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H_∞ Measurement Feedback Control for Time Delay Systems via Krein Space

Mei Liu, Huanshui Zhang and Guangren Duan

***Abstract*— H_∞ control problem for linear discrete-time systems with instantaneous and delayed measurements is studied. A sufficient and necessary condition for the existence of the H_∞ controller is derived by applying re-organized innovation analysis approach. The measurement-feedback controller is designed by performing two Riccati equations. The presented approach does not require the state augmentation.**

I. INTRODUCTION

H_∞ control has been one of the most important topics in control theory and has attracted much attention of numerous researchers in the past two decades. In 1981, Zames [1] originally proposed the H_∞ control problem in an input-output setting. It was an important advantage that Doyle introduced the state space method to H_∞ control problem and gave the state space solutions according to two Riccati equations for time-invariant system. Tadmor [4] extended the H_∞ control problem to the time-vary, finite horizon case by maximum principle. Plenty of results in frequency domain [1], [5], [6] and time domain [2]-[4] have been achieved.

Recently, increasing attention has been paid to the problem of H_∞ control for delay systems. Delays may exist in the state [8], [13], the control input [9], [10] and the measurements [14]-[16]. Such problems have been encountered in many practical control problems, such as process control. A lot of interesting results for this problem have been presented in [17]-[20] and references therein. However, only sufficient condition for the existence of the controller is given in most of the previous works.

In this paper, we study the H_∞ measurement feedback control problem for the systems with delayed measurement. A new approach is applied to derive the H_∞ controller. With the using of a re-organization innovation, we convert the delayed measurements into measurement delay-free. The H_∞ control problem is equivalent to an H_2 estimation problem in Krein space.

This paper is organized as follows: the problem statement is presented in Section II. The main results via indefinite quadratic form and the re-organized innovation analysis approach in Krein space are proposed in Section III. Some concluding remarks end the paper.

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M. Liu is with Center of Control Theory and Guidance Technology and Shenzhen Graduate School, Harbin Institute of Technology, P.R.China Mayliu@hit.edu.cn

H.S. Zhang is with Shenzhen Graduate School, Harbin Institute of Technology, Shenzhen, 518055, P.R.China h_s_zhang@hit.edu.cn

G.R. Duan is with Center of Control Theory and Guidance, Harbin Institute of Technology, Harbin, 150001, P.R.China Grduan@iee.org

II. PROBLEM STATEMENT AND PRELIMINARIES

A. Problem Statement

Consider the time-variant state-space model with instantaneous and delayed measurements

$$x(t+1) = F_t x(t) + G_{1,t} w(t) + G_{2,t} u(t) \quad (1)$$

$$y(t) = H_t x(t) + v(t) \quad (2)$$

$$z_{t-d}(t) = M_{t-d} x(t-d) + v_z(t) \quad (3)$$

$$s(t) = L_t x(t) \quad (4)$$

where $x(t) \in \mathrm{R}^n$, $w(t) \in \mathrm{R}^l$, $u(t) \in \mathrm{R}^r$, $y(t) \in \mathrm{R}^m$, $z_{t-d}(t) \in \mathrm{R}^p$ and $s(t) \in \mathrm{R}^q$ are respectively the state, process noise, control inputs, instantaneous measurement, delayed measurement and signals, $v(t) \in \mathrm{R}^m$ and $v_z(t) \in \mathrm{R}^p$ are the measurement noises. The matrices $F_t, G_{1,t}, G_{2,t}, H_t, M_{t-d}$ and L_t are known matrices of appropriate dimensions. For convenience, t_d denotes $t-d$ $(d>0)$ and $*$ stands for the transpose throughout this paper.

Let $\breve{u}(t)$ be a control strategy, we denote

$$\|\mathcal{T}(\mathcal{F})\|_\infty^2 = \sup_{x(0),w,v,v_z \in h_2} \frac{A}{B}, \quad (5)$$

where

$$A = x^*(N+1)P_{N+1}^c x(N+1) + \sum_{t=0}^{N} \breve{u}^*(t) Q_t^c \breve{u}(t) + \sum_{t=0}^{N} s^*(t) R_t^c s(t)$$

$$B = x^*(0)\Pi_0^{-1}x(0) + \sum_{t=0}^{N} w^*(t) Q_t^w w(t) + \sum_{t=0}^{N} v^*(t)(R_t^v)^{-1} v(t) + \sum_{t=d}^{N} v_z^*(t) R_{t_d}^{v_z} v_z(t)$$

The matrices $\Pi_0, P_{N+1}^c, Q_t^w, Q_t^c, R_t^v, R_t^c$ and $R_{t_d}^{v_z}$ are positive (semi) definite weighting matrices.

Now we formulate the problem which will be addressed in this paper.

Problem 1: Consider the system (1)-(4), find a suboptimal measurement-feedback H_∞ control strategy $\breve{u}(t) = \mathcal{F}_t(y(0), \cdots, y(t), z_0(d), \cdots, z_{t-d}(t))$ that achieves

$$\|\mathcal{T}(\mathcal{F})\|_\infty < \gamma,$$

where γ is a given positive scalar.

B. Preliminaries

In view of (5), it is not difficulty to observe that *Problem 1* is equivalent to J_N which has a minimum J_N^m over the variables $\{x(0), w(0), \cdots, w(N)\}$ and $\check{u}(t)$ can be chosen such that $J_N^m > 0$, where

$$J_N = x^*(0)\Pi_0^{-1}x(0) + \sum_{t=0}^{N} w^*(t)Q_t^w w(t) + \sum_{t=0}^{N}(y(t) - H_t x(t))^*(R_t^v)^{-1}(y(t) - H_t x(t)) + \sum_{t=d}^{N}(z_{t_d}(t) - M_{t_d}x(t_d))^*(R_{t_d}^{v_z})^{-1} \times (z_{t_d}(t) - M_{t_d}x(t_d)) - \gamma^{-2}\left(\sum_{t=0}^{N} s^*(t)R_t^c s(t) + x^*(N+1)P_{N+1}^c x(N+1) + \sum_{t=0}^{N}\check{u}^*(t)Q_t^c\check{u}(t)\right). \quad (6)$$

Let us rewrite J_N in the following fashion

$$J_N = x^*(0)\Pi_0^{-1}x(0) + \sum_{t=0}^{N}(y(t) - H_t x(t))^*(R_t^v)^{-1} \times (y(t) - H_t x(t)) + \sum_{t=d}^{N}(z_{t_d}(t) - M_{t_d}x(t_d))^* \times (R_{t_d}^{v_z})^{-1}(z_{t_d}(t) - M_{t_d}x(t_d)) - \gamma^{-2}\bar{J}_N \quad (7)$$

where

$$\bar{J}_N = x^*(N+1)P_{N+1}^c x(N+1) + \sum_{t=0}^{N} s^*(t)R_t^c s(t) + \sum_{t=0}^{N}\begin{bmatrix} w(t) \\ \check{u}(t)\end{bmatrix}^*\begin{bmatrix} -\gamma^2 Q_t^w & 0 \\ 0 & Q_t^c\end{bmatrix}\begin{bmatrix} w(t) \\ \check{u}(t)\end{bmatrix} \quad (8)$$

According to the discussion in [7], we get

$$\bar{J}_N = x^*(0)P_0^c x(0) + \sum_{t=0}^{N}\begin{bmatrix} w(t) - \tilde{w}(t) \\ \check{u}(t) - \tilde{u}(t)\end{bmatrix}^* R_{e,t}^c\begin{bmatrix} w(t) - \tilde{w}(t) \\ \check{u}(t) - \tilde{u}(t)\end{bmatrix}, \quad (9)$$

where $\tilde{w}(t)$ and $\tilde{u}(t)$ are given by

$$\begin{bmatrix}\tilde{w}(t) \\ \tilde{u}(t)\end{bmatrix} = -K_{c,t}x(t) = -\begin{bmatrix} K_{w,t} \\ K_{u,t}\end{bmatrix}x(t) \quad (10)$$

with

$$K_{c,t} = (R_{e,t}^c)^{-1}\begin{bmatrix} G_{1,t}^* \\ G_{2,t}^*\end{bmatrix}P_{t+1}^c F_t, \quad (11)$$

$$R_{e,t}^c = \begin{bmatrix} -\gamma^2 Q_t^w + G_{1,t}^*P_{t+1}^c G_{1,t} & G_{1,t}^*P_{t+1}^c G_{2,t} \\ G_{2,t}^*P_{t+1}^c G_{1,t} & Q_t^c + G_{2,t}^*P_{t+1}^c G_{2,t}\end{bmatrix} \quad (12)$$

and $P_t^c, t = 0, \cdots, N$ satisfies the backwards Riccati equation as

$$P_t^c = F_t P_{t+1}^c F_t + L_t^* R_t^c L_t - K_{c,t}^* R_{e,t}^c K_{c,t}, \quad P_{N+1}^c. \quad (13)$$

Now (9) allows us to write J_N as follows

$$J_N = x^*(0)(\Pi_0^{-1} - \gamma^{-2}P_0^c)x(0) + \sum_{t=0}^{N}\begin{bmatrix} w(t) - \tilde{w}(t) \\ \check{u}(t) - \tilde{u}(t) \\ y(t) - H_t x(t) \\ z_{t_d}(t) - M_{t_d}x(t_d)\end{bmatrix}^* \times \begin{bmatrix} -\gamma^{-2}\begin{bmatrix} R_{e,t}^c(1,1) R_{e,t}^c(1,2) \\ R_{e,t}^c(2,1) R_{e,t}^c(2,2)\end{bmatrix} & 0 & 0 \\ 0 & (R_t^v)^{-1} & 0 \\ 0 & 0 & (R_{t_d}^{v_z})^{-1}\end{bmatrix} \times \begin{bmatrix} w(t) - \tilde{w}(t) \\ \check{u}(t) - \tilde{u}(t) \\ y(t) - H_t x(t) \\ z_{t_d}(t) - M_{t_d}x(t_d)\end{bmatrix}, \quad (14)$$

where the $R_{e,t}^c(i,j), (i,j = 1,2)$ denote the (i,j) block entries of $R_{e,t}^c$ and $z_{t_d}(t) = M_{t_d} = R_{t_d}^{v_z} = 0$ for $0 \le t < d$.

Note that J_N is an indefinite quadratic form and includes the information of instantaneous and delayed measurement. A new approach termed as re-organized innovation analysis in Krein space shall be developed to deal with such a problem in the following discussion.

III. MAIN RESULTS

Denote

$$\bar{x}(t) = \begin{cases} x(t), 0 \le t < d \\ \begin{bmatrix} x(t) \\ x(t_d)\end{bmatrix}, t \ge d\end{cases}, Y_s(t) = \begin{cases} y(t), 0 \le t < d \\ \begin{bmatrix} y(t) \\ z_{t_d}(t)\end{bmatrix}, t \ge d\end{cases} \quad (15)$$

and

$$\begin{bmatrix}\Delta_t^{-1} & \bar{S}_t \\ \bar{S}_t^* & (\Delta_t^{'})^{-1}\end{bmatrix} = \begin{bmatrix} R_{e,t}^c(1,1) R_{e,t}^c(1,2) \\ R_{e,t}^c(2,1) R_{e,t}^c(2,2)\end{bmatrix}^{-1}. \quad (16)$$

Then (14) is easily rewritten as

$$J_N = x^*(0)(\Pi_0^{-1} - \gamma^{-2}P_0^c)x(0) + \sum_{t=0}^{N}\begin{bmatrix} w(t) - \tilde{w}(t) \\ \begin{bmatrix}\check{u}(t) \\ Y_s(t)\end{bmatrix} - \begin{bmatrix} -\bar{K}_{u,t} \\ \bar{H}_t\end{bmatrix}\bar{x}(t)\end{bmatrix}^* \times \begin{bmatrix} Q_t^{\tilde{w}} \; S_t \\ S_t^* \; Q_t^v\end{bmatrix}^{-1} \times \begin{bmatrix} w(t) - \tilde{w}(t) \\ \begin{bmatrix}\check{u}(t) \\ Y_s(t)\end{bmatrix} - \begin{bmatrix} -\bar{K}_{u,t} \\ \bar{H}_t\end{bmatrix}\bar{x}(t)\end{bmatrix}, \quad (17)$$

where

$$\bar{K}_{u,t} = \begin{cases} K_{u,t}, & 0 \le t < d \\ \begin{bmatrix} K_{u,t} 0\end{bmatrix}, & t \ge d\end{cases},$$

$$\bar{H}_t = \begin{cases} H_t, & 0 \le t < d \\ \begin{bmatrix} H_t & 0 \\ 0 & M_{t_d}\end{bmatrix}, & t \ge d\end{cases}, \quad Q_t^{\tilde{w}} = -\gamma^{-2}\Delta_t^{-1},$$

$$S_t = \begin{cases} -\gamma^2\begin{bmatrix}\bar{S}_t & 0\end{bmatrix}, & 0 \le t < d \\ -\gamma^2\begin{bmatrix}\bar{S}_t & 0 & 0\end{bmatrix}, & t \ge d\end{cases},$$

$$Q_t^v = \begin{cases} \begin{bmatrix} -\gamma^2(\Delta_t^{'})^{-1} & 0 \\ 0 & R_t^v \end{bmatrix}, & 0 \le t < d \\ \begin{bmatrix} -\gamma^2(\Delta_t^{'})^{-1} & 0 & 0 \\ 0 & R_t^v & 0 \\ 0 & 0 & R_{t_d}^{v_z} \end{bmatrix}, & t \ge d \end{cases}.$$

Note that $Y_s(t)$ is the observation of the system (1)-(4) at time t, which is given as

$$Y_s(t) = \begin{cases} H_t x(t) + v_s(t), & 0 \le t < d \\ \begin{bmatrix} H_t & 0 \\ 0 & M_{t_d} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t_d) \end{bmatrix} + v_s(t), & t \ge d \end{cases}, \tag{18}$$

where

$$v_s(t) = \begin{cases} v(t), & 0 \le t < d \\ \begin{bmatrix} v(t) \\ v_z(t) \end{bmatrix}, & t \ge d \end{cases}. \tag{19}$$

For the convenience of discussion, we introduce the Krein space state-space model associated with estimation quadratic form (17)

$$\mathbf{x}(t+1) = (F_t - G_{1,t}K_{w,t})\mathbf{x}(t) + G_{1,t}(\mathbf{w}(t) - \tilde{\mathbf{w}}(t)) + G_{2,t}\check{\mathbf{u}}(t) \tag{20}$$

$$\begin{bmatrix} \check{\mathbf{u}}(t) \\ \mathbf{Y}_s(t) \end{bmatrix} = \begin{bmatrix} -\bar{K}_{u,t} \\ \bar{H}_t \end{bmatrix} \bar{\mathbf{x}}(t) + \bar{\mathbf{v}}_s(t) \tag{21}$$

where

$$\bar{\mathbf{v}}_s(t) = \begin{bmatrix} \mathbf{v}_u(t) \\ \mathbf{v}_s(t) \end{bmatrix}$$

and $\mathbf{x}(0)$, $\mathbf{w}(t) - \tilde{\mathbf{w}}(t)$ and $\bar{\mathbf{v}}_s(t)$, in bold face, are Krein space variables with

$$\left\langle \begin{bmatrix} \mathbf{x}(0) \\ \mathbf{w}(t) - \tilde{\mathbf{w}}(t) \\ \bar{\mathbf{v}}_s(t) \end{bmatrix}, \begin{bmatrix} \mathbf{x}(\mathbf{0}) \\ \mathbf{w}(r) - \tilde{\mathbf{w}}(r) \\ \bar{\mathbf{v}}_s(r) \end{bmatrix} \right\rangle = \begin{bmatrix} (\Pi_0^{-1} - \gamma^{-2}P_0^c)^{-1} & 0 \\ 0 & \begin{bmatrix} Q_t^{\tilde{w}} & S_t \\ S_t^* & Q_t^v \end{bmatrix} \delta_{tr} \end{bmatrix}. \tag{23}$$

Recalling the discussion in [12], the minimum of J_N is given by

$$J_N^m = \sum_{t=0}^{N} \begin{bmatrix} \check{u}(t) + \bar{K}_{u,t}\widehat{\bar{x}}(t \mid t-1) \\ Y_s(t) - \bar{H}_t\widehat{\bar{x}}(t \mid t-1) \end{bmatrix}^* \times Q_{w_s}^{-1}(t) \begin{bmatrix} \check{u}(t) + \bar{K}_{u,t}\widehat{\bar{x}}(t \mid t-1) \\ Y_s(t) - \bar{H}_t\widehat{\bar{x}}(t \mid t-1) \end{bmatrix}, \tag{24}$$

where

$$\widehat{\bar{x}}(t \mid t-1) = \begin{cases} \hat{x}(t \mid t-1), & 0 \le t < d \\ \begin{bmatrix} \hat{x}(t \mid t-1) \\ \hat{x}(t_d \mid t-1) \end{bmatrix}, & t \ge d \end{cases}, \tag{25}$$

The value of $\hat{x}(t \mid t-1)$ and $\hat{x}(t_d \mid t-1)$ in (25) are obtained from the projection of $\mathbf{x}(t)$ and $\mathbf{x}(t_d)$ onto the linear space $\mathcal{L}\left\{ \begin{bmatrix} \check{\mathbf{u}}(i) \\ \mathbf{Y}_s(i) \end{bmatrix}_{i=0}^{t-1} \right\}$, respectively. In (24), $Q_{w_s}(t)$ is the covariance matrix of innovation $W_s(t)$, which is given as

$$W_s(t) = \begin{bmatrix} \check{\mathbf{u}}(t) \\ \mathbf{Y}_s(t) \end{bmatrix} - \begin{bmatrix} \hat{\mathbf{u}}(t \mid t-1) \\ \hat{\mathbf{Y}}_s(t \mid t-1) \end{bmatrix} = \begin{cases} \begin{bmatrix} -K_{u,t} & 0 \\ 0 & H_t \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) - \hat{\mathbf{x}}(t \mid t-1) \\ \mathbf{x}(t) - \hat{\mathbf{x}}(t \mid t-1) \end{bmatrix} + \bar{\mathbf{v}}_s(t), & 0 \le t < d \\ \begin{bmatrix} -K_{u,t} & 0 & 0 \\ 0 & H_t & 0 \\ 0 & 0 & M_{t_d} \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) - \hat{\mathbf{x}}(t \mid t-1) \\ \mathbf{x}(t) - \hat{\mathbf{x}}(t \mid t-1) \\ \mathbf{x}(t_d) - \hat{\mathbf{x}}(t_d \mid t-1) \end{bmatrix} + \bar{\mathbf{v}}_s(t), & t \ge d \end{cases} \tag{26}$$

where $\hat{\mathbf{x}}(t \mid t-1)$ ($\hat{\mathbf{x}}(t_d \mid t-1)$) is the projection of $\mathbf{x}(t)$ ($\mathbf{x}(t_d)$) onto the linear space $\mathcal{L}\left\{ \begin{bmatrix} \check{\mathbf{u}}(i) \\ \mathbf{Y}_s(i) \end{bmatrix}_{i=0}^{t-1} \right\}$. It should be seen that the estimator $\widehat{\bar{x}}(t \mid t-1)$ and innovation covariance matrix $Q_{w_s}(t)$ play important role for designing the controller. Note the observation $\begin{bmatrix} \check{\mathbf{u}}(i) \\ \mathbf{Y}_s(i) \end{bmatrix}$ contains time delay, the standard Kalman filtering formulation is not applicable to compute $\widehat{\bar{x}}(t \mid t-1)$ and $Q_{w_s}(t)$. To deal with such problems we shall re-organize the delayed measurements and define re-organization innovation. The estimator $\widehat{\bar{x}}(t \mid t-1)$ and innovation covariance matrix $Q_{w_s}(t)$ can be calculated by using innovation analysis method.

From (15), it is easy to verify that

$$\mathcal{L}\left\{ \begin{bmatrix} \check{\mathbf{u}}(i) \\ \mathbf{Y}_s(i) \end{bmatrix}_{i=0}^{t} \right\} = \begin{cases} \mathcal{L}\left\{ \begin{bmatrix} \check{\mathbf{u}}(t) \\ \mathbf{y}(t) \end{bmatrix} \right\}, & 0 \le t < d \\ \mathcal{L}\left\{ \begin{bmatrix} \check{\mathbf{u}}(0) \\ \mathbf{y}_f(0) \end{bmatrix}, \cdots, \begin{bmatrix} \check{\mathbf{u}}(t_d) \\ \mathbf{y}_f(t_d) \end{bmatrix}, \begin{bmatrix} \check{\mathbf{u}}(t_d+1) \\ \mathbf{y}(t_d+1) \end{bmatrix}, \cdots, \begin{bmatrix} \check{\mathbf{u}}(t) \\ \mathbf{y}(t) \end{bmatrix} \right\}, & t \ge d. \end{cases} \tag{27}$$

where

$$\mathbf{y}_f(i) = \begin{bmatrix} \mathbf{y}(i) \\ \mathbf{z}_i(i+d) \end{bmatrix} = \begin{bmatrix} H_i \\ M_i \end{bmatrix} \mathbf{x}(i) + \mathbf{v}_f(i), i = 0, \cdots, t-d \tag{28}$$

with

$$\mathbf{v}_f(i) = \begin{bmatrix} \mathbf{v}(i) \\ \mathbf{v}_z(i+d) \end{bmatrix}, 0 \le i < t-d.$$

When $0 \le t < d$, we have the relationships

$$\begin{bmatrix} \check{\mathbf{u}}(t) \\ \mathbf{y}(t) \end{bmatrix} = \begin{bmatrix} -K_{u,t} \\ H_t \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} \mathbf{v}_u(t) \\ \mathbf{v}(t) \end{bmatrix}. \tag{30}$$

When $t \ge d$, we have the relationship for $0 \le i < t-d$ and $t-d \le i \le t$

$$\begin{bmatrix} \check{\mathbf{u}}(i) \\ \mathbf{y}_f(i) \end{bmatrix} = \begin{bmatrix} -K_{u,i} \\ H_i \\ M_i \end{bmatrix} \mathbf{x}(i) + \begin{bmatrix} \mathbf{v}_u(i) \\ \mathbf{v}_f(i) \end{bmatrix}, \tag{31}$$

$$\begin{bmatrix} \check{\mathbf{u}}(i) \\ \mathbf{y}(i) \end{bmatrix} = \begin{bmatrix} -K_{u,i} \\ H_i \end{bmatrix} \mathbf{x}(i) + \begin{bmatrix} \mathbf{v}_u(i) \\ \mathbf{v}(i) \end{bmatrix}, \tag{32}$$

It should be noted that, after reorganizing the measurements, the new observation $\mathbf{y}_f(i)$ contains measurements of the state $\mathbf{x}(i)$ at time instants i and $i+d$. In other word, the delayed measurement has been applied to our design. $\begin{bmatrix} \breve{\mathbf{u}}(i) \\ \mathbf{y}_f(i) \end{bmatrix}$ and $\begin{bmatrix} \breve{\mathbf{u}}(i) \\ \mathbf{y}(i) \end{bmatrix}$ are delay-free and termed as *re-organized observation.* (20) and (30) or (20), (31) and (32) give a standard state space model without delay. Now define the innovation sequence associated with the re-organized observations for $i>0$ and $i=0$:

$$\begin{aligned} W(t+i,t)=&\begin{bmatrix} \breve{\mathbf{u}}(t+i) \\ \mathbf{y}(t+i) \end{bmatrix} - \begin{bmatrix} \widehat{\breve{\mathbf{u}}}(t+i \mid t+i-1,t) \\ \hat{\mathbf{y}}(t+i \mid t+i-1,t) \end{bmatrix} \\ =&\begin{bmatrix} -K_{u,t+i} \\ H_{t+i} \end{bmatrix} \mathbf{e}(t+i,t) + \begin{bmatrix} \mathbf{v}_u(t+i) \\ \mathbf{v}(t+i) \end{bmatrix}, \end{aligned} \tag{33}$$

$$\begin{aligned} W(t,t)=&\begin{bmatrix} \breve{\mathbf{u}}(t) \\ \mathbf{y}_f(t) \end{bmatrix} - \begin{bmatrix} \widehat{\breve{\mathbf{u}}}(t \mid t-1,t-1) \\ \hat{\mathbf{y}}(t \mid t-1,t-1) \end{bmatrix} \\ =&\begin{bmatrix} -K_{u,t} \\ H_t \\ M_t \end{bmatrix} \mathbf{e}(t,t) + \begin{bmatrix} \mathbf{v}_u(t) \\ \mathbf{v}_f(t) \end{bmatrix}, \\ &\begin{bmatrix} \widehat{\breve{\mathbf{u}}}(0 \mid -1,-1) \\ \hat{\mathbf{y}}_f(0 \mid -1,-1) \end{bmatrix} = 0 \end{aligned} \tag{34}$$

where

$$\begin{aligned} \mathbf{e}(t+i,t)=&\mathbf{x}(t+i) - \hat{\mathbf{x}}(t+i \mid t+i-1,t), i>0, \\ \mathbf{e}(t,t)=&\mathbf{x}(t) - \hat{\mathbf{x}}(t \mid t-1,t-1), \end{aligned}$$

while $\hat{\mathbf{x}}\ (j \mid t+i,t)\ (i \geq 0)$ is the estimate of $\mathbf{x}(j)$ given $\left\{ \begin{bmatrix} \breve{\mathbf{u}}(0) \\ \mathbf{y}_f(0) \end{bmatrix}, \ldots, \begin{bmatrix} \breve{\mathbf{u}}(t) \\ \mathbf{y}_f(t) \end{bmatrix}, \begin{bmatrix} \breve{\mathbf{u}}(t+1) \\ \mathbf{y}(t+1) \end{bmatrix}, \ldots, \begin{bmatrix} \breve{\mathbf{u}}(t+i) \\ \mathbf{y}(t+i) \end{bmatrix} \right\}$. It is easy to verify that $\{W(\cdot,\cdot)\}$ is the mutually uncorrelated sequence [11]. $\{W(\cdot,\cdot)\}$ is termed as *re-organization innovation* for observations $\begin{bmatrix} \breve{\mathbf{u}}(\cdot) \\ \mathbf{y}_f(\cdot) \end{bmatrix}$ and $\begin{bmatrix} \breve{\mathbf{u}}(\cdot) \\ \mathbf{y}(\cdot) \end{bmatrix}$.

A. Innovation Covariance Matrix and Optimal Estimator

In this subsection, we shall present the general optimal estimator $\hat{x}(t_l \mid t, t_d)$ (l is a positive integer) and innovation covariance matrix $Q_{w_s}(t)$ by using the re-organized innovation defined by (26). From (33)-(34), the covariance matrix for re-organization innovation

$$Q_w(t+i,t) = \langle W(t+i,t), W(t+i,t) \rangle, i \geq 0$$

is calculated by

$$\begin{aligned} Q_w(t,t)=&\begin{bmatrix} -K_{u,t} \\ H_t \\ M_t \end{bmatrix} P_{t,t-1}^t \begin{bmatrix} -K_{u,t} \\ H_t \\ M_t \end{bmatrix}^* \\ &+ \begin{bmatrix} Q_{\mathbf{v}_u}(t) & 0 \\ 0 & Q_{\mathbf{v}_f}(t) \end{bmatrix} \end{aligned} \tag{36}$$

$$\begin{aligned} Q_w(t+i,t)=&\begin{bmatrix} -K_{u,t+i} \\ H_{t+i} \end{bmatrix} P_{t+i,t}^{t+i} \begin{bmatrix} -K_{u,t+i} \\ H_{t+i} \end{bmatrix}^* \\ &+ \begin{bmatrix} Q_{\mathbf{v}_u}(t) & 0 \\ 0 & Q_{\mathbf{v}}(t+i) \end{bmatrix}, i>0 \end{aligned} \tag{37}$$

with $Q_{\mathbf{v}_u}(t) = -\gamma^{-2}(\Delta_t^{'})^{-1}$, $Q_{\mathbf{v}}(t) = R_t^v$, $Q_{\mathbf{v}_z}(t) = R_{t_d}^{v_z}$, $Q_{(}\mathbf{v}_f)(t) = \begin{bmatrix} Q_{\mathbf{v}}(t) & 0 \\ 0 & Q_{\mathbf{v}_z}(t) \end{bmatrix}$ and $P_{t+i,t}^{t+i} = \langle \mathbf{e}(t+i,t), \mathbf{e}(t+i,t) \rangle$ $(i \geq 0)$ is the covariance matrix of estimation error $\mathbf{e}(t+i,t)$, which can be computed by the following Lemma.

Lemma 1: Let $\phi_t = F_t - G_{1,t}K_{w,t}$. The cross-covariance matrix $P_{t+i,t}^{t+i}$ can be calculated as

1) For $i=1$, $P_{t+1,t}^{t+1}$ is calculated recursively by the following Riccati equation

$$\begin{aligned} P_{t+1,t}^{t+1}=&\phi_t P_{t,t-1}^t \phi_t^* + G_{1,t} Q_t^{\tilde{w}} G_{1,t}^* + G_{2,t} Q_{v_u}(t) G_{2,t}^* \\ &-\phi_t P_{t,t-1}^t \begin{bmatrix} -K_{u,t} \\ H_t \\ M_t \end{bmatrix}^* Q_w^{-1}(t,t) \begin{bmatrix} -K_{u,t} \\ H_t \\ M_t \end{bmatrix} \\ &\times P_{t,t-1}^t \phi_t^*, \qquad P_{0,-1}^0 = (\Pi_0^{-1} - \gamma^{-2} P_0^c)^{-1}, \end{aligned} \tag{38}$$

where $Q_w(t,t)$ is the same as in (36).

2) For $i>1$, $P_{t+i,t}^{t+i}$ is calculated as

$$\begin{aligned} P_{t+i+1,t}^{t+i+1}=&\phi_{t+i} P_{t+i,t}^{t+i} \phi_{t+i}^* + G_{2,t+i} Q_{v_u}(t+i) \\ &\times G_{2,t+i}^* + G_{1,t+i} Q_{t+i}^{\tilde{w}} G_{1,t+i}^* \\ &-\phi_{t+i} P_{t+i,t}^{t+i} \begin{bmatrix} -K_{u,t+i} \\ H_{t+i} \end{bmatrix}^* \\ &\times Q_w^{-1}(t+i,t) \begin{bmatrix} -K_{u,t+i} \\ H_{t+i} \end{bmatrix} P_{t+i,t}^{t+i} \phi_{t+i}^*, \end{aligned} \tag{39}$$

where $P_{t+1,t}^{t+1}$ and $Q_w(t+i,t)$ are as in (38) and (37).

Proof: The proof is straightforward from [11]. ■

Further, let

$$R_{t+i,t}^{t+j} \triangleq \langle \mathbf{x}(t+j), \mathbf{e}(t+i,t) \rangle, i,j \geq 0 \tag{40}$$

be the cross-covariance matrix of the state $\mathbf{x}(t+j)$ and the state estimation error $\mathbf{e}(t+i,t)$, then we have following result

$$R_{t+i,t}^{t+j} = \begin{cases} P_{t+j,t}^{t+j} A^*(t+j,t) \cdots A^*(t+i-1,t), & i \geq j \\ \phi_{t+j-1} \cdots \phi_{t+i} P_{t+i,t}^{t+i}, & i<j \end{cases}. \tag{41}$$

where $A(t+k,t), k>0$ is given by

$$\begin{aligned} &A(t+k,t) = \phi_{t+k} \\ &\times \left\{ I_n - P_{t+k,t}^{t+k} \begin{bmatrix} -K_{u,t+k} \\ H_{t+k} \end{bmatrix}^* Q_w^{-1}(t+k,t) \begin{bmatrix} -K_{u,t+k} \\ H_{t+k} \end{bmatrix} \right\} \end{aligned}$$

For $k \leq 0$, $P_{t+k,t}^{t+k} = P_{t+k,t+k}^{t+k}$ and $A(t+k,t) = A(t+k,t+k)$, the matrix $A(t+k,t+k)$ is given by

$$\begin{aligned} A(t+k,t+k)=&\phi_{t+k} \left\{ I_n - P_{t+k,t+k}^{t+k} \begin{bmatrix} -K_{u,t+k} \\ H_{t+k} \\ M_{t+k} \end{bmatrix}^* \right. \\ &\left. \times Q_w^{-1}(t+k,t+k) \begin{bmatrix} -K_{u,t+k} \\ H_{t+k} \\ M_{t+k} \end{bmatrix} \right\}. \end{aligned}$$

Next we will calculate the optimal estimator $\hat{x}(t_l \mid t, t_d)$ and innovation covariance matrix $Q_{w_s}(t)$.

Theorem 1: For the Krein space state space model (20)-(21), the innovation covariance matrix $Q_{w_s}(t)$ and the optimal estimator $\hat{x}(t_l \mid t) = \hat{x}(t_l \mid t, t_d)$ (l is an integer) are given by

1) The innovation covariance matrix $Q_{w_s}(t) = \langle W_s(t), W_s(t)\rangle$ is given by

$$Q_{w_s}(t) = \begin{cases} \begin{bmatrix} Q_{w_s}(1,1) Q_{w_s}(1,2) \\ Q_{w_s}(2,1) Q_{w_s}(2,2) \end{bmatrix}, \\ \qquad 0 \le t < d, \\ \begin{bmatrix} Q_{w_s}(1,1) Q_{w_s}(1,2) Q_{w_s}(1,3) \\ Q_{w_s}(2,1) Q_{w_s}(2,2) Q_{w_s}(2,3) \\ Q_{w_s}(3,1) Q_{w_s}(3,2) Q_{w_s}(3,3) \end{bmatrix}, \\ \qquad t \ge d, \end{cases}$$

where

$$\begin{aligned} &Q_{w_s}(1,1) = K_{u,t} P^t_{t,t_d-1} K^*_{u,t} - \gamma^2 (\Delta'_t)^{-1}, \\ &Q_{w_s}(1,2) = -K_{u,t} P^t_{t,t_d-1} H^*_t, \\ &Q_{w_s}(2,1) = -H_t P^t_{t,t_d-1} K^*_{u,t}, \\ &Q_{w_s}(2,2) = H_t P^t_{t,t_d-1} H^*_t + R^v_t, \\ &Q_{\mathbf{w_s}}(1,3) = -K_{u,t} \left(R^{t_d}_{t,t_d-1}\right)^* M^*_{t_d}, \\ &Q_{\mathbf{w_s}}(2,3) = H_t \left(R^{t_d}_{t,t_d-1}\right)^* M^*_{t_d}, \\ &Q_{\mathbf{w_s}}(3,1) = -M_{t_d} R^{t_d}_{t,t_d-1} K^*_{u,t}, \\ &Q_{\mathbf{w_s}}(3,2) = M_{t_d} R^{t_d}_{t,t_d-1} H^*_t, \\ &Q_{\mathbf{w_s}}(3,3) = M_{t_d} P(t_d) M^*_{t_d} + R^{v_z}_t, \\ &P(t_d) = P^{t_d}_{t_d,t_d-1} - \sum_{i=0}^{d-1} R^{t_d}_{t_d+i,t_d-1} H^*_{t_d+i} \\ &\quad \times Q_w^{-1}(t_d+i, t_d-1) H_{t_d+i} \left(R^{t_d}_{t_d+i,t_d-1}\right)^*. \end{aligned}$$

2) The optimal estimator $\hat{x}(t_l \mid t, t_d)$ can be calculated as:

a) For $d > l \ge 0$, the optimal estimator $\hat{x}(t_l \mid t, t_d)$ is calculated by

$$\begin{aligned} &\hat{x}(t_l \mid t, t_d) = \hat{x}(t_l \mid t_l - 1, t_d) \\ &+ \sum_{i=0}^{l} R^{t_l}_{t_l+i,t_d} \begin{bmatrix} -K_{u,t_l+i} \\ H_{t_l+i} \end{bmatrix}^* \\ &\times Q_w^{-1}(t_l+i, t_d) \left(\begin{bmatrix} \breve{u}(t_l+i) \\ y(t_l+i) \end{bmatrix} \right. \\ &\left. - \begin{bmatrix} -K_{u,t_l+i} \\ H_{t_l+i} \end{bmatrix} \hat{x}(t_l+i \mid t_l+i-1, t_d) \right), \end{aligned}$$

where $\hat{x}(t_l+i \mid t_l+i-1, t_d)$, $i = 0, \cdots, l$ in the above equation is calculated recursively as

$$\begin{aligned} &\hat{x}(t_l+i+1 \mid t_l+i, t_d) \\ &= \phi_{t_l+i} \hat{x}(t_l+i \mid t_l+i-1, t_l) \\ &+ \phi_{t_l+i} P^{t_l+i}_{t_l+i,t_d} \\ &\times \begin{bmatrix} -K_{u,t_l+i} \\ H_{t_l+i} \end{bmatrix}^* Q_w^{-1}(t_l+i, t_d) \left(\begin{bmatrix} \breve{u}(t_l+i) \\ y(t_l+i) \end{bmatrix} \right. \\ &\left. - \begin{bmatrix} -K_{u,t_l+i} \\ H_{t_l+i} \end{bmatrix} \hat{x}(t_l+i \mid t_l+i-1, t_d) \right), \\ &\qquad \hat{x}(t_d+1 \mid t_d, t_d), \quad i = 1, \cdots, d-1, \end{aligned}$$

and where $Q_w(t_l + i, t_d)$ and $P^{t_l+i}_{t_l+i,t_d}$ ($i = 2, \cdots, d$) are computed by (37) and (39) respectively. The initial value $\hat{x}(t_d+1 \mid t_d, t_d)$ can be computed recursively as:

$$\begin{aligned} &\hat{x}(t_d+1 \mid t_d, t_d) \\ &= \phi_{t_d} \hat{x}(t_d \mid t_d-1, t_d-1) + \phi_{t_d} P^{t_d}_{t_d,t_d-1} \\ &\times \begin{bmatrix} -K_{u,t_d} \\ H_{t_d} \\ M_{t_d} \end{bmatrix}^* Q_w^{-1}(t_d, t_d) \left(\begin{bmatrix} \breve{u}(t_d) \\ y_f(t_d) \end{bmatrix} \right. \\ &\left. - \begin{bmatrix} -K_{u,t_d} \\ H_{t_d} \\ M_{t_d} \end{bmatrix} \hat{x}(t_d \mid t_d-1, t_d-1) \right), \\ &\qquad \hat{x}(0 \mid -1, -1) = 0, \end{aligned}$$

where $Q_w(t_d, t_d)$ and $P^{t_d}_{t_d,t_d-1}$ are as in (36) and (38) respectively.

b) For $l < 0$, the optimal estimator $\hat{x}(t_l \mid t, t_d)$ is given by

$$\hat{x}(t_l \mid t, t_d) = \phi_{t_l-1} \cdots \phi_{t+1} \hat{x}(t+1 \mid t, t_d), \quad (42)$$

where $\hat{x}(t+1 \mid t, t_d)$ has been given by *a*).

Proof: The proof can be obtained by applying a similar discussion as in [12] and [11]. ■

B. Solution to H_∞ Control Problem

Now we are in the position to present the main result of this paper.

Theorem 2: Consider the state-space model (1)-(4). Then, for any given $\gamma > 0$, a measurement-feedback H_∞ controller $\breve{u}(t) = \mathcal{F}_t(y(0), \cdots, y(t), z_0(d), \cdots, z_{t-d}(t))$ that achieves $\|\mathcal{T}(\mathcal{F})\|_\infty < \gamma$ exists if and only if

1) $\Pi_0^{-1} - \gamma^{-2} P_0^c > 0$,
2) $\triangle_t = -\gamma^2 Q^w_t + G^*_{1,t} P^c_{t+1} G_{1,t} - G^*_{1,t} P^c_{t+1} G_{2,t} R^{-1}_{G^c,t} \times G^*_{2,t} P^c_{t+1} G_{1,t} < 0$, for all $t = 0, 1, \cdots, N$ and
3) the matrices $Q^v_t - S_t \left(Q^{\tilde{w}}_t\right)^{-1} S^*_t$ and $Q_{w_s}(t)$ have the same inertia for all $t = 0, 1, \cdots, N$,

where P^c_{t+1} satisfies (13) and

$$R_{G^c,t} = Q^c_t + G^*_{2,t} P^c_{t+1} G_{2,t}.$$

Then the central controller is given by

$$\begin{aligned} \breve{u}(t) &= -\bar{K}_{u,t} \hat{\bar{x}}(t \mid t-1) - K_{k,t} \\ &\times \bar{Q}^{-1}_{w_s}(t) \left(Y_s(t) - \bar{H}_t \hat{\bar{x}}(t \mid t-1)\right). \end{aligned} \quad (44)$$

where

$$K_{k,t} = \begin{cases} K_{u,t} P^t_{t,t_d-1} H^*_t, & 0 \le t < d \\ \left[K_{u,t} P^t_{t,t_d-1} H^*_t K_{u,t} R^{t_d}_{t,t_d-1} M^*_{t_d}\right], & t \ge d \end{cases} \quad (45)$$

and the optimal estimate $\hat{\bar{x}}(t \mid t-1)$ is given by

$$\hat{\bar{x}}(t \mid t-1) = \begin{cases} \hat{x}(t \mid t-1), & 0 \le t < d \\ \begin{bmatrix} \hat{x}(t \mid t-1) \\ \hat{x}(t_d \mid t-1) \end{bmatrix}, & t \ge d \end{cases}, \quad (46)$$

while $\hat{x}(t \mid t-1)$ and $\hat{x}(t_d \mid t-1)$ can be calculated by *Theorem 1* for $l = -1$ and for $l = d-1$ respectively.

Proof: According to the preliminaries in Section II , we know that $\|\mathcal{T}(\mathcal{F})\|_\infty < \gamma$ is equivalent to J_N has a minimum J^m_N over the variables $\{x(0), w(0), \cdots, w(N)\}$

and $\check{u}(t)$ can be chosen such that $J_N^m > 0$. From (17), a necessary condition for J_N to be positive for all variables $\{x(0), w(0), \cdots, w(N)\}$ is 1), 2) and 3). The minimum of the J_N is given by (24). By using UDL factorization of $Q_{w_s}(t)$, J_N^m can be written as

$$J_N^m = \sum_{t=0}^{N} (\check{u}(t) - \bar{u}(t))^* \triangle_{R,t}^{-1} (\check{u}(t) - \bar{u}(t)) + \sum_{t=0}^{N} \left(Y_s(t) - \bar{H}_t \widehat{\bar{x}}(t \mid t-1)\right)^* \bar{Q}_{w_s}^{-1}(t) \times \left(Y_s(t) - \bar{H}_t \widehat{\bar{x}}(t \mid t-1)\right), \quad (47)$$

where

$$\triangle_{R,t} = \begin{cases} -\gamma^2 \left(\triangle_t'\right)^{-1} + K_{u,t} P_{t,t-1}^t K_{u,t}^* - K_{u,t} P_{t,t-1}^t H_t^* \\ \quad \times Q_{w_s}^{-1}(t) H_t P_{t,t-1}^t K_{u,t}^*, \quad 0 \le t < d, \\ -\gamma^2 \left(\triangle_t'\right)^{-1} + K_{u,t} P_{t,t-1}^t K_{u,t}^* \\ \quad - \left[K_{u,t} P_{t,t-1}^t H_t^* K_{u,t} \left(R_{t,t_d-1}^{t_d}\right)^* M_{t_d}^*\right] \\ \quad \times \bar{Q}_{w_s}^{-1}(t) \begin{bmatrix} H_t P_{t,t-1}^t K_{u,t}^* \\ M_{t_d} R_{t,t_d-1}^{t_d} K_{u,t}^* \end{bmatrix}, \quad t \ge d, \end{cases},$$

$$\triangle_t' = Q_t^c + G_{2,t}^* P_{t+1}^c G_{2,t} - G_{2,t}^* P_{t+1}^c G_{1,t} \left(R_{G^c,t}'\right)^{-1} \times G_{1,t}^* P_{t+1}^c G_{2,t},$$

$$R_{G^c,t}' = -\gamma^2 Q_t^w + G_{1,t}^* P_{t+1}^c G_{1,t},$$

$$\bar{Q}_{w_s}(t) = \begin{cases} H_t P_{t,t-1}^t H_t^* + R_t^v, & 0 \le t < d \\ \begin{bmatrix} H_t P_{t,t-1}^t H_t^* + R_t^v & H_t \left(R_{t,t_d-1}^{t_d}\right)^* M_{t_d}^* \\ M_{t_d} R_{t,t_d-1}^{t_d} H_t^* & M_{t_d} P(t_d) M_{t_d}^* + R_t^{v_z} \end{bmatrix}, & t \ge d. \end{cases}$$

and where $\triangle_{R,t}$ is the Schur complement of $\bar{Q}_{w_s}(t)$ in $Q_{w_s}(t)$. We have

$$\bar{u}(t) = -\bar{K}_{u,t} \widehat{\bar{x}}(t \mid t-1) - K_{k,t} \times \bar{Q}_{w_s}^{-1}(t) \left(Y_s(t) - \bar{H}_t \widehat{\bar{x}}(t \mid t-1)\right) \quad (48)$$

with $K_{k,t}$ is as in (45). In view of condition 3) on $Q_{w_s}(t)$, we have $\bar{Q}_{w_s}(t) > 0$ and $\triangle_{R,t} < 0$.

Thus we choose the control signal to be $\check{u}(t) = \bar{u}(t)$ which renders J_N^m positive. At the same time, the above necessary condition is also sufficient. From (25), we use re-organized innovation sequence to calculate the value of $\widehat{\bar{x}}(t \mid t-1)$ by *Theorem 1*. This control strategy $\check{u}(t)$ which satisfies the H_∞ performance requirement is referred to as the central controller. ■

IV. CONCLUSION

The H_∞ measurement-feedback control problem for linear discrete-time systems with delayed measurement has been studied in this paper. By introducing a Krein space state model, the delayed H_∞ control problem has been transformed into a full information control and an H_2 optimal estimation problem for measurement delayed systems. A necessary and sufficient condition is derived by using the re-organized innovation analysis. The measurement-feedback controller is calculated by performing two Riccati equations with the same dimension as the original system. The presented approach can be easily extended to the H_∞ control problems for the continuous-time systems with multiple-time delays and for continuous-time systems with delayed measurements. The necessary and sufficient condition for the existence of the H_∞ controller will be derived by using a similar discussion.

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