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Robust Direct Adaptive Control of Nonlinear Uncertain Systems with Unknown Disturbances

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Abstract— **This paper addresses a direct adaptive control approach to the problems of adaptive stabilization of a class of multivariable nonlinear systems affected by time-varying uniformly bounded affine perturbations, which are confined to a characterized class. The proposed framework is Lyapunov-based, and the nominal control has augmented with a robustifying component, guaranteed asymptotic stability of the closed-loop systems. Furthermore, the framework can also extended to solve the problems with exogenous disturbances, uncertain nonlinearities and unmodeled dynamics. As applications of our results we address the problems of control of chaotic oscillator, and the tracking control of one-link rigid robot under gravitation field and flexible joint robot.**

Keywords:Direct adaptive control, adaptive stabilization, exogenous disturbances

I. Introduction

In the rapidly growing research on nonlinear control theory, much work has focused in the problems of uncertainties exist in the system model or systems with unknown disturbances and nonlinearities. A direct adaptive control framework for adaptive stabilization, disturbance rejection, and command following of multivariable nonlinear uncertain systems with exogenous disturbances, where the bounded disturbances were assumed to be a known vector, has developed in [1] and guarantees partial stability of the closed-loop system. However, it is worth to note that the disturbances may be the result of unmodeled dynamics, noisy measurements, parameter uncertainty, or non dissipative forces affecting the plant, and most of time not available for the control design.

Motivated by the result of robust stabilization of nonlinear systems affected by time-varying uniformly bounded affine disturbances [2], where a sliding mode technique and passive-based control framework has formulated and achieved global uniform convergence. With direct adaptive scheme, our framework guarantees that the closed-loop system is Lyapunov stable. In addition, the asymptotic stable of solution x with respect to origin can be proved.

In this paper, we applied a Lyapunov-based direct adaptive control framework for multivariable nonlinear uncertain systems with time-varying uniformly bounded affine disturbances, where the disturbances can be characterized by combination of unknown constants and known continuous function. Our feedback control design, the nominal control has augmented with a robustifying component, guaranteed asymptotic stability. In addition, the states associated with the adaptive controller gains is Lyapunov stable. Finally, several numerical examples are presented to demonstrate the efficacy of the proposed framework.

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II. Adaptive Control for Uncertain Nonlinear Systems with Unknown Disturbances

Our main concern in this paper is to deal with uncertain nonlinear systems perturbed by affine disturbances, we begin by considering the problem of characterizing adaptive feedback control laws for nonlinear uncertain MIMO systems $\mathcal{G}$ given by

$$\dot{x}(t) = f(x(t)) + G(x(t))u(x(t)) + J(x(t))w(t, x(t)), \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $x(0) = x_0$, $u : \mathbb{R}^n \to \mathbb{R}^m$ is the control vector, $f : \mathbb{R}^n \to \mathbb{R}^n$ characterize system dynamics with uncertain entries, and $f(0) = 0$. $G : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ and $J : \mathbb{R}^n \to \mathbb{R}^{n\times d}$ are the input and disturbance weighting matrix functions, respectively, with unknown entries. In addition, the disturbance vector $w : \mathbb{R}\times\mathbb{R}^n \to \mathbb{R}^{d\times d}$ is directly applied from [2], and given as following

Assumption 2.1: The vector function w is bounded, and can be characterized by

$$w(t, x) \le \overline{w}(x, t)\theta_1 + \theta_2, \quad (2)$$

where $\theta_1 \in \mathbb{R}^d$ and $\theta_2 \in \mathbb{R}^d$ are unknown constants, and $\overline{w} : \mathbb{R}\times\mathbb{R}^n \to \mathbb{R}^{d\times d}$ is a known continuous matrix function. It is important to note that the disturbance $w(t, x)$ may be the result of unmodeled dynamics, noisy measurement, parameter uncertainty or exogenous disturbances. For the nonlinear system $\mathcal{G}$, we assume that the existence and uniqueness of solutions are satisfied and zero-state observability of (1) with $w(t) \equiv 0$. Furthermore, assume there exits $F : \mathbb{R}^n \to \mathbb{R}^s$ with $F(0) = 0$, $K_\mathrm{g} \in \mathbb{R}^{m\times s}$, and $\hat{G} : \mathbb{R}^n \to \mathbb{R}^{m\times m}$ such that

$$f_\mathrm{c}(x(t)) \triangleq f(x(t)) + G(x(t))\hat{G}(x(t))K_\mathrm{g}F(x(t)), \quad (3)$$

is globally asymptotically stable, and a scalar function $V_\mathrm{s} : \mathbb{R}^n \to \mathbb{R}$ is Lyapunov function, and

$$V_\mathrm{s}'(x)f_\mathrm{c}(x) = -\ell^\mathrm{T}(x)\ell(x), \quad \forall x \in \mathbb{R}^n, \quad (4)$$

where, $\ell : \mathbb{R}^n \to \mathbb{R}^t$. Next, we state and prove the main results of this paper.

Theorem 2.1: Consider the nonlinear uncertain system $\mathcal{G}$ given by (1) is zero state observable with $w(x, t) \equiv 0$, where the disturbances $w(x,t)$ satisfy Assumption 2.1. Assume that the zero solution of (1) defined in (3) is globally asymptotically stable. Furthermore, there exists a matrix $\Psi \in \mathbb{R}^{m\times d}$ and a function $\hat{J}:\mathbb{R}^n \to \mathbb{R}^{m\times m}$ such that the matching condition $G(x)\hat{J}(x)\Psi = J(x)$ is satisfied. Then the adaptive feedback control law

$$\begin{aligned} u(x(t)) &= \hat{G}(x(t))K(t)F(x(t)) \\ &\quad +\hat{J}(x(t))\Phi(t)(\overline{w}(x,t)\hat{\theta}_1+\hat{\theta}_2), \end{aligned} \tag{5}$$

where $K(t)\in\mathbb{R}^{m\times n}$, $\Phi(t)\in\mathbb{R}^{m\times d}$, $\tilde{\theta}_1 \triangleq \hat{\theta}_1-\theta_1$, and $\tilde{\theta}_2 \triangleq \hat{\theta}_2-\theta_2$. Now, let design matrices $P_1>0$, $P_2>0$, $Q_1>0$, $Y>0$, $Q_2>0$, and $Z>0$ with the update laws

$$\dot{K} = -\frac{1}{2}Q_1\hat{G}^{\mathrm{T}}(x)G^{\mathrm{T}}(x)V_{\mathrm{s}}'^{\mathrm{T}}(x)F^{\mathrm{T}}(x)Y, \tag{6}$$

$$\begin{aligned} \dot{\Phi} &= -\frac{1}{2}Q_2\hat{J}^{\mathrm{T}}(x)G^{\mathrm{T}}(x)V_{\mathrm{s}}'^{\mathrm{T}}(x)(\overline{w}(x,t)\hat{\theta}_1 \\ &\quad +\hat{\theta}_2)^{\mathrm{T}}Z, \end{aligned} \tag{7}$$

and

$$\dot{\hat{\theta}}_1 = \frac{1}{2}P_1^{-1}\overline{w}^{\mathrm{T}}(x,t)J^{\mathrm{T}}(x(t))V_{\mathrm{s}}'^{\mathrm{T}}(x(t)), \tag{8}$$

$$\dot{\hat{\theta}}_2 = \frac{1}{2}P_2^{-1}J^{\mathrm{T}}(x(t))V_{\mathrm{s}}'^{\mathrm{T}}(x(t)), \tag{9}$$

where $V_{\mathrm{s}}'(x) \triangleq \frac{\partial V_{\mathrm{s}}(x)}{\partial x}$ guarantees that the closed-loop system, given by (1), (5), and (6) to (9) is Lyapunov stable. Furthermore, if (4) applied and let the output $y(t)\triangleq \ell(x)$, then $\ell(x)\to 0$ as $t\to\infty$, and asymptotic stable of solution x with respect to origin will arrive when $\ell^{\mathrm{T}}(x)\ell(x)>0$.

Proof: To show Lyapunov stability of the closed-loop system (1), (5), and (6) to (9), we first consider the Lyapunov function candidate

$$\begin{aligned} V(x,K,\Phi,\tilde{\theta}_1,\tilde{\theta}_2) &= V_{\mathrm{s}}(x)+\tilde{\theta}_1^{\mathrm{T}}P_1\tilde{\theta}_1+\tilde{\theta}_2^{\mathrm{T}}P_2\tilde{\theta}_2 \\ &+ \operatorname{tr} Q_1^{-1}(K-K_{\mathrm{g}})Y^{-1}(K-K_{\mathrm{g}})^{\mathrm{T}} \\ &+ \operatorname{tr} Q_2^{-1}(\Phi+\Psi)Z^{-1}(\Phi+\Psi)^{\mathrm{T}}, \end{aligned} \tag{10}$$

where $V_{\mathrm{s}}(x)$ satisfies (4). Note that $V(0,K_{\mathrm{g}},-\Psi,0,0)=0$ and $V(x,K,\Phi,\tilde{\theta}_1,\tilde{\theta}_2)>0$ for all $(x,K,\Phi,\tilde{\theta}_1,\tilde{\theta}_2)\neq(0,K_{\mathrm{g}},-\Psi,0,0)$. In addition, $V(x,K,\Phi,\tilde{\theta}_1,\tilde{\theta}_2)$ is radially unbounded. Furthermore, $V(\cdot,K,\Phi,\tilde{\theta}_1,\tilde{\theta}_2)$ and K are continuous in x for $t\geq 0$. The corresponding Lyapunov derivative is given by

$$\begin{aligned} \dot{V} &= V_{\mathrm{s}}'(x)[f(x)+G(x)u(t)+J(x(t))(\overline{w}(x,t)\theta_1+\theta_2)] \\ &\quad +2\mathrm{tr}\, Q_1^{-1}(K-K_{\mathrm{g}})Y^{-1}\dot{K}^{\mathrm{T}}+2\dot{\hat{\theta}}_1^{\mathrm{T}}P_1\tilde{\theta}_1 \\ &\quad +2\mathrm{tr}\, Q_2^{-1}(\Phi+\Psi)Z^{-1}\dot{\Phi}^{\mathrm{T}}+2\dot{\hat{\theta}}_2^{\mathrm{T}}P_2\tilde{\theta}_2 \\ &= V_{\mathrm{s}}'(x)f_{\mathrm{c}}(x)+V_{\mathrm{s}}'(x)G(x)[u-\hat{G}(x)KF(x) \\ &\quad -\hat{J}(x)\Phi(\overline{w}(x,t)\hat{\theta}_1+\hat{\theta}_2)]+2\mathrm{tr}\, Q_2^{-1}(\Phi+\Psi)Z^{-1}\dot{\Phi}^{\mathrm{T}} \\ &\quad +V_{\mathrm{s}}'(x)G(x)\hat{G}(x)(K-K_{\mathrm{g}})F(x) \\ &\quad +2\mathrm{tr}\, Q_1^{-1}(K-K_{\mathrm{g}})Y^{-1}\dot{K}^{\mathrm{T}} \\ &\quad +V_{\mathrm{s}}'(x)G(x)\hat{J}(x)\Phi(\overline{w}(x,t)\hat{\theta}_1+\hat{\theta}_2)+2\dot{\hat{\theta}}_1^{\mathrm{T}}P_1\tilde{\theta}_1 \\ &\quad -V_{\mathrm{s}}'(x)J(x)\overline{w}(x,t)\tilde{\theta}_1-V_{\mathrm{s}}'(x)J(x)\tilde{\theta}_2+2\dot{\hat{\theta}}_2^{\mathrm{T}}P_2\tilde{\theta}_2 \\ &= V_{\mathrm{s}}'(x)f_{\mathrm{c}}(x). \end{aligned} \tag{11}$$

Next, by (4) the resulting Lyapunov derivative along the system trajectory is

$$\dot{V}(x,K,\Phi,\tilde{\theta}_1,\tilde{\theta}_2) = -\ell^{\mathrm{T}}(x)\ell(x)\leq 0. \tag{12}$$

This complete the proof. Furthermore, if $\ell(x)\to 0$ as $t\to\infty$, and asymptotic stable of solution x with respect to origin will arrive when $\ell^{\mathrm{T}}(x)\ell(x)>0$. ■

We further extend the above result to the case where the entries of the system matrix and the input matrix are uncertain. Note that the adaptive control law (5) does not require explicit knowledge of the desire gain matrix K_{g}, disturbances $w(x.t)$, system dynamics f, and matching matrix Ψ. Theorem 2.1 also requires that the zero solution to (3) is globally asymptotically stable. Next, we consider the case where $f(x)$, input weighting matrix $G(x)=B$ and disturbance weighting matrix $J(x)=D$ are uncertain. Specifically, given as the following

$$\dot{x}(t) = f(x(t))+Bu(t)+Dw(t,x(t)), \tag{13}$$

where, $w(t,x)$ satisfies Assumption 2.1, and let there exists a $K_{\mathrm{g}}\in\mathbb{R}^{m\times s}$ such that

$$f_{\mathrm{c}}(x(t)) \triangleq f(x(t))+BK_{\mathrm{g}}F(x(t)), \tag{14}$$

is global asymptotically stable. We assume that $B=[0_{m\times(n-m)},B_{\mathrm{s}}]^{\mathrm{T}}$, where $B_{\mathrm{s}}\in\mathbb{R}^{m\times m}$ is the sign definite matrix with unknown entries; that is, $B_{\mathrm{s}}>0$ or $B_{\mathrm{s}}<0$. Let B_{s} is symmetric and sign definite matrix with the Schur decomposition $B_{\mathrm{s}}=UD_{B_{\mathrm{s}}}U^{\mathrm{T}}$, where U is orthogonal and $D_{B_{\mathrm{s}}}$ is real diagonal, where $|B_{\mathrm{s}}|=(B_{\mathrm{s}}^2)^{1/2}$. Hence, $|B_{\mathrm{s}}|^{-1}B^{\mathrm{T}}=[0_{m\times(n-m)},\mathcal{I}_m]=B_0^{\mathrm{T}}$, where $\mathcal{I}_m=I_m$. We then define $B_0\triangleq[0_{m\times(n-m)},I_m]^{\mathrm{T}}$ for $B_{\mathrm{s}}>0$, and $B_0\triangleq[0_{m\times(n-m)},-I_m]^{\mathrm{T}}$ for $B_{\mathrm{s}}<0$, as in [1]. By Similar characterization, assume that there exists a D_{s}, which is a symmetric and sign definite matrix. It follows that the Schur decomposition $D_{\mathrm{s}}=U_{\mathrm{d}}D_{D_{\mathrm{s}}}U_{\mathrm{d}}^{\mathrm{T}}$, where U_{d} is orthogonal and $D_{D_{\mathrm{s}}}$ is real diagonal matrix. Hence, $D|D_{\mathrm{s}}|^{-1}=[0_{d\times(n-d)},\mathcal{I}_d]=D_0$, where $\mathcal{I}_d=I_d$ for $D_{\mathrm{s}}>0$ and $\mathcal{I}_d=-I_d$ for $D_{\mathrm{s}}<0$.

Corollary 2.1: Consider the nonlinear uncertain system given by (13) is zero state observable. Let the sign definiteness matrices B_{s} and D_{s} are known, and (14) is satisfied. Then, the adaptive feedback control law

$$u(t) = K(t)F(x(t))+\Phi(t)(\overline{w}(x,t)\hat{\theta}_1+\hat{\theta}_2), \tag{15}$$

with the update laws

$$\dot{K}(t) = -\frac{1}{2}B_0^{\mathrm{T}}V_{\mathrm{s}}'^{\mathrm{T}}(x(t))F^{\mathrm{T}}(x(t))Y, \tag{16}$$

$$\dot{\Phi}(t) = -\frac{1}{2}B_0^{\mathrm{T}}V_{\mathrm{s}}'^{\mathrm{T}}(x(t))(\overline{w}(x,t)\hat{\theta}_1+\hat{\theta}_2)^{\mathrm{T}}Z, \tag{17}$$

and

$$\dot{\hat{\theta}}_1 = \frac{1}{2}\overline{w}^{\mathrm{T}}(x,t)D_0^{\mathrm{T}}V_{\mathrm{s}}'^{\mathrm{T}}(x(t)), \tag{18}$$

$$\dot{\hat{\theta}}_2 = \frac{1}{2}D_0^{\mathrm{T}}V_{\mathrm{s}}'^{\mathrm{T}}(x(t)), \tag{19}$$

guarantees that the closed-loop system, given by (13), (15), and (16) to (19) is Lyapunov stable. Furthermore, if (14) applied and let the output $y(t) \triangleq \ell(x)$, then $\ell(x) \to 0$ as $t \to \infty$, and asymptotic stable of solution x with respect to origin will arrive when $\ell^{\mathrm{T}}(x)\ell(x) > 0$.

Proof: The result is a direct extension of Theorem 2.1. Let $\hat{G}(x) = I_m$ and $\hat{J}(x) = I_m$, and the matching condition be $B\hat{J}(x)\Psi = D$. In addition, since P_1 is positive definite symmetric matrix, assume that $P_1^{-1}\overline{w}^{\mathrm{T}}(x,t) = \overline{w}^{\mathrm{T}}(x,t)P_1^{-1}$. Next, since Q_1, Q_2, P_1, and P_2 are arbitrary positive definite matrices, let Q_1 be replaced by $q_1|B_{\mathrm{s}}|^{-1}$, Q_2 be replaced by $q_2|B_{\mathrm{s}}|^{-1}$, P_1 be replaced by $q_3|D_{\mathrm{s}}|$, and P_2 be replaced by $q_4|D_{\mathrm{s}}|$, where $q_i > 0$, $i = 1, 2, 3, 4$ are arbitrary real numbers. Finally, (6) and (7), with q_1Y and q_2Z replaced by Y and Z, implies (16), and (17), respectively. Let (8) and (9) with $q_3 = q_4 = 1$, implies (18), and (19), respectively. ■

Note that the above frameworks of Theorem 2.1 and Corollary 2.1 can applied to linear system when $f(x) = Ax$ and $f_{\mathrm{c}}(x) = A_{\mathrm{c}}x$, where A_{c} is an asymptotically stable matrix. Also, Theorem 2.1 and Corollary 2.1 are valid for nonlinear time-varying uncertain systems, specifically

$$\dot{x}(t) = f(t,x) + G(t,x)u(x) + J(t,x)w(t,x), \qquad (20)$$

and tracking problems, given by

$$\dot{e}(t) = f(e(t)) + G(e(t))u(x) + J(e(t))w(t,e(t)), \quad (21)$$

where $e(t) \triangleq x(t) - r_{\mathrm{d}}(t)$ is tracking error.

III. Numerical Examples

In this section we illustrate the utility of the proposed direct adaptive control framework in the control problems of chaotic oscillator [2], one-link rigid robotic manipulator given by [3], and flexible joint robot manipulator [1], [4].

A. The van der Pol oscillator

The first example is a well known perturbed van der Pol equation used to model electrical circuit with triode valve [2], and given as following

$$\ddot{v} + \mu(1 - v^2)\dot{v} + v = u + q\cos(\omega t), \qquad (22)$$

where particular chosen $\mu = 5$, $q = 5$, and $\omega = 2.463$, which exhibits chaotic behaviour, and u is control input. Next (22) rewritten as state space form with $x = [v, \dot{v}]^{\mathrm{T}} = [x_1, x_2]^{\mathrm{T}}$, then

$$\begin{aligned} f(x) &= \begin{bmatrix} x_2 \\ -\mu(1 - x_1^2)x_2 - x_1 \end{bmatrix}, \quad G(x) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ J(x) &= 1, \quad \theta_1 = \begin{bmatrix} 0 \\ q \end{bmatrix}, \end{aligned}$$

and $\overline{w}(x,t) = \cos(\omega t)$. Next, let

$$F(x) = \begin{bmatrix} x_2 \\ x_1^2 x_2 \end{bmatrix}, \quad K_{\mathrm{g}} = \begin{bmatrix} \mu - \beta \\ -\mu \end{bmatrix}^{\mathrm{T}}. \qquad (23)$$

$$A_{\mathrm{c}} = \begin{bmatrix} 0 & 1 \\ -1 & -\beta \end{bmatrix}. \qquad (24)$$

Specifically, we chose

$$\begin{aligned} R &= \mathrm{diag}[0.005, 12.5], \quad Z = 0.5, \quad \beta = 0.8, \\ Y &= 1, \quad P_1 = \mathrm{diag}[100, 25], \end{aligned}$$

and P is the solution of Lyapunov equation

$$A_{\mathrm{c}}^{\mathrm{T}}P + PA_{\mathrm{c}} + R = 0. \qquad (25)$$

The initial conditions given $x(0) = [1, 1]^{\mathrm{T}}$ and $K(0) = [0, 0]$. It follows from Corollary 2.1 that the closed-loop system (15) guarantees $x(k) \to 0$, as $k \to \infty$, if $\overline{w}(x,t) = 0$. Figure 1 shows the phase portrait of the controlled system. The adaptive controller regulate the perturbed system to the origin under no knowledge of system dynamics, matrix K_{g}, and disturbance, while the disturbance exist. Figures 2 illustrates the time response of the feedback gain $K(k)$ and the control inputs.

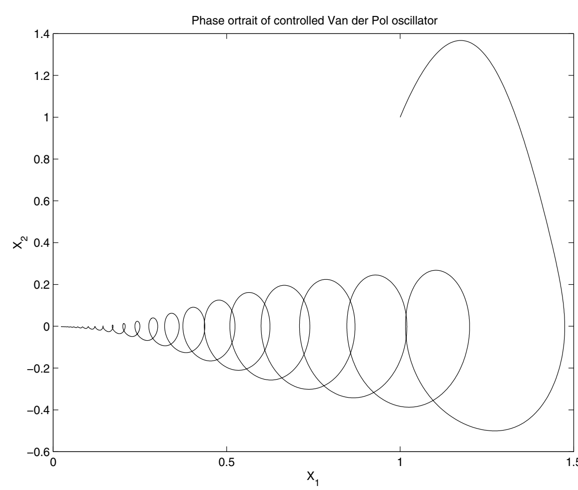

Fig. 1. Phase Plot of perturbed van der pol equation

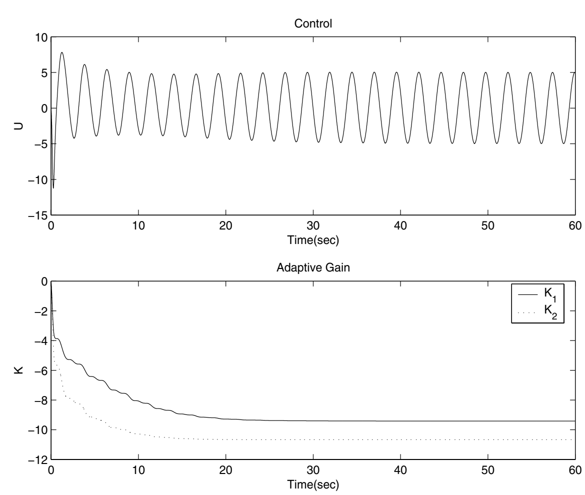

Fig. 2. Control Signal and Adaptive gains

B. One-Link Rigid Robot under Gravitation Field

The dynamic equation of the one-link rigid robot in state space form is given by [3], and placed on a tilted surface with an fixed angle ϕ

$$\begin{bmatrix} \dot{q} \\ \ddot{q} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{d}{ml^2} \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{ml^2} \end{bmatrix} u - \begin{bmatrix} 0 \\ \frac{g}{l} \end{bmatrix} \cos(q+\phi). \quad (26)$$

The reference model is defined as

$$\begin{bmatrix} \dot{q}_r \\ \ddot{q}_r \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -16 & -8 \end{bmatrix} \begin{bmatrix} q_r \\ \dot{q}_r \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r. \quad (27)$$

Next, we define the tracking error as

$$\begin{bmatrix} e \\ \dot{e} \end{bmatrix} \triangleq \begin{bmatrix} q \\ \dot{q} \end{bmatrix} - \begin{bmatrix} q_r \\ \dot{q}_r \end{bmatrix}. \quad (28)$$

Then, the tracking model can be formulated as

$$\begin{bmatrix} \dot{e} \\ \ddot{e} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{d}{ml^2} \end{bmatrix} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{ml^2} \end{bmatrix} u + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 16 & 1 & 1 & 1 & 1 \end{bmatrix} \overline{w} \begin{bmatrix} 1 \\ 8-\frac{d}{ml^2} \\ -\frac{g}{l}\cos(\phi) \\ \frac{g}{l}\sin(\phi) \\ 1 \end{bmatrix}, \quad (29)$$

where

$$\overline{w} = \begin{bmatrix} q_r & 0 & 0 & 0 & 0 \\ 0 & \dot{q}_r & 0 & 0 & 0 \\ 0 & 0 & \cos(q) & 0 & 0 \\ 0 & 0 & 0 & \sin(q) & 0 \\ 0 & 0 & 0 & 0 & r \end{bmatrix}. \quad (30)$$

Specifically, we chose

$$\begin{aligned} R &= \mathrm{diag}[100, 500], \quad Z = 8000, \quad Y = 6000, \\ P_1 &= \mathrm{diag}[10, .1, 0.02, 0.05, 0.02], \quad A_c = \begin{bmatrix} -2 & 1 \\ 0.5 & -3 \end{bmatrix}, \\ B_0 &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad r = \sin(20t). \end{aligned}$$

and P is the solution of Lyapunov equation (26). In addition, let $m = l = d = 1$ and $g = 9.8$. Here, the form of (30) is identical to (13), and Corollary 2.1 can be directly applied. The initial conditions given $e(0) = [0, 0]^{\mathrm{T}}$ and $K(0) = [0, 0]$. It follows from Corollary 2.1 that the closed-loop system (15) guarantees $x(t) \to 0$, as $t \to \infty$, if $\overline{w}(x, t) = 0$. Figure 3 shows the states for each time step. The adaptive controller regulate the perturbed system to the origin under no knowledge of system dynamics, matrix K_{g}, and disturbance, while the disturbance exist. Figures 4 illustrates the time response of the the control input, a constant force is applied to compensate the gravitation field. To demonstrate the robustness of the controller handle the uncertainty of the system dynamics, we introduce a changed to $m = 0.8$ at time $t = 0.5$ second. It shows that the controller readapt this sudden change and stabilize the system.

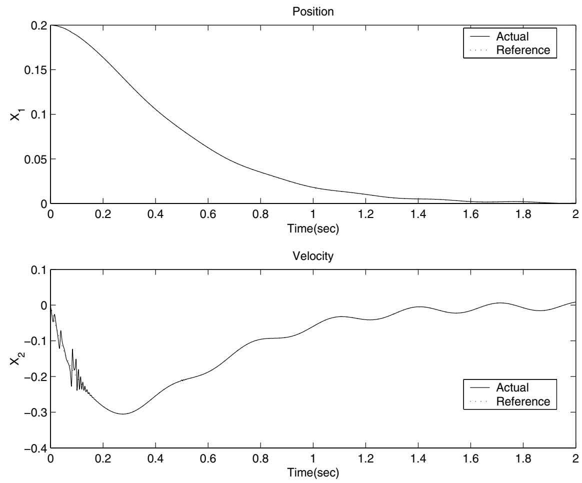

Fig. 3. The states of one-link Rigid Robot

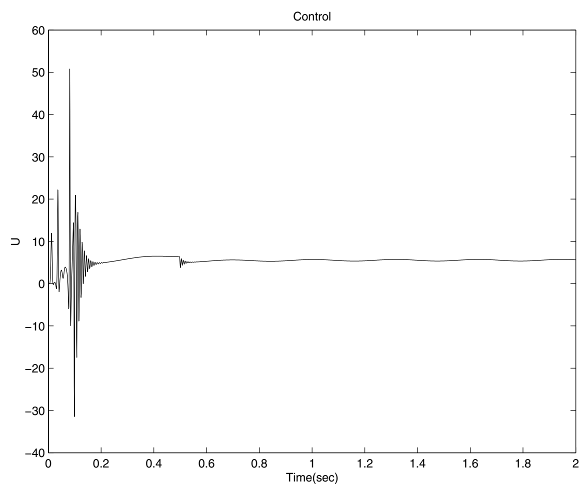

Fig. 4. Control signals

C. Flexible Joint Robot

Finally, we consider a robot manipulator with elastic joint and coupled damping which is represented by [5]

$$\begin{aligned} I\ddot{\theta}_1 + mgl\sin(\theta_1) - K(\theta_2 - \theta_1) &= 0, \quad (31) \\ J\ddot{\theta}_2 + \beta\dot{\theta}_2 + K(\theta_2 - \theta_1) &= u, \quad (32) \end{aligned}$$

where θ_1 and θ_2 are the link position and the actuator rotor position, as reflected through the gear ratios, respectively, u is control input (motor torque), I and J are the mass moments of inertia of the link and motor, respectively, K is spring constant, β is motor viscous friction, l is the distance from the joint axis to the link center of mass, and g is gravitational constant. Let r_{d} be the desired trajectory, and

$x_1 \triangleq \theta_1$, the state space form of (32) and (33) is given as

$$\begin{aligned}
\dot{x}_1 &= x_2, & (33)\\
\dot{x}_2 &= x_3, & (34)\\
\dot{x}_3 &= x_4, & (35)\\
\dot{x}_4 &= -\frac{K\beta}{I^2}x_2 - (\frac{K}{I}+\frac{K}{J})x_3 - \frac{\beta}{I}x_4 - \alpha\sin(x_1)\\
&\quad +\gamma x_2\cos(x_1) - \lambda x_3\cos(x_1) + \lambda x_2^2\sin(x_1)\\
&\quad +\sigma\sin(x_1), & (36)
\end{aligned}$$

where $\alpha = \frac{mgl}{I}(\frac{K}{I}+\frac{K}{J})$, $\gamma = \frac{\beta mgl}{I^2}$, $\lambda = \frac{mgl}{I}$, and $\sigma = \frac{Kmgl}{I^2}$, with $I = 0.00067478$, $J = 0.007362$, $K = 3$, $\beta = 0.020308318$, $l = 1$, and $mg = 0.5$. Furthermore, let e be tracking error and can be formulated as following

$$e \triangleq \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \\ \dot{e}_4 \end{bmatrix} \triangleq \begin{bmatrix} x_2 \\ x_3 \\ x_4 \\ \dot{x}_4 \end{bmatrix} - \begin{bmatrix} \dot{r}_{\rm d}(t) \\ \ddot{r}_{\rm d}(t) \\ \frac{{\rm d}\ddot{r}_{\rm d}(t)}{{\rm d}t} \\ \frac{{\rm d}^2\ddot{r}_{\rm d}(t)}{{\rm d}t^2} \end{bmatrix}, \qquad (37)$$

and

$$e = \begin{bmatrix} e_2 \\ e_3 \\ e_4 \\ \hat{f}(e_1,e_2,e_3,e_4) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{K}{IJ} \end{bmatrix} u$$

$$+ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -s_{r_{\rm d}} & \dot{r}_{\rm d}c_{r_{\rm d}} & -\ddot{r}_{\rm d}c_{r_{\rm d}} & \dot{r}_{\rm d}^2 s_{r_{\rm d}} & s_{r_{\rm d}} \end{bmatrix} \begin{bmatrix} \alpha \\ \gamma \\ \lambda \\ \lambda \\ \sigma \end{bmatrix}, \quad (38)$$

where

$$\begin{aligned}
s_{r_{\rm d}} &= \sin(r_{\rm d}), \quad c_{r_{\rm d}} = \cos(r_{\rm d}),\\
\hat{f}(e) &= -\frac{K\beta}{I^2}e_2 - (\frac{K}{I}+\frac{K}{J})e_3 - \frac{\beta}{I}e_4 - \alpha\sin(e_1+r_{\rm d})\\
&+ \gamma(e_2+\dot{r}_{\rm d})\cos(e_1+r_{\rm d}) - \lambda(e_3+\ddot{r}_{\rm d})\cos(e_1+r_{\rm d})\\
&+ \lambda(e_2+\dot{r}_{\rm d})^2\sin(e_1+r_{\rm d}) + \sigma\sin(e_1+r_{\rm d}), \quad (39)
\end{aligned}$$

and (39) is correct form of the nonlinear uncertain MIMO systems $\mathcal{G}$ given by (1) with Assumption 2.1 directly applied. Next, we select $F(e) = [e_1, e_2, e_3, e_4, \sin(e_1 + r_{\rm d}), (e_3 + \ddot{r}_{\rm d})\cos(e_1 + r_{\rm d}), (e_2 + \dot{r}_{\rm d})\cos(e_1 + r_{\rm d}), (e_2 + \dot{r}_{\rm d})^2\sin(e_1 + r_{\rm d})]^{\rm T}$ and $\hat{G}(e) = \frac{IJ}{K}$. Here $J(e) = \hat{J}(e) = I_4$. By proper selection of $K_{\rm g}$, then $f_{\rm c}(e)$ is given by

$$f_{\rm c}(e) = A_{\rm c}e = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \eta_1 & \eta_2 & \eta_3 & \eta_4 \end{bmatrix} e, \quad \eta_i \in \mathbb{R}, \quad (40)$$

Specifically, let $V_{\rm m} = 2.38$, $T_{\rm a} = 0.333$, $T_{\rm b} = 0.666$, and $T_{\rm c} = 0.999$, we let

$$r_{\rm d} = \begin{cases} 1.19(t - \frac{T_{\rm a}}{\pi})\sin(t\frac{\pi}{T_{\rm a}}) & t < 0.333\text{sec} \\ V_{\rm m}(t - \frac{T_{\rm a}}{2}) & 0.333\text{sec} \geq t < 0.666\text{sec} \\ \hat{V}_{\rm m} & 0.666\text{sec} \geq t < 0.999\text{sec} \\ V_{\rm m}(\frac{T_{\rm c}+T_{\rm b}-T_{\rm a}}{2}) & t \geq 0.999\text{sec} \end{cases} \quad (41)$$

where

$$\begin{aligned}
\hat{V}_{\rm m} &= \frac{V_{\rm m}}{2}(t + \frac{T_{\rm c}-T_{\rm b}}{\pi})\sin(\frac{(t-T_{\rm b})\pi}{T_{\rm c}-T_{\rm b}})\\
&\quad +V_{\rm m}(\frac{T_{\rm b}-T_{\rm a}}{2}), \quad (42)
\end{aligned}$$

and careful selection the design matrices

$$\begin{aligned}
R &= 0.0002\text{diag}[2, 0.1, 0.1, 0.1], Z = 10I_4, Y = 50I_8,\\
P_1 &= 1000\text{diag}[10, 2, 5, 1, 1], \eta_1 = -3, \quad \eta_2 = -15,\\
\eta_3 &= -10, \quad \eta_4 = -5, \quad B_0 = [0, 0, 0, 1]^{\rm T},
\end{aligned}$$

and P is the solution of Lyapunov equation (26). Here, Corollary 2.1 can be directly applied. The initial conditions given $e(0) = [0, 0, 0, 0]^{\rm T}$ and $K(0) = 0$. It follows from Corollary 2.1 that the closed-loop system (15) guarantees $e(t) \to 0$, as $t \to \infty$, if $\overline{w}(x, t) = 0$. Figure 5 and 6 show the states for each time step. The adaptive controller regulate the perturbed system to the origin under no knowledge of system dynamics, matrix $K_{\rm g}$, and disturbance, while the disturbance exist. Figures 7 illustrate the time response of the the control inputs, and Figures 8 and 9 be the time response of the feedback gain $K(t)$.

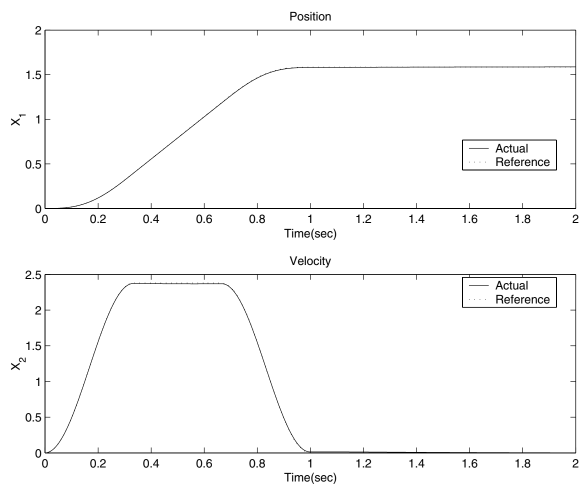

Fig. 5. The States of Flexible Joint Robot

IV. CONCLUSION

We have illustrated in this framework applications of direct adaptive control of uncertain nonlinear systems with unknown bounded disturbances, where the disturbances may be the result of unmodeled dynamics, noisy measurements, parameter uncertainty, or non dissipative forces affecting the plant. Our feedback control design guarantees robustness with respect to this unknown bounded disturbances and system uncertainties, while without the knowledge of system dynamics, matrices $K_{\rm g}$ and Ψ. In addition, the states associated with the adaptive controller gains are assured to be Lyapunov stable.

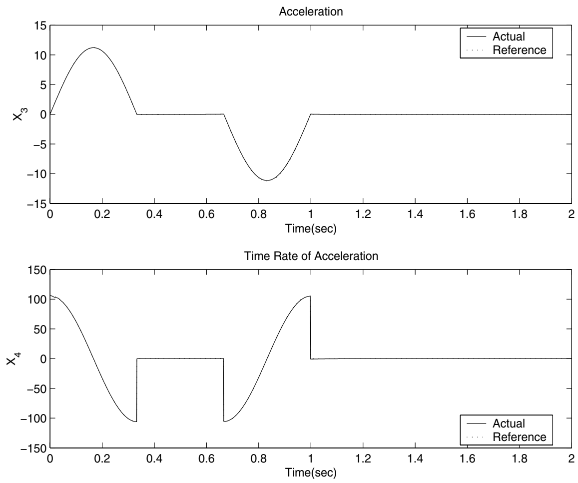

Fig. 6. The States of Flexible Joint Robot

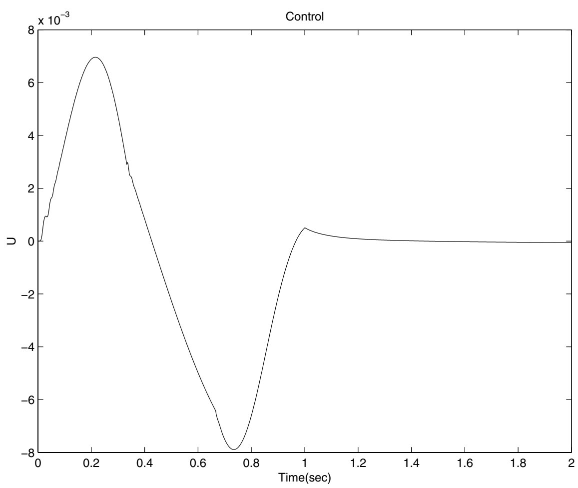

Fig. 7. Control Signal Flexible Joint Robot

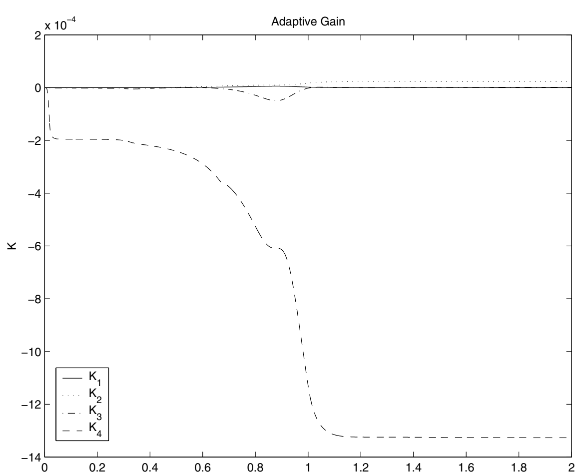

Fig. 8. Adaptive gains

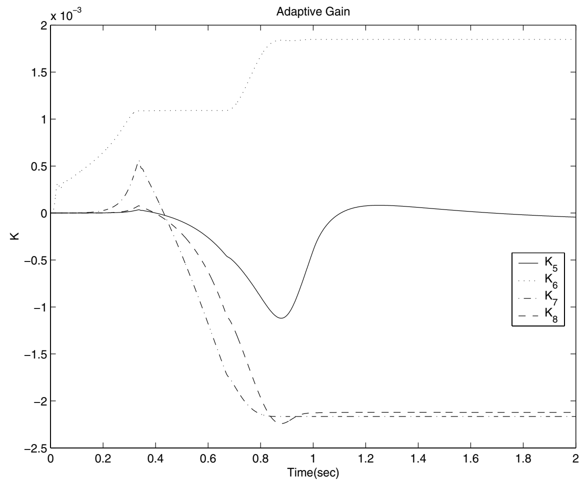

Fig. 9. Adaptive gains

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