Heavy Traffic Control Policies for Wireless Systems with Time-Varying Channels

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Abstract— **Heavy traffic methods are well-studied in wireline queueing networks but only recently have been applied to wireless queueing systems with their random environments (due to channel variations, multi-access interference, etc.). Under the heavy traffic method, one can obtain a limit model approximating the queueing dynamics and it typically has the form of a stochastic differential equation with reflection (SDER). In one of the first works applied to wireless ([2]), an important assumption was made on the reflection process in order to get existence and uniqueness of a SDER limit model. Here we obtain the optimal control policies in a wireless system with the SDER limit model using numerical methods for continuous-time stochastic control problems. The simulation results indicate the influence of the reflection process on the optimal control polices which motivates further investigation into the strength of the reflection assumption. The optimal policies have the form of a "Max Weight" discipline and are similar to results obtained analytically under simplifying assumptions.**

I. Introduction

Heavy traffic methods for analyzing wireline queueing networks have been extensively studied (see the comprehensive exposition in [1] and the references therein). In the heavy traffic method for these applications, one applies a central limit-type scaling to the queue balance equations using a parameter n, where time is sped up by a factor of n and the state-space is scaled by a factor of $1/\sqrt{n}$. By heavy traffic we mean, loosely speaking, that the system is operating at near capacity. One applies weak convergence methods ([4]) to obtain a limit process (as $n \to \infty$) approximating the queueing dynamics, where the limit process often satisfies a stochastic differential equation with reflection (SDER) (we say "often" since the limit model may also have jump terms, which we do not consider here, but see [5]). In particular, for our example (described in section II)

$$x(t) = x(0) + \int_0^t b(u, x(s))\, ds + w(t) + z(t), \qquad (1)$$

where x is the (scaled) queue size vector, $b(u, x)$ is the drift term, u is a reserve control, and w is a Weiner process and z is the reflection term which constrains $x \geq 0$. The power of the heavy traffic method is that the limit model eliminates inessential detail. Another feature is that the optimal policies obtained by applying stochastic control methods with the dynamics in (1) often work well in the actual system when far from the heavy traffic condition.

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Only a few works have applied heavy traffic modeling to wireless systems where the main new feature is the random environment due to the channel variations, multi-access interference, etc. ([2], [3]). For the model in [2], outlined in section II, one applies a scaling dependent on the channel process to get the limit dynamics in (1), under an assumption on the reflection process, namely the completely-$\mathcal{S}$ condition. Stochastic control methods are applied to determine how to allocate resource u (which can have several meanings , see [2]). Here, u represents a "reserve" power whereas most of the power (the "nominal" power, $\bar{p}$) is preallocated such that mean arrival rate equals the mean departure rate; this is our heavy traffic condition (see equation (2)). For a cost rate penalizing (weighted) queue size, under some simplifying assumptions, one can analytically obtain the optimal policy (see [2]) which states that all u is allocated to the queue which maximizes a (weighted) product of queue size with a term incorporating the channel quality; i.e. the optimal policy is a Max Weight discipline. For the model in [3], *all* of the power can be allocated and a Max Weight policy is shown to induce a state-space collapse and is optimal in the sense of minimizing workload.

In section II we summarize the model in [2], noting the completely-$\mathcal{S}$ assumption made on the reflection process. In section III, under this assumption on the reflection, we obtain the optimal policy for the reserve power u using the numerical methods in [5]. The optimal policy has a Max Weight structure and the reflection process is seen to influence its form. This motivates further investigation of the strength of the assumption on the reflection directions for various cases in order to better understand the limitations of the control policies obtained using these stochastic control methods. The optimal policy is also used in initial simulations of a real time queueing system and shows good performance compared to a heuristic policy.

II. Heavy traffic model

The highlights of the wireless system model, the scaled queue balance equations, and the limit SDER are given here; for full detail see [2]. We consider a one-cell forward-link wireless system with K independent queues at the base station with random data arrivals each dedicated to a mobile. The *vector* channel state is indexed $j \in \mathcal{J}$; each j specifies the channel for all K mobiles. We suppose that j specifies a vector $\bar{\lambda}^d(j)$, the achievable transmission rate/unit power and we suppose that j evolves according to a semi-Markov

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process. The system is in heavy traffic in the sense that most of the available power (i.e. nominal power, $\bar{p}$) balances the mean arrival rate:

$$\Lambda_i^a = \sum_{j\in\mathcal{J}} \bar{\lambda}_i^d(j)\bar{p}_i(j)\pi(j), \tag{2}$$

where i indexes the queues and π is the stationary distribution of the channel. The remaining power u is the reserve and we are interested in allocating it optimally among the queues (see section III). In heavy traffic, a small reserve power can have a large affect on the queues.

We will scale the queue balance equations with parameter n giving an embedded set of systems indexed by n. Assume that the mean arrival rate of the data is $O(n)$. The channel process is denoted $L^n(t)$ and it changes at a rate $O(n^\nu)$, $\nu \in (0,1)$. Let $\gamma = 1-\nu/2$, $x_i^n(t)$ be $1/n^\gamma$ times the content in queue i, and $A_i^n(t)$ be $1/n^\gamma$ times the number of arrivals to queue i by time t. When a queue is empty we allow reallocation of its power to the other queues. The scaled queue balance equations are given by

$$x_i^n(t) = x_i^n(0) + A_i^n(t) - D_i^n(t) - \sum_k y_{ki}^n(t),$$

where $y_{ki}^n(t)$ represents the service due to reallocation of power from empty queue k to queue i. The departure process is given by

$$\begin{aligned} D_i^n(t) = \frac{n}{n^\nu}\int_0^t \sum_{j\in\mathcal{J}} I_{\{L^n(t)=j\}}\bar{\lambda}_i^d(j) \\ \times\left[\bar{p}_i(j) + \frac{u_i(j,x^n(t))}{n^{\nu/2}}\right] \times I_{\{x_i^n(s)>0\}}\, ds, \end{aligned} \tag{3}$$

where $I_{\{\cdot\}}$ is the indicator function and $I_{\{x_i^n(s)>0\}}$ constrains the queues from being negative. Centering about the mean dynamics under the nominal power $\bar{p}$ and applying the heavy traffic condition (2), one can obtain (see [2])

$$\begin{aligned} x_i^n(t) &= x_i^n(0) + M_i^{a,n}(t) - M_i^{d,n}(t) + z_i^n(t) \\ &\quad - \int_0^t \sum_{j\in\mathcal{J}} I_{\{L^n(s)=j\}}\bar{\lambda}_i^d(j)u_i(j,x^n(s))\, ds, \\ z_i^n(t) &= y_i^n(t) - \sum_{j,k\neq i} y_{ki}^n(j,t), \end{aligned} \tag{4}$$

where the $M_i^{a,n}(t)$ and $M_i^{d,n}(t)$ terms are the centered (about the mean flow) arrival and departure process. In $M_i^{d,n}(t)$ the queues potentially are allowed to go negative (i.e. the term $I_{\{x_i^n(s)>0\}}$ in (3) is removed) but this is compensated for by the the $y_i^n(t)$ which represents the work that could have been done using the nominal power $\bar{p}$, had the queues not been empty. We have

$$\begin{aligned} M_i^{d,n}(t) = \frac{1}{n^{\nu/2}}\int_0^{tn^\nu} \Big[\sum_{j\in\mathcal{J}} I_{\{L(s)=j\}} \\ -\pi(j)\Big] \times \bar{\lambda}_i^d(j)\bar{p}_i(j)\, ds. \end{aligned}$$

Comments on the Reflection Process. Both terms in (4) depend on the scaled idle time for queue i given by

$$T_i^n(j,t) = \frac{n}{n^\gamma}\int_0^t I_{\{L^n(s)=j\}} I_{\{x_i^n(s)=0\}}\, ds,$$

i.e. the reflection directions depend on the channel conditions when a queue goes empty. This is different from the wireline case in that now we can have multiple reflection directions on a boundary. For example, Figure 1 shows the reflection directions for a two-queue, two-channel case when all the reserve power is assigned to one of the queues at a given time. All of the reserve power is reallocated to the other (nonempty) queue when the other queue empties. In the left of Figure 1, for $j=1$ the power is reassigned to a less efficient queue (i.e. $\bar{\lambda}_1^d(1) < \bar{\lambda}_2^d(1)$) when queue 2 goes empty; similary when queue 1 empties and $j = 2$. Fixing these directions, the completely-$\mathcal{S}$ condition is satisfied (which roughly says the reflection direction point hard enough into the state space, but see [6], [2] for precise statements). These directions are the most probable since the queue is most likely to empty in its best channel state. However, it may be possible that the queues empty in opposite states, shown in the right side of Figure 1. Here the completely-$\mathcal{S}$ condition is not satisfied but one would like to quantify the probability of this occurrence.

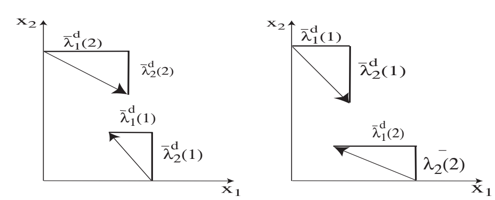

Fig. 1. Reflections. Left: Completely-$\mathcal{S}$, Right: Not Completely-$\mathcal{S}$

Weak convergence. One can apply weak convergence techniques to (4) to obtain the limiting form (1). We will only comment on the results; full detail is in [2]. $M_i^{a,n}(t)$ is shown to weakly converge to the zero process due to the tightness of the arrival process under a conventional (wireline) heavy traffic scaling of $1/\sqrt{n}$ of the state space along with the fact that here the state space is scaled by $1/n^\gamma$, $\gamma > 0.5$. The $M_i^{d,n}(t)$ can be shown to weakly converge to a Weiner process using perturbed test function methods provided there is strong enough mixing in the channel process, $L^n(t)$ [7]. The term with the reserve power $u_i(j,x)$ weakly converges to the drift term using the same techniques as for $M_i^{d,n}(t)$.

III. Stochastic Control Problem

A stochastic control problem is solved to get the optimal reserve power: the dynamics are given by (1) and we consider a cost function to minimize with respect to u:

$$W(x,u) = E_x \int_0^\infty e^{-\beta s} \Big[\sum_i \alpha_i x_i^{p_i}(s) + c'(x(s))\, dz(s) \Big] ds,$$
$$\beta > 0 \text{ and small}, \ \ \alpha_i > 0, \ \ p_i > 1, \quad (5)$$

and $c'(\cdot)$ specifies the boundary cost. An analytical solution is not available and so the numerical methods (in particular, approximation in policy space) in [5] were used to obtain the optimal control. The reflection directions were either specified to be orthogonal to the boundary or, when reallocation of power was allowed, a randomization of the directions was implemented to obtain a single direction associated with a boundary while satisfying the completely-$\mathcal{S}$ condition.

In Figure 2 are policies obtained for a simple two-queue, two-channel case with buffer size $B = 5$, $\beta = 0.01$, $\alpha_1 = \alpha_2 = 1$, $p_1 = p_2 = 2$, and the component of c in (5) for the upper boundaries ($B = 5$) is 250 and for the lower boundaries ($B = 0$) it is 10. The policies have the Max Weight form—all of the reserve power is applied to a single queue at any point in the queue state space. This is the form derived analytically in [2] but under simplifying assumptions which included neglecting the reflection; in [2] the optimal policies formed "wedges" eminating from the origin. Here the reflection results in deviations from this wedge: note in the lower right plot in Figure 2 that near the origin the reserve power is allocated to queue 2 even when queue 1 is smaller. On can surmise that this may be due to the reallocation since the power is reallocated to queue 1 when queue 2 empties. This simple example highlights the importance of the reflection directions on the control policy and motivates further investigation of affects of the reflection directions and the strength of the completely-$\mathcal{S}$ assumption.

Some initial real-time simulations with Poisson arrivals and departures specified by the rates $\bar{\lambda}_i^d(j) \times (\bar{p}_i(j) + u_i(j,x))$ using the heavy traffic policies show promising results. Figure 3 is a case comparing against "wedge policy" (i.e., for any j, the boundary is the 45-degree line in Figure 2). There are many variables in this simulation (arrival process, channel process, closeness to the heavy traffic condition) and full scale simulation study is planned for a future paper.

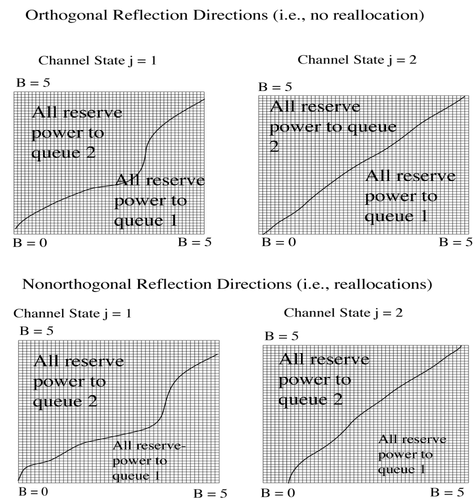

Fig. 2. Optimal reserve control policies as a function of the queue state space and channel state. The channel state is given by $(\bar{\lambda}_1^d(1), \bar{\lambda}_2^d(1), \bar{\lambda}_1^d(2), \bar{\lambda}_2^d(2)) = (0.06, 0.1, 0.08, 0.08)$.

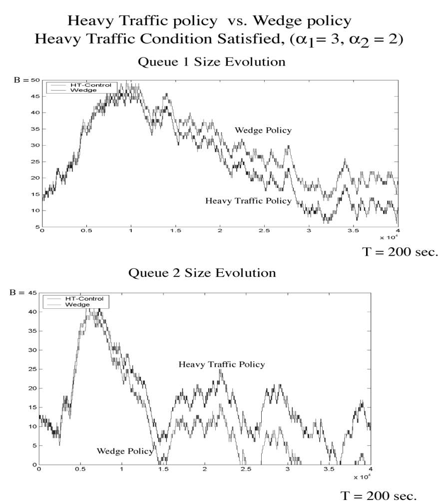

Fig. 3. Heavy traffic vs. wedge policy. Pathwise cost (analog to (5) with $\alpha_1 = 3, \alpha_2 = 2$) 8.5% lower with heavy traffic than wedge policy

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