Robust Sliding Mode Control Based on Integral Sliding Surfaces

Xiaoyu Zhang, Hongye Su, Lingfei Xiao and Jian Chu

Abstract—**Based on a novel robust integral sliding mode surface (RISM), method of designing nonlinear robust sliding mode control is presented. Firstly, the closed-loop system under the sliding mode is proved to be asymptotically stable by using Lyapunov stability theories. Then variable structure control (VSC) is constructed to ensure the reachability of the sliding surface. Robustness of the closed-loop system in the whole state space is obtained. Finally, numerical simulation results are proposed to test the validity and feasibility of the method.**

I. Introduction

SLIDING mode control (SMC) as the dominant method in theories of variable structure control system (VSS) is an excellent robust control approaches to resolve the control problems of nonlinear systems[1]-[2]. However, for some nonlinear dynamic systems, the designing process of sliding mode control may be not easy. Thus switching surface in nonlinear form is a better way to deal with this difficulty.

Generally, sliding mode surfaces are in linear forms. If we consider designing a sliding surface in nonlinear form, there may be some difficulties, such as the determinations of the mathematic form (of the sliding mode), the dynamic orders, and the structure etc. For example we analyze two local problems during SMC synthesis process. Firstly, the system behaviors under the sliding mode must be stable. This point may be easy if the sliding surface is in linear form, but for the sliding surface in nonlinear form, the process of verifying its stability must be proposed strictly. Secondly, the controller designing becomes not easy because of the nonlinear mathematic form of the sliding surface. Nevertheless many sliding surfaces in nonlinear forms are presented from 1990s[3]-[13], such as the three-segment nonlinear sliding mode (TSNSM)[3], the integral sliding mode (ISM)[4]-[10] and the time varying sliding mode (TVSM)[11]-[14] etc.

In this paper a novel robust integral sliding mode surface (RISM) is developed. The mathematic form of the surface is very concise and easy for implementation. Moreover the uncertainties are not required to satisfy the matching condition. The variable structure control (VSC) which satisfies the reaching condition is easy to be designed. Furthermore, the method is suitable for many nonlinear systems with approximative mathematics model, especially with unmatched uncertainty or external disturbance.

The paper is organized as the following. In section 2, problem formulations are presented. In section 3, a robust integral sliding mode surface is constructed and the system stability in the sliding surface is verified. In section 4, the SMC controller is constructed and the reaching condition is proved to be satisfied. Numerical simulation examples are shown in section 5 and finally, conclusions are generalized in section 6.

II. Problem Formulation

Consider nonlinear system with single-input described as

$$\dot{x} = f(x,t) + g(x,t)u \tag{1}$$

where $x = [x_1 \ \cdots \ x_n]^T$ is the state vector and u is the single control input of the system.

According to *Taylor* series of an arbitrary function, there is the equation

$$f(x,t) = [\frac{\partial f}{\partial x^T}\bigg|_{x=0} + O(x,t)]x \tag{2}$$

where $x = 0$ is the quilibrium position of the system (1) and $O(x,t)$ represents the residual series. Thereby the following assumptions are natural.

Assumption 1. $f(x,t) \in R^{n\times 1}$, $f(x,t) = [\boldsymbol{A}_0 + \Delta\boldsymbol{A}(x,t)]x$,where $\boldsymbol{A}_0$ is a constant matrix and $\Delta\boldsymbol{A}(x,t) \in R^{n\times n}$ satisfies that $\|\Delta\boldsymbol{A}(x,t)\|_2 \le \tau$ with τ is a known positive scalar. Here $\|\cdot\|_2$ is induce norm operator of matrices.

Assumption 2. $g(x,t) = \boldsymbol{B}_0 + \Delta\boldsymbol{B}(x,t)\boldsymbol{E}_b \in R^{n\times 1}$ and the uncertainty $\Delta\boldsymbol{B}$ satisfies $\eta < \|\Delta\boldsymbol{B}(x,t)\|_2 \le \psi$, where η, ψ are

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suitable positive scalar quantities and $\boldsymbol{B}_0$, $\boldsymbol{E}_b$ are all constant matrices.

Define the nonlinear sliding surface s as the following,

$$s(t) = \boldsymbol{C}_0 x - \int_0^t \{\boldsymbol{C}_0\boldsymbol{A}_0 - \boldsymbol{K}\}x dt \tag{3}$$

where $\boldsymbol{C}_0 = [c_1 \ \cdots \ c_n] \in R^{1\times n}$ is the coefficient vector of the linear part of the sliding mode with $\boldsymbol{C}_0\boldsymbol{B}_0 \neq 0$ and the parameter vector $\boldsymbol{K} \in R^{1\times n}$ is to be designed later. In order to guarantee the existence of the sliding surface s and the synthesis process, the next assumption is necessary.

Assumption 3. The uncertainty $\Delta\boldsymbol{B}(x,t)$ also satisfies that $\|\boldsymbol{C}_0\Delta\boldsymbol{B}\boldsymbol{E}_b\|_2 < |\boldsymbol{C}_0\boldsymbol{B}_0|$.

The problem: A proper control u should be constructed to impel the state variable x of the closed-loop system moving to the surface defined by (3) from an arbitrary initial point in the state space. Furthermore, the system states converge to the origin after the system stepped into the sliding mode surface.

For the convenience of the analysis in the following section, several lemmas are introduced.

Lemma 1. For arbitrary matrix $\boldsymbol{A} \in \boldsymbol{R}^{n\times n}$, if $\|\boldsymbol{A}\| < 1$, then $\boldsymbol{I} - \boldsymbol{A}$ is not singular and satisfies that $\|(\boldsymbol{I}-\boldsymbol{A})^{-1}\| \leq \frac{\|\boldsymbol{I}\|}{1-\|\boldsymbol{A}\|}$.

Lemma 2. For arbitrary matrix $\boldsymbol{A} \in \boldsymbol{R}^{n\times n}$ and $\boldsymbol{B}$, $\boldsymbol{C}$ with suitable dimensions, if $\boldsymbol{A}$ and $\boldsymbol{I}+\boldsymbol{C}\boldsymbol{A}^{-1}\boldsymbol{B}$ are all not singular, then $(\boldsymbol{A}+\boldsymbol{B}\boldsymbol{C})^{-1} = \boldsymbol{A}^{-1} - \boldsymbol{A}^{-1}\boldsymbol{B}(\boldsymbol{I}+\boldsymbol{C}\boldsymbol{A}^{-1}\boldsymbol{B})^{-1}\boldsymbol{C}\boldsymbol{A}^{-1}$.

Lemma 3. For arbitrary matrix $\boldsymbol{U}, \boldsymbol{V}$ with suitable dimensions, there exists the matrix inequality equation $\boldsymbol{U}\boldsymbol{V} + \boldsymbol{V}^T\boldsymbol{U}^T \leq \boldsymbol{V}^T\boldsymbol{V} + \boldsymbol{U}\boldsymbol{U}^T$.

Here $\boldsymbol{I}$ is unit matrix with suitable dimensions.

III. Robust Sliding Mode Surface

The sliding mode (3) is expected to be converging to zero. However, once it converged to zero, the movements of closed-loop system under the sliding mode must be studied. Thereby the system behaviors under the sliding mode are firstly analyzed in this section. By using the equation (1), (3) and $\dot{s} = 0$, we have the system equations under sliding mode,

$$\begin{cases} \dot{x} = (\boldsymbol{A}_0 + \Delta\boldsymbol{A})x + (\boldsymbol{B}_0 + \Delta\boldsymbol{B}\boldsymbol{E}_b)u_{eq} \\ \boldsymbol{C}_0\Delta\boldsymbol{A}x + \boldsymbol{C}_0(\boldsymbol{B}_0 + \Delta\boldsymbol{B}\boldsymbol{E}_b)u_{eq} + \boldsymbol{K}x = 0 \end{cases} \tag{4}$$

where u_{eq} is equivalent control. From the second line in the equation (4), the equivalent control can be achieved easily,

$$u_{eq} = -[\boldsymbol{C}_0\boldsymbol{B}_0 + \boldsymbol{C}_0\Delta\boldsymbol{B}\boldsymbol{E}_b]^{-1}[\boldsymbol{C}_0\Delta\boldsymbol{A}x + \boldsymbol{K}x] \tag{5}$$

Then according to the **Assumption 3** and *Lemma 1*, $\boldsymbol{I} + \boldsymbol{C}_0(\boldsymbol{C}_0\boldsymbol{B}_0)^{-1}\Delta\boldsymbol{B}\boldsymbol{E}_b$ is not singular and satisfies that

$$\|(\boldsymbol{I} + \boldsymbol{C}_0(\boldsymbol{C}_0\boldsymbol{B}_0)^{-1}\Delta\boldsymbol{B}\boldsymbol{E}_b)^{-1}\|_2 \leq \frac{1}{1-|\boldsymbol{C}_0\boldsymbol{B}_0|^{-1}\|\boldsymbol{C}_0\Delta\boldsymbol{B}\boldsymbol{E}_b\|_2} \tag{6}$$

Well then according to *Lemma 2* the system equation under sliding mode becomes

$$\begin{aligned}\dot{x} &= (\boldsymbol{A}_0 + \Delta\boldsymbol{A})x - (\boldsymbol{B}_0 + \Delta\boldsymbol{B}\boldsymbol{E}_b)[\boldsymbol{C}_0\boldsymbol{B}_0 + \boldsymbol{C}_0\Delta\boldsymbol{B}\boldsymbol{E}_b]^{-1}[\boldsymbol{C}_0\Delta\boldsymbol{A}x + \boldsymbol{K}x] \\ &= (\boldsymbol{A}_0 + \Delta\boldsymbol{A})x - (\boldsymbol{B}_0 + \Delta\boldsymbol{B}\boldsymbol{E}_b)(\boldsymbol{C}_0\boldsymbol{B}_0)^{-1}\cdot\{\boldsymbol{I} - \boldsymbol{C}_0[\boldsymbol{I} + \\ &\qquad \boldsymbol{C}_0(\boldsymbol{C}_0\boldsymbol{B}_0)^{-1}\Delta\boldsymbol{B}\boldsymbol{E}_b]^{-1}\Delta\boldsymbol{B}\boldsymbol{E}_b(\boldsymbol{C}_0\boldsymbol{B}_0)^{-1}\}\cdot\{\boldsymbol{C}_0\Delta\boldsymbol{A}x + \boldsymbol{K}x\} \\ &= (\boldsymbol{A}_0 + \Delta\boldsymbol{A})x - (\boldsymbol{B}_0 + \Delta\boldsymbol{B}\boldsymbol{E}_b)(\boldsymbol{C}_0\boldsymbol{B}_0)^{-1}(\boldsymbol{I} - \boldsymbol{M})\{\boldsymbol{C}_0\Delta\boldsymbol{A}x + \boldsymbol{K}x\}\end{aligned}$$

where $\boldsymbol{M} = \boldsymbol{C}_0[\boldsymbol{I} + \boldsymbol{C}_0(\boldsymbol{C}_0\boldsymbol{B}_0)^{-1}\Delta\boldsymbol{B}\boldsymbol{E}_b]^{-1}\Delta\boldsymbol{B}\boldsymbol{E}_b(\boldsymbol{C}_0\boldsymbol{B}_0)^{-1}$ and $\boldsymbol{I}$ is unit matrix with suitable dimensions. Continuing to deduce the state equation, we have

$$\begin{aligned}\dot{x} &= (\boldsymbol{A}_0 + \Delta\boldsymbol{A})x - \{\boldsymbol{B}_0(\boldsymbol{C}_0\boldsymbol{B}_0)^{-1} + \Delta\boldsymbol{B}\boldsymbol{E}_b(\boldsymbol{C}_0\boldsymbol{B}_0)^{-1} + \\ &\qquad \boldsymbol{B}_0(\boldsymbol{C}_0\boldsymbol{B}_0)^{-1}\boldsymbol{M} + \Delta\boldsymbol{B}\boldsymbol{E}_b(\boldsymbol{C}_0\boldsymbol{B}_0)^{-1}\boldsymbol{M}\}\{\boldsymbol{C}_0\Delta\boldsymbol{A}x + \boldsymbol{K}x\} \\ &= (\boldsymbol{A}_0 + \Delta\boldsymbol{A})x - \{\boldsymbol{B}_0(\boldsymbol{C}_0\boldsymbol{B}_0)^{-1} + \boldsymbol{N}\}\{\boldsymbol{C}_0\Delta\boldsymbol{A}x + \boldsymbol{K}x\}\end{aligned}$$

where $\boldsymbol{N} = \Delta\boldsymbol{B}\boldsymbol{E}_b(\boldsymbol{C}_0\boldsymbol{B}_0)^{-1} + \boldsymbol{B}_0(\boldsymbol{C}_0\boldsymbol{B}_0)^{-1}\boldsymbol{M} + \Delta\boldsymbol{B}\boldsymbol{E}_b(\boldsymbol{C}_0\boldsymbol{B}_0)^{-1}\boldsymbol{M}$. Then the equation

$$\begin{aligned}\dot{x} &= [\boldsymbol{A}_0 - \boldsymbol{B}_0(\boldsymbol{C}_0\boldsymbol{B}_0)^{-1}\boldsymbol{K}]x + [\boldsymbol{I} - \\ &\qquad \boldsymbol{B}_0(\boldsymbol{C}_0\boldsymbol{B}_0)^{-1}\boldsymbol{C}_0]\Delta\boldsymbol{A}x - \boldsymbol{N}\boldsymbol{C}_0\Delta\boldsymbol{A}x - \boldsymbol{N}\boldsymbol{K}x \qquad (7) \\ &= \hat{\boldsymbol{A}}_0 x + \hat{\boldsymbol{C}}_0\Delta\boldsymbol{A}x - \boldsymbol{N}\boldsymbol{C}_0\Delta\boldsymbol{A}x - \boldsymbol{N}\boldsymbol{K}x\end{aligned}$$

is obtained where

$$\hat{\boldsymbol{A}}_0 = \boldsymbol{A}_0 - \boldsymbol{B}_0(\boldsymbol{C}_0\boldsymbol{B}_0)^{-1}\boldsymbol{K} \tag{8}$$

$$\hat{\boldsymbol{C}}_0 = \boldsymbol{I} - \boldsymbol{B}_0(\boldsymbol{C}_0\boldsymbol{B}_0)^{-1}\boldsymbol{C}_0 \tag{9}$$

According to the equation (6) and the **Assumption 2**, the bounds of the matrix $\boldsymbol{M}$, $\boldsymbol{N}$ can be estimated as,

$$|\boldsymbol{M}| \leq \frac{\psi\|\boldsymbol{C}_0\boldsymbol{E}_b\|_2}{|\boldsymbol{C}_0\boldsymbol{B}_0| - \eta\|\boldsymbol{C}_0\boldsymbol{E}_b\|_2} = M_{\max} \tag{10}$$

$$\|\boldsymbol{N}\| \leq \psi(1 + M_{\max})\|\boldsymbol{E}_b\| + \|\boldsymbol{B}_0\|M_{\max} = N_{\max} \tag{11}$$

Now select a Lyapunov function $V = x^T P x$, where $P = P^T, P > 0$. Seek the time derivate of V along the trajectory of the system (7),

$$\begin{aligned}\dot{V} &= x^T\{\hat{\boldsymbol{A}}_0^T P + P\hat{\boldsymbol{A}}_0 + \Delta\boldsymbol{A}^T[\hat{\boldsymbol{C}}_0 - \boldsymbol{N}\boldsymbol{C}_0]^T P + \\ &\qquad P[\hat{\boldsymbol{C}}_0 - \boldsymbol{N}\boldsymbol{C}_0]\Delta\boldsymbol{A} - [\boldsymbol{N}\boldsymbol{K}]^T P - P[\boldsymbol{N}\boldsymbol{K}]\}x\end{aligned}$$

Then because of *Lemma 3*, there is the inequality

$$\begin{aligned}\dot{V} &\leq x^T\{\hat{\boldsymbol{A}}_0^T P + P\hat{\boldsymbol{A}}_0 + \Delta\boldsymbol{A}^T\Delta\boldsymbol{A} + \boldsymbol{K}^T\boldsymbol{K} + \\ &\qquad [\hat{\boldsymbol{C}}_0 - \boldsymbol{N}\boldsymbol{C}_0]^T P^T P[\hat{\boldsymbol{C}}_0 - \boldsymbol{N}\boldsymbol{C}_0] + \boldsymbol{N}^T P^T P\boldsymbol{N}\}x \\ &\leq x^T\{\hat{\boldsymbol{A}}_0^T P + P\hat{\boldsymbol{A}}_0 + \hat{\boldsymbol{C}}_0^T P^T P\hat{\boldsymbol{C}}_0 + \boldsymbol{K}^T\boldsymbol{K} + \tau^2 + \\ &\qquad [\boldsymbol{C}_0^T\boldsymbol{N}^T\boldsymbol{N}\boldsymbol{C}_0 + 2N_{\max}\|\hat{\boldsymbol{C}}_0\|\cdot\|\boldsymbol{C}_0\| + N_{\max}^2]\|P\|_2^2\}x \\ &\leq x^T\{\hat{\boldsymbol{A}}_0^T P + P\hat{\boldsymbol{A}}_0 + \hat{\boldsymbol{C}}_0^T P^T P\hat{\boldsymbol{C}}_0 + \boldsymbol{K}^T\boldsymbol{K} + \tau^2 + \\ &\qquad [2\|\boldsymbol{C}_0\|\cdot\|\hat{\boldsymbol{C}}_0\|N_{\max} + (1 + \|\boldsymbol{C}_0\|^2)N_{\max}^2]\cdot\|P\|_2^2\}x\end{aligned}$$

Thereby the following theorem 1 can be acquired.

Theorem 1. The state vector of the nonlinear system described by the equation (4), (5) is asymptotically robust stable if there exist matrix $P = P^T$, $P > 0$ and the sliding mode parameter vectors $\boldsymbol{C}_0$, $\boldsymbol{K}$ in the equation (3) which satisfy the following inequality

$$\hat{A}_0^T P + P\hat{A}_0 + \hat{C}_0^T P^T P\hat{C}_0 + K^T K + \tau^2 + \left\{2\|C_0\| \cdot \|\hat{C}_0\| N_{\max} + (1+\|C_0\|^2) N_{\max}^2\right\} \cdot \|P\|_2^2 < 0 \tag{12}$$

Here it is verified that the sliding mode surface (3) is a robust integral sliding mode (RISM) if its parameters are selected as those depicted by the theorem 1. The uncertainties do not satisfy the matching condition. Following the reaching property of the RISM (3) will be analyzed.

IV. Robust Integral Sliding Mode Control

In this section, a just control u will be constructed to ensure that from arbitrary initial values, the state variable of closed-loop system steps into the sliding mode defined by the equation (3).

Firstly the control u is constructed as the following,

$$u = -(C_0 B_0)^{-1}[Kx + k\frac{\|x\|}{\|s(t)\|}s(t)] \tag{13}$$

where k satisfies that

$$k > \frac{\|C_0\|(\tau|C_0 B_0| + \psi\|E_b\| \cdot \|K\|)}{1 - \psi\|C_0\| \cdot \|E_b\|} \tag{14}$$

and then the following theorem 2 will be proofed.

Theorem 2. For nonlinear systems described by the equation (1) which dynamics satisfy the assumptions 1,2 and 3, the state variable of the closed-loop system steps into the sliding mode surface defined by the equation (3) if the control is designed as the equations (13),(14).

Proof: by considering the time derivate of the sliding mode surface s, we can obtain that

$$\begin{aligned} s\dot{s} &= s\{C_0 \Delta A x + (C_0 B_0 + C_0 \Delta B E_b)u + Kx\} \\ &= s\Big\{C_0 \Delta A x - C_0 \Delta B E_b (C_0 B_0)^{-1} Kx - \\ &\qquad [1 + C_0 \Delta B E_b (C_0 B_0)^{-1}]k\frac{\|x\|}{\|s(t)\|}s(t)\Big\} \\ &\le s\Big\{\tau\|C_0\| \cdot \|x\| + \frac{\psi\|C_0\| \cdot \|E_b\| \cdot \|K\|}{|C_0 B_0|}\|x\| - \\ &\qquad [1 + C_0 \Delta B E_b (C_0 B_0)^{-1}]k\frac{\|x\|}{\|s(t)\|}s(t)\Big\} \\ &\le s\Big\{(\tau + \frac{\psi\|E_b\| \cdot \|K\|}{|C_0 B_0|})\|C_0\| \cdot \|x\| - k \cdot \frac{1 - \psi\|C_0\| \cdot \|E_b\|}{|C_0 B_0|}\frac{\|x\|}{\|s(t)\|}s(t)\Big\} \\ &\le -\Big\{k - \frac{\|C_0\|(\tau|C_0 B_0| + \psi\|E_b\| \cdot \|K\|)}{1 - \psi\|C_0\| \cdot \|E_b\|}\Big\}\|x\| \cdot \|s(t)\| \end{aligned} \tag{15}$$

Obviously, $s\dot{s} < 0$ when $s \neq 0$. So the sliding mode surface s must be reachable.

In the next section, a numerical example is proposed to validate the controller design.

V. Numerical Example

Consider a concrete nonlinear system described as the equation (16). For this system, its canonical form is that $\dot{x} = A(x) + B(x)u$ where the matrix $A(x) = A_0 + \Delta A$ and the matrix $B(x) = B_0 + \Delta B$. Obviously, the system dynamics satisfy the Assumption 1-3 (in section 2). Thus a robust integral sliding mode surface (RISM) can be designed according to **theorem 1**. Then supposing that the initial value of the state is $x_0 = [2 \quad -0.8 \quad 1 \quad 0.5]^T$, the sliding mode parameter vector C_0 is designed as $C_0 = [0.25 \quad 1.25 \quad 0 \quad 1]$.

$$\dot{x} = \begin{bmatrix} 0 & 1 & \sin x_3 & 0 \\ \sin x_1 & 0 & 0 & 4 \\ \cos x_1 & 0 & 0 & 1 \\ 0 & 3 & -12 & 0 \end{bmatrix} x + \begin{bmatrix} \sin x_1 \\ 1 + \exp(-\cos x_2) \\ 0 \\ 1.01 \end{bmatrix} u \tag{16}$$

After the simulation, the response results of the state vector and the control input signal are shown in Fig 1 and Fig 2. The value of the sliding mode is shown in Fig 3.

From Fig 3, it can be seen that although the unmatched uncertainties exist, the system state vector rests on the sliding surface. Notice that the value of the sliding mode is always very small initially. This point owes to the selection of the parameter C_0, because we can let $s_0 = C_0 x_0 = 0$. The sliding mode value is limited in the neighbor region of the origin by the control law. And then the state variables converge to the origin along.

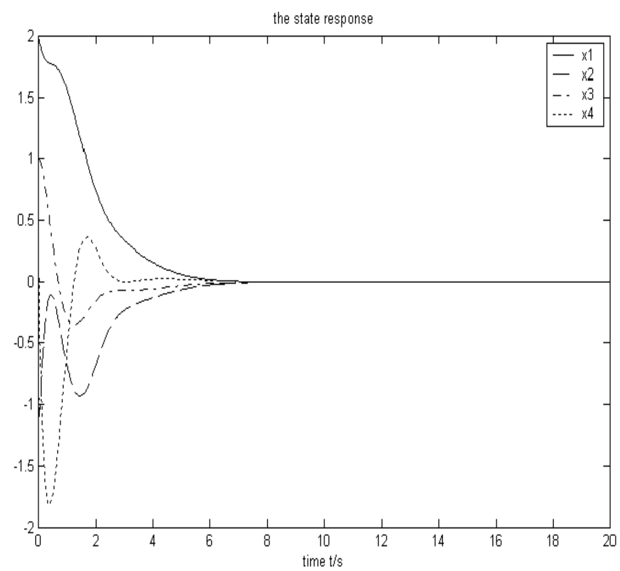

Fig 1. The state response of the system (16) which controller is designed based on RISM depicted by the theorem 1.

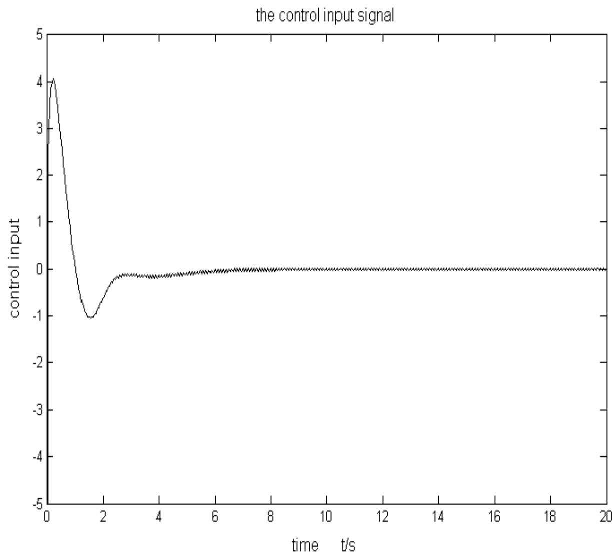

Fig 2. The control input signal of the closed-loop system (16).

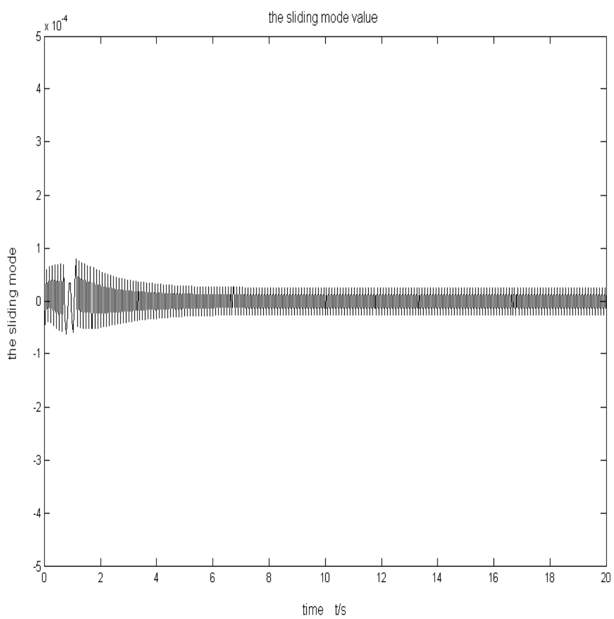

Fig 3. The value of the RISM of the closed-loop system (16). Note that the value is less than 10^{-4}. This point owes to the selection of the parameter $\boldsymbol{C}_0$ which leads to $s_0 = \boldsymbol{C}_0 x_0 = 0$.

VI. CONCLUSION

A robust integral sliding mode control (RISM) is designed firstly and then the variable structure control (VSC) is constructed based on it. The state vector of the closed-loop system under the sliding mode moves to the origin along the switching surface, and furthermore the RISM is robust while the uncertainties are not required to satisfy the matching condition. The sliding mode can be reached from the initial point by selecting the sliding mode parameters according to the proposed theorem. Systemic designed approach provided in the two theorems is easy for implementation. Finally numerical example validates the performance of the controller design.

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