polynomials $a^i(z,n)$ are stable (they have an extra root in the origin). To prove the stability of the $(n+1)$-dimensional polytope $S^{a(1)}(n+1)$ we have to prove stability of edges

$conv\{a^{n+1}(n+1), a^i(n+1)\}$, $i=1,...,n$. It can be done by the Perez stability condition [9]:
Every polynomial in the polygon will be stable if and only if all the corner polynomials of the following polygon are stable:

1) each pair of coefficients (a_i, a_j) , $0 \le i \le n$, $n-i \le j \le n$ is varying inside a polytope with edges sloped in closed interval $[\pi/4, 3\pi/4]$
2) each a_i can only be combined with one a_j and vice-versa.

Indeed,according to (7) Perez condition holds for the following pairs of coefficients :

1) (a_0, a_2) and (a_1, a_3) for the edges $conv\{a^{n+1}, a^{n-j+1}\}$, $j=1,2$;

2) (a_0, a_j) and (a_1, a_{j+1}) for the edges $conv\{a^{n+1}, a^{n-j+1}\}$, $j=3,...,n-2$.

All the other coefficients are fixed for the edges considered.

The stability of edges $conv\{a^{n+1}, a^1\}$ can be proved similarly to the proof of Theorem 2 by using the stable line segments

$$conv\{0^1(1), 0^{1i}(-1,\pm 1), i=1,...,n\},$$

$$conv\{0^1(-1), 0^{1i}(1,\pm 1), i=1,...,n\}.$$

Obviously, for $n=2$ the polytope $S^{a(1)}(2)$ is stable. We have proved the stability of polytopes $S^{a(1)}(n)$ for $n>2$.

Corollary 1. The inner points of the union of the polytopes $S^{a(1)}$ are Schur stable

$$S_n = \bigcup_{j=1}^{N_n} S_j^{a(n)} \subset \mathcal{A}.$$

Corollary 2. The inner points of the union of the polytopes $S^{a(n)}$ are Schur stable

$$S_1 = \bigcup_{j=1}^{N_1} S_j^{a(1)} \subset \mathcal{A}.$$

Corollary 3. The inner points of the union of the polytopes $S^{a(1)}$ and $S^{a(n)}$ are Schur stable

$$S_{1,n} = S_1 \cup S_n \subset \mathcal{A}.$$

Example 2: Let $n=2$. Then the stability region in the polynomial coefficient space $a = (a_1, a_0)$ is the triangle *FGH* (Fig.1). Let us find some stability region inside approximations according to Theorems 2 and 3.

1) Let us start from the polynomial $a'(z) = z^2 + 0.5$ (point *A'* in Fig.1a) with reflection coefficients $k_1(a') = 0$, $k_2(a') = -0.5$. According to Lemma 1 we can draw 2 stable line segments through the point *A'*. By varying the first reflection coefficient k_1, $-1 < k_1 < 1$, we get the line segment *FB'* and by varying the second reflection coefficient k_2,

$-1 < k_2 < 1$, we get the line segment *C'D'*. By definition the second order polynomial $a(z)$ has 4 reflection vectors :

$$\begin{array}{lcl} a'^1(1) &=& [\;\; -1.5 \quad 0.5 \;\;], \\ a'^1(-1) &=& [\;\; 1.5 \quad 0.5 \;\;], \\ a'^2(1) &=& [\;\; 0 \quad -1 \;\;], \\ a'^2(-1) &=& [\;\; 1 \quad 0 \;\;]. \end{array}$$

The requirements of Theorem 2 are fulfilled for the polynomial $a(z)$. So the inner points of the polytope *B'C'D'F* of reflection vectors are stable. It is easy to see that for the vertices $a'^1(1)$ and $a'^1(-1)$ the Cohn stability condition (6) does not hold because

$$\sum_i |a_i^1(\pm 1)| = 2 > 1.$$

2) For the polynomial $a'(z) = z^2 + 0.5z$ (point *A"* in Fig.1b) with reflection coefficients $k_1(a'') = -0.5$, $k_2(a'') = 0$ the requirements of Theorem 3 are fulfilled. So the inner points of the polytope *B"C"D"F* of reflection vectors

$$\begin{array}{lcl} a''^1(1) &=& [\;\; -1 \quad 0 \;\;], \\ a''^1(-1) &=& [\;\; 1 \quad 0 \;\;], \\ a''^2(1) &=& [\;\; 0 \quad -1 \;\;], \\ a''^2(-1) &=& [\;\; 1 \quad 1 \;\;] \end{array}$$

are stable.

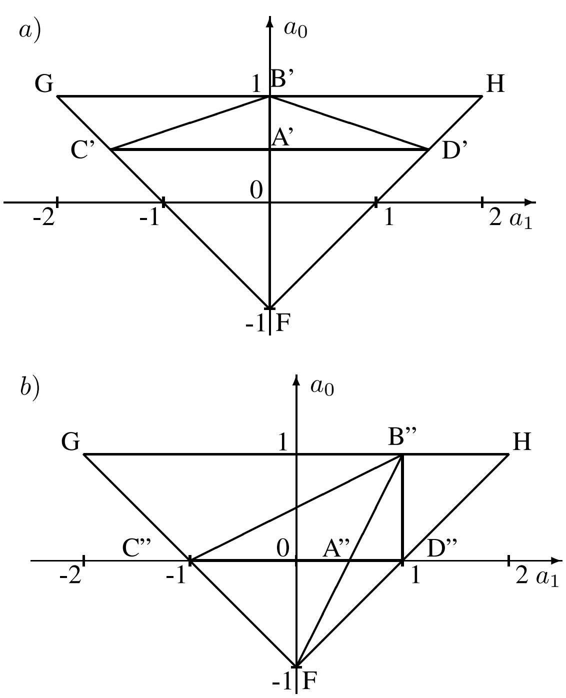

Fig.1 Stable polytopes of reflection vectors ($n = 2$)

V. Conclusions

The geometry of stable discrete polynomials using their coefficients and their reflection coefficients is investigated. By splitting the unit hypercube of reflection coefficients you can improve barycentric simplex (necessary stability condition) in terms of unions of polytopes. It is interesting to mention that If $N_i \to \infty$ for $i = 1, ..., n$ then $\mathcal{A} = \cap_i \mathcal{A}_i$. A generalization of Cohn's sufficient stability criterion is obtained in terms of reflection vectors of special stable polynomials .

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